

Title: TBA

Speakers: Christina Giarmatzi

Collection: Indefinite Causal Structure

Date: December 10, 2019 - 2:00 PM

URL: <http://pirsa.org/19120029>



THE UNIVERSITY
OF QUEENSLAND
AUSTRALIA

OUR QUANTUM SWITCH

— — — — —

WITNESSING QUANTUM MEMORY

Christina Giarmatzi and Fabio Costa

Templeton Grant

Perimeter Institute, Waterloo, December 2019

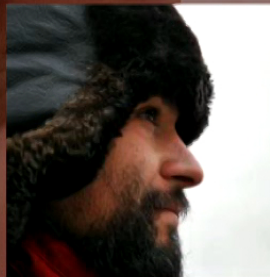
Context



Andrew White



Fabio Costa



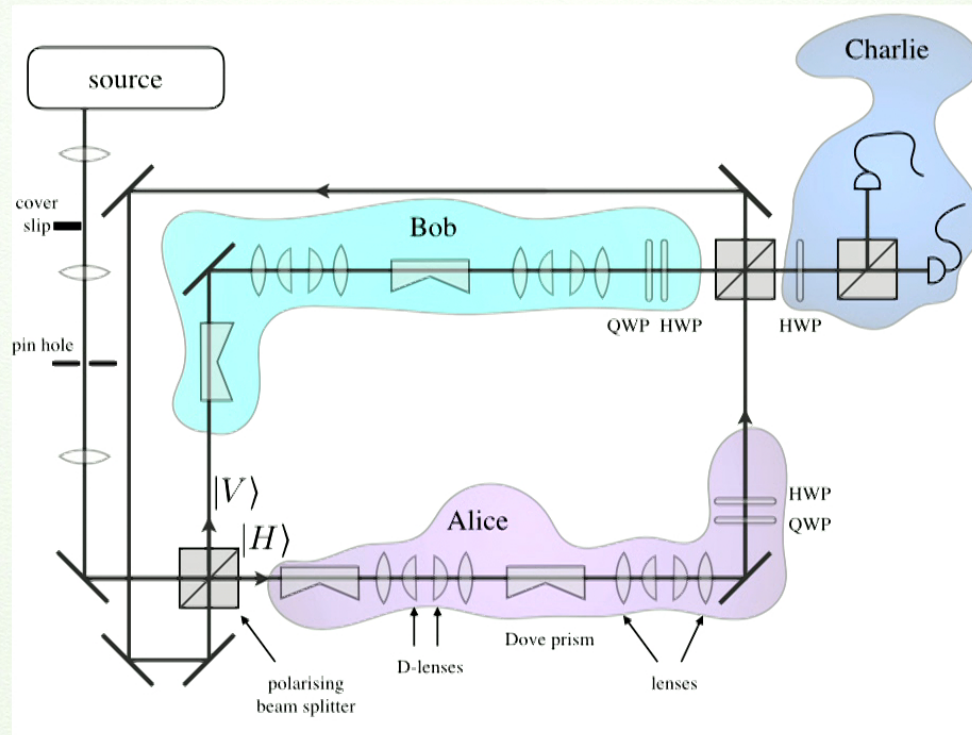
Gerard Milburn



Outline

- Quantum switch - experiment - what have we really achieved?
- Witnessing quantum memory - algorithm - SDP

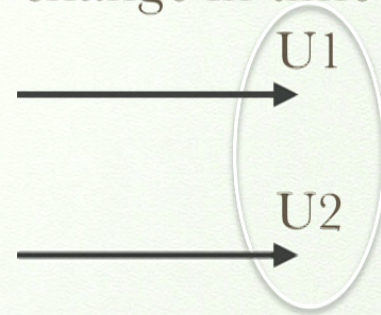
Quantum switch



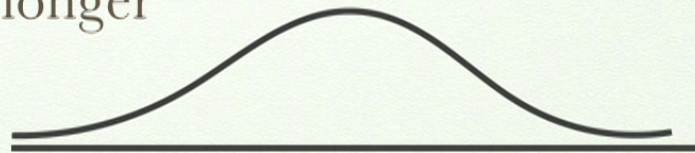
K. Goswami, C. Giarmatzi, M. Kewming, F. Costa, C. Branciard, J. Romero, and A. G. White.
Indefinite causal order in a quantum switch. *Physical Review Letters*, 121(9):090503–, 08 2018

The improvement

- Optical elements are less likely to change in time vs position of input photon



- Our photons are longer



K. Goswami, C. Giarmatzi, M. Kewming, F. Costa, C. Branciard, J. Romero, and A. G. White.
Indefinite causal order in a quantum switch. *Physical Review Letters*, 121(9):090503–, 08 2018

Achievement

- Why did we do it?
- Was it indefinite causal order?
- There are verifications.

O. Oreshkov, “*On the whereabouts of the local operations in physical realizations of quantum processes with indefinite causal order*”, <http://arxiv.org/abs/1801.07594>.

K. Goswami, C. Giarmatzi, M. Kewming, F. Costa, C. Branciard, J. Romero, and A. G. White.
Indefinite causal order in a quantum switch. *Physical Review Letters*, 121(9):090503–, 08 2018

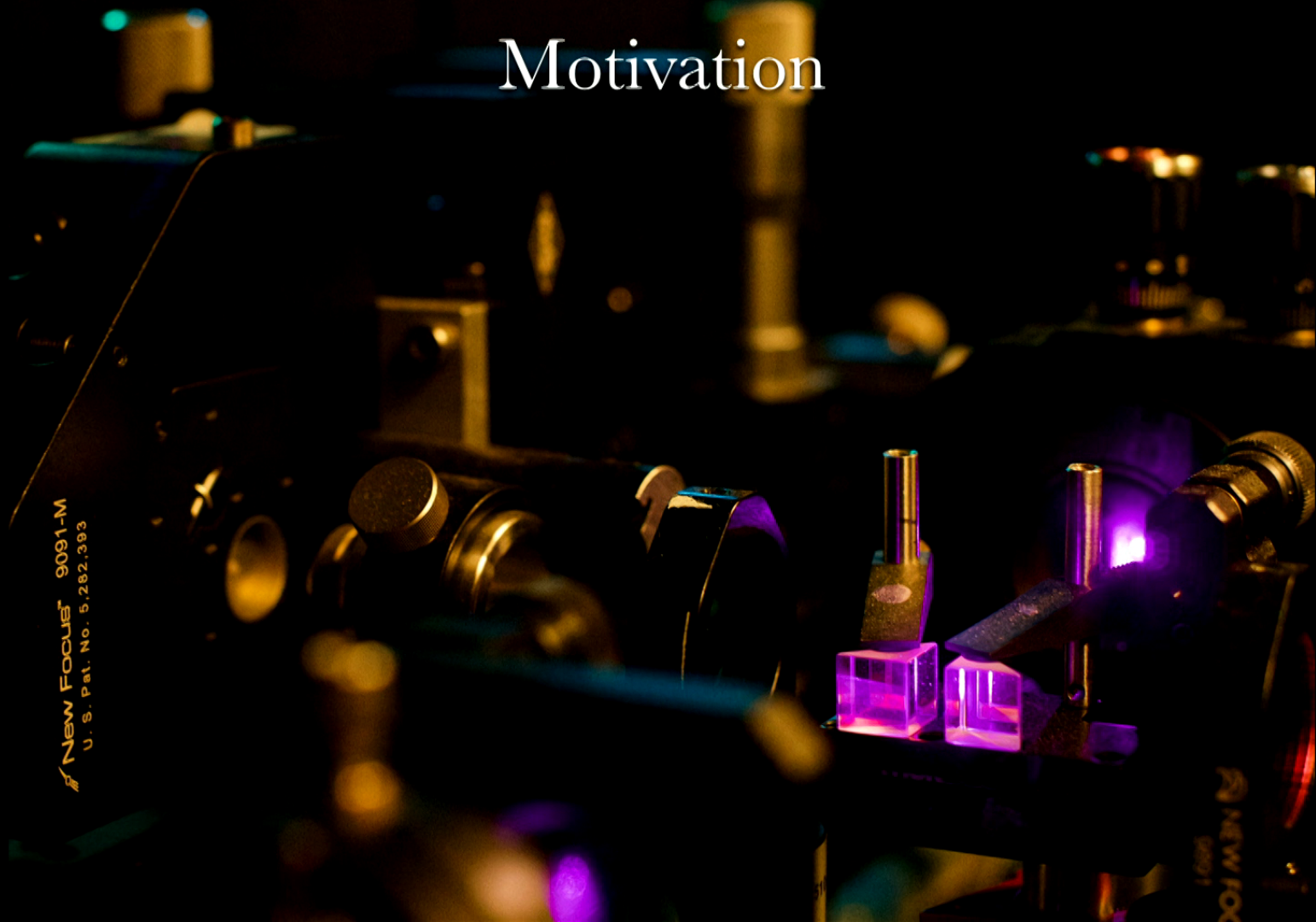
Anything else to say?

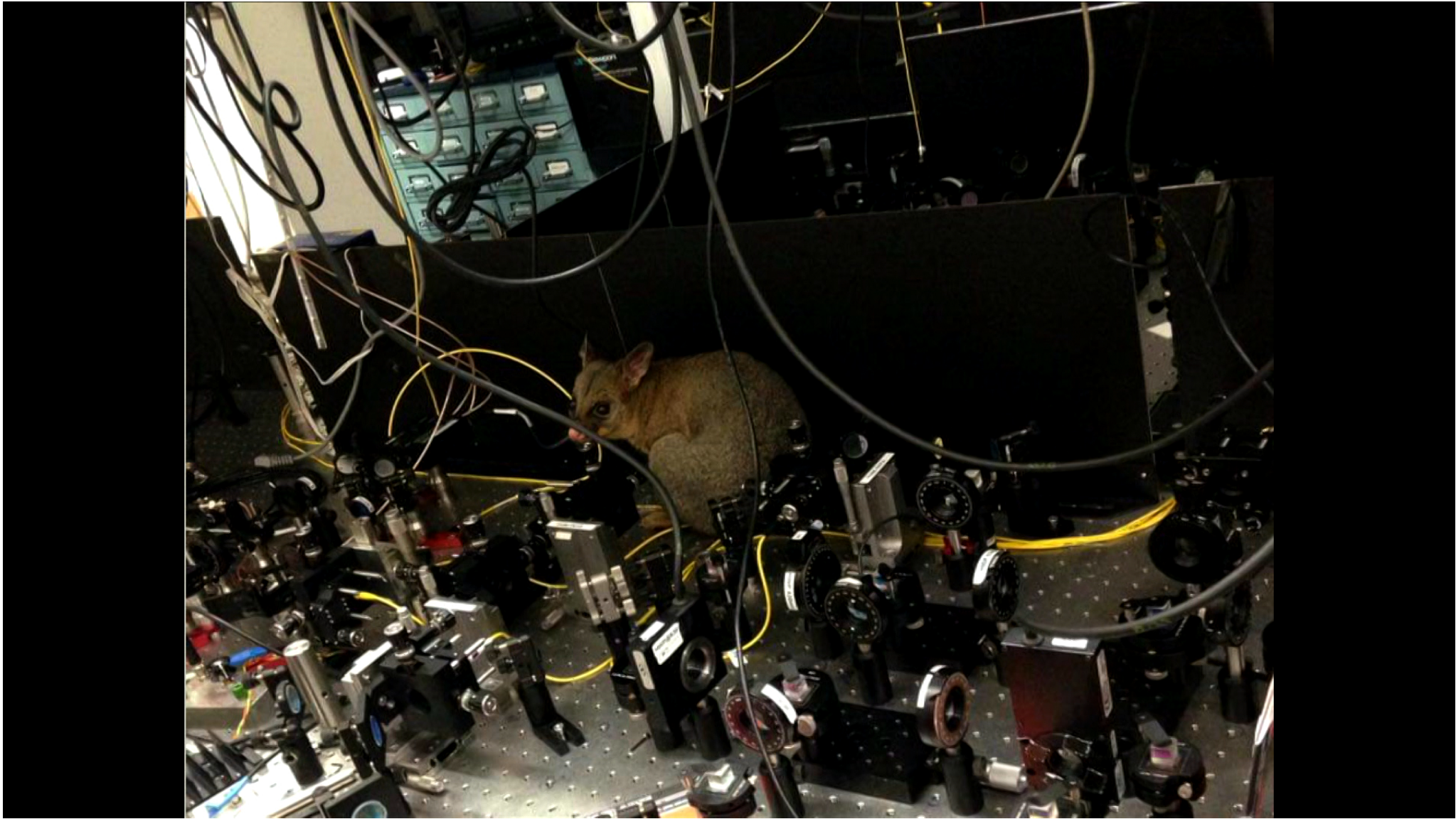
C. T. M. Ho, F. Costa, C. Giarmatzi, and T. C. Ralph. Violation of a causal inequality in a spacetime with definite causal order. arXiv:1804.05498 [quant-ph], 2018

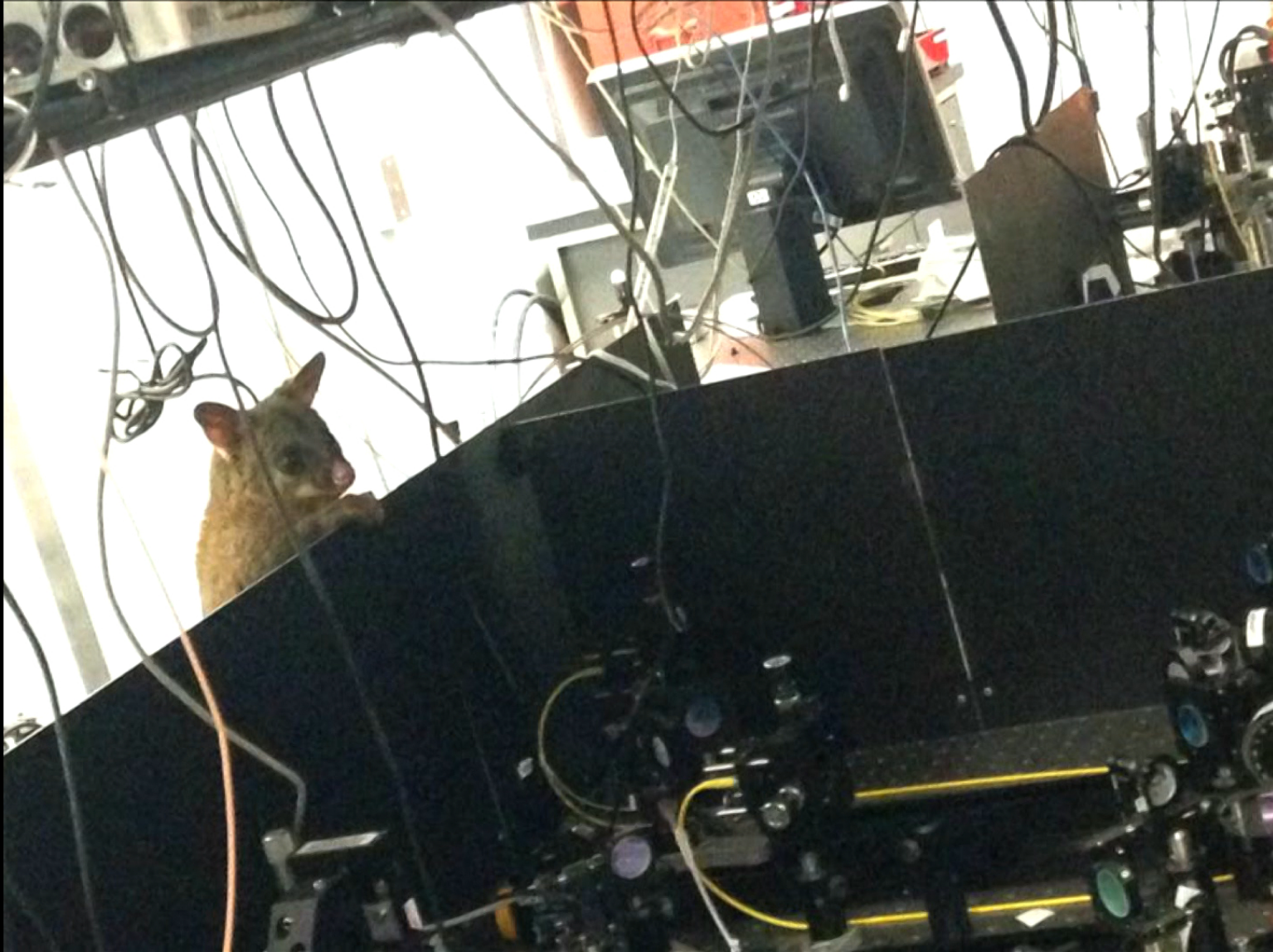
Outline

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- Witnessing quantum memory - algorithm - SDP

Motivation





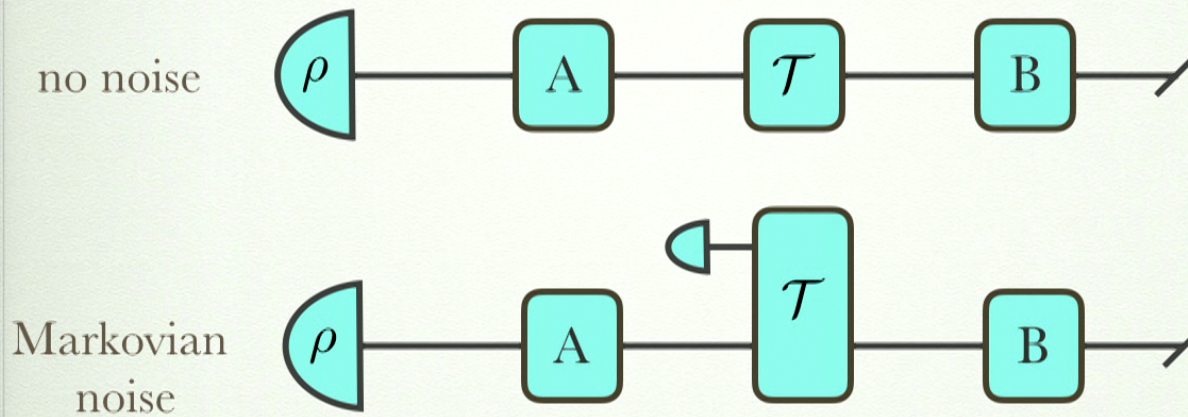


A sunset over a beach with a person carrying a surfboard. The sun is low on the horizon, casting a golden glow across the sky and reflecting on the wet sand. A person is walking away from the camera, carrying a surfboard under their arm. The sky is filled with soft, golden clouds.

Outline

- Markovianity of a process
- Non-Markovianity - classical and quantum memory
- Detecting a quantum memory
- Results on an example

Markovianity



G. Lindblad, "Non-Markovian quantum stochastic processes and their entropy", *Comm. Math. Phys.* 65, 281 (1979)

L. Accardi, A. Frigerio, and J.T. Lewis, "Quantum stochastic processes", *Publications of the Research Institute for Mathematical Sciences* 18, 97 (1982)

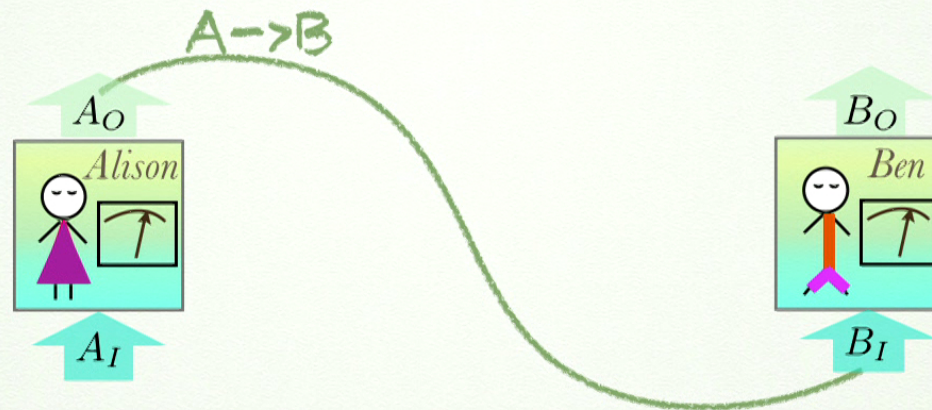
F.A. Pollock et al., "Non-Markovian quantum processes: Complete framework and efficient characterization", *Physical Review A* 97, 012127 (2018)

C. Giarmatzi and F. Costa, "Witnessing quantum memory in non-Markovian processes", arXiv: 1811.03722 (2018)

Process matrix formalism

O. Oreshkov, F. Costa, Č. Brukner, "Quantum correlations with no causal order", Nat. Commun. **3** 1092 (2012)
O. Oreshkov and C. Giarmatzi, *New Journal of Physics* **18**, 093020 (2016)

Example of a W

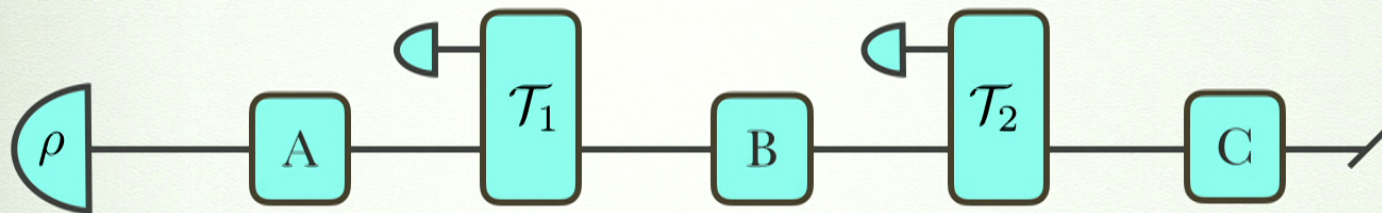


$$A \rightarrow B \quad W^{A \prec B} = \rho^{A_I} \otimes T^{A_O B_I} \otimes \mathbb{1}^{B_O}$$

O. Oreshkov, F. Costa, Č. Brukner, "Quantum correlations with no causal order", Nat. Commun. 3 1092 (2012)

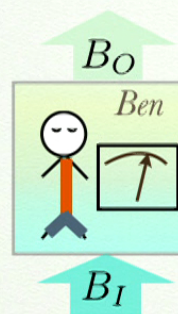
O. Oreshkov and C. Giarmatzi, New Journal of Physics 18, 093020 (2016)

Markovianity



Process matrix formalism

Closed lab
Quantum instrument



CP map $\mathcal{M}_{oA}^{A_I \rightarrow A_O}$

Result

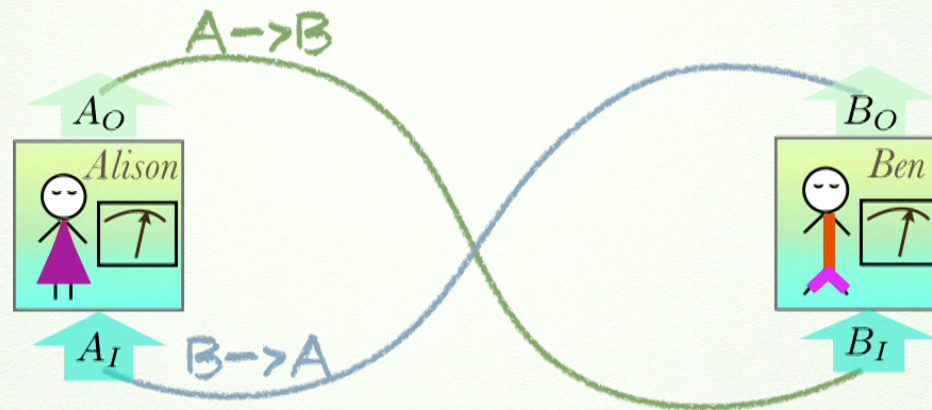
$$p(\mathcal{M}^A, \mathcal{M}^B) = \text{Tr}(W^{A_I A_O B_I B_O} M_{CJ}^{A_I A_O} \otimes M_{CJ}^{B_I B_O})$$

probabilities environment operations

O. Oreshkov, F. Costa, Č. Brukner, "Quantum correlations with no causal order", Nat. Commun. **3** 1092 (2012)

O. Oreshkov and C. Giarmatzi, New Journal of Physics **18**, 093020 (2016)

Example of a W



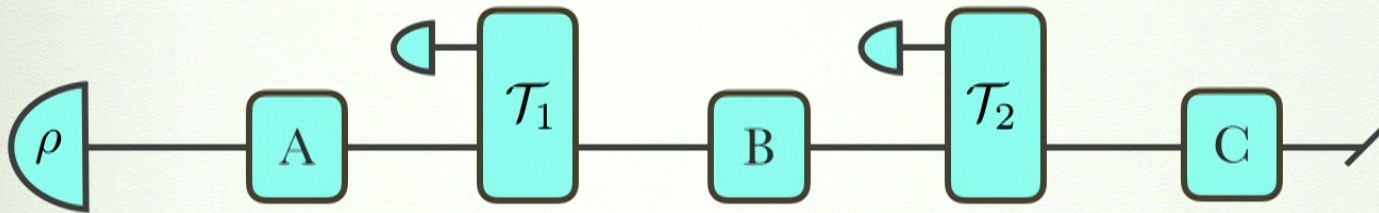
$$A \rightarrow B \quad W^{A \prec B} = \rho^{A_I} \otimes T^{A_O B_I} \otimes \mathbb{1}^{B_O}$$

$$B \rightarrow A \quad W^{B \prec A} = \rho^{B_I} \otimes T^{B_O A_I} \otimes \mathbb{1}^{A_O}$$

O. Oreshkov, F. Costa, Č. Brukner, "Quantum correlations with no causal order", Nat. Commun. 3 1092 (2012)

O. Oreshkov and C. Giarmatzi, New Journal of Physics 18, 093020 (2016)

Markovianity



Definition

$$W_M^{ABC} = \rho^{A_I} \otimes T_1^{A_O B_I} \otimes T_2^{B_O C_I} \otimes \mathbb{1}^{C_O}$$

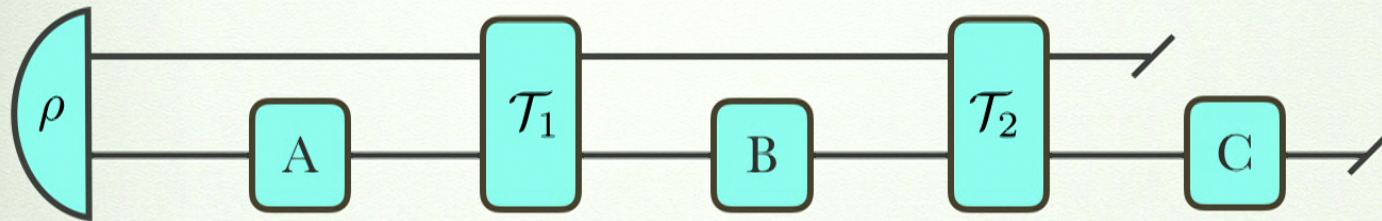
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non-Markovianity

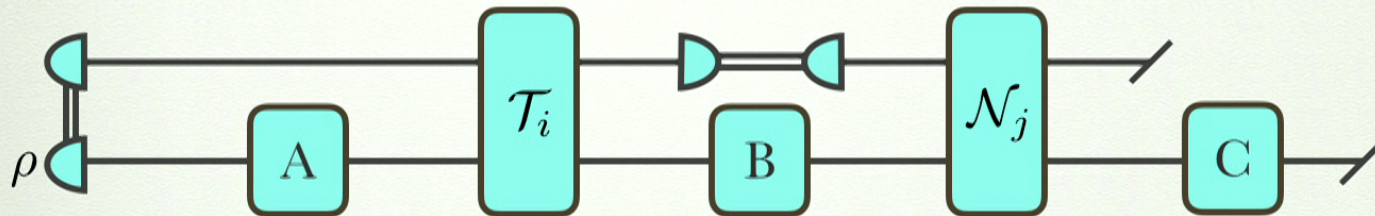
There is a memory that carries correlations



Is it classical or quantum?

Classical memory

Environment obtains classical info and can affect future interactions



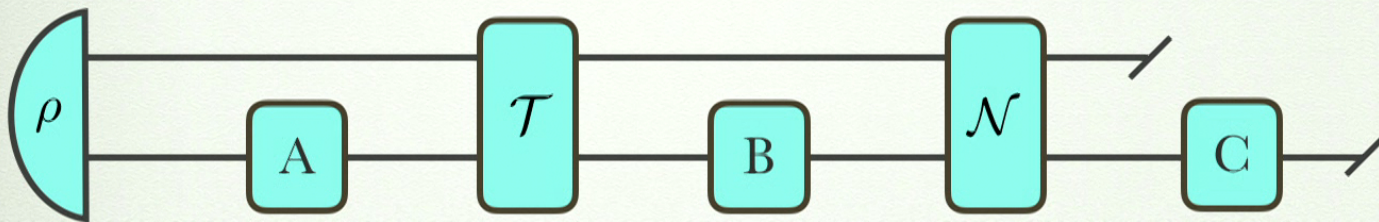
$$W_{\text{Cl}}^{AB\dots} = \sum_j \rho_j^{A_I} \otimes T_j^{A_O B_I} \otimes \dots,$$

Choi of the channels

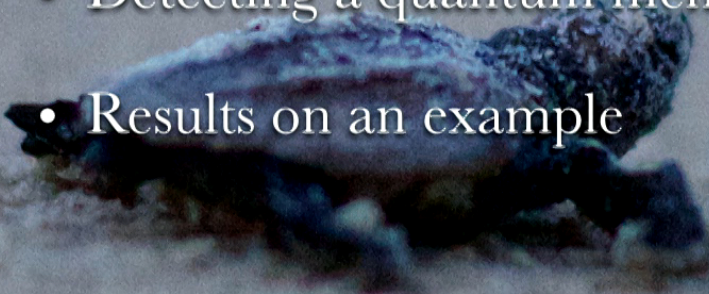
But they have to sum up to CPTP

Quantum memory

Everything else



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Detecting quantum memory

$$\overset{\text{Markovian}}{W_M^{AB\dots}} = \rho^{A_I} \otimes T^{A_O B_I} \otimes \dots ,$$

$$\overset{\text{classical memory}}{W_{Cl}^{AB\dots}} = \sum_j q_j \rho_j^{A_I} \otimes T_j^{A_O B_I} \otimes \dots ,$$

Detecting quantum memory

Markovian

$$W_M^{AB\dots} = \rho^{A_I} \otimes T^{A_O B_I} \otimes \dots,$$

classical memory

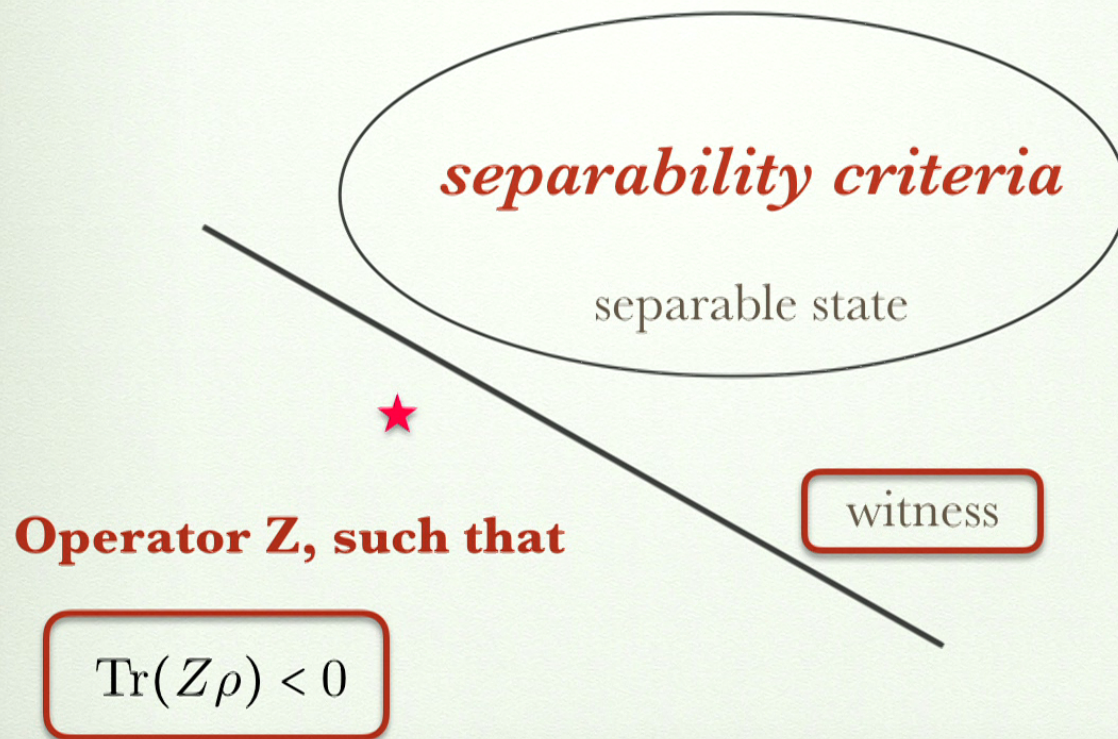
$$W_{Cl}^{AB\dots} = \sum_j q_j \rho_j^{A_I} \otimes T_j^{A_O B_I} \otimes \dots,$$

Separable states

witness

★
quantum memory

Detecting entanglement



separability criteria for ρ^{AB}

I

Positive Partial Transpose criterion
(PPT) on the state

$$\rho_{sep} \Rightarrow \rho^{T_A} > 0$$

A. Peres, “Separability criterion for density matrices”, *Physical Review Letters* **77**, 1413 (1996)

Finding a witness

I

$$\rho_{sep} \Rightarrow \rho^{T_A} > 0$$

Let's say $\rho^{T_A} < 0 \longrightarrow |\psi\rangle^{T_A} = \sum_i \epsilon_i E_i \quad \exists j \quad \epsilon_j < 0$

$$\text{Tr}(|E_j\rangle\langle E_j|\rho^{T_A}) = \epsilon_j < 0 \quad \Rightarrow \quad \text{Tr}(\underbrace{|E_j\rangle\langle E_j|^{T_A}}_{\text{witness}}\rho) < 0$$

OR

SemiDefinite Program



separability criteria for ρ^{AB}

II

PPT on symmetric extensions

If ρ_{sep} there is a $\tilde{\rho} = \rho^{ABA'}$

such that $\tilde{\rho} > 0$

$$\tilde{\rho}^{T_A} > 0$$

$$\tilde{\rho}^{T_B} > 0$$

A. C. Doherty, P. A. Parrilo, and F. M. Spedalieri, "Complete family of separability criteria", *Physical Review A* **69**, 022308 (2004)

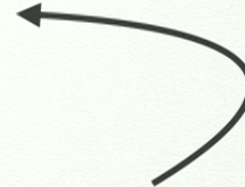
Finding a witness

II

$$\tilde{\rho} > 0$$

$$\tilde{\rho}^{T_A} > 0$$

$$\tilde{\rho}^{T_B} > 0$$



SDP: Primal: “Find an extension of ρ ,
such that ...”

If the problem is infeasible,
the state is entangled.

Dual: find a witness for an extension of ρ .



A. C. Doherty, P. A. Parrilo, and F. M. Spedalieri, “Complete family of separability criteria”, *Physical Review A* **69**, 022308 (2004)

separability criteria for ρ^{AB}

III

$$\rho_{sep} \Rightarrow \text{for } \tilde{\rho} = \rho^{ABA} \quad \begin{aligned} \tilde{\rho} &> 0 \\ \tilde{\rho}^{T_A} &> 0 \\ \tilde{\rho}^{T_B} &> 0 \end{aligned}$$

Keep extending the state

$$\rho_{sep} \Rightarrow \text{for } \tilde{\rho} = \rho^{ABA'A''} \quad \begin{aligned} \tilde{\rho} &> 0 \\ \tilde{\rho}^{T_A} &> 0 \\ \tilde{\rho}^{T_B} &> 0 \\ \tilde{\rho}^{T_{A'}} &> 0 \end{aligned}$$

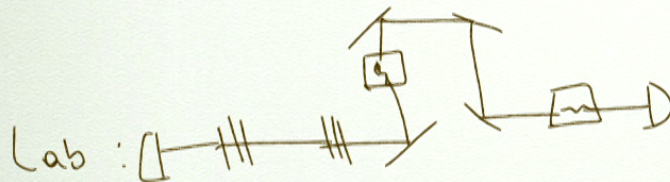
A. C. Doherty, P. A. Parrilo, and F. M. Spedalieri, "Complete family of separability criteria", *Physical Review A* **69**, 022308 (2004)

What to do with a witness

Operator Z , such that

$$\text{Tr}(Z\rho) < 0$$

process matrix



Guess: W (or ρ)

Find a witness Z .

Go to Lab, obtain $\text{Tr}(Z \cdot \rho) < 0$

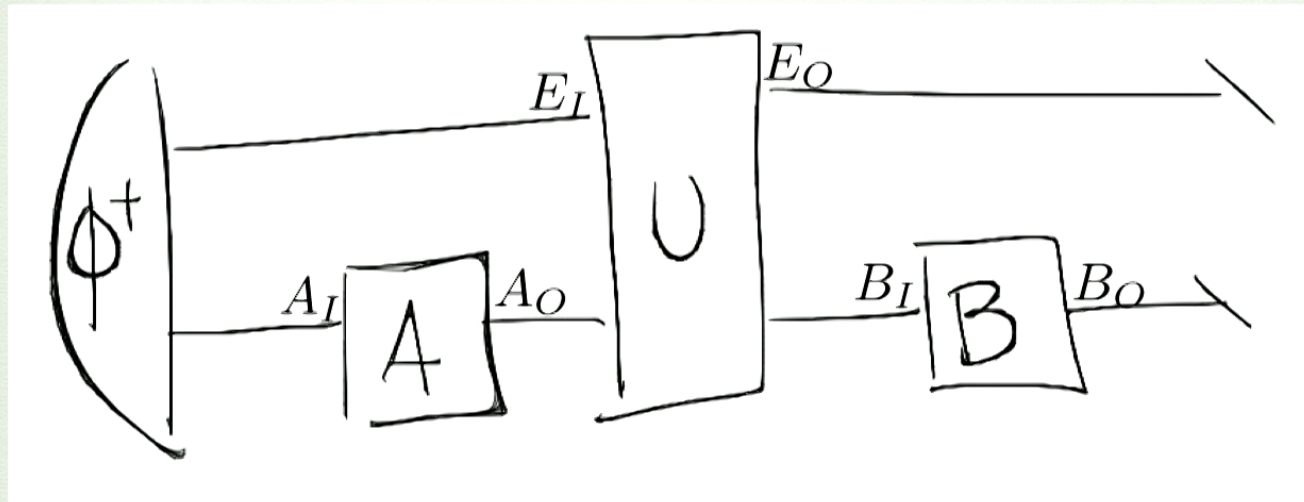


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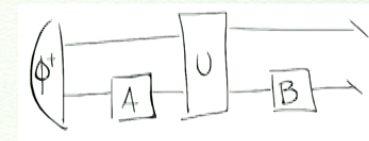
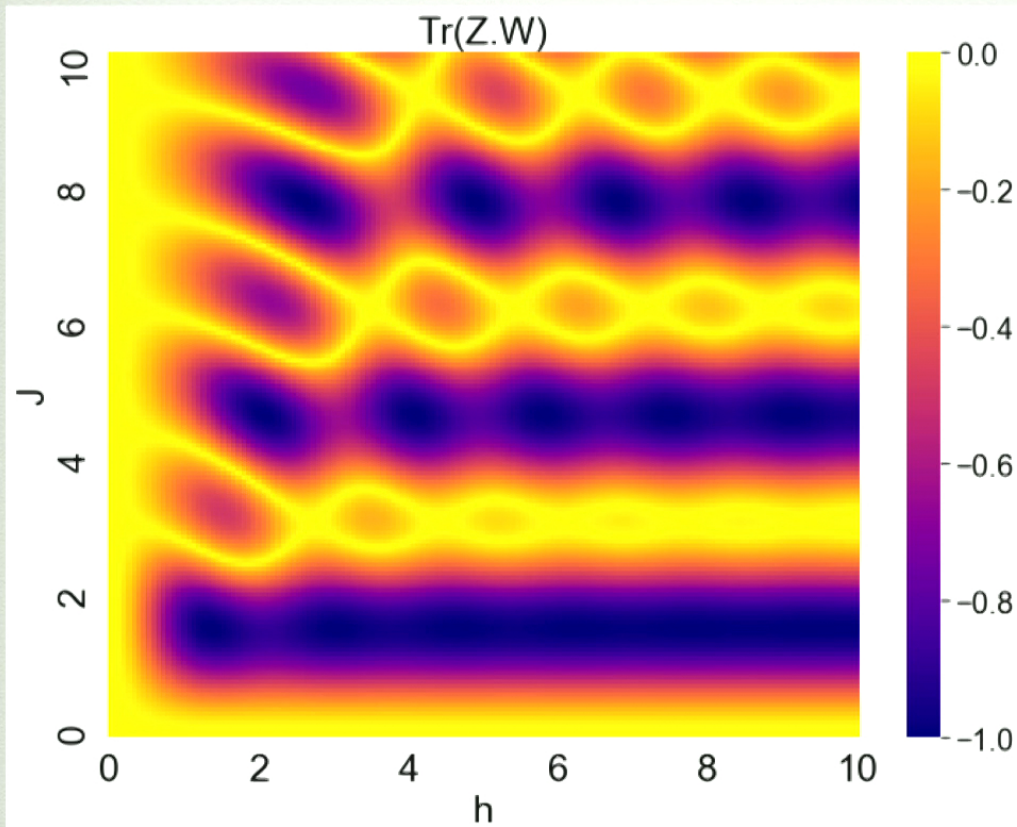
Example: Φ^+ and Ising model



$$\Phi = 1/\sqrt{2}(|00\rangle + |11\rangle) \quad U(J, h, t) = e^{-iH(J, h)t} \quad \mathcal{H} = -JXX - h(ZI + IZ)$$

$$W^{A_I A_O B_I}(J, h, t) = \text{Tr}_{E_O} [[U(J, h, t)]]$$

Results - quantum memory



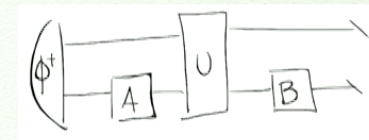
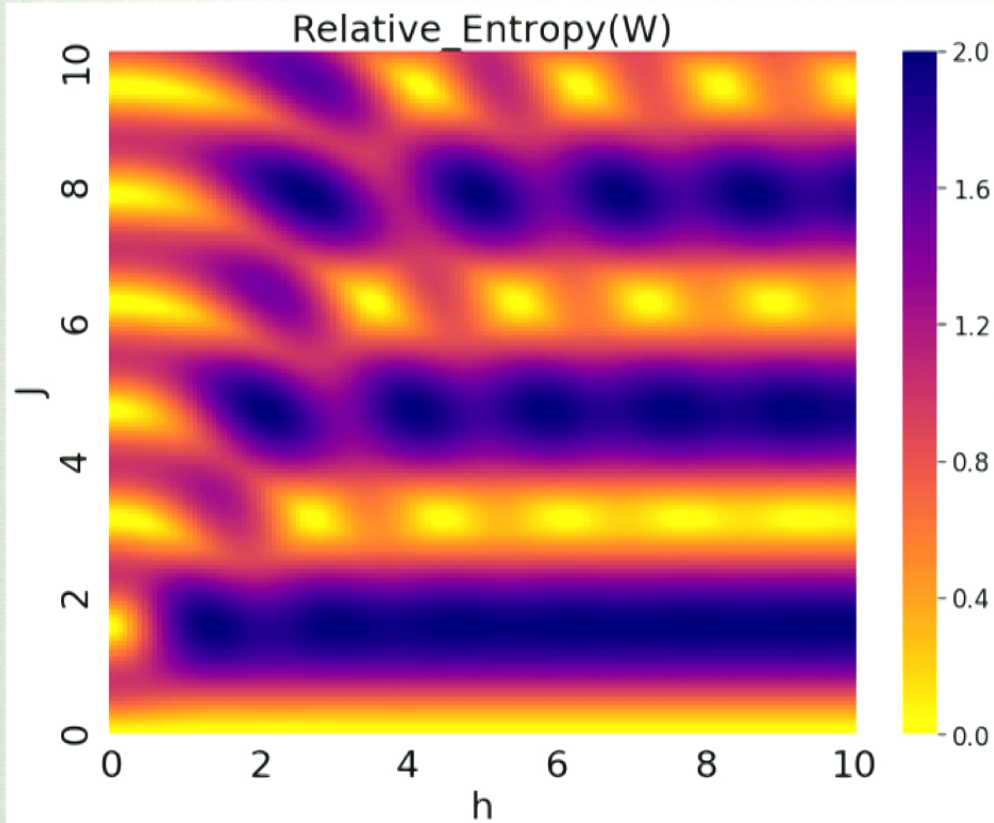
$$t = 1$$

$$W^{A_I A_O B_I}(J, h)$$

PPT

C. Giarmatzis and F. Costa, "Witnessing quantum memory in non-Markovian processes", arXiv: 1811.03722 (2018)

Results - Markovianity

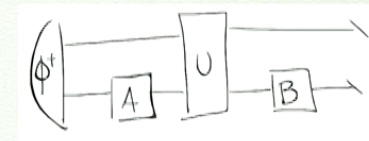
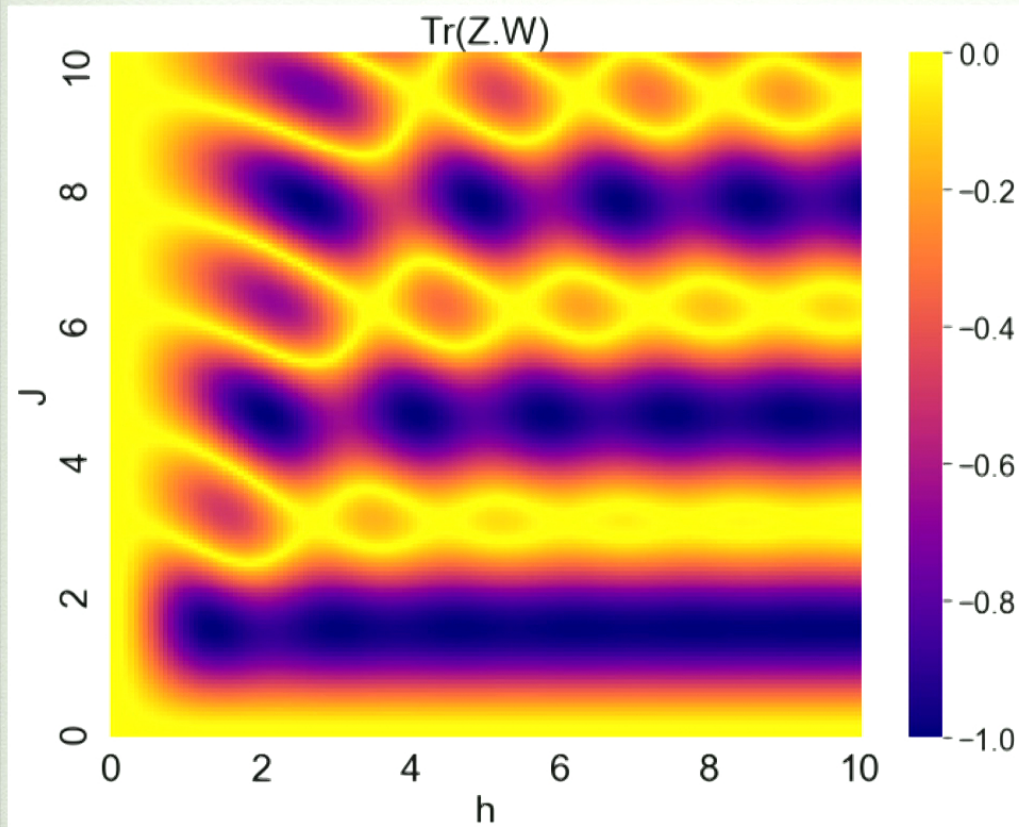


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Results - quantum memory



$$t = 1$$

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PPT

C. Giarmatzis and F. Costa, "Witnessing quantum memory in non-Markovian processes", arXiv: 1811.03722 (2018)

Work in progress

Detecting quantum memory in the lab

Start with classical memory

Machine Learning for detecting it

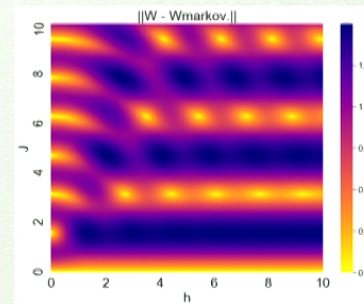
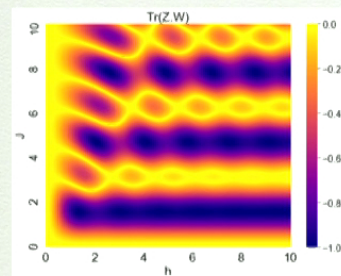
We train it

Perform the experiment

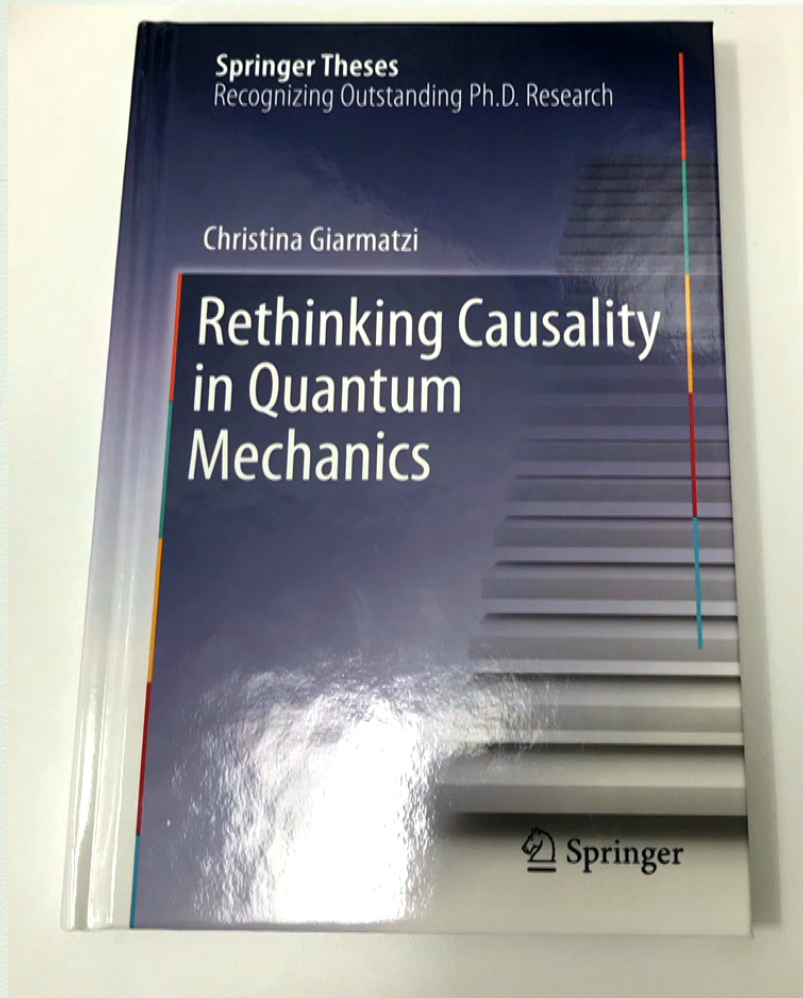
Summary

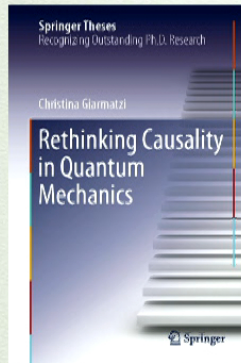
[arXiv:1811.03722](https://arxiv.org/abs/1811.03722)

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C. Giarmatzi and F. Costa, "Witnessing quantum memory in non-Markovian processes", arXiv: 1811.03722 (2018)





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Physics : Quantum Physics

Giarmatzi, Christina, The University of Queensland, Brisbane, QLD, Australia

Rethinking Causality in Quantum Mechanics

- Nominated as an outstanding Ph.D. thesis by the University of Queensland, Brisbane, Australia,
- One of the first comprehensive introductions to quantum causality
- In-depth description of concepts and methods
- Investigates quantum causality in theory and experiments

Causality is central to understanding the mechanisms of nature: some event "A" is the cause of another event "B". Surprisingly, causality does not follow this simple rule in quantum physics: due to quantum superposition we might be led to believe that "A causes B" and that "B causes A". This idea is not only important to the foundations of physics but also leads to practical advantages: a quantum circuit with such indefinite causality performs computationally better than one with definite causality. This thesis provides one of the first comprehensive introductions to quantum causality, and presents a number of advances. It provides an extension and generalization of a framework that enables us to study causality within quantum mechanics, thereby setting the stage for the rest of the work. This comprises: mathematical tools to define causality in terms of probabilities; computational tools to prove indefinite causality in an experiment; means to experimentally test particular causal structures; and finally an algorithm that detects the exact causal structure in a quantum experiment.

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Next chapter...



Why *another* framework?

...because previous frameworks scrambled causal and inferential concepts.



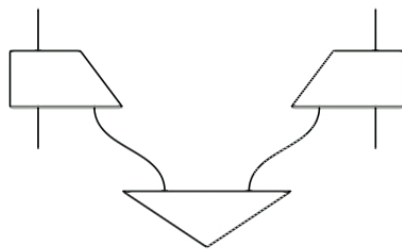
Remarks on operationalism

Operational frameworks often aim to be “minimalist descriptions of an experiment that everyone agrees upon”

What do circuits in such a framework represent?

1. Manifest structure

Causal structure must then be encoded in the probabilistic structure of the theory...



Hitchcock
(Alex's talk)

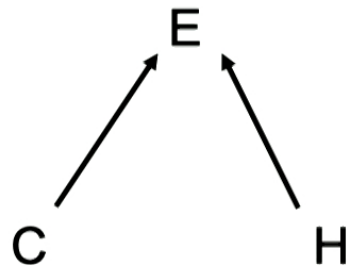
C is a cause of E just in case $P(E|C) > P(E|\sim C)$

Causality Axiom
(Pavia approach)

The probability of preparations is independent of the choice of observations.

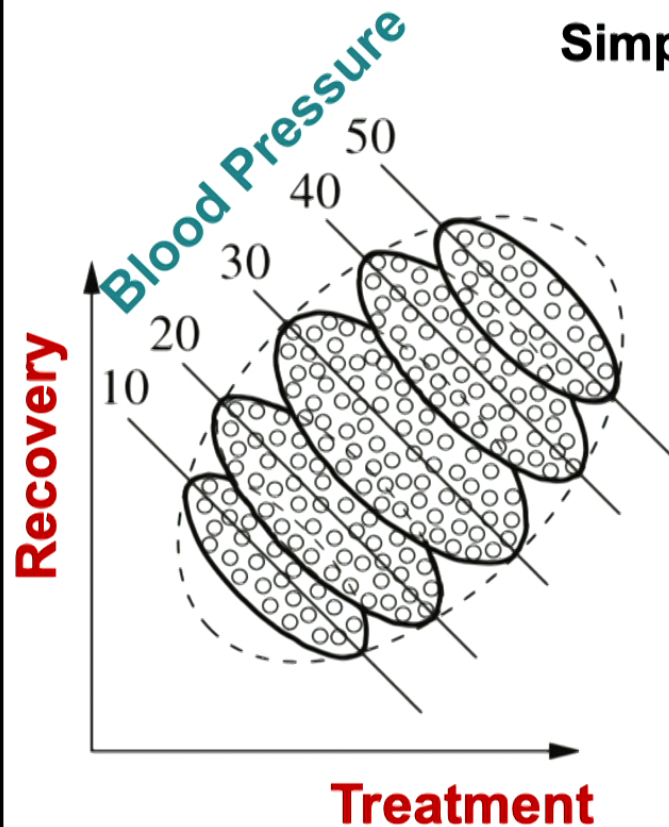
Problems with this approach:

Vernam
cypher:

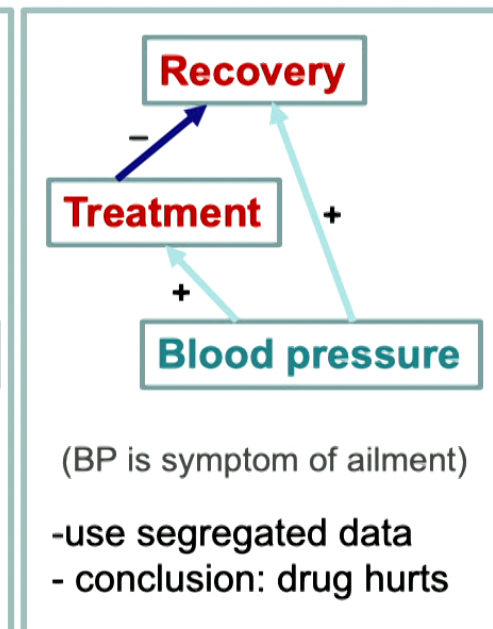
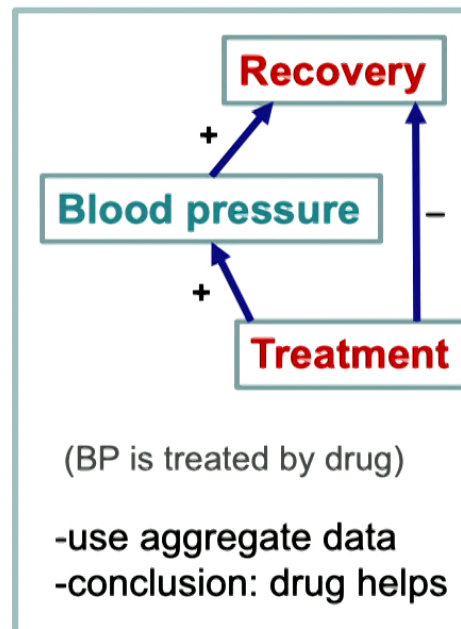


$$E = C + H \pmod{2}$$
$$p(H=0) = p(H=1) = 1/2$$

Simpson's paradox



operational data is not enough!



Pearl's approach to causality:

- causation is primitive
- causal assumptions constrain probability assignments

Remarks on operationalism

Operational frameworks often aim to be “minimalist descriptions of an experiment that everyone agrees upon”

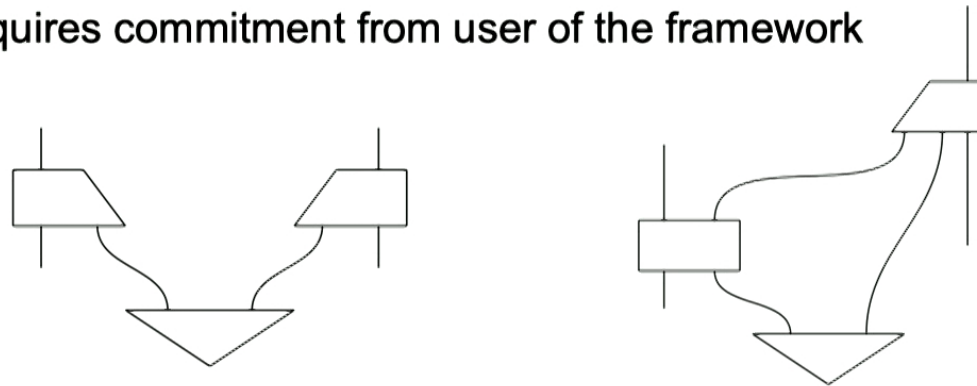
What do circuits in such a framework represent?

1. Manifest structure

Causal structure must then be encoded in the probabilistic structure of the theory...

2. Causal structure

Requires commitment from user of the framework



Key insight of classical causal modeling framework:
separating causation and inference

Our foundational frameworks have not
achieved this separation.

Mixtures of procedures (lists of lab instructions)

Physical

vs

Inferential

Procedure P is defined as: sample a binary variable X according to weights $(w, 1-w)$; set up a mechanism that varies between implementing P_1 and P_2 according to the value of X, and do not keep a record of the value of X.

- requires autonomy of X and P_1, P_2
- requires one to erase *all* records of X
- why must all mixtures be allowed?
- set of procedures has no intrinsic convex structure

Probability distribution $\sigma = (w, 1-w)$ describes one's degree of belief about whether the procedure was P_1 or P_2 .

$$\sigma = w\delta_{P,P_1} + (1-w)\delta_{P,P_2}$$

- no need for assumption of autonomy
- no need to erase any records
- explains why all mixtures are allowed
- set of states of knowledge has intrinsic convex structure

Past work took the physical attitude to defining a mixture, and claimed that the representation of P must be the mixture of representations of P_1 and P_2

- follows from fact that law of total probability is linear

Other inferential concepts

	$(\rho_1, \rho_2) \rightarrow p\rho_1 + (1-p)\rho_2$	$(E_1, E_2) \rightarrow E_1 + E_2$	$\rho \rightarrow \text{Tr}[\rho]$	$\rho \rightarrow \text{Tr}[E\rho]$	ρ	E
Physical interpretation	physical mixture: "flip a coin..."	physical coarse-graining: "rewire your mmt device..."	throw away the system	post-selection	preparation procedure	possible event in a mmt procedure
Inferential interpretation	ignorance	logical disjunction	ignore the system	conditioning	inferential equivalence class of states of knowledge about preparation procedures	physically-permitted inference about an equivalence class of {propositions about mmt procedures about which one has a given state of knowledge}

Inferential point of view:

- is more general
- avoids problems of autonomy, erasure
- resolves formal issues with definitions regarding ontological modeling

The Framework

Two quick preliminaries:

Process Theory G

collection of processes (on some systems)
which is closed under forming diagrams

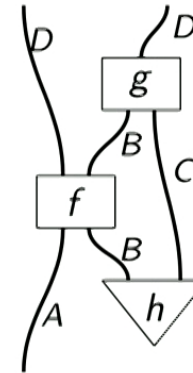
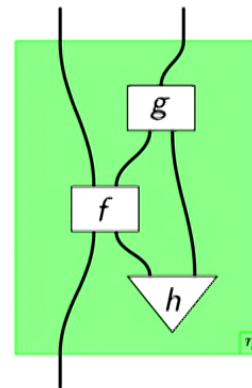


Diagram Preserving map

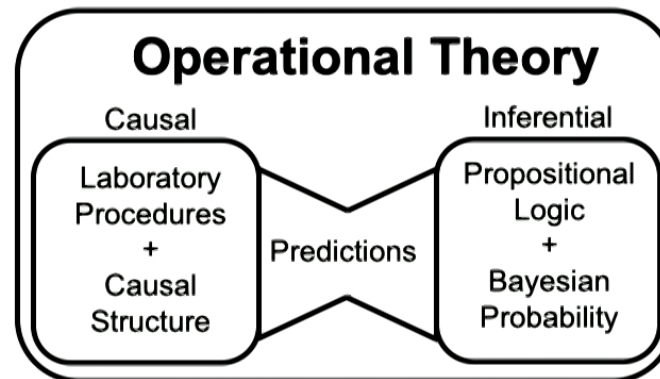
$$\eta : G \rightarrow G'$$

takes processes from one theory to
processes of another such that
diagrams are preserved

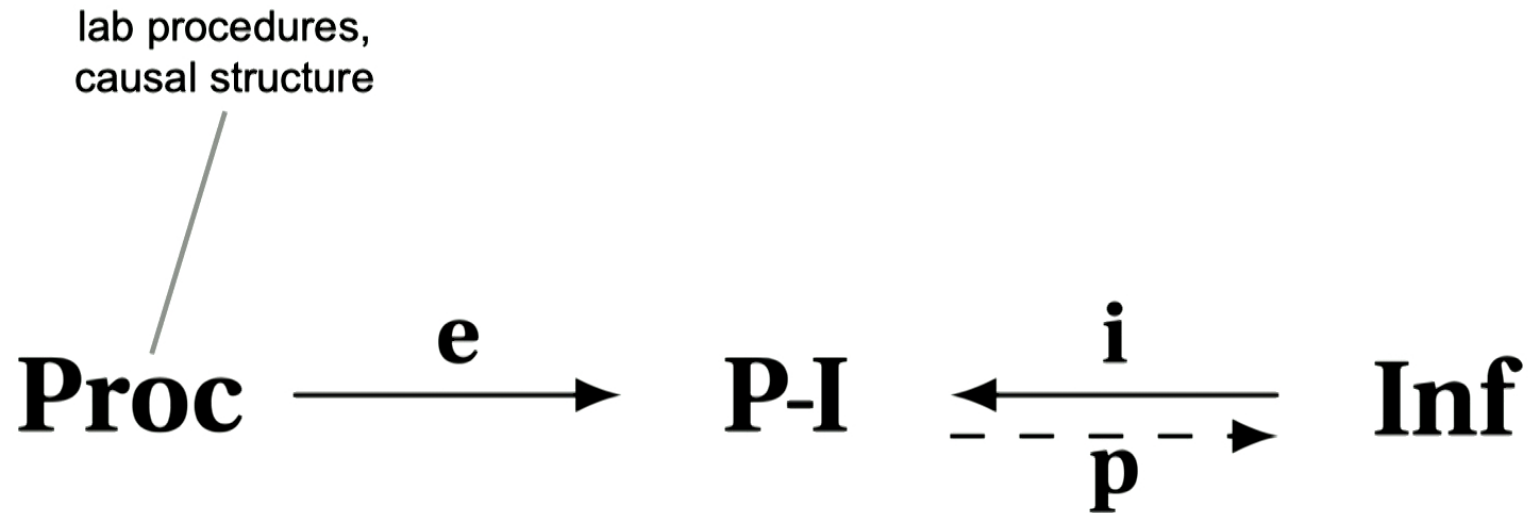


Causal-inferential theory of laboratory procedures

Operational Theories



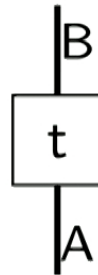
Causal-inferential theory of laboratory procedures



Proc —the process theory of lab procedures

Systems: classical, quantum, ...

Processes: lists of lab instructions with an associated input and output

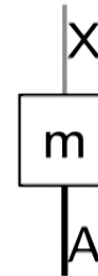


transformation
procedure



preparation procedure

(classical = grey wire)



measurement
procedure

Proc —the process theory of lab procedures

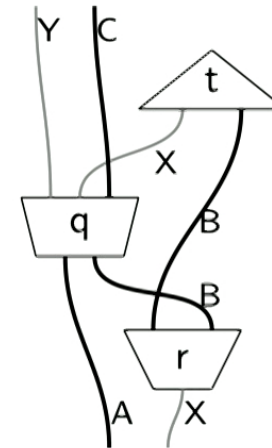
Compose processes to form larger processes:

No equalities
(e.g. closed diagrams are not probabilities)

Diagrammatic connectivity represents causal structure...



prepare-measure



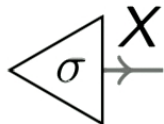
more general

NOT inferential elements such as:

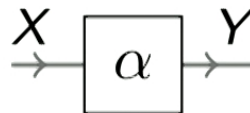
- pre/post-processings
- trace/marginalization
- post-selection
- states of knowledge
- etc

Inf —the process theory of classical inference (horizontal systems/processes)

States of knowledge



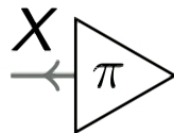
their stochastic updating



marginalization



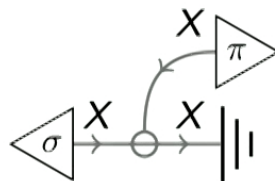
Propositions



Attaching propositions to inferential systems



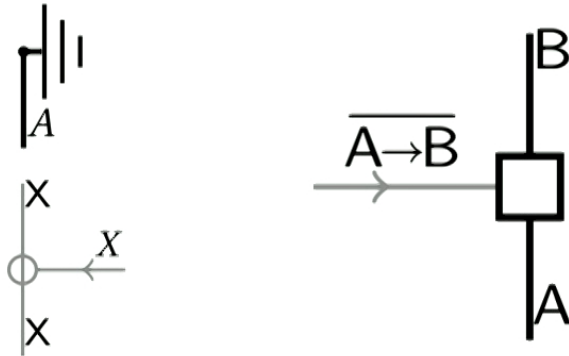
Probabilities



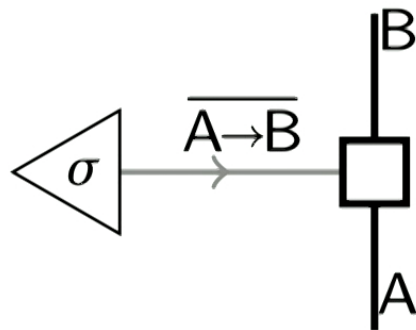
$$= \text{Prob}(\pi|\sigma)$$

P-I —the procedural-inferential process theory

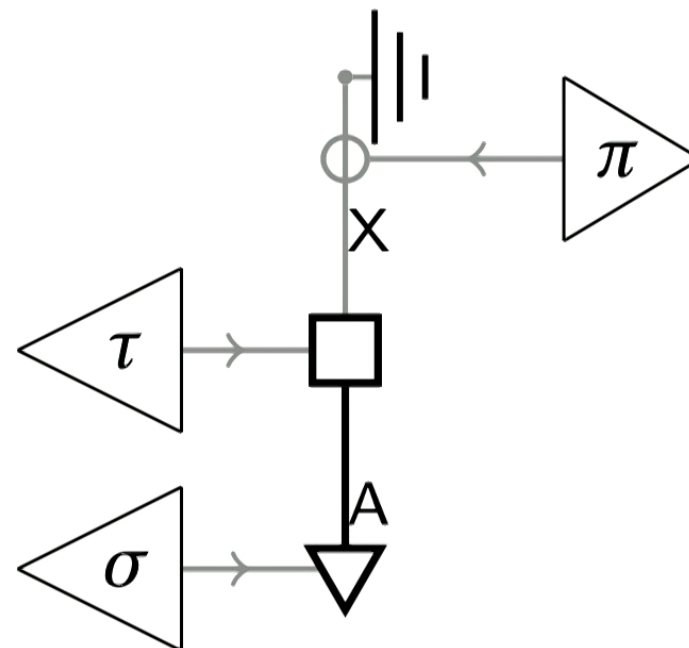
Generators:



States of knowledge about procedures:

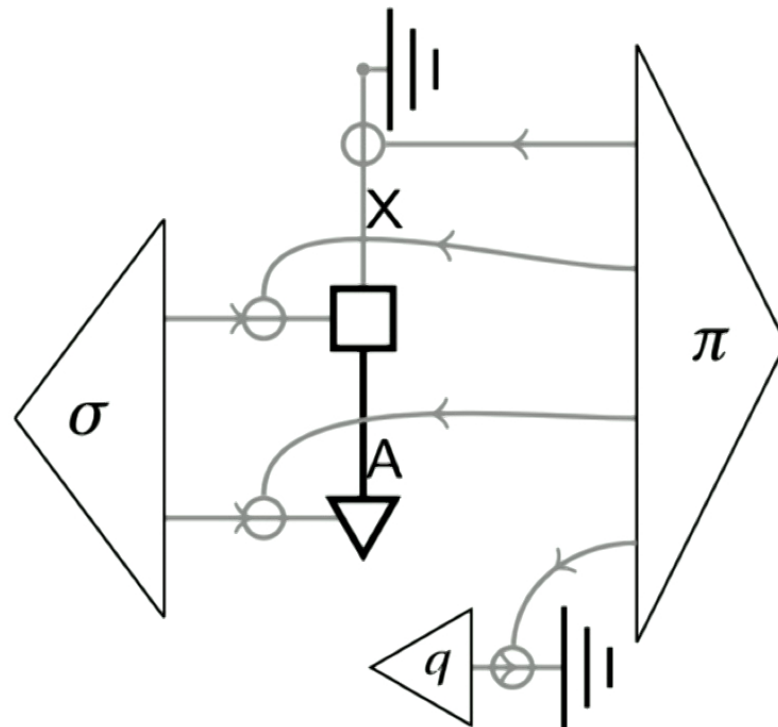


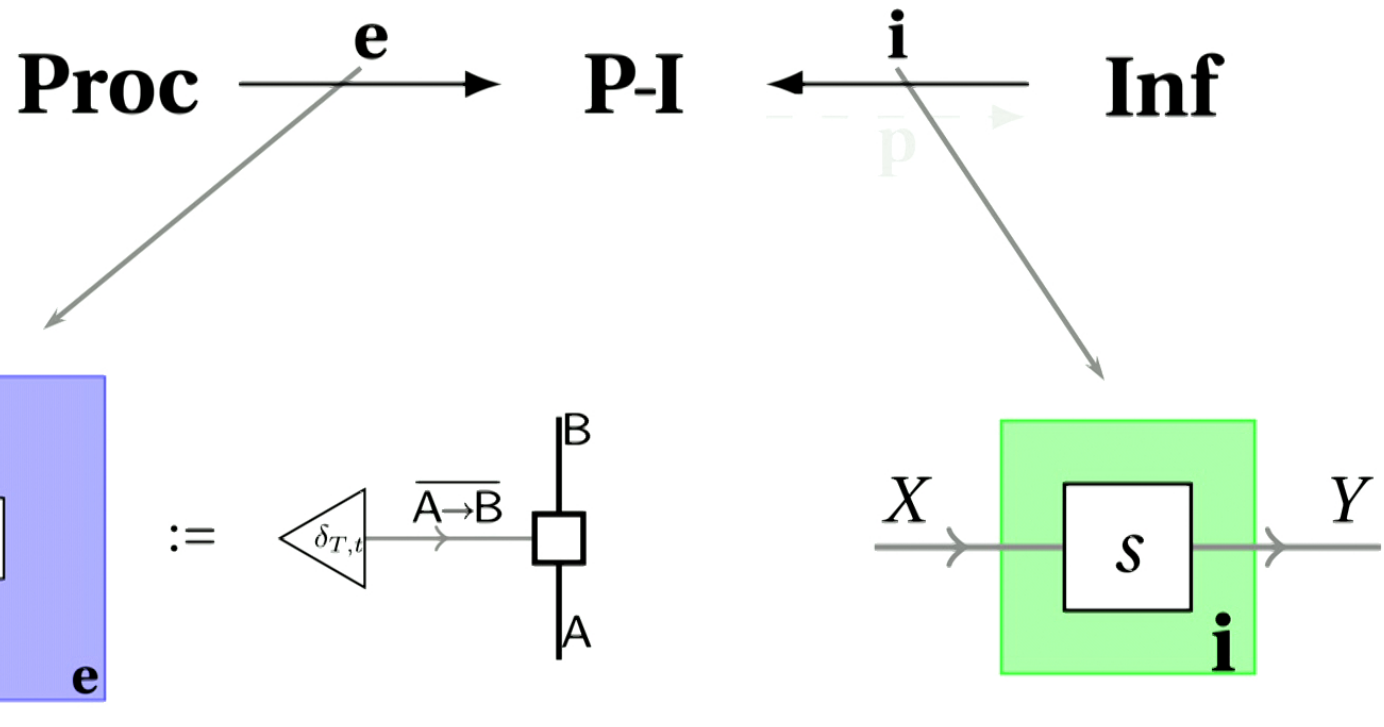
Ex: prepare-measure scenario



P-I —the procedural-inferential process theory

More generally:



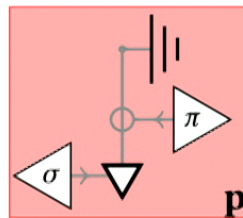


*with appropriate rewrite rules, e is a valid diagram-preserving map

Making Predictions

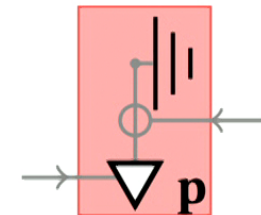


probability:

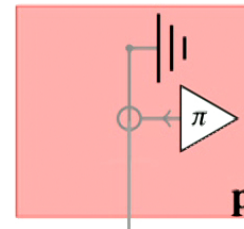


$$= \text{Prob}(\pi|\sigma)$$

stochastic map:



partial map: acts only on processes with no vertical inputs/outputs



probability assigned to the proposition is not unique (depends on causal past)

The map $\mathbf{p}$ is not unique; it varies with the physical theory.

$\mathbf{p}$ is fully specified by the probabilities it assigns to perfect states of knowledge and atomic propositions

this explains why previous formalisms got away with just specifying $p(k|M,P)$

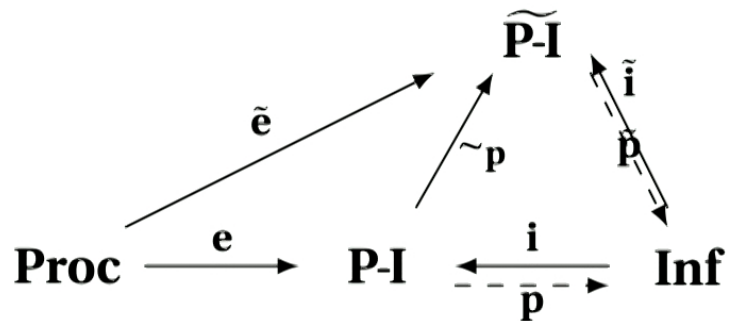
We innovate not the way probabilities are computed under mixings/coarse-grainings, but rather the *structure of the objects* in the theory (prior to considering probabilities).

GPTs are what is left of our framework when you quotient with respect to inferential equivalence

Inferential Equivalence

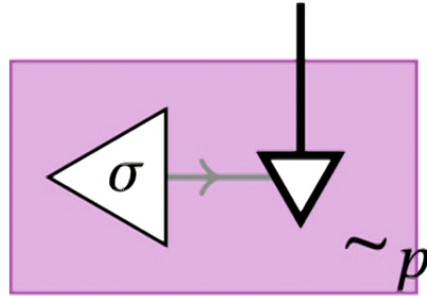
$$\begin{array}{c}
 \begin{array}{ccc}
 \triangleleft \sigma \rightarrow \square \begin{array}{c} |B \\ |A \end{array} & \sim_p & \triangleleft \sigma' \rightarrow \square \begin{array}{c} |B \\ |A \end{array} \\
 \text{iff} & &
 \end{array} \\
 \begin{array}{ccc}
 \begin{array}{c} \text{Diagram 1: } \triangleleft \sigma \rightarrow \square \begin{array}{c} |B \\ |A \end{array} \text{ inside } \mathcal{T} \text{ box } p \end{array} & = & \begin{array}{c} \text{Diagram 2: } \triangleleft \sigma' \rightarrow \square \begin{array}{c} |B \\ |A \end{array} \text{ inside } \mathcal{T} \text{ box } p \end{array} \\
 \forall \mathcal{T} \in \mathbf{P-I} & &
 \end{array}
 \end{array}
 \quad (135)$$

This is a valid process theory congruence relation. $\sim_p$
 Can define quotiented theory: $\widetilde{\mathbf{P-I}}$



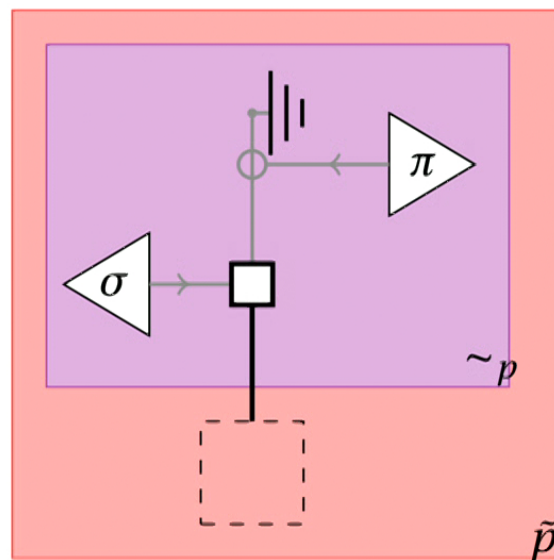
elements of the quotiented theory
 contain all and only information
 needed to make predictions

What is a GPT state?

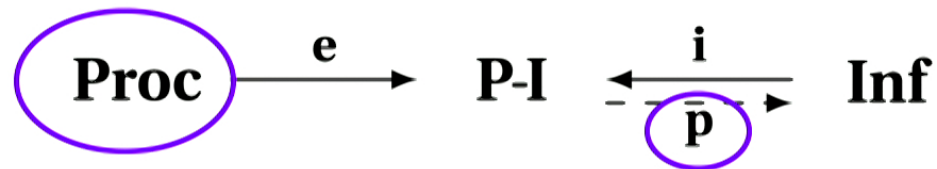


inferential equivalence class of states of
knowledge about preparation procedure

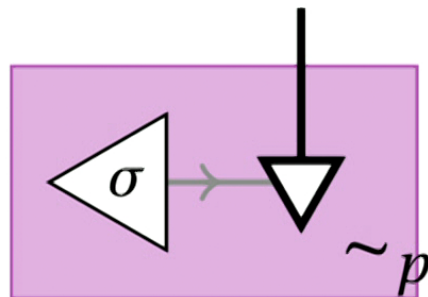
What is a GPT effect?



What varies from one physical theory to the next?

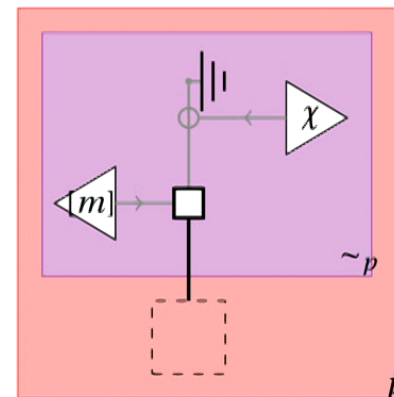


Assuming a classical theory of inference, then everything else is fixed!



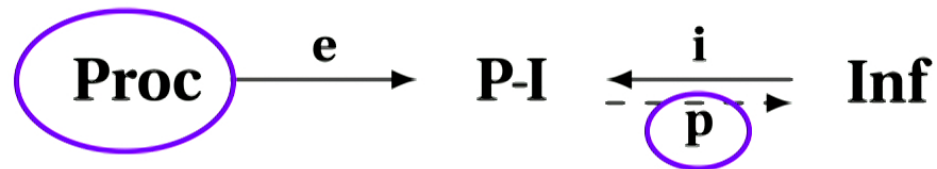
convex structure

This implies that all operational theories share certain features.

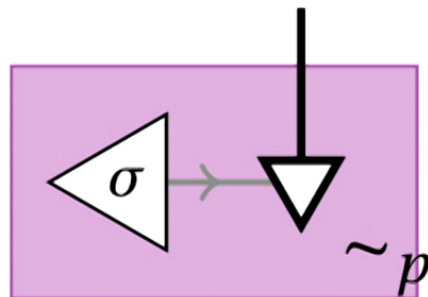


zonotope structure

What varies from one physical theory to the next?

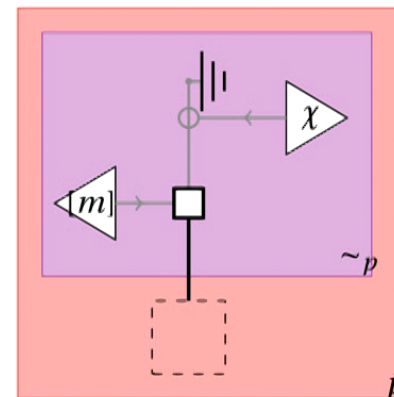


Assuming a classical theory of inference, then everything else is fixed!



convex structure

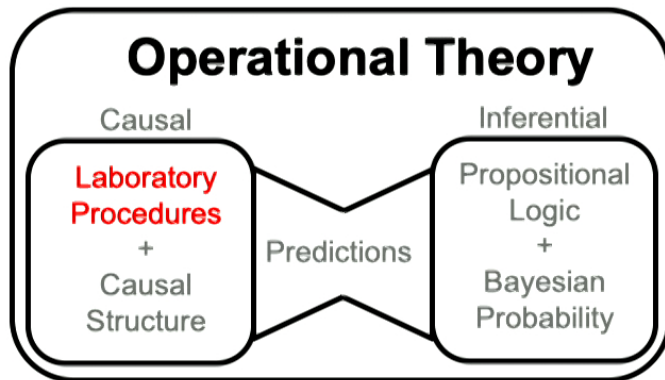
This implies that all operational theories share certain features.



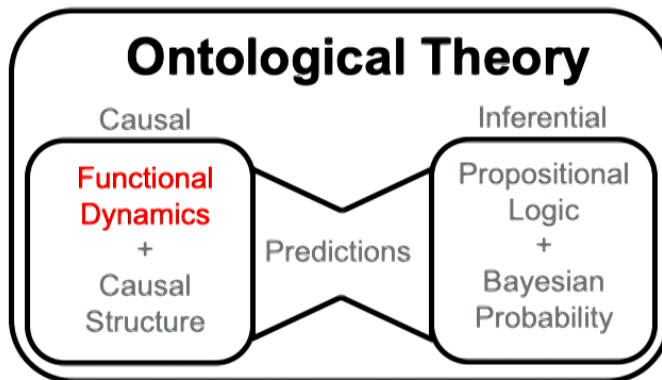
zonotope structure

Ontological theories

Just as much of an omelette!

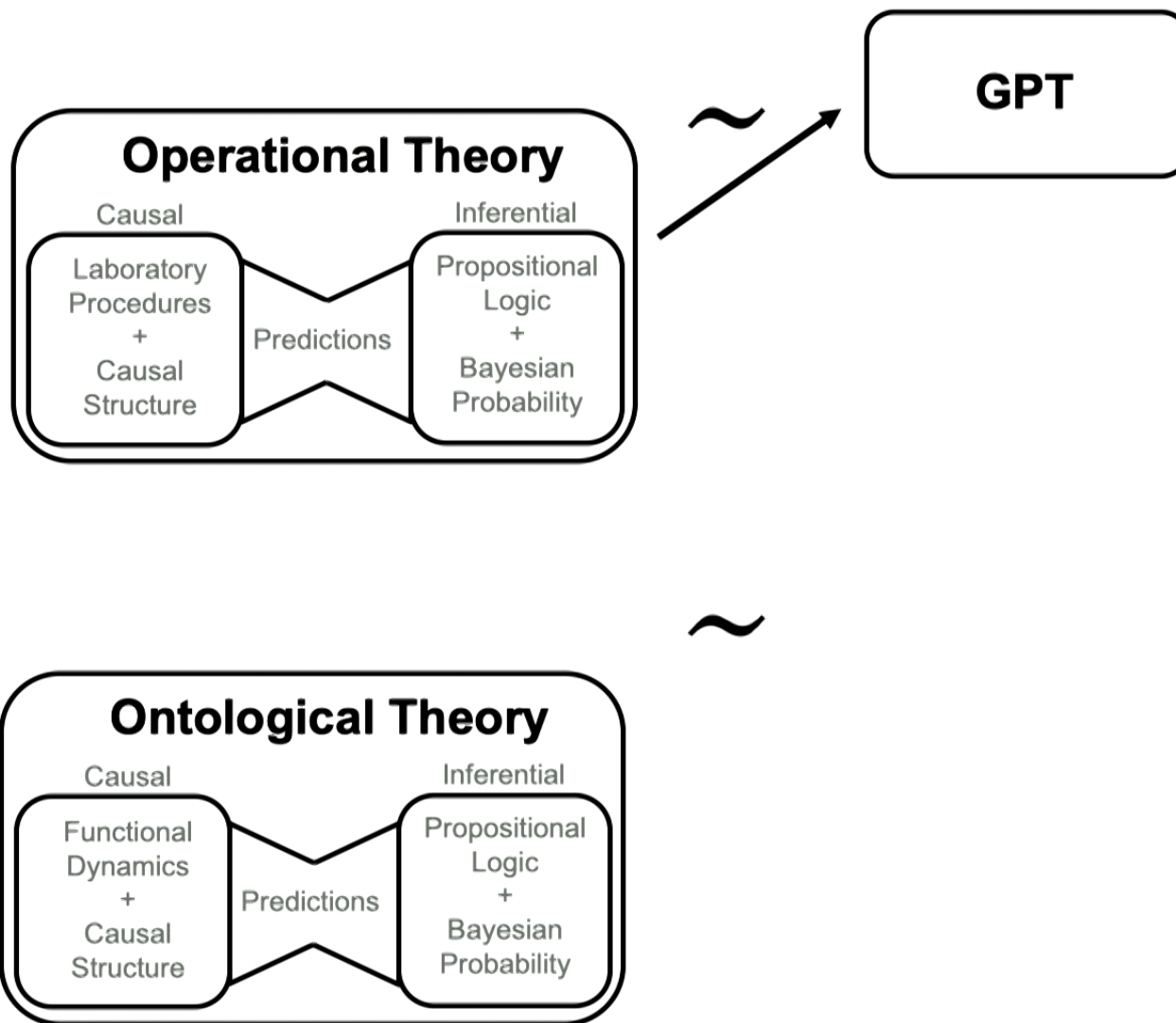


$$\text{Proc} \xrightarrow{e} \text{P-I} \xrightleftharpoons[\bar{p}]{i} \text{Inf}$$

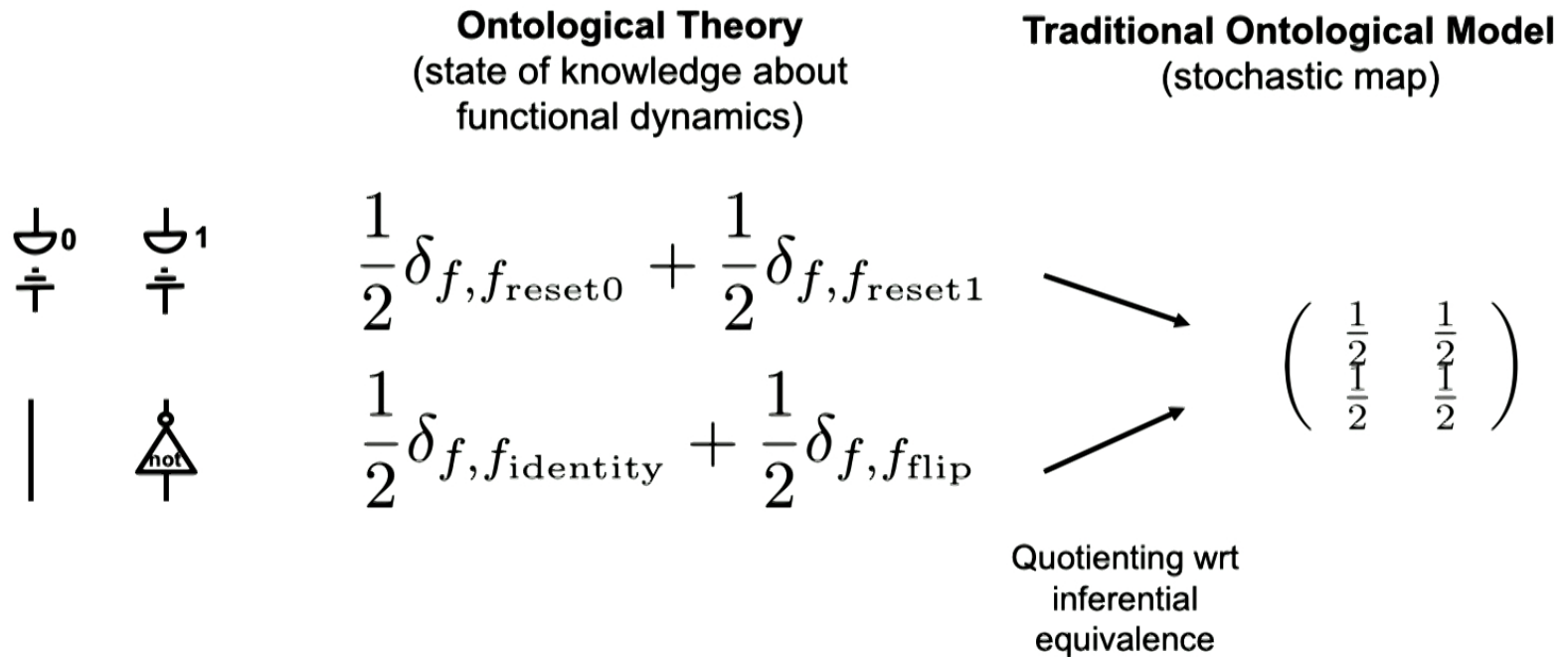


$$\text{Func} \xrightarrow{e'} \text{F-I} \xrightleftharpoons[p^*]{i'} \text{Inf}$$

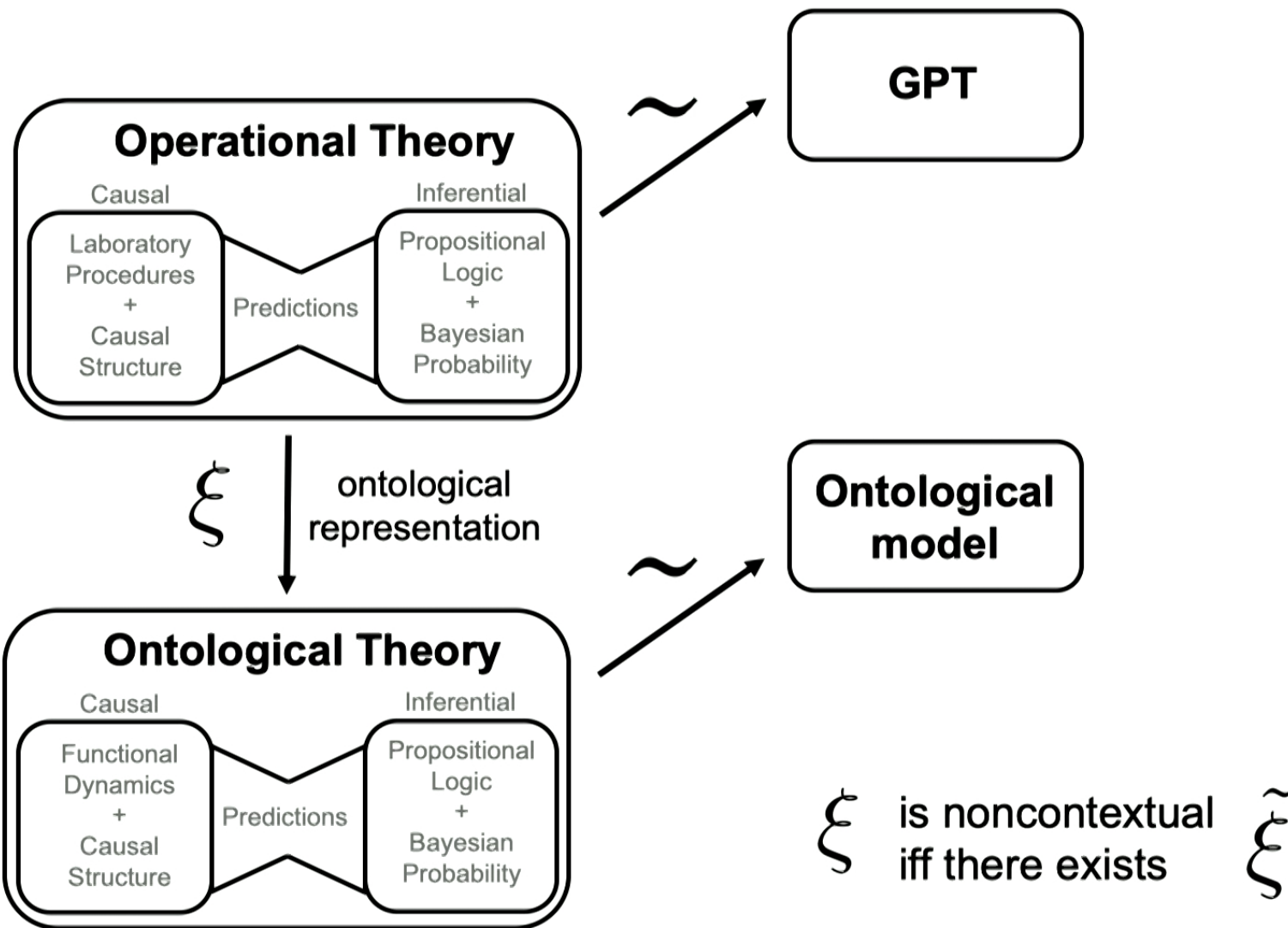
Func has equalities (**Proc** did not)...
Can derive that there is a unique ontological theory!



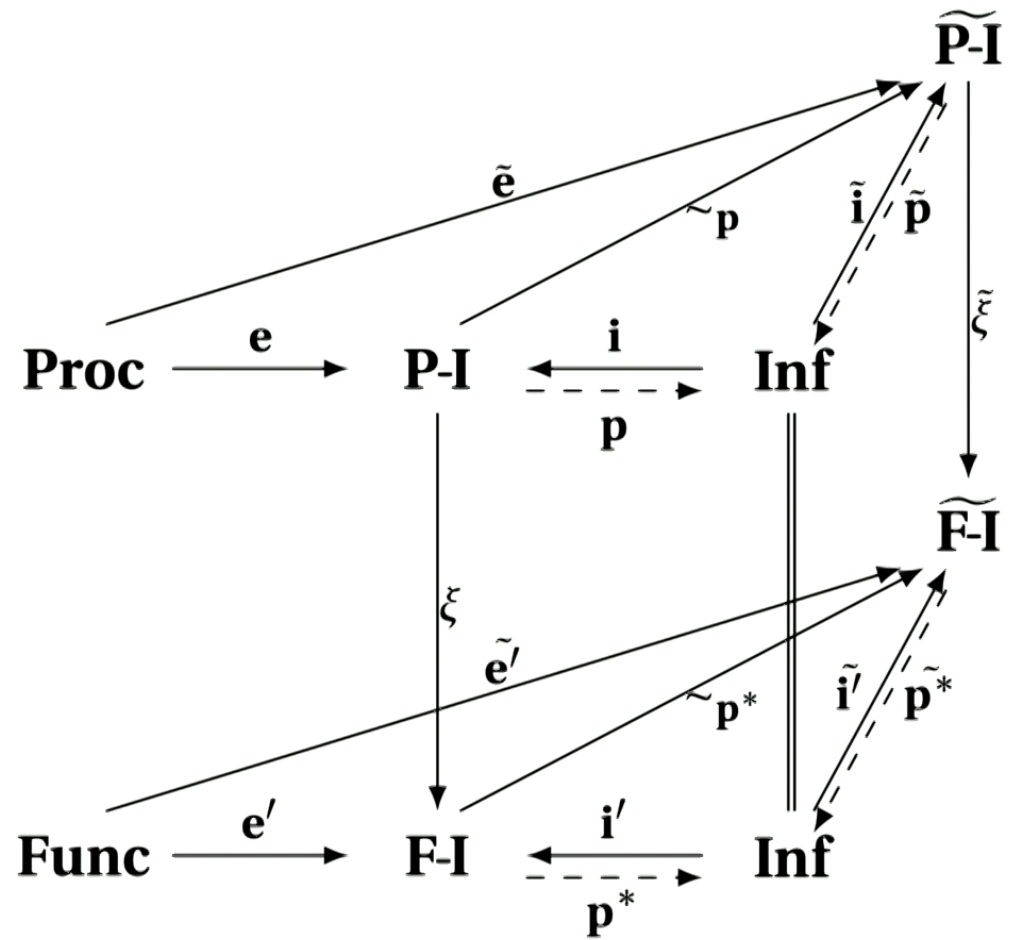
Example



(Implications for noncontextuality)



Full Framework (formally)



A few key takeaways

- Separation of inferential and causal aspects of the theory
 - can consider manipulating independently! (we changed only the latter here)
- Causal relations are fundamental, inference supervenes on them; not vice versa
 - causal structure encoded in vertical circuit, constrains probability rule
- States of knowledge about procedures are the key concepts
 - GPT/quantum/ontological represent equivalence classes of these

(draft in preparation)

Thanks for your attention!