Title: TBA

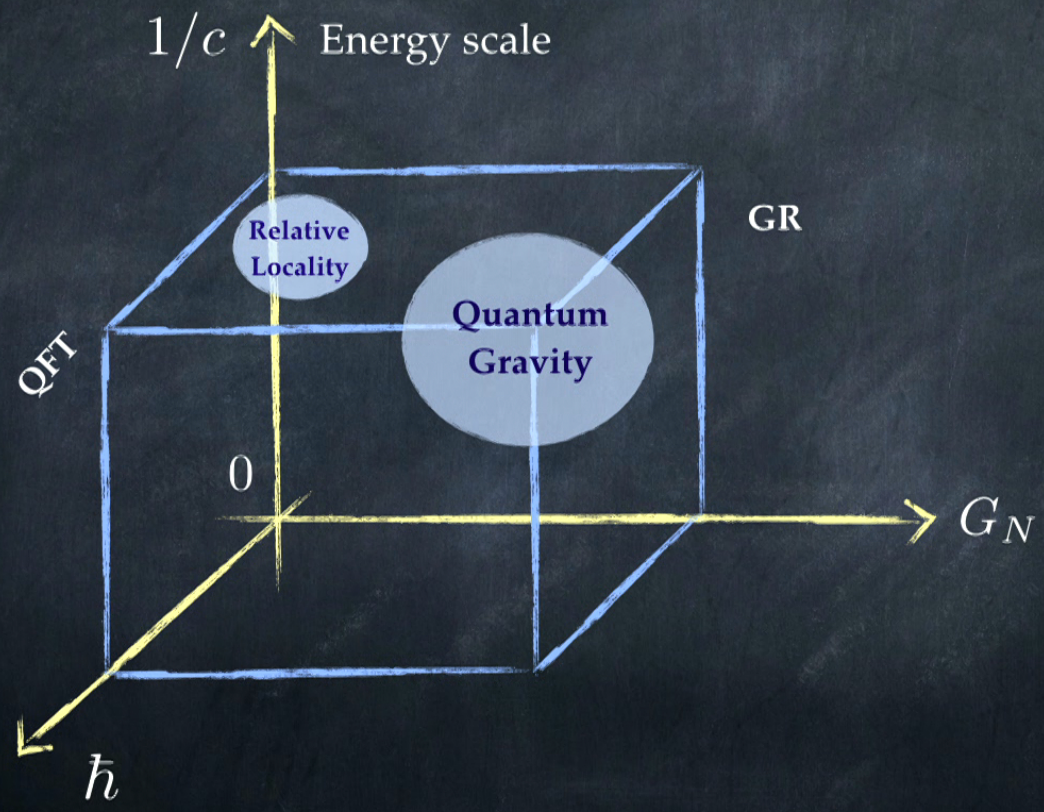
Speakers: Lin-Qing Chen

Collection: Indefinite Causal Structure

Date: December 12, 2019 - 2:50 PM

URL: http://pirsa.org/19120041

- Approaching quantum gravity from different directions:



(I didn't include cosmological constant and number of d.o.f. here)

Ideas behind the construction of Relative Locality:

G.Amelino-Camelia, L.Freidel, J.Kowalski-Glikman and L.Smolin, PRD, 2011

- A deepening of relativistic principles between interaction and observer
- Our knowledge of spacetime geometry is **constructed** from the measurement of probes -- energy, momenta and times of events.
- Allowing momentum space to have nontrivial geometry is a natural realization of the Born reciprocity.
- Taking account of possible "microcausality" of interaction vertex
- Possible Planck scale modification of Lorentz symmetry.

New physics in the regime $\hbar \to 0$ $G_N \to 0$ but $M_p = \sqrt{\hbar c / G_N}$ fixed.


Taking account of
Possible Planck scale
New physics in the
CAUTION

Relative Locality is a proposal for describing the Planck scale modifications to relativistic dynamics resulting from non-trivial momentum space geometry

G.Amelino-Camelia, L.Freidel, J.Kowalski-Glikman and L.Smolin, PRD, 2011

- Taking momentum space $(\mathcal{P}, g^{ab}, \Gamma^{ab}_{c})$ as primary, and formulating classical dynamics on the phase space $T^*(\mathcal{P})$.

- There is no universal spacetime. Spacetimes are cotangent spaces attach on each momentum point $x_I \subset T^*_{p^I}$, canonical conjugate variable $\{x^a_I, p^J_b\} = \delta^a_b \delta^J_I$

Relative Locality is a proposal for describing the Planck scale modifications to relativistic dynamics resulting from non-trivial momentum space geometry

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- The addition rule of momenta is defined as a map:

$$\oplus : \mathcal{P} \times \mathcal{P} \to \mathcal{P}$$
$$(p, q) \to p \oplus q \qquad \text{with inverse } (\ominus p) \oplus p = 0$$

- The form of vertex reflects "microscopic causal orders": $\mathcal{K}_a = (p \oplus q) \ominus k \equiv 0$
- Momentum space $\mathcal{P}$: Connection $\frac{\partial}{\partial p_a} \frac{\partial}{\partial q_b} (p \oplus_k q)_c \big|_{p=q=k} = -\Gamma^{ab}_c(k)$
  curvature: lack of associativity
  torsion: lack of commutativity of the momenta's combination

- The dynamics of particles is defined by the action:

$$S = \sum_J S^J_{free} + \sum_i S^i_{int}$$
$$= \sum_J \int_{s_i}^{s_i'} ds(x^a_J \dot{p}^J_a + \mathcal{N}_J \mathcal{C}^J(p^J)) + \sum_i \mathcal{K}^i_a(p^J(s_i)) z^a_i$$

mass shell — momenta conservation on vertex

$$\mathcal{C}^J(p) \equiv \mathcal{D}^2(p^J) - m_J^2$$

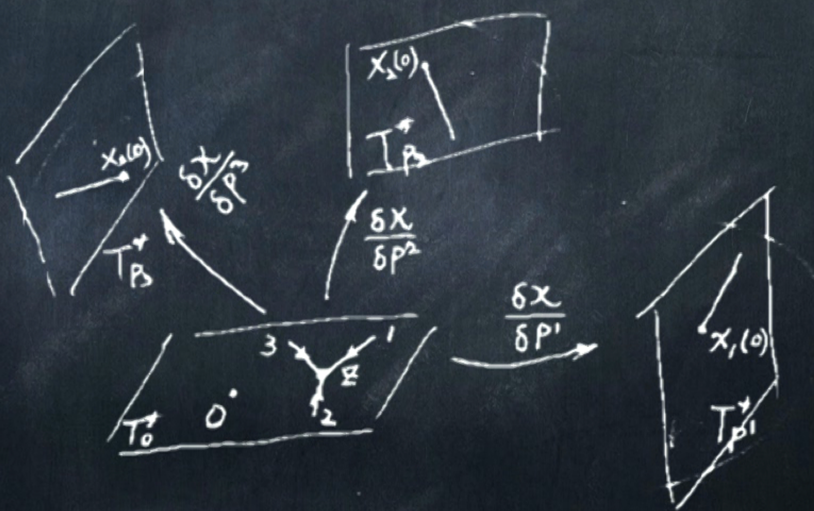
- Equation of motion

$$u^a_J \equiv \dot{x}^a_J = \mathcal{N}_J \frac{\partial \mathcal{C}(k^J)}{\partial k^J_a}$$

$$x^a_J(s_i) = \pm z^b_i \frac{\partial \mathcal{K}^i_b}{\partial k^J_a}$$

$$\frac{\partial \mathcal{K}_b}{\partial k_a} : T^*_0 \to T^*_k$$

How an interaction connects the end/starting points of different particles' worldlines.

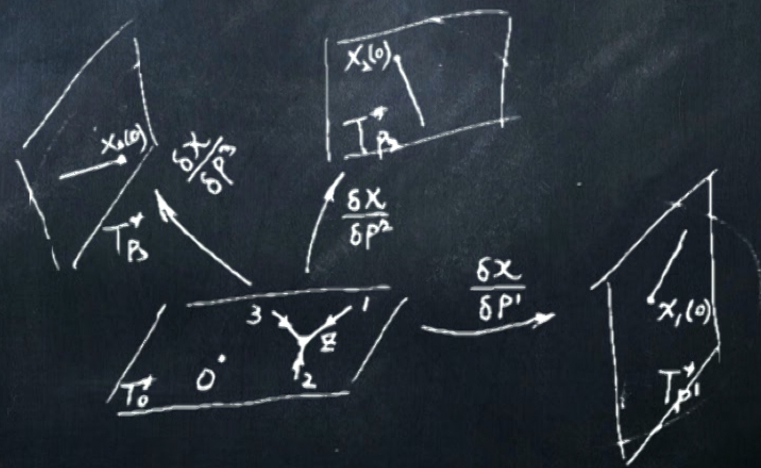


What is the world view given by the action?

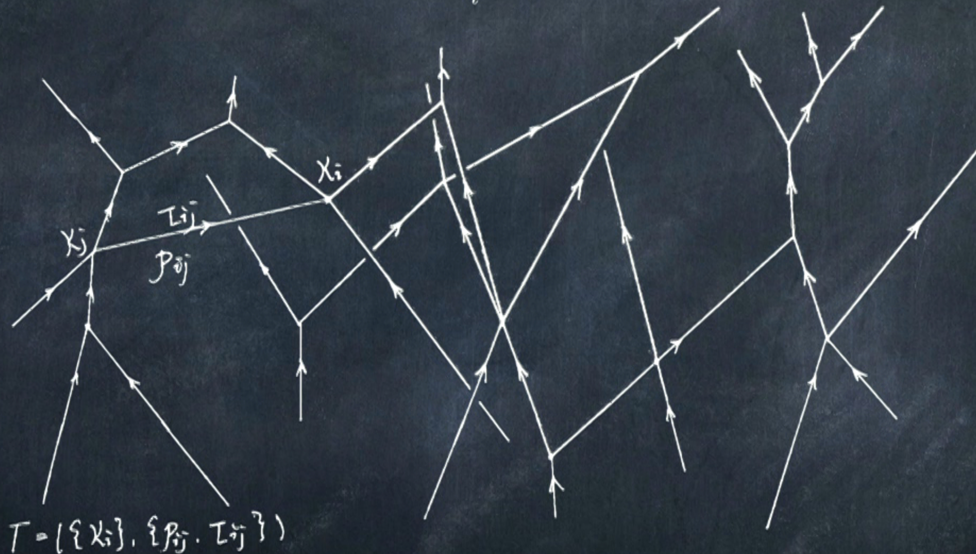
- There is no invariant global projection of the physical process into a unique classical spacetime, the projection is observer dependent.

$$x^a_J(s_i) = z^b_i \frac{\partial \mathcal{K}^i_b}{\partial k^J_a} = z^a - z^b \sum\nolimits_J C_J \Gamma^{ac}_b k^J_c + \ldots \qquad x_J \in T^*_{p^I} \;\; z \in T^*_0$$

- If one **assumes** that x and z are coordinates of the same spacetime, then **distant observer** would see non-locality of high-energy collisions!



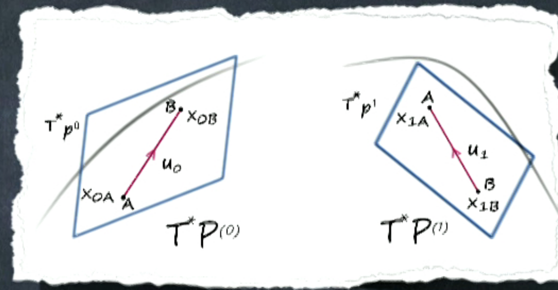
$$
\begin{aligned}
S &= \sum_J S^J_{free} + \sum_i S^i_{int} \\
&= \sum_J \int_{s_i}^{s_i'} ds(x^a_J \dot{p}^J_a + \mathcal{N}_J \mathcal{C}^J(p^J)) + \sum_i \mathcal{K}^i_a(p^J(s_i)) z^a_i
\end{aligned}
$$



- Causal relationship: Event B is in the causal past of event A if $\exists$ a sequence of events $B, B_1, ...B_n, A, n \geq 0$ s.t. from each event there exists an outgoing free-propagating particle coming to the next event. We write it as $B \prec A$. Vice versa we can define causal future $C \succ A$.

Causal loops

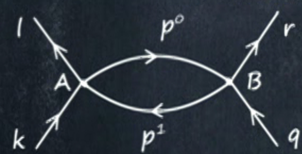
[L. Q. Chen, Phys.Rev.D 88, 024052 (2013)]



- The simplest case (two collisions) $A \prec B, B \prec A$

Particle 0 with momentum p^0 is created from event A and then collides with another particle at event B.
The twist is now to consider particle 1 with momentum p^1 created at event B and then colliding with another particle at event A, which creates the particle 0.

- Equations $(\mathcal{M}_A - \mathcal{M}_B)^\nu_\mu \, x^\mu_{0A} = \tau_0 \, (\mathcal{M}_B)^\nu_\mu u^\mu_0 + \tau_1 u^\nu_1, \;\; \tau_0, \tau_1 \in \mathbb{R}_+$



$$(\mathcal{M}_A)^\nu_\mu := (\partial \mathcal{K}^A_\alpha / \partial p^1_\nu) \cdot (-\partial \mathcal{K}^A_\alpha / \partial p^0_\mu)^{-1}$$

- In the limit of Special Relativity

$$\tau_0 u^a_0 + \tau_1 u^a_1 = 0$$

- Invariant under momentum space diffeomorphism

- Straightforward to generalize to loops that have more events $A \prec B, B \prec \ldots N, N \prec A$

$$(\mathcal{M}_A - \mathcal{M}_B \mathcal{M}_C \ldots \mathcal{M}_n)^\nu_\mu \, x^\mu_{1A} = \tau_1 \mathcal{M}_B \ldots \mathcal{M}_n u^\nu_1 +$$
$$+ \tau_2 \mathcal{M}_C \ldots \mathcal{M}_n u^\nu_2 + \ldots + \tau_{n-1} \mathcal{M}_n u^\nu_{n-1} + \tau_n u^\nu_n$$

Kappa Poincare Momentum Space

- Kappa-Poincare Hopf algebra, coming from a dimensionful deformation of the Poincare group, can describe a momentum space with de Sitter metric, torsion and nonmetricity.
  [G.Gubitosi & F. Mercati 2011, G.Amelino-Camelia, M.Arzano, J.Kowalski-Glikman, G.Rosati & G.Trevisan 2011]

- Line element of the momentum space in comoving coordinates:
  $$ds^2 = dp_0^2 - e^{2p_0/\kappa}\delta^{ij}dp_i dp_j \quad i,j = 1,2,3$$

- Mass-shell condition $m(p) = \kappa Arccosh(\cosh(p_0/\kappa) - e^{p_0/\kappa}|\vec{p}|^2/2\kappa^2)$

- Momenta addition
  $$(p\oplus q)_0 = p_0 + q_0 \quad (p\oplus q)_i = p_i + e^{-p_0/\kappa}q_i$$
  $$(p\ominus q)_0 = p_0 - q_0 \quad (p\ominus q)_i = p_i - e^{q_0 - p_0/\kappa}q_i$$

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- For notation convenience, define **left and right transport operator**
$$\frac{\partial(p \oplus q)}{\partial q} := U^q_{p\oplus q}, \quad \frac{\partial(p \oplus q)}{\partial p} := V^p_{p\oplus q} \quad \frac{\partial(\ominus p)}{\partial p} := I_p$$

- Important properties

$(q \oplus p) \ominus p = q \;\longrightarrow\; U^0_q \cdot V^p_0 = V^{q\oplus p}_q U^p_{q\oplus p} = -U^{\ominus p}_q I_p$ Right inverse property

$(\ominus p) \oplus (p \oplus q) = q \longrightarrow V^0_p \cdot U^q_0 = U^{p\oplus q}_q V^p_{p\oplus q} = -V^{\ominus p}_q I_p$ Left inverse property

$U^p_k \cdot U^k_q = U^k_q \cdot U^p_k = U^p_q, \quad V^p_k \cdot V^k_q = V^k_q \cdot V^p_k = V^p_q$ Chain rule (for associative addition rule)

Then let us solve the equation of the simplest causal loop:

$$(\mathcal{M}_A - \mathcal{M}_B)^{\nu}_{\ \mu}\, x^{\mu}_{0A} = \tau_0\, (\mathcal{M}_B)^{\nu}_{\ \mu} u^{\mu}_0 + \tau_1 u^{\nu}_1,\ \ \tau_0, \tau_1 \in \mathbb{R}_+$$

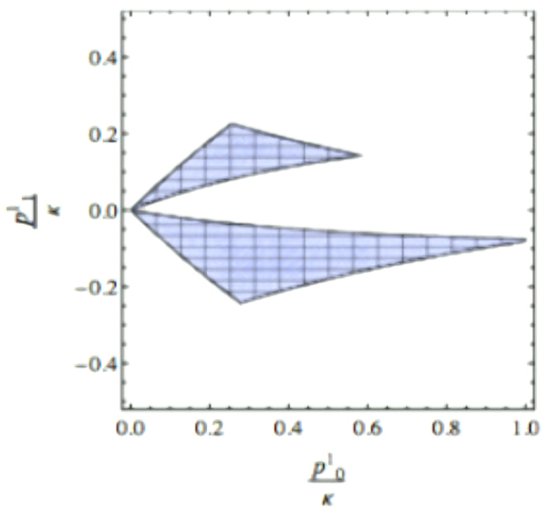
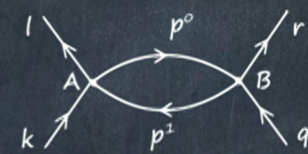


FIG.: The region of (p_0^1, p_1^1) that possiblly leads to a causal loop by events A & B when $p_0^0 = p_0^1, p_1^0 = -p_1^1, r_0 = 1.1k_0 = 0.88\kappa, k_1 = -r_1 = 0.1\kappa$.



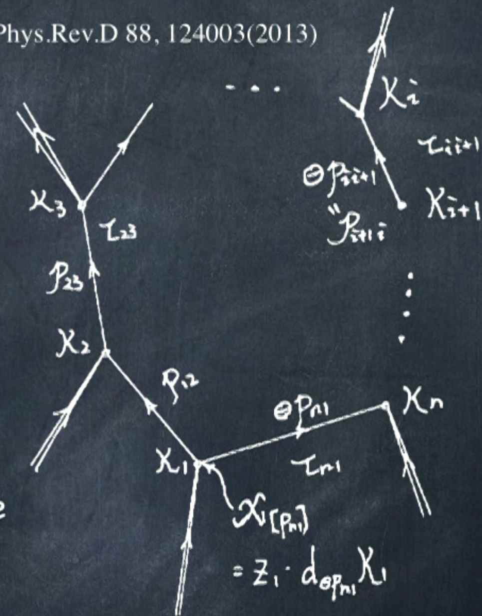
$$\mathcal{K}_A = (k \oplus p^1) \ominus (p^0 \oplus l) \equiv 0$$

$$\mathcal{K}_B = (p^0 \oplus q) \ominus (r \oplus p^1) \equiv 0$$

- x-dependence? $\quad x^1_{0A} \approx \dfrac{(p^1_0 p^0_1 - p^0_0 p^1_1)\tau_0 \kappa}{[p^1_0(k_0 - r_0) + p^1_1(r_1 - k_1)]m}$

A general loop process:

L. Q. Chen, ``Orientability of loop processes in Relative Locality'' Phys.Rev.D 88, 124003(2013)



- Consider a loop with n vertices, in which each node is associated with an equation of the momenta conservation $\mathcal{K}_1, \mathcal{K}_2 ... \mathcal{K}_n \equiv 0$

- Define **transport operator** on vertex $\mathcal{K}_i$

$$H_i := \left(\frac{\partial \mathcal{K}_i}{\partial p_{i-1,i}}\right)^{-1} \left(-\frac{\partial \mathcal{K}_i}{\partial p_{i,i+1}}\right) \qquad T^*_{p_{i-1,i}} \to T^*_{p_{i,i+1}}$$

- Imposing equations of motion around the loop (from the $x_{1_{[p_{n,1}]}} \in T^*_{p_{n,1}}$ and finally coming back to the same point), we will get

$$x_{1_{[p_{n,1}]}} \cdot \left[\mathbb{1} - \prod_{i=1}^{n} H_i\right] = \sum_{i=1}^{n} \tau_{i,i+1} v_{i,i+1} \prod_{i<j\leq n} H_j \quad (i = n+1 := 1)$$

(Using $x_{1_{[p_{n,1}]}}$ to label the endpoint which corresponds to event $\mathcal{K}_1$ on particle $p_{n,1}$'s worldline.)

- The general condition of x-independence: Effective flatness!

$$x - independent \Longleftrightarrow H_{tot} := \prod_{i=1}^{n} H_i = \mathbb{1}$$

The difference between flat loop and the one has "effective curvature":

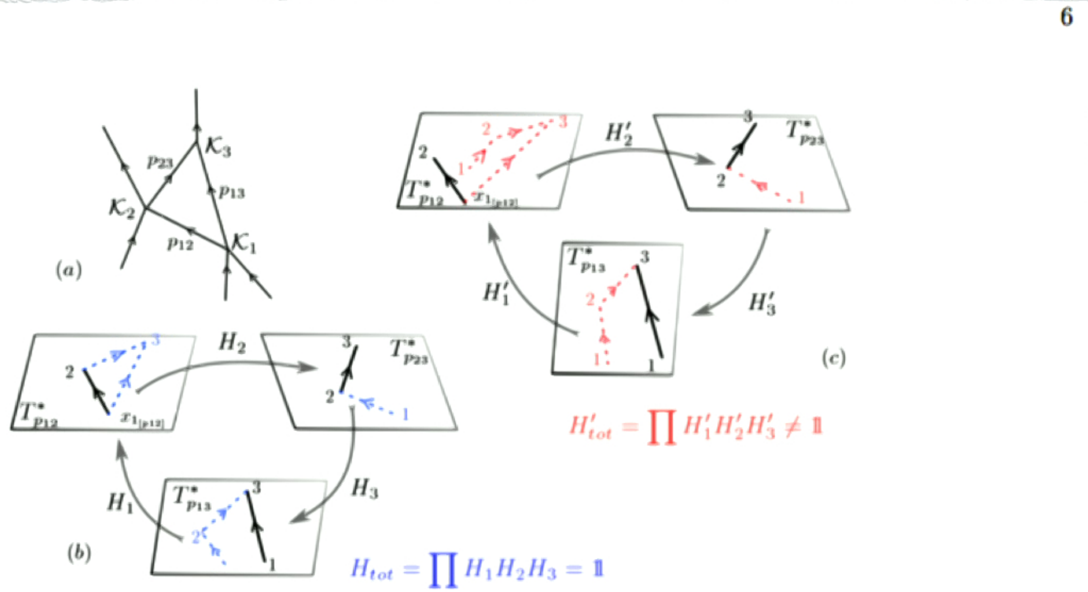


FIG. 3: Comparison between a flat loop (b) and a loop with effective curvature (c). Both of them have the same event-net (a).


FIG. 3: Comparison between a flat loop

Orientable loops

- For the momenta conservation of a vertex $\mathcal{K}_i$ in a loop, if the order of adding internal momenta and external momenta has an orentation, i.e. clockwise or anti-clockwise, we say that the vertex is orientable.

- Three-vertices are always orientable.

- Above three-vertices (more than two external momenta), the vertices are orientable only when the external momenta can be grouped as a whole up to permutations.

  Non-orientable: $\mathcal{K} \equiv p_1 \oplus l_1 \oplus p_2 \oplus l_2 = 0$ ℓ: external momenta

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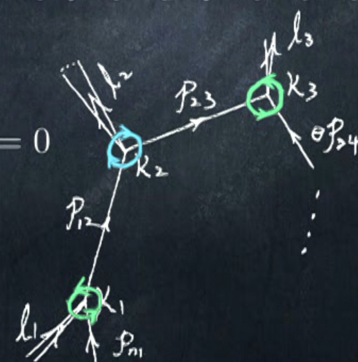
- A loop is orientable: if all the vertices have the same orientation after embedding the loop in a 2-d surface.

- A nice property:

  $\mathcal{K}_A \equiv p_1 \oplus l_1 \oplus p_2 \oplus l_2 = 0 \Rightarrow \mathcal{K}_{A'} \equiv l_1 \oplus p_2 \oplus l_2 \oplus p_1 = 0$

  though $d_{p_1}\mathcal{K}_A \neq d_{p_1}\mathcal{K}_{A'}$

  $H'_A = (d_{p_1}\mathcal{K}_{A'})^{-1}(-d_{p_2}\mathcal{K}_{A'}) = H_A \qquad H'_{tot} = H_{tot}$



$K_1 = p_{n1} \ominus p_{12} \oplus l_1$

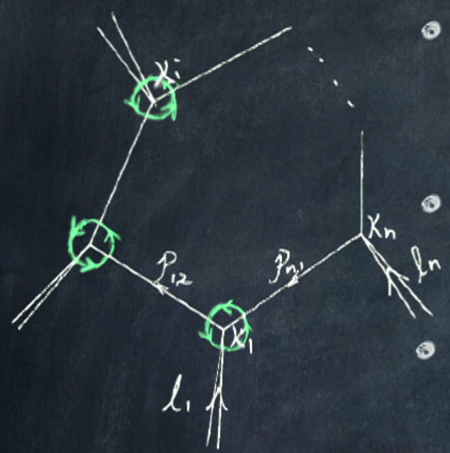
$K_2 = p_{12} \ominus l_2 \ominus p_{23}$

$K_3 = p_{23} \oplus p_{43} \ominus l_3$

- Thus we just need to consider two cases

  $\mathcal{K}_i = l_i \oplus p_{i-1,i} \oplus (\ominus p_{i,i+1}) \qquad \tilde{\mathcal{K}}_i = l_i \oplus (\ominus p_{i,i+1}) \oplus p_{i-1,i}$

Orientable $\longleftrightarrow$ x-independent



- Consider an orientable loop with n vertices and the conservation law on the vertex given by

$$\mathcal{K}_i = l_i \oplus p_{i-1,i} \ominus p_{i,i+1}$$

- Edge transport operator

$$H_i = -(U_0^{\ominus l_i} V_{\ominus l_i}^{p_{i-1,i}})^{-1}(U_0^{\ominus p_{i,i+1}} I_{p_{i,i+1}}) = U_{p_{i-1,i}}^{p_{i,i+1}}$$

- Around the whole loop:

$$H_{tot} = \prod_{i=1}^{n} U_{p_{i-1,i}}^{p_{i,i+1}} = \mathbb{1}$$

two kinds of non-orientable loops:

- E.g. the vertex m becomes of the opposite orientation compared with other vertices:

$$\tilde{H}_{tot} = U_{p_{n,1}}^{p_{m-1,m}} \cdot V_{p_{m-1,m}}^{p_{m,m+1}} \cdot U_{p_{m,m+1}}^{p_{n,1}}$$

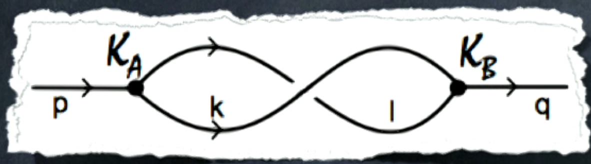
$$(d_{p_\mu}\left[V^{q_\nu}_{(q\oplus p)_\rho} - U^{q_\nu}_{(p\oplus q)_\rho}\right]\Big|_{p,q=0} = T^{\mu\nu}_{\rho} = \frac{1}{\kappa}\delta_0^{[\mu}\delta_i^{\nu]}\delta^i_\rho, \quad i=1,2,3)$$

- Some of the vertices do not have orientation, e.g. $\mathcal{K}'_i = l_i \oplus p_{i-1,i} \oplus k_i \oplus (\ominus p_{i,i+1})$

$$H'_i = V_{p_{i-1,i}}^{p_{i-1,i}\oplus k_i} U_{p_{i-1,i}\oplus k_i}^{p_{i,i+1}} \quad \Rightarrow H'_{tot} \neq \mathbb{1}$$

A twisted loop could break global momentum concervation

[A.~Banburski, Phys. Rev.D 88, 076012 (2013)]



A twisting loop with $p \neq q$

$$\mathcal{K}_A = p \ominus (k \oplus l) \qquad \mathcal{K}_B = (l \oplus k) \ominus q$$

- There are loop processes that locally momenta are conserved, but there is no global momentum conservation.
- Also has x-dependence -- breaking of translation invariance

Summary of results

[L. Q. Chen, Phys.Rev.D 88, 024052 (2013)]
[L. Q. Chen, Phys.Rev.D 88, 124003(2013)]

- The Lagrangian in the theory of Relative Locality allows causal loops, but does not lead to any logical paradox.
- For loop processes in the momentum space with associative addition rule , for example Kappa-Poincare momentum space,

$$\text{Causal} \Leftarrow \text{The loop is orientable} \Leftrightarrow \text{X-independent}$$

$$\Updownarrow$$

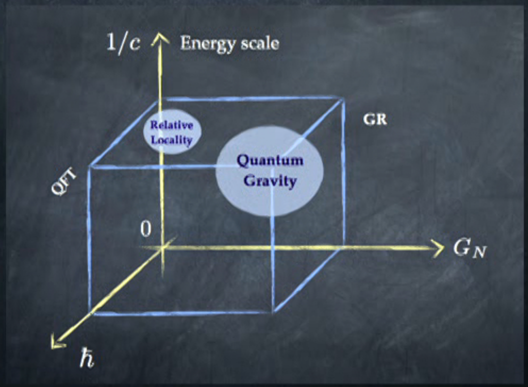
$$\text{Global momenta conservation}$$

- Non-orientable loops contain an "effective curvature" caused by a combination of nonlinear interactions, and these loops have strange features.
- For non-associative momentum space (such as Snyder momentum space), "orientablility" will not lead to effective flatness, we expect weird loops to happen in general.

The mathematical result is consistent with the work in Rome in 2014:
G.Amelino-Camelia, S.Bianco, F.Brighenti and R.J.Buonocore, `
`Causality and momentum conservation from relative locality,'' Phys. Rev. D 91, no. 8, 084045 (2015)

- In the quantum gravity regime, we expect that the notion of locality and causality to be nontrivial and deviate from our current knowledge.
- As we know in classical gravity, causality is as rigid as classical geometry, and diffeormophism invariant observables, conserved quantities are essentially quasi-local.
- In the framework of quantum field theory, we know that locality, causality, the spin statistics are tightly and precisely related to each other.
- In quantum reference frame, we just heard the new result of observer-dependent locality and entanglement; In quantum information, we learn that the superposition of channels lead to interesting notion of causal structure.

It is very exciting to combine these different regimes, and see how our fundamental notion of causality, locality and structure of spacetime deviated from what has been known.



Thank you !