Title: Two views of relative locality

Speakers: Lee Smolin

Collection: Indefinite Causal Structure

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Abstract: Relative locality is a quantum gravity phenomenon in which whether an event is local or not-and the degree of non-locality-is dependent on the position and motion of the observer, as well as on the energy of the observerâ€™s probes. It was first discovered and studied, beginning in 2010, in a limit in which h and G both go to zero, with their ratio, which is the Planck energy-squared, and c held fixed (arXiv:1101.0931, arXiv:1103.5626).

Relative locality was also found in a different, non-relativistic limit, involving quantum reference frames, in which c is taken to infinity while h and G are held fixed. I describe some of what we learned in the first studies, in the hope it might be useful to people developing the quantum reference frame approach.

Two views of Relative Locality

Lee Smolin
PI

Indefinite causal structure workshop Dec 2019

with Giovanni Amelino-Camelia, Laurent Freidel, Jerzy Kowalski-Glikman

arXiv:1101.0931, arXiv:1103.5626, arXiv:1104.2019,arXiv:1108.0910, rXiv:1110.0521.

Also Energetic Causal Sets are joint work with Marina Cortes.

Many thanks to Sabine Hossenfelder and to R Schutzhold and Bill Unruh for raising the issue of non-locality in theories with deformed lorentz invariance.

Thanks also to Flaminia Giacomini and Thomas Gallery for current work in progress

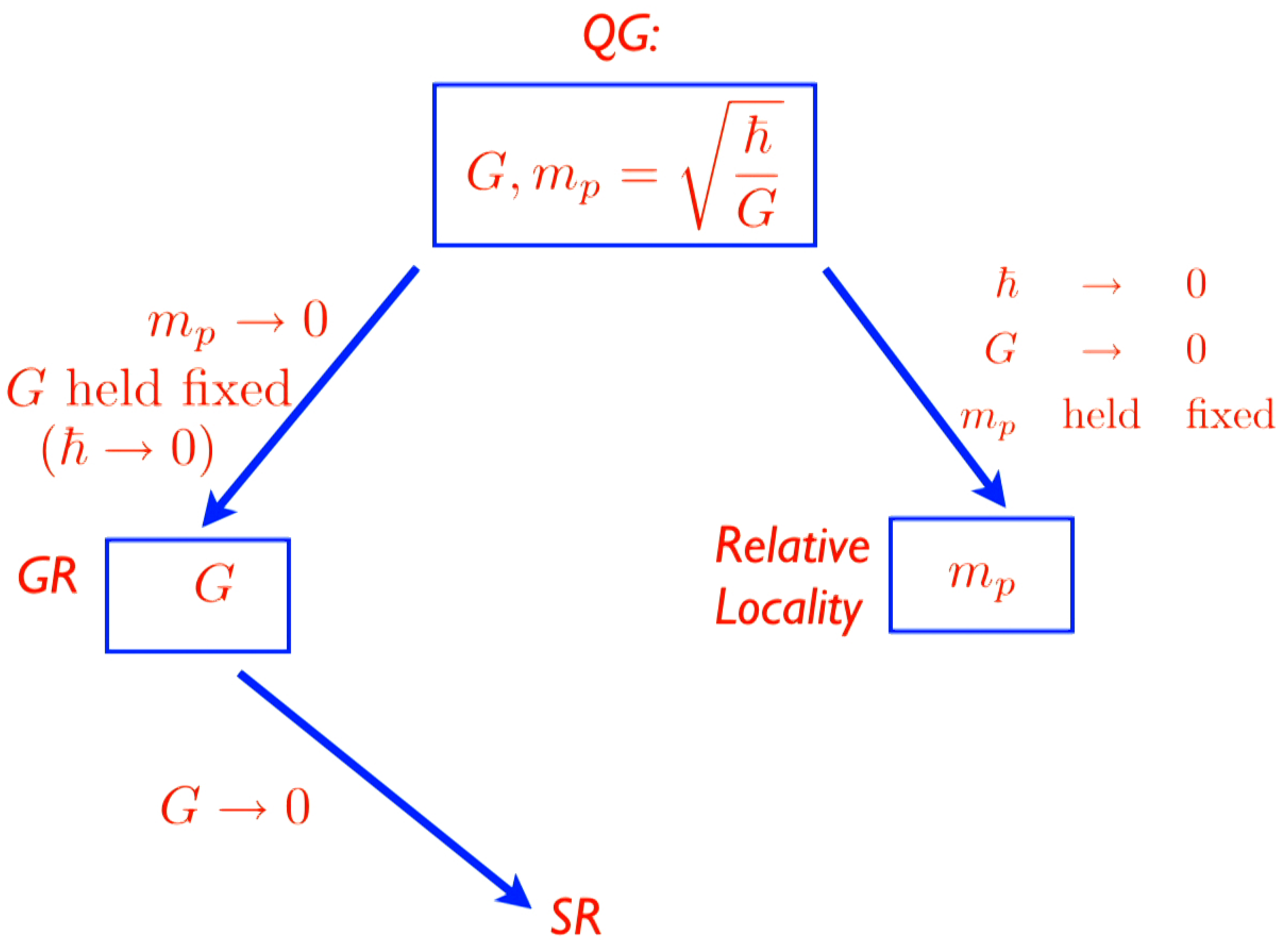
Energy and momentum are fundamental, intrinsic properties.

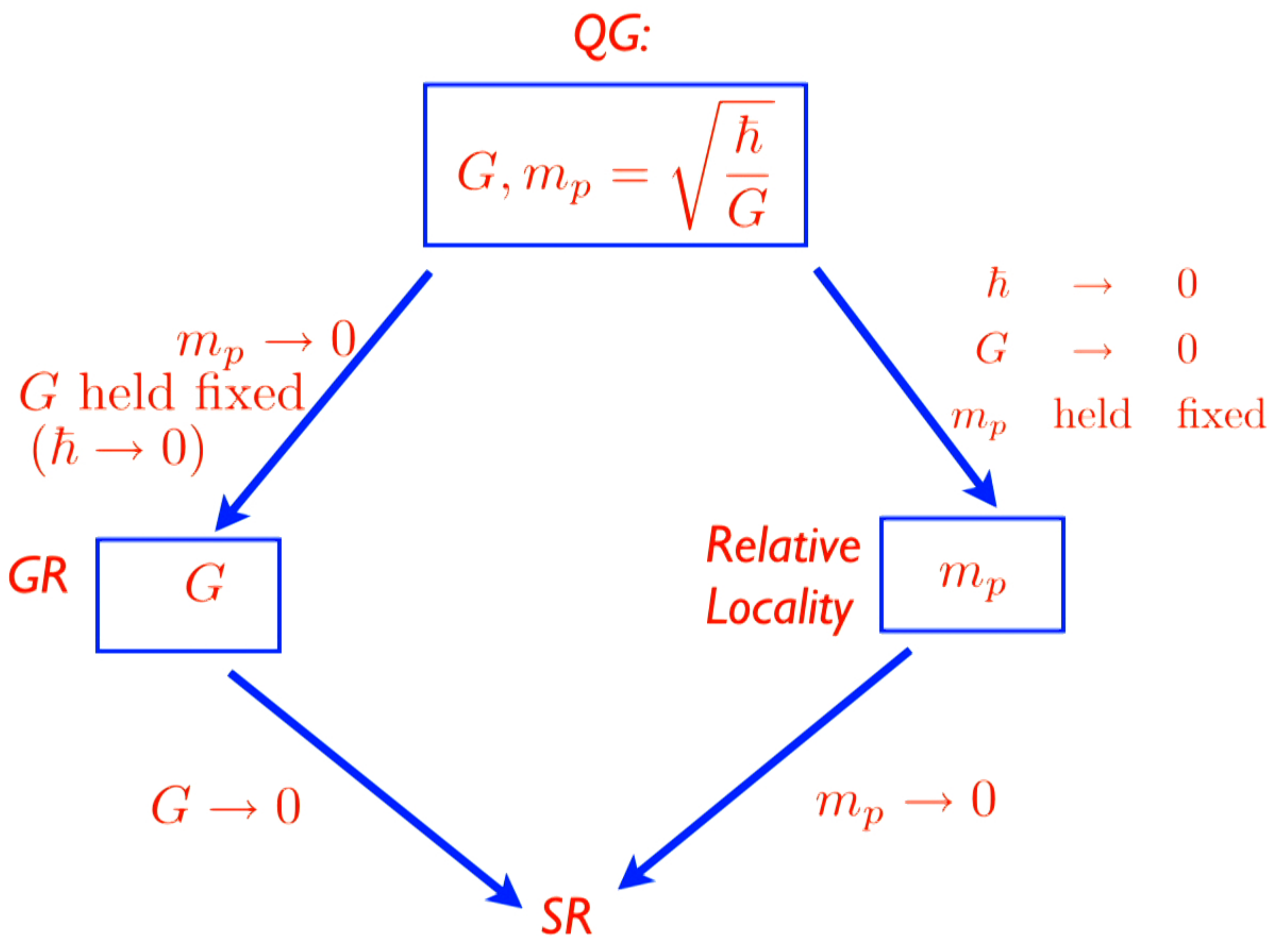
Relative locality teaches us that the primary geometry is the geometry of momentum spacetime.

Causality is also primary.

Spacetime and its geometry are not primary: they are secondary and emergent.

Einstein taught us that concepts like simultaneity and locality are constructed from primary observations of energy and momentum.


QG:
$G, m_p = \sqrt{\frac{\hbar}{G}}$
$m_p \rightarrow 0$
G held fixed
$(\hbar \rightarrow 0)$
$\hbar \rightarrow 0$
$G \rightarrow 0$
m_p held fixed
GR
G
Relative Locality
m_p
$G \rightarrow 0$
SR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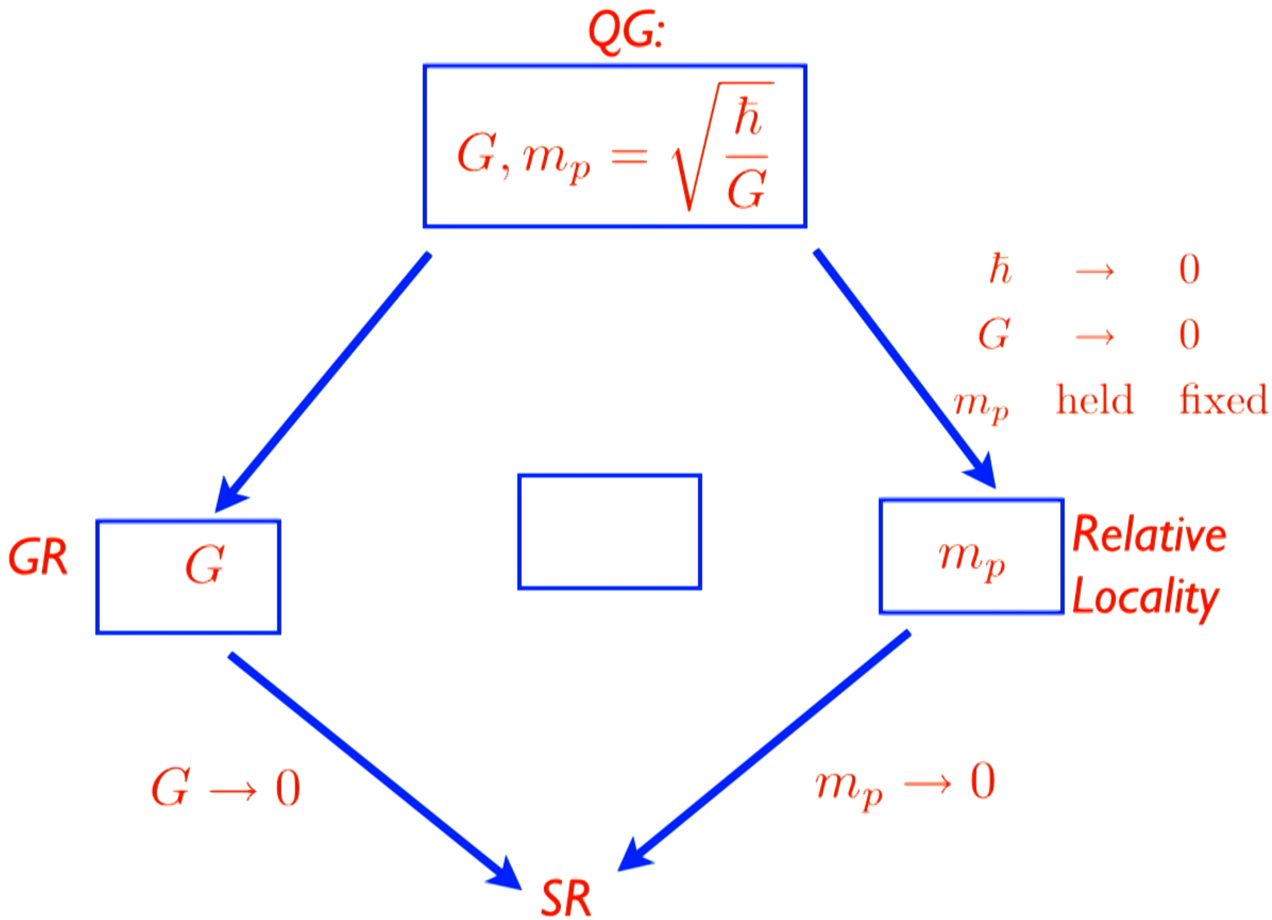
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Relative Locality
$G \rightarrow 0$
$m_p \rightarrow 0$
SR

QG:

$$\boxed{G, m_p = \sqrt{\frac{\hbar}{G}}}$$

$m_p \rightarrow 0$

$(\hbar \rightarrow 0)$

$\hbar \rightarrow 0$

$G \rightarrow 0$

m_p held fixed

GR $\boxed{G}$ curved spacetime

Relative Locality $\boxed{m_p}$ curved momentum space

$$\Gamma^{GR} = \mathcal{T}_*(\mathcal{M})$$

$$\Gamma^{RL} = \mathcal{T}_*(\mathcal{P})$$

What is this regime good for? What is the role of m_p?

l_p has gone to zero, so there is no modification of spacetime geometry.

Notice that operationally, particle physicists and astronomers make measurements in momentum space. They measure the energy and momentum of incoming and outgoing photons and other particles and the proper times the detectors registered those quanta.

Most fundamentally, we physicists are calorimeters with clocks.

Everything else is inferred. ***This includes spacetime geometry.***

We then take mp as measuring a deformation of 4-momentum space, P.

We are used to thinking that spacetime is fundamental, and momenta are auxiliary variables for describing motion within spacetime. But this is the opposite of experimental practice. Momenta and energy are fundamental. *Spacetime coordinates are auxiliary variables for describing dynamics in momentum space.*

Relative locality is experimentally testable.

What happens to Poincare symmetry for energies~ m_p?

This is the basic question that can bring QG in contact with experiment.

Experimental probe $O(E/m_p)$ deformations of momentum space:

- *Gamma Ray burst time of flight measurements at Fermi etc*
- *Tests of GZK cutoff at AUGER*
- *Birefringence of photons, ie polarized radio galaxies, Gamma rays etc.*

Why are quantum gravity effects being probed by astrophysical measurements?

Because with l_p , *G, h =0,* dimensionally quantum gravity effects can only show up at very large scales, as apparent ambiguities in the localization of distant events inferred by measurements of the energy and momentum of probes.

$$\Delta x \approx x \left(\frac{E}{m_p}\right)^p$$

p=1,2

Is it consistent to have violations of locality for distant observations?

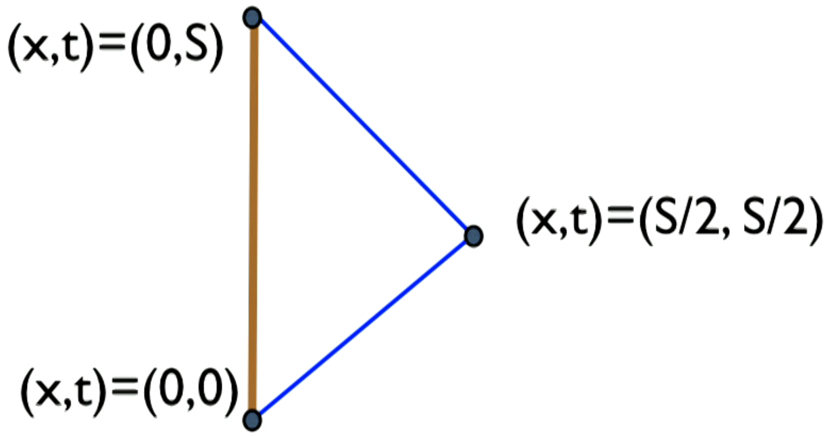
Like special relativity there are apparent paradoxes that are solved by thinking carefully.

Like special relativity these lead to new experiments.

Why should curvature of momentum space lead to issues with locality in spacetime?

Because spacetime geometry is inferred from momentum space.

Spacetime is inferred. As Einstein taught us, distant spacetime coordinates are inferred from momentum space measurements.



Do we all infer the same spacetime?
Do we infer the same spacetime at different energies?

In special relativity the answers are yes. Why?

- The conservation laws that generate transformations between observers are linear in momenta.

$$\mathcal{P}_c^{tot} = \sum_I p_c^I$$

- Total momentum generates translations:

$$\delta x_I^a = \{x_I^a, b^c \mathcal{P}_c^{tot}\} = b^a$$

- How much a worldline is translated, does not depend on how much momentum and energy it carries.
- Hence we all construct the same spacetime.
- If an interaction is local for one observer it is local for all observers

What if the conservation laws are non-linear?

Do we still all infer the same spacetime?

Do we still infer the same spacetime at different energies?

•Suppose the conservation laws that generate transformations between observers are *non-linear* in momenta:

$$\mathcal{P}^{tot}_c = \sum_I p^I_c + \mathcal{G}(I,;J,K)^{de}_c p^J_d p^K_e + \ldots$$

•Total momentum generates translations, which now depend on the momenta:

$$\delta x^a_I = \{x^a_I, b^c \mathcal{P}^{tot}_c\} = b^a + b^c \mathcal{G}(I;I,K)^{ad}_c p^K_d + \ldots$$

For every interaction, observers local to it will infer it to be local. Distant observers may infer that the same interaction appears non-local in the spacetime coordinates they construct.

We call this the principle of relative locality.

There is a simple and coherent mathematical framework for it, based on the geometry of momentum space.

$$\Delta x \approx x \left(\frac{E}{m_p} \right)^p$$

Geometry of momentum space

Operational point of view: an observer is equipped with a calorimeter and a clock.

From her measurement she conclude that each isolated system $\mathcal{P}$ possess 4 conserved quantities: Energy momentum space

She can realise two type of measurements:

One particle measurements: measurement of the mass and kinetic energy *determines the metric*

Multi particle measurements: sacttering processes, interactions, merging. *determines the connection.*

Kikkawa, Sabinin, Freidel

Geometry of momentum space

One postulate that single particle measurements determine the geometry of $\mathcal{P}$

$\mathcal{P}$ is a lorentzian metric manifold

The mass is interpreted as the timelike distance from the origin

$$D^2(p) \equiv D^2(p,0) = m^2.$$

The kinetic energy defines the geodesic spacelike distance between a particle p at rest and a particle p' of identical mass $D(p) = D(p') = m$

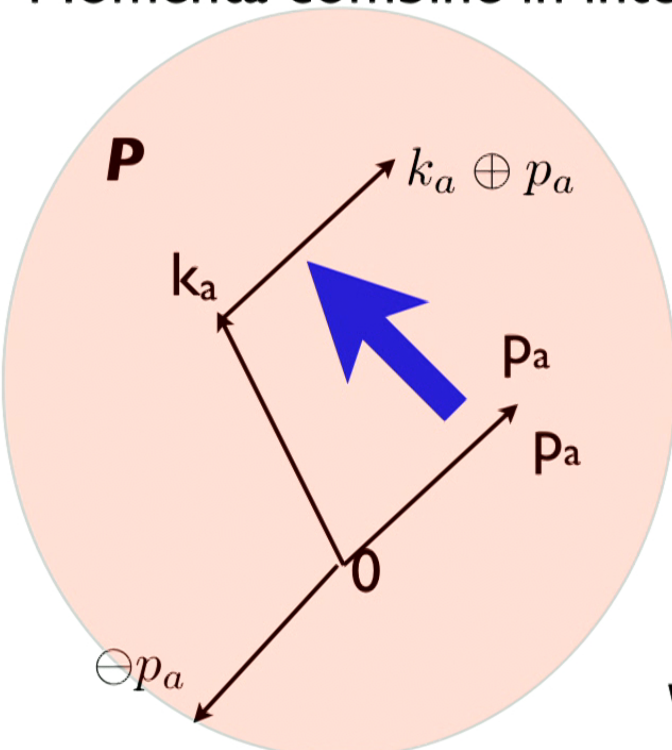
$$D^2(p,p') = -2mK.$$

from these measurements we can reconstruct the metric on $\mathcal{P}$

$$dk^2 = h^{ab}(k)dk_a dk_b$$

Geometry of momentum space

Momenta combine in interactions: we need a rule:



$$(k,q) \longrightarrow k'_a = k_a \oplus q_a$$

This is a rule for combining geodesics on a curved manifold, so it defines a connection or parallel transport.

$$\begin{aligned} k_a \oplus dp_a &= k_a + U(k)_a^b dp_b \\ &= k_a + dp_a + \Gamma_a^{bc} k_b dp_c + \ldots \end{aligned}$$

Complicated process are built up:

$$(k_a \oplus q_a) \oplus p_a$$

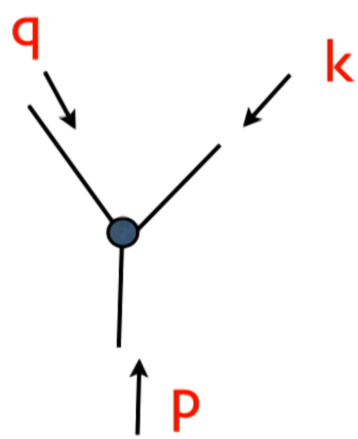
We assume neither commutativity nor associativity.

We do assume there is an inverse

$$p_a \longrightarrow \ominus p_a, \qquad (\ominus p_a) \oplus p_a = 0$$

$$(\ominus p_a) \oplus (p_a \oplus k_a) = k_a$$

The non-linear composition rule is used to define conservation laws at interactions.



$$\mathcal{K}(k,p,q)_a = (k_a \oplus p_a) \oplus q_a = 0$$

This requires choices when the composition rule is non-commutative or non-associative.

Geometry of momentum space

The composition rules defines an affine connection on $\mathcal{P}$

$$\frac{\partial}{\partial p_a}\frac{\partial}{\partial q_b}(p\oplus q)_c|_{q,p=o} = -\Gamma_c^{ab}(0)$$

transform as an affine connexion

Torsion measure non commutativity

$$-\frac{\partial}{\partial p_a}\frac{\partial}{\partial q_b}((p\oplus q)_c - (p\oplus q)_c)_{q,p=o} = T_c^{ab}(0)$$

What about associativity?

Geometry of momentum space

The composition rules defines an affine connection on $\mathcal{P}$

$$\frac{\partial}{\partial p_a}\frac{\partial}{\partial q_b}(p \oplus q)_c|_{q,p=o} = -\Gamma^{ab}_c(0)$$

transform as an affine connexion

Curvature measure non associativity

$$2\frac{\partial}{\partial p_{[a}}\frac{\partial}{\partial q_{b]}}\frac{\partial}{\partial k_c}((p \oplus q) \oplus k - p \oplus (q \oplus k))_d\,|_{q,p,k=o} = R^{abc}{}_d(0)$$

Three aspects of geometry, which can be measured:

$$p_a \oplus q_a = p_a + q_a + \Gamma_a^{bc} p_b q_c + \ldots$$

•Torsion: measures non-commutativity of interactions.

$$T_a^{bc} = \Gamma_a^{bc} - \Gamma_a^{cb}$$

•Curvature: measures non-associativity of interactions.

$$R^{abc}{}_d = \partial^a \Gamma_d^{bc} - \partial^b \Gamma_d^{ac} + \Gamma\Gamma$$

•Non-metricity: if the connection defined by interactions is not the metric connection defined from propagation.

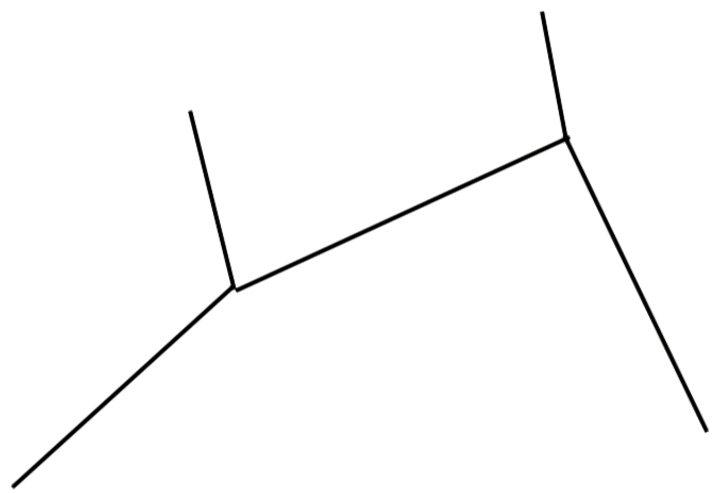
$$N^{abc} = \nabla^a g^{bc}$$

Dynamics

Dynamics

- Spacetime emerges from dynamics on momentum space.
- In our limit, we study first classical particle dynamics
- Each process has an action principle

$$S^{process} = \sum_{trajectories, I} S_I^{free} + \sum_{interactions, \alpha} S_\alpha^{int}$$



The free relativistic particle:

• Canonical coordinates, x^a, and canonical momenta k_b

$$S_{free} = \int ds \left(x^a \dot{k}_a + \mathcal{N}\mathcal{C}(k) \right)$$

Energy-momentum relations expressed as a constraint: $\mathcal{C}(k) = -k_0^2 + \vec{k}\cdot\vec{k} + m^2 = 0$

Canonical Poisson brackets: $\{x_I^a, k_b^J\} = \delta_b^a \delta_I^J$

Equations of motion: N=lagrange multiplier

$$\begin{aligned} \dot{k}_a^J &= 0 \\ \dot{x}_J^a &= \mathcal{N}_J \frac{\delta \mathcal{C}^J}{\delta k_a^J} = \mathcal{N}_J p^a \\ \mathcal{C}^J(k) &= 0 \end{aligned}$$

The free action in curved momentum space:

- Just one change: *introduce a metric on momentum space*

$$S_{free} = \int ds \left(x^a \dot{k}_a + \mathcal{N}\mathcal{C}(k) \right)$$

Energy-momentum relations expressed as a constraint: $\boxed{\mathcal{C}(k) = D^2(k) + m^2 = 0}$

Canonical Poisson brackets: $\{x_I^a, k_b^J\} = \delta_b^a \delta_I^J$

Equations of motion:

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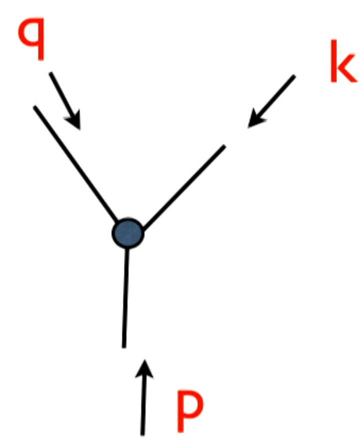
$$\begin{aligned} \dot{k}_a^J &= 0 \\ \dot{x}_J^a &= \mathcal{N}_J \frac{\delta \mathcal{C}^J}{\delta k_a^J} \\ \mathcal{C}^J(k) &= 0 \end{aligned}$$

The interaction imposes a conservation law at each node

$$\boxed{S^{int} = \mathcal{K}(k(o))_a z^a} \qquad \frac{\delta S^{int}}{\delta z^a} = \mathcal{K}_a = 0$$

$$\mathcal{K}(k,p,q)_a = (k_a \oplus p_a) \oplus q_a = 0$$

z^a is a lagrange multiplier that enforces the conservation law $K_a = 0$.



But, in turn, z^a become the point representing the interaction in spacetime.

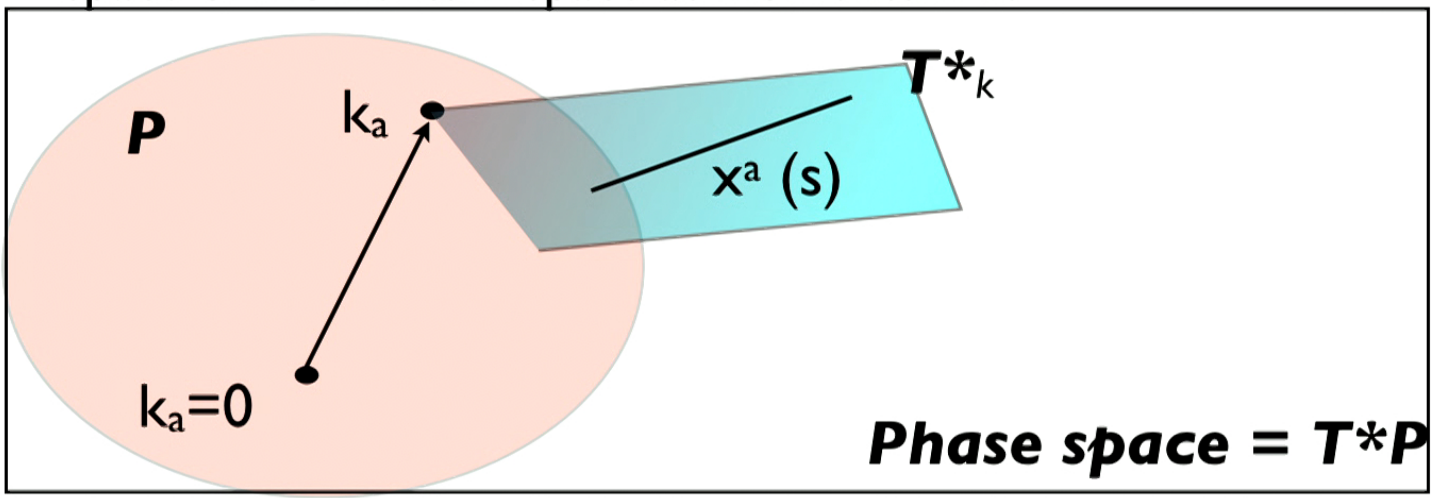
Two kinds of spacetime coordinates: $x^{a}{}_{I}$ and $z^{a}{}_{I}$

•Canonical coordinates, $x^a{}_I$, from the variation of the free action

$$S^I_{free} = \int ds \left(x^a_J \dot{k}^J_a + \mathcal{N}_J \mathcal{C}^J(k) \right)$$

$$\{x^a_I, k^J_b\} = \delta^a_b \delta^J_I \qquad \{x^a_I, x^b_J\} = 0$$

These are momentum dependent. They live in the cotangent space of momentum space at momentum k.

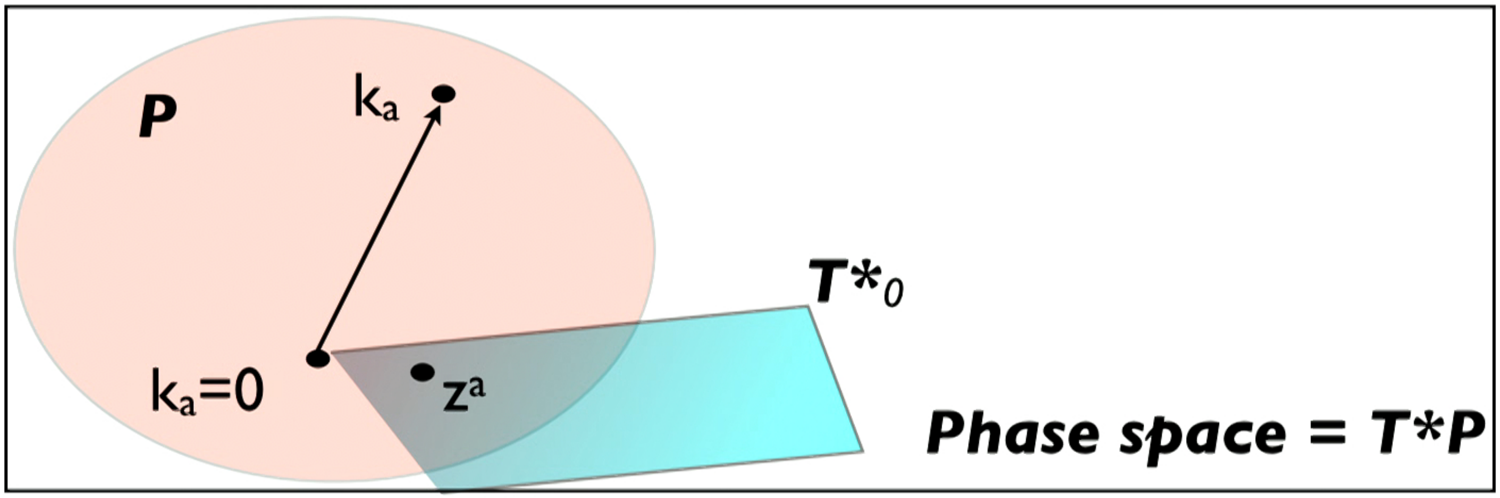


•Interaction coordinates, z^a, from the variation of the interaction

$$S^{int} = \mathcal{K}(k(o))_a z^a \qquad \frac{\delta S^{int}}{\delta z^a} = \mathcal{K}_a = 0$$

These are non-commutative. They live in the cotangent space of momentum space at momentum k=0.

$$\{z^a, z^b\} = T^{ab}_d z^d + R^{abc}{}_d p_c z^d + \cdots$$



Relating the two kinds of spacetime coordinates:

- Is a consequence of the equations of motion at the endpoints

$$\delta S = \left(\frac{\delta \mathcal{K}(k(o))_a}{\delta k^I_a(0)} z^a - x^a(0)\right)\delta k_a(0)$$

The interaction point is related to the endpoint of the worldline by a parallel transport between the spaces where they live.

$$\boxed{x^a(0) = U(k)^a_b z^b,} \qquad U(k)^a_b = \frac{\delta \mathcal{K}_b}{\delta k_a}$$

If the conservation K_a is linear, U=I and $x^a = z^a$.
Then the interaction is local.

When K_a is non-linear, the interaction is relatively local

ie $x^a = 0$ when $z^a = 0$.

Relating the two kinds of spacetime coordinates:

• Is a consequence of the equations of motion at the endpoints

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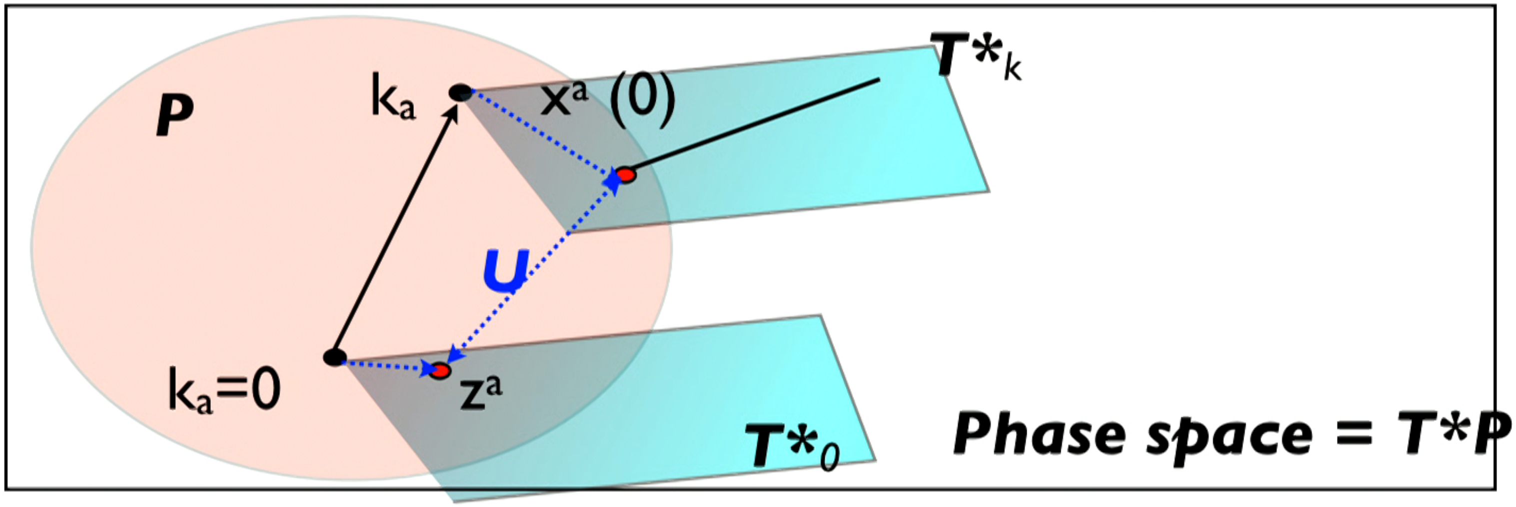
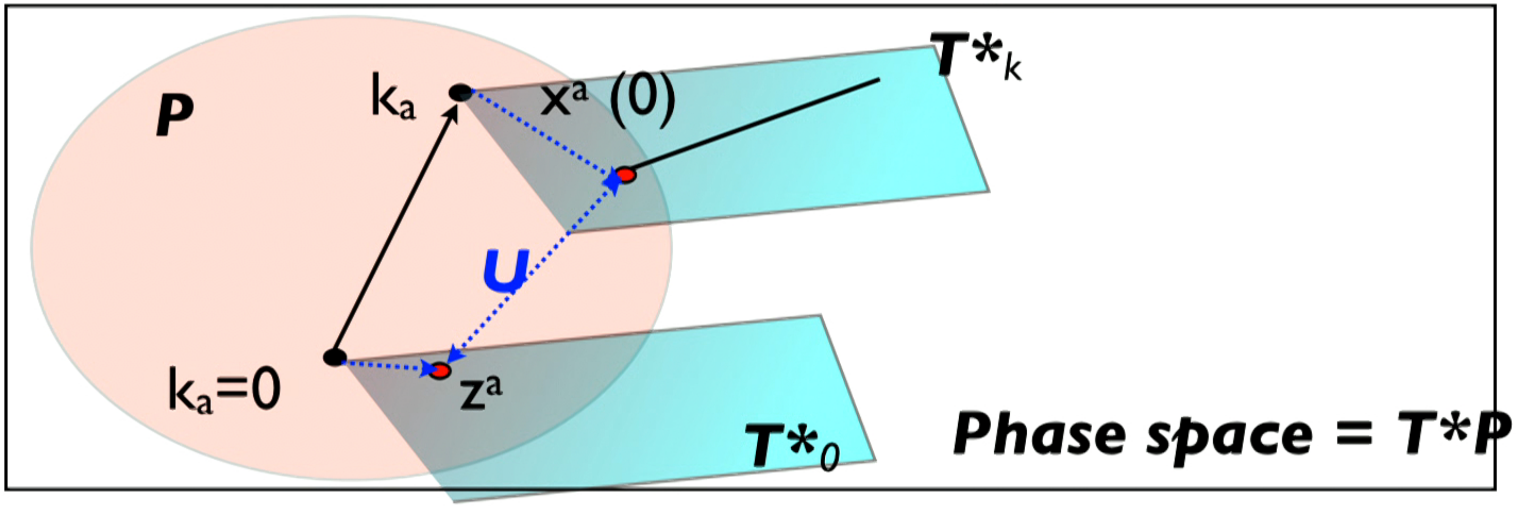


There is no invariant momentum independent spacetime:

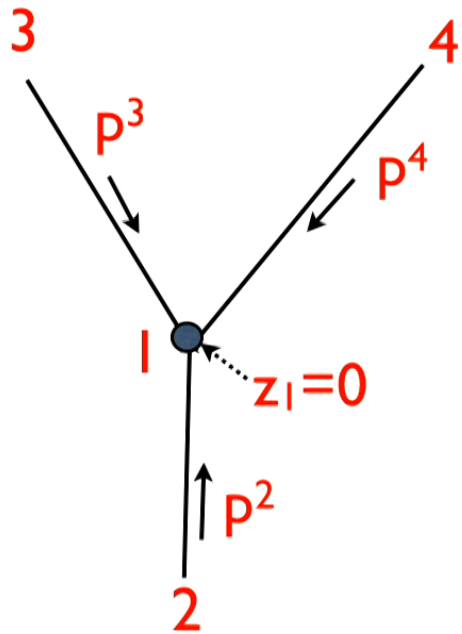
When U=I you can identify all the cotangent spaces to give a universal spacetime. This is the case when momentum conservation is linear. When momentum conservation is nonlinear, U is non trivial and there is a copy of spacetime for each energy.

The interactions are as local as possible given this: relative locality.

$$\boxed{x^a(0) = U(k)^a_b z^b,} \qquad U(k)^a_b = \frac{\delta K_b}{\delta k_a}$$

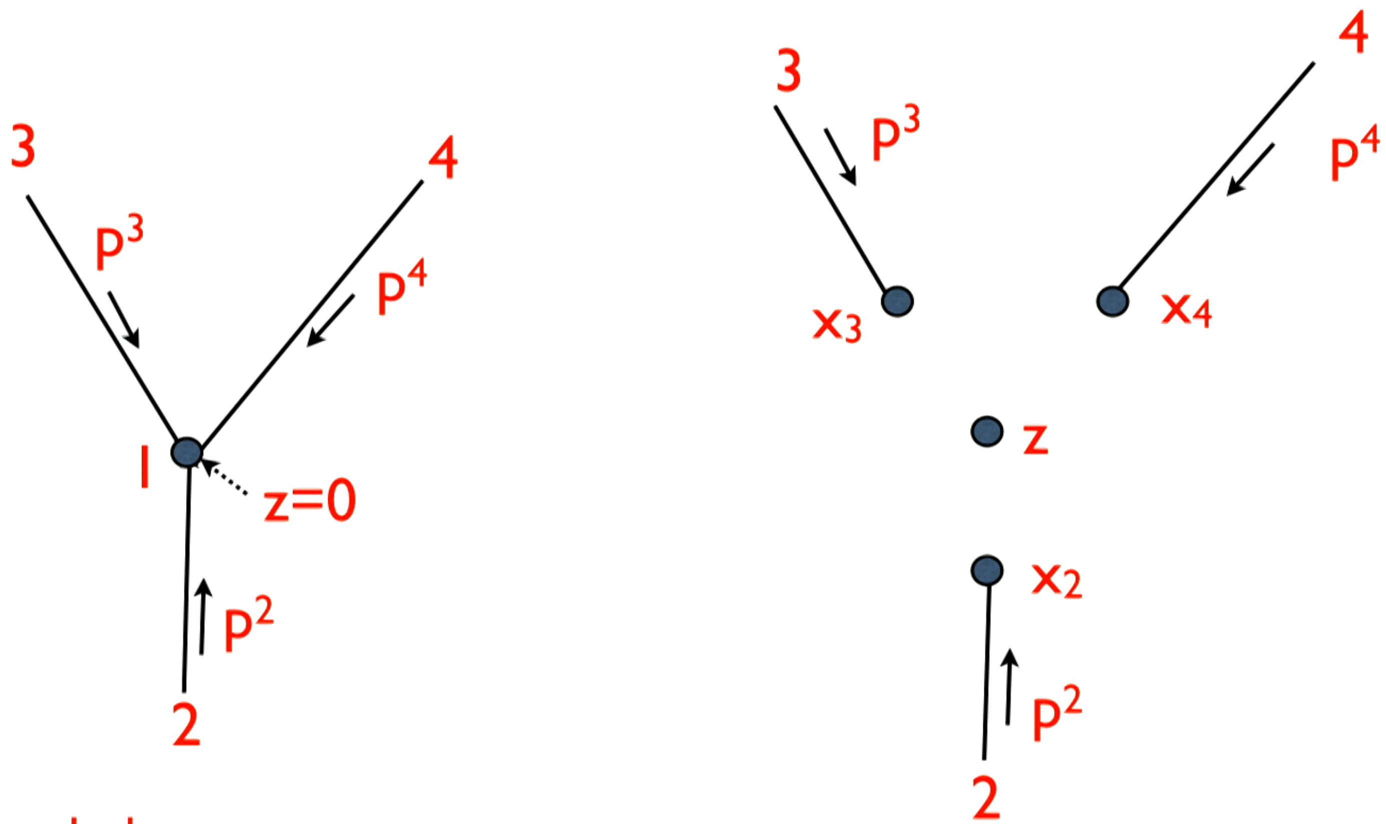


Vertices look local to local observers, for which $z^a = 0$



local observer

Vertices look non-local to distant observers



local observer

distant observer

$$\delta x^a_I = \pm\{b^c \mathcal{K}_c, x^a_I\} = b^a + \Gamma^{ac}_b b^a p^I_c + \ldots$$

Examples: κ-*Poincare in 2+1 and 3+1*

- Metric on ***P*** is deSitter spacetime
- ***P*** is a group: AN(3), so ⊕ is associative, curvature vanishes.
- antipode is group inverse

$$(p \oplus q)_0 = p_0 + q_0\,, \quad (p \oplus q)_i = p_i + e^{-p_0/\kappa}\, q_i\,,$$

$$(\ominus p)_0 = -p_0\,, \quad (\ominus p)_i = -e^{p_0/\kappa}\, p_i\,,$$

- Torsion and non-metricity are non-zero.
- In 3+1:

$$T_i^{0j} = \frac{1}{\kappa}\delta_i^j\,, \qquad N^{0ij} = \frac{1}{\kappa} g^{ij}$$

- In 2+1 this is the effective dynamics of spin foams.

Theorists propose but experiments decide.

The Gamma Ray Burst (GRB) problem

Long ago and far away there was a GRB.

Two photons were created simultaneously (according to a local observer there) but with very different energies.

Are they detected by the Fermi satellite simultaneously?

Naive (wrong) argument: you can choose coordinates on curved momentum space so that the speed of light is energy dependent.

$$c(E) = \frac{dE}{dp} = c\left(1 - \frac{E}{M_{QG}} + \ldots\right)$$

Hence there is a time delay

$$\Delta T = T_{flight} \frac{\Delta E}{M_{QG}} = 1sec \frac{m_P}{M_{QG}} \frac{T_{flight}}{10^{10}\ years} \frac{\Delta E}{10\ Gev}$$

The problem with this: you can also choose coordinates on momentum space so the speed of light is a constant!

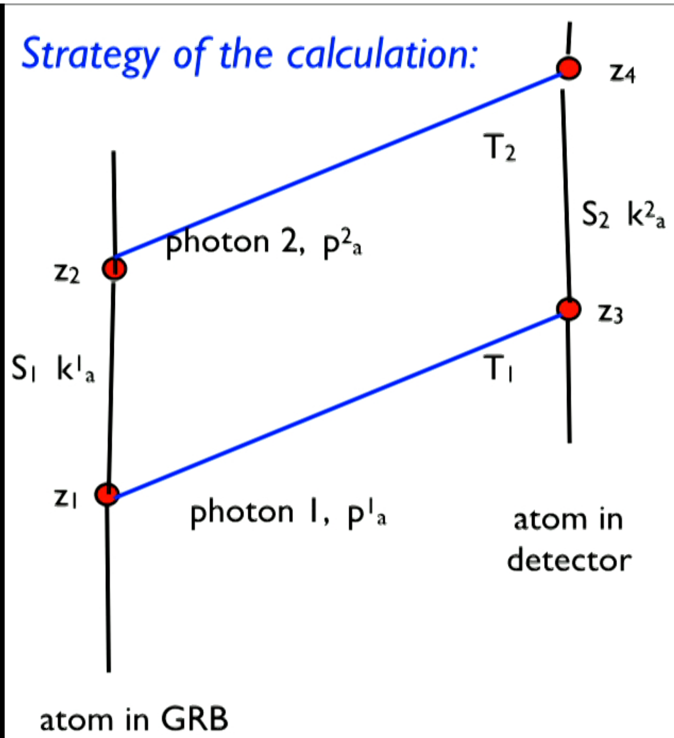
These are Riemann normal coordinates:

$$D(p) = \eta^{ab} p_a p_b$$

$$\partial^b g^{bc}|_{p=0} = 0 \rightarrow \Gamma = T + N$$

So is there no time delay??

To find out you have to compute the proper time between detections of the two photons.



Start with a trivial remark:

$$z_2^a - z_1^a + z_4^a - z_2^a = z_3^a - z_1^a + z_4^a - z_3^a$$

In special relativity:

$$z_2^a - z_1^a = S_1 \frac{k_1^a}{m_1}$$

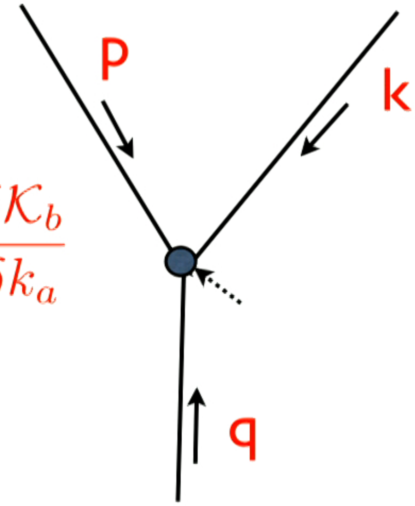
In relative locality:

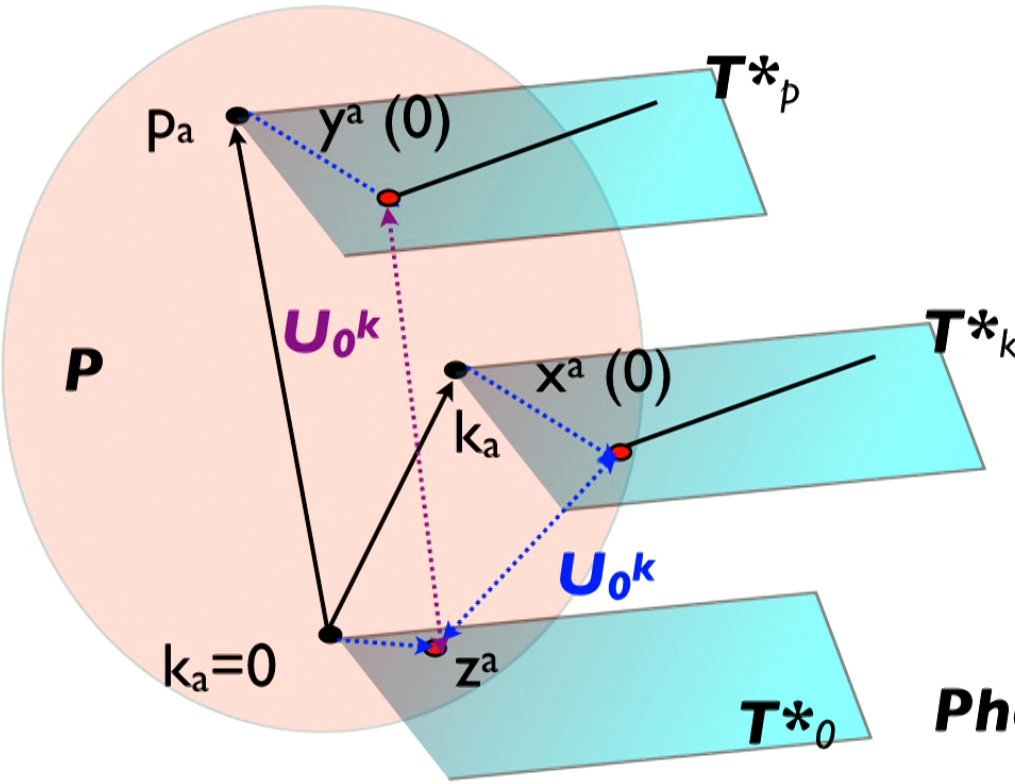
$$z_2^a - z_1^a = S_1 \frac{k_1^b}{m_1} U(1)_b^a$$

Parallel transport from cotangent plane over k_1 where the worldline lives to the z plane about 0 where the interaction point lives

The particles of different momenta travel on different cotangent planes on the phase space. To bring their end points together to interact requires parallel transport in the phase space.

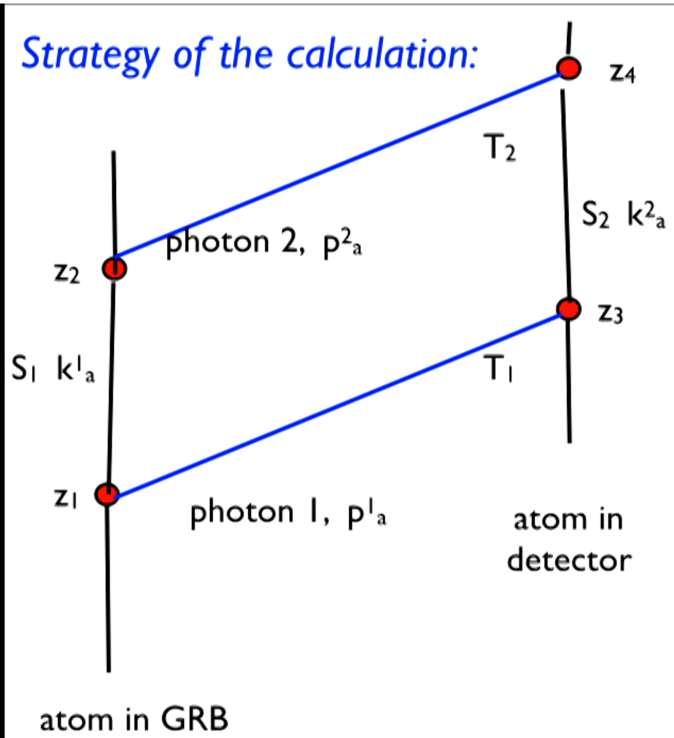
$$x^a(0) = U(k)^a_b z^b, \qquad U(k)^a_b = \frac{\delta \mathcal{K}_b}{\delta k_a}$$





Phase space = T*P

Strategy of the calculation:



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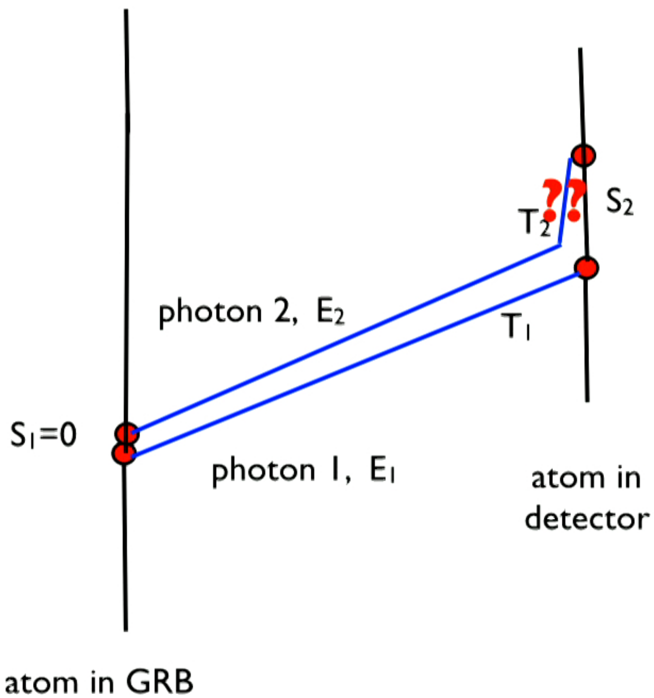
In relative locality:

$$z_2^a - z_1^a = S_1 \frac{k_1^b}{m_1} U(1)_b^a$$

In special relativity: $$S_1 \frac{k_1^a}{m_1} - S_2 \frac{k_2^a}{m_1} + T_2 \hat{p}_2 - T_1 \hat{p}_1 = 0$$

In relative locality:

$$S_1 \frac{k_1^b}{m_1} U(k_1)_b^a - S_2 \frac{k_2^b}{m_1} U(k_2)_b^a + T_2 \hat{p}_2^b U(p_2)_b^a - T_1 \hat{p}_1^b U(p_2)_b^a = \frac{1}{m_p^2} \text{curvatures}$$



Neglect all energies except E_2

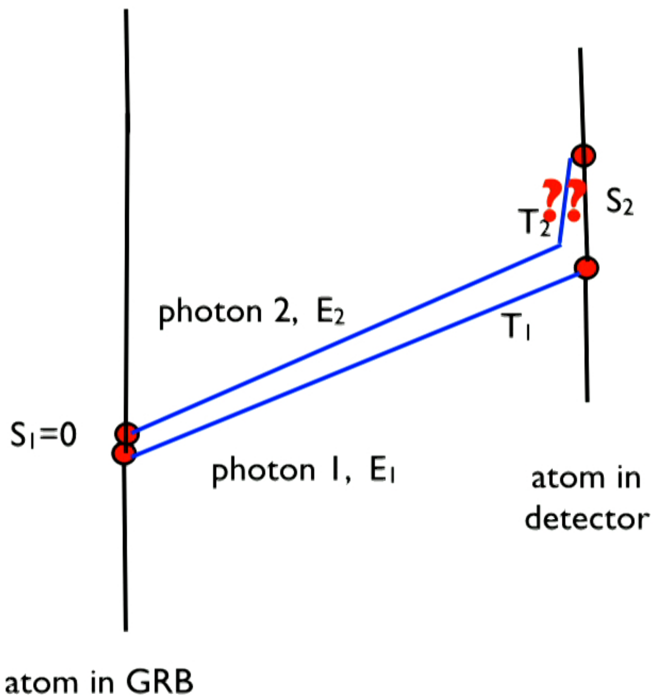
$T_1 \sim T_2 = T >> S_{1,2}$

neglect curvatures

$$\boxed{\begin{aligned} S_2 - S_1 &= -\frac{1}{2} E_2 \, T \, \Gamma^{++}_{-} \\ &= -\frac{1}{2} E_2 \, T \, N^{+++} \end{aligned}}$$

The leading order effect is due to non-metricity.

If emission is simultaneous in the GRB frame, so S_1=0, *there is still a time delay!*



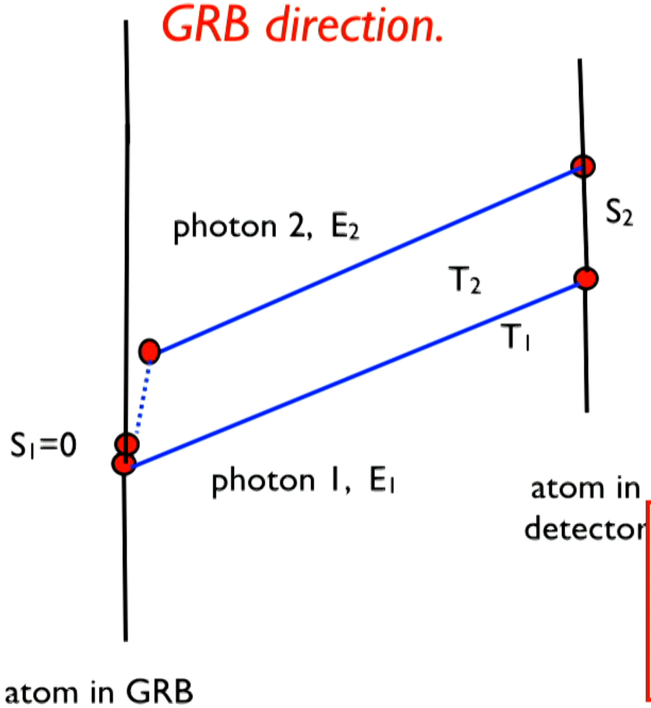
Neglect all energies except E_2
$T_1 \sim T_2 = T >> S_{1,2}$

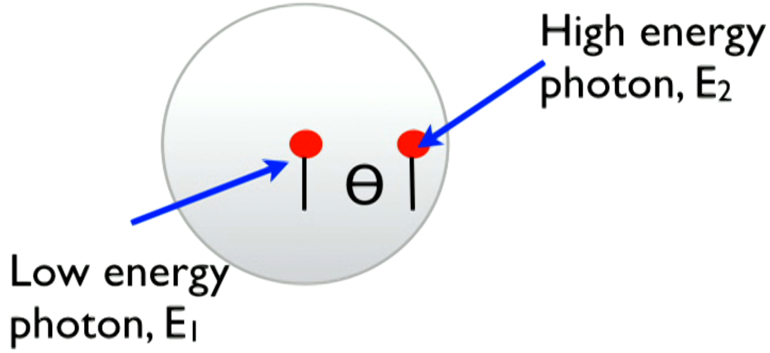
$$S_2 - S_1 = -\frac{1}{2} E_2 \, T \, \Gamma^{++}_{-} = -\frac{1}{2} E_2 \, T \, N^{+++}$$

The Fermi event **GRB 090510** bounds the non-metricity tensor:

$$N^{+++} < \frac{1}{.6 M_{planck}}$$

There is also a transverse effect: *torsion can make the photons appear to come from a direction slightly away from the GRB direction.*





$$\boxed{\theta = \frac{E_1 + E_2}{2}\sqrt{|T_-^{+a}\eta_{ab}T_-^{+b}|}}$$

"Dual gravitational lensing!"

If there was more time...

- The soccer ball problem is solved
- Interferometry in momentum space
- Analogue to Thomas precession (Girelli-Livine)
- More on localization
- The effects of curvature in momentum space
- Speculative remarks on the black hole information puzzle
- OPERA?

Conclusions:

Physics takes place in Hilbert space.
There is an experimental regime, in which the arena is a phase space

$$\begin{aligned} G_{Newton} &\rightarrow 0 \\ \hbar &\rightarrow 0 \\ m_p &= \sqrt{\frac{\hbar}{G_{Newton}}} \rightarrow \text{constant} \end{aligned}$$

m_p can measure the geometry of momentum space, *P*.

- If momentum space is curved there is no invariant notion of spacetime.
- There is only an invariant phase space, $T^*(P)$

If so, spacetime is as misleading a concept as space is in special relativity.
O(energy/m_p) phenomena appear paradoxical if one attempts to describe them using a notion of invariant spacetime.