

Title: Dueling Arrows of Causality, Causal Uncertainty and Quadratic Gravity

Speakers: John Donoghue

Collection: Indefinite Causal Structure

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Abstract: Quadratic gravity is a renormalizeable theory of quantum gravity which is unitary, but which violates causality by amounts proportional to the inverse Planck scale. To understand this, I will first discuss the arrow of causality in quantum field theory (with a detour concerning the arrow of time), and then discuss theories with dueling arrows of causality. But the causality violation might be better described by causality uncertainty. This is discussed both in quadratic gravity and in the effective field theory of general relativity.

# Dueling Arrows of Causality, Causal Uncertainty and Quadratic Gravity

- 1) Causality and the arrow of causality
- 2) Philosophic interlude – the arrow of time
- 3) Dueling arrows of causality
- 4) Quartic propagators in interacting theories
- 5) Quadratic Gravity
- 6) Causal Uncertainty – QG and Effective Field Theory

QFT in  
Minkowski



Work with Gabriel Menezes  
arXiv:1712.04468 , arXiv:1804.04980, arXiv:1812.03603  
arXiv:1908.02416, arXiv:1908.04170, ...

John Donoghue  
Perimeter  
December, 2019

## **Brief motivation:**

Quadratic gravity:

$$S_{\text{quad}} = \int d^4x \sqrt{-g} \left[ \frac{2}{\kappa^2} R + \frac{1}{6f_0^2} R^2 - \frac{1}{2\xi^2} C_{\mu\nu\alpha\beta} C^{\mu\nu\alpha\beta} \right]$$

**Renormalizeable QFT** for quantum gravity

- the most conservative version of quantum gravity

**BUT:**

$$R \sim \partial^2 g \qquad R^2 \sim \partial^2 g \partial^2 g$$

Higher derivative theories have “issues” and mythology

**Bottom line:** Find unitary theory, appears stable near Minkowski

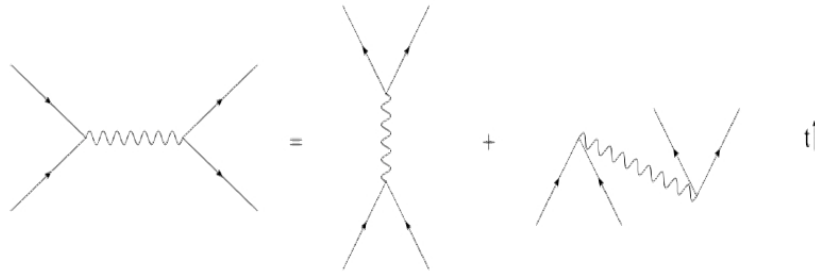
-but with Planck scale causality violation/uncertainty

## Where we are going:

- 1) Causal arrow of QFT
- 2) Easy to have **dueling causal arrows**
  - modes in theory with opposite causality
  - heavy modes pick up **decay width**
  - **limits problems with causality** (Lee + Wick, Coleman~1969)
- 3) Quantum gravity may have this feature
  - even near Minkowski
- 4) Causality **uncertainty** of particle reactions
  - certainly product of causality violating theories
  - plausibly product of gravity in general
  - **lack of definite light cones** in the Effective Field Theory



# I. Causality is not really “cause before effect”



$$iD_F(q) = \frac{i}{q^2 - m^2 + i\epsilon}$$

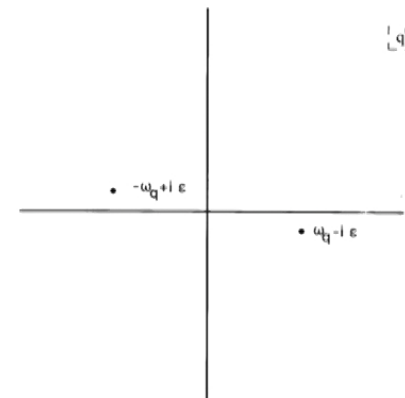
Decompose into time orderings:

$$iD_F(x) = D_F^{\text{for}}(x)\theta(t) + D_F^{\text{back}}(x)\theta(-t)$$

Positive energies propagate forward in time

$$D_F^{\text{for}}(x) = \int \frac{d^3q}{(2\pi)^3 2E_q} e^{-i(E_q t - \vec{q} \cdot \vec{x})}$$

$$D_F^{\text{back}}(x) = (D_F^{\text{for}}(x))^*$$

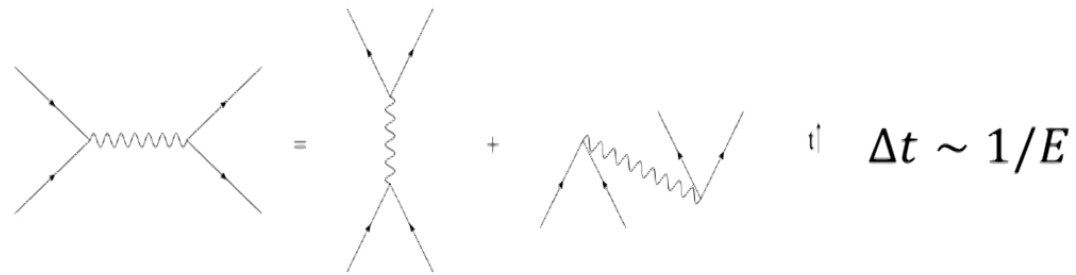


## Aside:

**Can we see effect before cause?**

- precisely defined pulses at LHC
- precision vertex detectors

**No – uncertainty principle:**



## Operators commute for spacelike separation

$$[\mathcal{O}(x), \mathcal{O}(x')] = 0 \quad \text{for} \quad (x - x')^2 < 0.$$

Note: metric is  
(+, -, -, -)

PHYSICAL REVIEW

VOLUME 95, NUMBER 6

SEPTEMBER 15, 1954

### Use of Causality Conditions in Quantum Theory

M. GELL-MANN, *Institute of Nuclear Studies and Department of Physics, University of Chicago, Chicago, Illinois*  
M. L. GOLDBERGER, \* *Princeton University, Princeton, New Jersey*

AND

W. E. THIRRING, † *Institute for Advanced Study, Princeton, New Jersey*  
(Received May 24, 1954)

The limitations on scattering amplitudes imposed by causality requirements are deduced from the demand that the commutator of field operators vanish if the operators are taken at points with space-like separations. The problems of the scattering of spin-zero particles by a force center and the scattering of photons by a quantized matter field are discussed. The causality requirements lead in a natural way to the well-known dispersion relation of Kramers and Kronig. A new sum rule for the nuclear photoeffect is derived and the scattering of photons by nucleons is discussed.

## But also – Arrow of causality

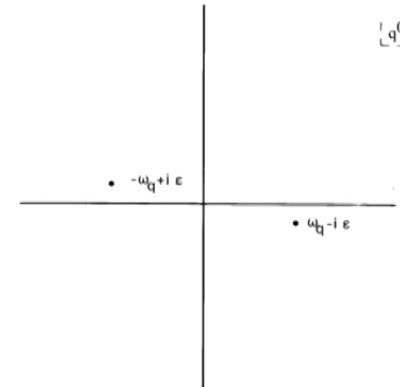
Commutator compatible with either direction

Extra ingredient of “arrow”

Enforced by analyticity properties

$$iD_F(q) = \frac{i}{q^2 - m^2 + i\epsilon}$$

↖  
\*



## What if we used $e^{-iS}$ instead of $e^{iS}$ ?

Consider generating functions:

$$\begin{aligned} Z_{\pm}[J] &= \int [d\phi] e^{\pm iS(\phi, J)} \\ &= \int [d\phi] e^{\pm i \int d^4x [\frac{1}{2}(\partial_{\mu}\phi\partial^{\mu}\phi - m^2\phi^2) + J\phi]} \end{aligned} \quad \hbar = c = 1$$

Need to make this better defined – add

$$e^{-\epsilon \int d^4x \phi^2 / 2}$$

Solved by completing the square:

$$Z_{\pm}[J] = Z[0] \exp \left\{ -\frac{1}{2} \int d^4x d^4y J(x) iD_{\pm F}(x-y) J(y) \right\}$$

Yield propagator with specific analyticity structure

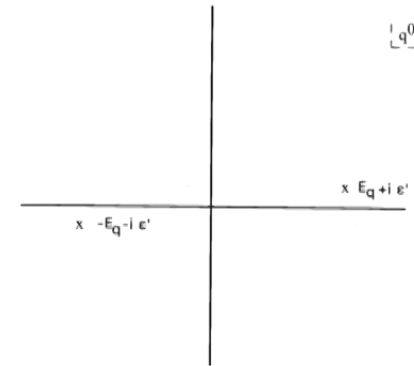
$$iD_{\pm F}(x-y) = \int \frac{d^4q}{(2\pi)^4} e^{-iq \cdot (x-y)} \frac{\pm i}{q^2 - m^2 \pm i\epsilon}$$

## Result is time-reversed propagator

$$iD_{-F}(x) = D_{-F}^{\text{for}}(x)\theta(t) + D_{-F}^{\text{back}}(x)\theta(-t)$$

Positive energy propagates backwards in time

$$D_{-F}^{\text{back}}(x) = \int \frac{d^3q}{(2\pi)^3 2E_q} e^{-i(E_q t - \vec{q} \cdot \vec{x})}$$



Use of this generating functional yields time reversed scattering processes

## **Opposite arrow of causality**

**Note:** This derivation made no intrinsic assumption about causal direction

## Time reversal is anti-unitary

**Lagrangian can be invariant, but PI is not**

$$Z_+[J] \rightarrow Z_-[J]$$

This can be seen in other ways also:

**In canonical quantization**, note that with  $\mathbf{t} = -\boldsymbol{\tau}$ ,

$$H\psi = -i\hbar \frac{\partial}{\partial \tau} \psi$$

**and canonical quantization rules change**

$$[\phi(t, x), \pi(t, x')] = i\hbar \delta^3(x - x')$$

to

$$[\phi(t, x), \bar{\pi}(t, x')] = -i\hbar \delta^3(x - x') \quad \text{with} \quad \bar{\pi} = \frac{\partial \mathcal{L}}{\partial(\partial_\tau \phi)}.$$

## Direction of time as a parameter is a convention



← Time measured by decreasing amounts

← Time measured by increasing amounts

Countdown:



**Different factors of  $i$  for different conventions**



## How quantum rules determine causal arrow:

Example:  $A + B \rightarrow \text{Resonance} \rightarrow A + B$

- emission always occurs later than absorption

**Bedrock convention: kinetic energy and mass are positive**

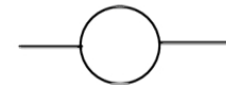
- initial state carries positive energy
- propagator with usual  $i\epsilon$  propagates positive energy forward in time

**Resonances and  $i\epsilon$**

$$iD_F(q) = \frac{i}{q^2 - m^2 + \Sigma(q)}$$

Imaginary part of self energy determined by  $i\epsilon$ , i.e. neglecting masses

$$\Sigma(q) = -\frac{\gamma}{\pi} \log \left( \frac{-q^2 - i\epsilon}{\mu^2} \right) = \left[ -\frac{\gamma}{\pi} \log \left( \frac{|q^2|}{\mu^2} \right) + i\gamma\theta(q^2) \right]$$



Can show generally  $\text{Im } \Sigma(q) \geq 0$  (unitarity)

Determines resonance propagator

$$iD_F \sim_{q^2 \sim m^2} \frac{i}{q^2 - m^2 + i\gamma_m} \sim \frac{i}{q^2 - (m_r - i\frac{1}{2}\Gamma)^2}$$

This then determines causal features – scattering wavepackets:

$$\begin{aligned} \langle \psi_{\text{out}} | \psi_{\text{in}} \rangle &= -ig^2 \int \frac{d^4q}{(2\pi)^4} \hat{F}(q) \hat{G}(q) e^{-iq \cdot (z_f - z_i)} \frac{1}{q^2 - M^2 + iM\Gamma} \\ &= -g^2 \int_0^\infty ds \int \frac{d^4q}{(2\pi)^4} \hat{F}(q) \hat{G}(q) e^{-iq \cdot (z_f - z_i)} \\ &\quad \times e^{is(q^2 - M^2 + iM\Gamma)} \end{aligned}$$

Follows:  
Grinstein,  
O'Connell,  
Wise 2009

After stationary phase approximation

$$\langle \psi_{\text{out}} | \psi_{\text{in}} \rangle \sim \theta(t_f - t_i) \hat{F}(M) \hat{G}(M) e^{-iM(t_f - t_i)} e^{-\Gamma(t_f - t_i)/2}$$

**There is a direct connection between the factors of  $i$  and the direction of causal behavior**

Changing these would change direction but remain causal

## **Basic conclusion:**

**Quantum physics is causal and has an arrow of causality**

**With usual time convention:**

use  $e^{iS}$  or usual commutators  
causal in the direction of increasing time

**Or with decreasing time convention**

use  $e^{-iS}$  or opposite commutator  
causal in the opposite direction  
time reversed theory

**But just a difference in convention of in flow of time**

**Our conventions are  $e^{iS}$  , so all causal flow is towards increasing time**

## **Aside: “Arrow of time” discussions:**

Typical motivations:

*"The laws of physics at the fundamental level don't distinguish between the past and the future."*

*"The difference between past and future, between cause and effect, between memory and hope ... in the elementary laws that describe the mechanisms of the world, there is no such difference."*

*"The key point is that things happen irreversibly and time asymmetrically at the macro scale....even though the foundational dynamics at the microscale (based in Hamiltonians) is in principle reversible and time symmetric."*

**But these are not correct!**

- “Laws of physics” are more than the classical EoM

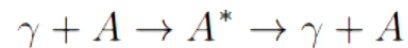
## This is not the “Thermodynamic Arrow of Time”

Often: “Laws do not distinguish arrow, but Thermodynamics does“

But this is also not correct:

Microscopic processes have causal arrow

As we saw: absorption and emission



Elementary processes run forward in time

Emission is always later than absorption

**Rather:** Arrow of thermodynamics follows arrow of causality

## **This is not spontaneous symmetry breaking**

Often: “Laws of physics have time-reversal symmetry (or CPT)  
but our ground state does not  $\Rightarrow$  SSB”

But this is really a “hard” breaking (or really lack of symmetry)  
**Lagrangian has symmetry, but full PI does not**

### **Similar to anomalies:**

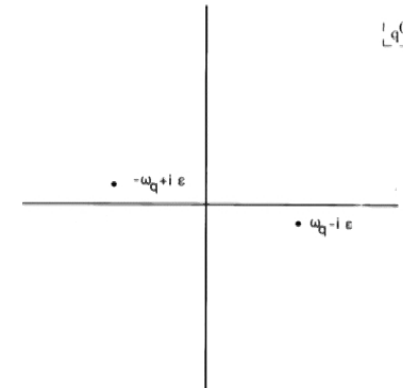
Anomaly is when classical physics (Lagrangian) has symmetry  
but the quantum theory (PI) does not.

Fujikawa: PI measure not invariant for standard anomalies

# How time reversal symmetry works in a causal theory

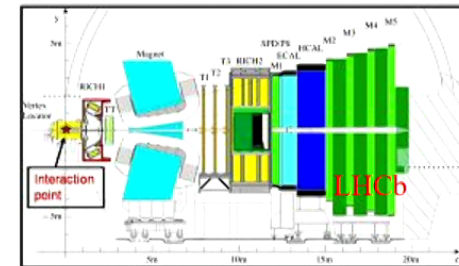
Enforced by analyticity properties

$$iD_F(x) = D_F^{\text{for}}(x)\theta(t) + D_F^{\text{back}}(x)\theta(-t)$$



**Example:** long-lived resonance production

- production  $A+B \rightarrow R$
- decay  $R \rightarrow C+D$
- decay always happens later
  - this is the arrow of causality



Note: Time reversal relates  $A+B \rightarrow C+D$  and  $C+D \rightarrow A+B$

- but experiment runs both reactions forward in time

## II. Dueling arrows of causality

JFD +GM PRL 2019

Quartic propagators have opposing arrows

$$\mathcal{L} = \frac{1}{2} [\partial_\mu \phi \partial^\mu \phi - m^2 \phi^2] - \frac{1}{2M^2} \square \phi \square \phi.$$

$$iD(q) = \frac{i}{q^2 - q^4/M^2} = \frac{i}{q^2} - \frac{i}{q^2 - M^2}$$



No tachyons allowed

**Who wins?**

- massive state decays
- stable states win



## The all-in-one propagator (including self-energy)

$$iD(q) = \frac{i}{q^2 - m^2 + \Sigma(q) - q^4/M^2 + i\epsilon}$$

Above some threshold

$$\text{Im } \Sigma(q) = \gamma(q) \qquad \gamma(q) \geq 0$$

If threshold is above  $m^2$ , stable particle at  $q^2=m^2$

If threshold is below  $m^2$ , this is a normal resonance

$$iD_F \sim_{q^2 \sim m^2} \frac{i}{q^2 - m^2 + i\gamma_m} \sim \frac{i}{q^2 - (m_r - i\frac{1}{2}\Gamma)^2}$$

The high mass pole carries two minus sign differences:

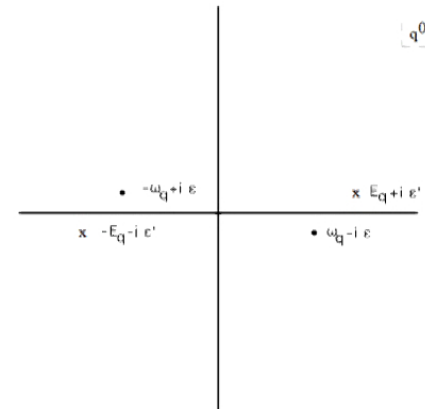
$$\begin{aligned} iD_F(q) &= \frac{i}{q^2 - \frac{q^4}{M^2} + i\gamma(q)} \\ &= \frac{i}{\frac{q^2}{M^2} [M^2 - q^2 + i\gamma(q)(M^2/q^2)]} \\ &\sim \frac{-i}{q^2 - M^2 - i\gamma_M} \end{aligned}$$

This is a finite width version of  $D_{-F}$

## Propagation in both directions:

$$D^{\text{for}}(t, \vec{x}) = \int \frac{d^3q}{(2\pi)^3} \left[ \frac{e^{-i(\omega_q t - \vec{q} \cdot \vec{x})}}{2\omega_q} - \frac{e^{i(E_q t - \vec{q} \cdot \vec{x})}}{2(E_q + i\frac{\gamma}{2E_q})} e^{-\frac{\gamma t}{2E_q}} \right]$$

$$D^{\text{back}}(t, \vec{x}) = \int \frac{d^3q}{(2\pi)^3} \left[ \frac{e^{i(\omega_q t - \vec{q} \cdot \vec{x})}}{2\omega_q} - \frac{e^{-i(E_q t - \vec{q} \cdot \vec{x})}}{2(E_q + i\frac{\gamma}{2E_q})} e^{-\frac{\gamma |t|}{2E_q}} \right]$$



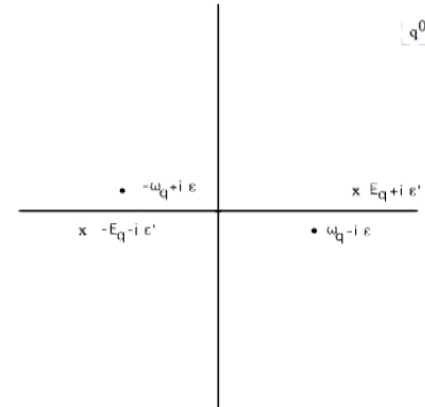
## Quartic propagators carry two arrows of causality

- stable states win over large times
- massive states decay
- backwards propagation over scale of width

## Propagation in both directions:

$$D^{\text{for}}(t, \vec{x}) = \int \frac{d^3q}{(2\pi)^3} \left[ \frac{e^{-i(\omega_q t - \vec{q} \cdot \vec{x})}}{2\omega_q} - \frac{e^{i(E_q t - \vec{q} \cdot \vec{x})}}{2(E_q + i\frac{\gamma}{2E_q})} e^{-\frac{\gamma t}{2E_q}} \right]$$

$$D^{\text{back}}(t, \vec{x}) = \int \frac{d^3q}{(2\pi)^3} \left[ \frac{e^{i(\omega_q t - \vec{q} \cdot \vec{x})}}{2\omega_q} - \frac{e^{-i(E_q t - \vec{q} \cdot \vec{x})}}{2(E_q + i\frac{\gamma}{2E_q})} e^{-\frac{\gamma |t|}{2E_q}} \right]$$



## Quartic propagators carry two arrows of causality

- stable states win over large times
- massive states decay
- backwards propagation over scale of width

## **Merlin modes:**

-Merlin (the wizard in the tales of King Arthur) ages backwards



*“Now ordinary people are born forwards in Time, if you understand what I mean, and nearly everything in the world goes forward too. (...) But I unfortunately was born at the wrong end of time, and I have to live backwards from in front, while surrounded by a lot of people living forwards from behind.”*

T. H. White *Once and Future King*

Note, there is a key distinction with usual nomenclature “ghosts”

- ghost is anything with a minus sign in the numerator
- these Merlin modes refer to crucial sign  $-i\gamma$  in denominator in addition

## At the Lagrangian level:

Sample interacting theory (notation for convenience in QG)

$$\mathcal{L} = \frac{1}{2} \partial_\mu \phi \partial^\mu \phi - \frac{\kappa^2}{4\xi^2} \square \phi \square \phi - \frac{\kappa}{2} (\square \phi) \sum_i \chi_i^2$$

Use auxiliary field

$$\mathcal{L} = \frac{1}{2} \partial_\mu \phi \partial^\mu \phi - \square \phi \eta + \frac{\xi^2}{\kappa^2} \eta^2 - \frac{\kappa}{2} (\square \phi) \sum_i \chi_i^2$$

Then shift  $\phi = h - \eta$ ,

$$\begin{aligned} \mathcal{L} = & \left[ \frac{1}{2} \partial_\mu h \partial^\mu h - \frac{\kappa}{2} \square h \sum_i \chi_i^2 \right] \\ & - \left[ \frac{1}{2} \left( \partial_\mu \eta \partial^\mu \eta - \frac{2\xi^2}{\kappa^2} \eta^2 \right) - \frac{\kappa}{2} \square \eta \sum_i \chi_i^2 \right] \end{aligned}$$

Two modes propagating in opposite directions (coupled by matter)

## Or keep single field description

Overall propagator (notation to match quadratic gravity)

$$D_2(q) = \left\{ q^2 + i\epsilon - \frac{\kappa^2 q^4}{2\xi^2(\mu)} - \frac{\kappa^2 q^4 N_{\text{eff}}}{640\pi^2} \left[ \ln\left(\frac{|q^2|}{\mu^2}\right) - i\pi\theta(q^2) \right] \right\}^{-1}$$

$N_{\text{eff}} = 20 \text{ N}$  (or  $20(N+1)$  if we include self interaction)

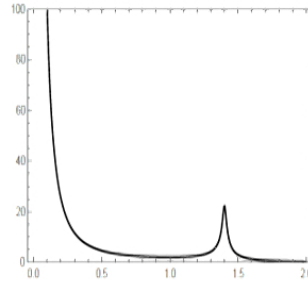


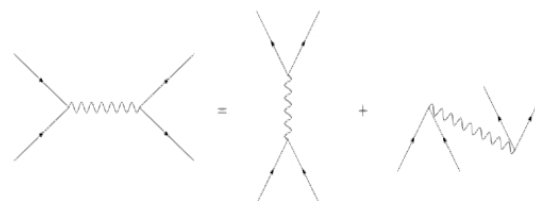
FIG. 1: The absolute value of the spin-two propagator for  $\xi^2 = 1$ , showing the high mass resonance. The x-axis is the momentum  $|q|$  in the time-like region, in units where  $\kappa = 1$ . The imaginary parts have been calculated with loops of Standard Model particles and gravitons.

## Phenomenology

Lee, Wick  
Coleman  
Grinstein, O'Connell, Wise  
Alvarez, Da Rold, Schat, Szykman

### Vertex displacements: (ADSS)

- look for final state emergence (LHC)
- before beam collision



### Form wavepackets – early arrival of signal (LW, GOW)

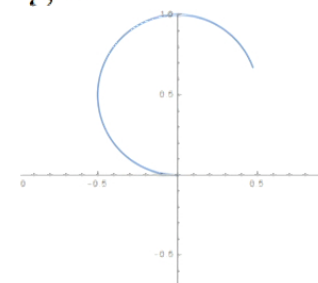
- wavepacket description of scattering process
- some components arrive at detector early

### Resonance - Wigner time delay reversal

- normal resonances counterclockwise on Argand diagram

$$\Delta t \sim \frac{\partial \delta}{\partial E} \sim > 0$$

- Merlin modes are clockwise resonance



### For gravity, all are Planck scale

- no conflict with experiment

## Ghost stories:

We have been discussing “ghosts”

- lots of conflicting lore

Ghosts can be quantized as free fields

- indefinite metric (Lee-Wick, Mannheim-Bender, Slavio-Strumia)
- but we **do not have** to do so
  - interactions make them decay, remove from physical spectrum
  - value of PI treatment

Ostrogradski instability of classical Hamiltonian

- seems to be absent in interacting QFT

Worries about unitarity?

- we have proven unitarity (JFD + GM, PRD100)
- unstable states do not appear in unitarity sum



## Lesson for quantum theories

Usual rules give causal theory

But you can break this through a simple modification

- higher derivatives
- learn to deal with ghosts

QM can have causality violating realizations

Obvious question is: **Why would we do this?**

- higher derivative theories can be **finite** (Lee-Wick)
- it is reasonably natural in gravity

## **Gravity is the most likely place for this to occur:**

### **Power counting theorem:**

- quantize gravity normally
  - $R \sim \partial g \partial g + g \partial g \partial g + \dots$
  - propagators  $\sim 1/q^2$
- consider loops
- expansion in energy  $\sim$  derivatives

### **Power counting – dimensional coupling G**

- tree level  $\sim (\partial g)^2$
- one loop  $\sim (\partial g)^4$
- two loop  $\sim (\partial g)^6$

**Gravity naturally has higher dimension propagators**

## Quadratic gravity - renormalizeable

$$S_{\text{quad}} = \int d^4x \sqrt{-g} \left[ \frac{2}{\kappa^2} R + \frac{1}{6f_0^2} R^2 - \frac{1}{2\xi^2} C_{\mu\nu\alpha\beta} C^{\mu\nu\alpha\beta} \right]$$

$$\kappa^2 = 32\pi G$$

Free-field mode decomposition depends on gauge fixing.

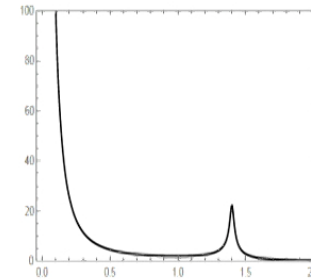
-all contain a scalar mode and tensor mode

Scalar has massive non-ghost and massless ghost – due to  $R^2$

$$D_{\mu\nu\alpha\beta}^{(0)}(q^2) = \left( \frac{q^4}{f_0^2} - \frac{2q^2}{\kappa^2} \right)^{-1} \mathcal{P}_{\mu\nu\alpha\beta}^{(0)} = \frac{\kappa^2}{2} \left( \frac{1}{q^2 - M_0^2} - \frac{1}{q^2} \right) \mathcal{P}_{\mu\nu\alpha\beta}^{(0)}$$

Spin 2 mode has massive ghost

$$D_{\mu\nu\alpha\beta}^{(2)}(q^2) = \left( \frac{q^2}{\kappa^2} - \frac{q^4}{2\xi^2} \right)^{-1} \mathcal{P}_{\mu\nu\alpha\beta}^{(2)} = \kappa^2 \left( \frac{1}{q^2} - \frac{1}{q^2 - M_2^2} \right) \mathcal{P}_{\mu\nu\alpha\beta}^{(2)}$$



## **Also: Asymptotic Safety**

Basic idea: UV fixed point in Euclidean PI

- Use RG to run to the real world (i.e. including all quantum effects)

Result is a special Lagrangian

- infinite number of terms

$$\mathcal{L} = \sqrt{-g} \left[ -\Lambda_{vac} - \frac{1}{16\pi G} R + c_1 R^2 + c_2 C_{\mu\nu\alpha\eta} C^{\mu\nu\alpha\eta} + d_1 R^3 + d_2 R \square R + \dots \right]$$

- infinite number of parameters
- but most are determined by UV constraints
- in practice, truncations like that of quadratic gravity
- each truncation has causality violation like quadratic gravity

### **Advertisement (off-topic):**

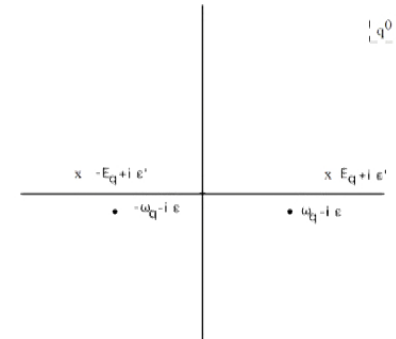
see JFD “A Critique of the Asymptotic Safety Program” Nov 2019

## Stability:

See also Salvio;  
Reis, Chapiro, Shapiro

Consider propagator with retarded BC:

$$\log(-(q_0 + i\epsilon)^2 - \vec{q}^2) = \log(-q^2 - i\epsilon q_0) = \log|q^2| - i\pi\theta(q^2)(\theta(q_0) - \theta(-q_0))$$



Again propagation in both directions:

$$D_{\text{ret}}(t > 0, \vec{x}) = D_{\text{ret}}^{(0)}(t > 0, \vec{x})$$

$$D_{\text{ret}}(t < 0, \vec{x}) \equiv D_{\text{ret}}^{<}(t, \vec{x}) = i \int \frac{d^3q}{(2\pi)^3} \left[ \frac{e^{-i(E_q t - \vec{q} \cdot \vec{x})}}{2(E_q + i\frac{\gamma}{2E_q})} e^{-\frac{\gamma|t|}{2E_q}} - \frac{e^{i(E_q t - \vec{q} \cdot \vec{x})}}{2(E_q - i\frac{\gamma}{2E_q})} e^{-\frac{\gamma|t|}{2E_q}} \right]$$

Backwards perturbations have finite lifetime:

$$h_{\mu\nu}(t, x) = \int d^3x' \left[ \int_{-\infty}^t dt' D_{\text{ret}}^{(0)}(t - t', x - x') + \int_t^{\infty} dt' D_{\text{ret}}^{<}(t - t', x - x') \right] J_{\mu\nu}(t', x')$$

## Causality Uncertainty

Wavepackets are an idealization:

- really formed by previous interactions

Likewise beam construction from previous scattering

- and measurement due to final scattering

The timing of scattering will become **uncertain**



**Uncertainty principle of causality?**

## **But also from the opposite extreme:**

Quadratic gravity is a possible UV completion for gravity

- only one of many possibilities
- has hard causality violation

The **Effective Field Theory of General Relativity** is rigorous

- low energy predictions

EFT also suggests causal uncertainty

This comes from basic QFT with gravity

## Quantum gravity makes sense at ordinary scales

GR makes a fine QFT

- quantized by Feynman – DeWitt , via Path Integrals
- just like Yang-Mills

We have learned how to deal with ‘non-renormalizable’ theories

- Effective Field Theory
- we do this every day now
  - including comparison with experiment

Gravity fits EFT framework perfectly

Still need UV completion for more extreme scales

- possibly new DOF
- but shows **issue is not really QM vs GR**



## How the EFT works:

(Extra brief)

High energy effects appear **local** at low energy

- general local Action – ordered in the energy expansion

$$S_{grav} = \int d^4x \sqrt{-g} \left\{ \Lambda + \frac{2}{\kappa^2} R + c_1 R^2 + c_2 R_{\mu\nu} R^{\mu\nu} + \dots \right\}$$

Low energy effects must be **dynamical**

- use low energy propagator – no Merlin modes
- non-local in coordinate space
- non-analytic in momentum space

$$V(q^2) = \frac{GMm}{q^2} \left[ 1 + a'G(M+m)\sqrt{-q^2} + b'G\hbar q^2 \ln(-q^2) + c'Gq^2 \right]$$

$$V(r) = -\frac{GMm}{r} \left[ 1 + a\frac{G(M+m)}{rc^2} + b\frac{G\hbar}{r^2c^3} \right] + cG^2Mm\delta^3(r)$$

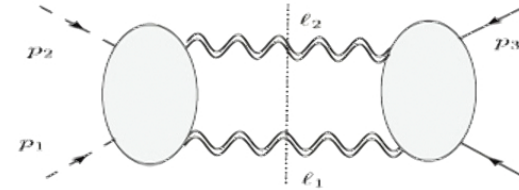
Low energy effects uniquely determined by **couplings of GR**

$$V(r) = -\frac{Gm_1m_2}{r} \left[ 1 + 3\frac{G(m_1+m_2)}{r} + \frac{41}{10\pi} \frac{G\hbar}{r^2} \right]$$

## Light bending at one loop

- Using unitarity methods
- Gravity Compton as square of EM Compton
- Compare massless spin 0 and photon

Bjerrum-Bohr, JFD, Holstein  
Plante, Vanhove



$$\begin{aligned}
 i\mathcal{M}_{[\phi(p_3)\phi(p_4)]}^{[\eta(p_1)\eta(p_2)]} &\simeq \frac{\mathcal{N}^\eta}{\hbar} (M\omega)^2 \left[ \frac{\kappa^2}{t} + \kappa^4 \frac{15}{512} \frac{M}{\sqrt{-t}} + \hbar\kappa^4 \frac{15}{512\pi^2} \right. \\
 &\quad \times \log\left(\frac{-t}{M^2}\right) - \hbar\kappa^4 \frac{bu^\eta}{(8\pi)^2} \log\left(\frac{-t}{\mu^2}\right) \\
 &\quad + \hbar\kappa^4 \frac{3}{128\pi^2} \log^2\left(\frac{-t}{\mu^2}\right) \\
 &\quad \left. + \kappa^4 \frac{M\omega}{8\pi} \frac{i}{t} \log\left(\frac{-t}{M^2}\right) \right], \quad (11)
 \end{aligned}$$

$$\begin{aligned}
 bu^0 &= \frac{371}{120}, & bu^\gamma &= \frac{113}{120}, \\
 bu^{\{grav\}} &= -29/8
 \end{aligned}$$

$bu^i$  is different  
coefficient for  
spin 0 , 1 and 2

Amplitude turned into bending angle through eikonal method  
 - saddle point at large impact parameter  $b$

$$\mathcal{M}^{0+1}(\Delta^\perp) = \frac{1}{2(s - M_\sigma^2)} \int d^2\mathbf{b}^\perp e^{-i\Delta^\perp \cdot \mathbf{b}^\perp} [e^{i(\chi_0 - i \ln[1 + i\chi_2])} - 1]$$

Using:  
 Akhoury  
 Ryo  
 Sterman

$$\theta \simeq \frac{4G_N M}{b} + \frac{15}{4} \frac{G_N^2 M^2 \pi}{b^2} + \left( 8bu^S + 9 - 48 \log \frac{b}{2b_0} \right) \frac{\hbar G_N^2 M}{\pi b^3} + \dots$$

### **Massless particles deviate from null geodesics**

- irreducible tidal effects from loops
- also non-universal – violation of some forms of EP

## Why this is relevant for causality uncertainty in gravity:

This calculation is **fully causal** – no real violation

### **But no longer any practical geodesics**

- massless particles deviate from classical null geodesics
- irreducible tidal effects from quantum loops
- also **non-universal** – violation of some forms of EP

In curved spacetimes, one loses clarity on causal effects

- lightcones lose their meaning
- can't prepare or detect with certainty

Further effects of superposition of metrics, but this is a simple calculable effect

## **Summary:**

Quantum physics has an arrow of causality

Easy to have dueling arrows of causality

- higher order terms in propagator

Quadratic gravity exhibits this phenomenon

- appears stable and unitarity
- Planck scale causality violation

Gravity may have causality uncertainty

- in quadratic gravity, real violation  
plus difficulty in preparing and detecting
- in EFT, lack of precise geodesics