Title: TBA

Speakers: Ding Jia

Collection: Indefinite Causal Structure

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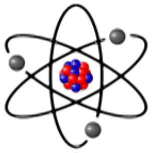

World Quantum Gravity

and the choices leading to it

arXiv: 1909.05322

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Indefinite Causal Structure

December 10, 2019

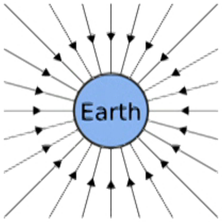
Perimeter Institute, Waterloo

Trailer

Some questions that will be touched on

- At which step does indefinite causal structure enter QG?
- Why single out causal structure in studying QG?
- What to consider when constructing a theory of QG?

Work with some gravitational d.o.f. g



Alternatives

- Entanglement
- Thermodynamics
- Holography
- Matter
- Agents/operations
- Spin chain
- Etc.

Not necessarily distinct:
Take as g whatever expresses gravity

e.g., Agents -> signal -> signalling strength measured by M_{signal}

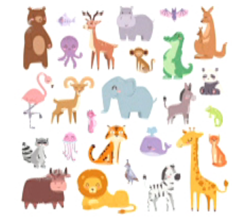
=> $g = M_{signal}$

Choice II

Path integral

$$\int \mathcal{D}[\text{Zoo}] =$$

Path integral approach

$$\int D[\mathrm{Zoo}] =$$ 

$$A = \sum_{g} A_{QG}[g] \sum_{m} A_M[g, m]$$

Example: QFT of GR

$$A = \int Dg_{ab}\ e^{iS_{EH}[g_{ab}]} \int D\phi\ e^{iS_M[g_{ab},\phi]}$$

Path integral approaches

- Metric approaches
  -Euclidean, Higher order actions, asymptotic safety etc.
- Quantum Regge calculus
- (Causal) dynamical triangulation
- Covariant LQG (spin-foam)
- Causal set
- Etc.

-> different choices of g

$$A = \sum_{g} A_{QG}[g] \sum_{m} A_M[g, m]$$

Canonical approaches

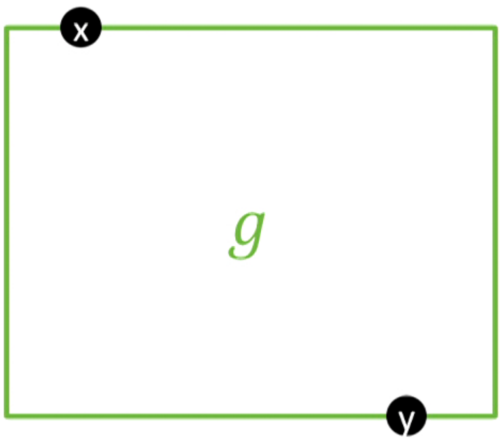
- Canonical LQG
- Etc.

Others

- Bohmian type theories
- Etc.

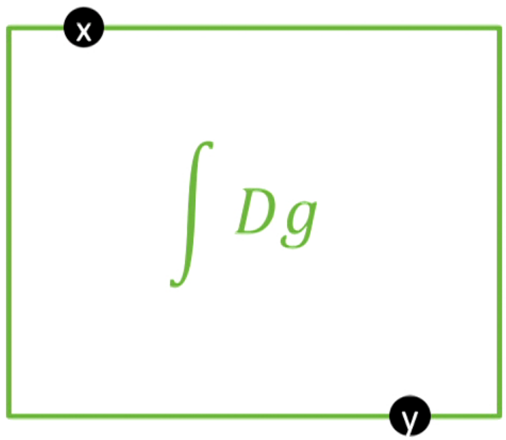
Path integral -> Indefinite causal structure

Classical



$x \to y$ or
$x \leftarrow y$ or
$x - y$

Quantum



$$A_{x\to y} + A_{x\leftarrow y} + A_{x-y}$$

The milk flavor ice cream question:

If approaches to QG generically incorporate indefinite causal structure,

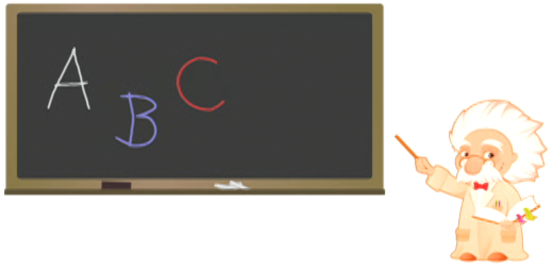
why single it out for study?

Choice III

Basic variable

g?

$$A = \sum_g A_{QG}[g] \sum_m A_M[g, m]$$



Basic variables in relative approaches

Path integral approaches	Basic variable
Metric approaches	g_{ab} - metric
Quantum Regge calculus	l, θ - invariant length, deficit angle
(Causal) dynamical triangulation	N_i - number of simplicies
Covariant LQG (spin-foam)	$SU(n)$ - gauge variables
Causal set	N, γ – Number and order
Etc.	Etc.

Factors to consider

It is not difficult to write down a theory of QG…

Difficulties comes from

- Quantitative analysis
  -Dealing with many body/field systems
- Identifying useful observables
  -What predictions?
- Including matter
  -Subtleties of relativistic matter
- Signature of spacetime
  -Euclidean is unrealistic but practical, Lorentzian is realistic but hard

Choice: world function

$$ds^2 = g_{ab} dx^a dx^b$$

$\sigma(x,y) = \frac{1}{2}\int_x^y ds^2$ - Synge world function

$$g_{ab}(x) = -\lim_{y\to x} \frac{\partial}{\partial x^a}\frac{\partial}{\partial y^b}\sigma(x,y)$$

Why σ?

1. Practical

   $\sigma(x,y) = \sigma(x',y')$

2. Matter-friendly

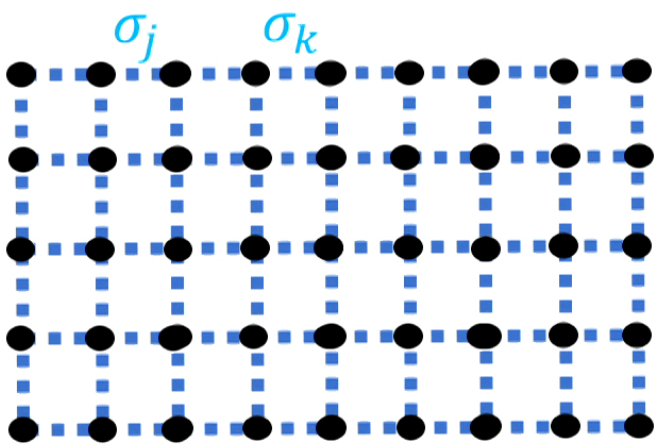
   $A_M \to e^{i\sigma/2l - im^2 l}$

3. Causal structure manifest

   $\sigma \quad =,<,> \quad 0$

Quantitative analysis - 1
Identifying useful observables - 2
Including matter - 2
Signature of spacetime -3

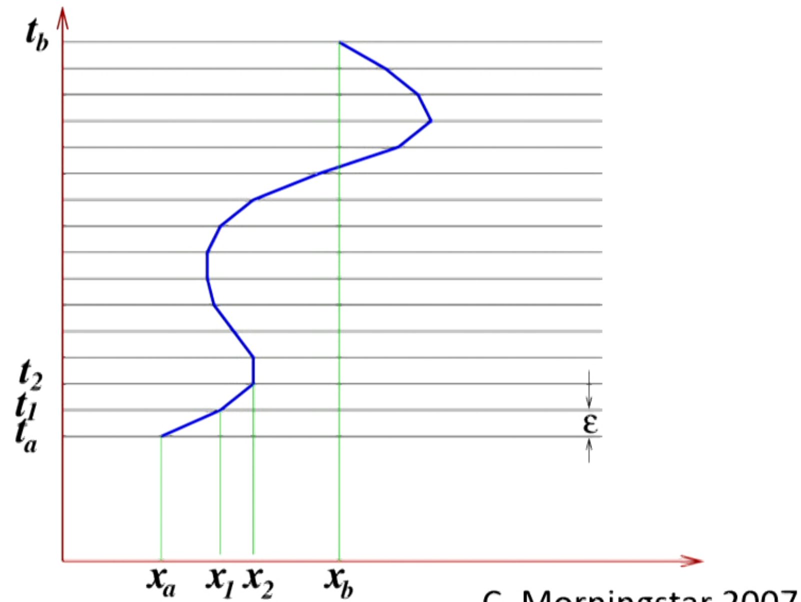
Locating σ on a skeleton



For concreteness, think of a 4D lattice graph

Fine-grain the graph to improve calculation

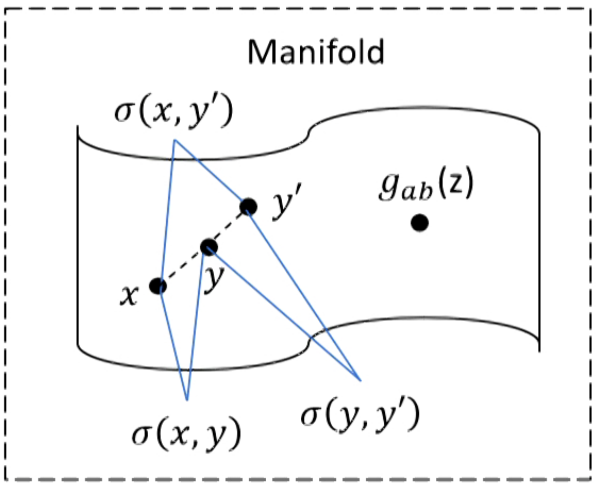
Algorithmic discreteness,
not necessarily fundamental!



Think of the standard algorithm for
particle path integral

Why skeleton?

- Because σ is relational and "local"



$$\sigma(x,y) = \frac{1}{2}\int_x^y ds^2$$

$$\sigma(x,y) + \sigma(y,y') = \sigma(x,y')$$

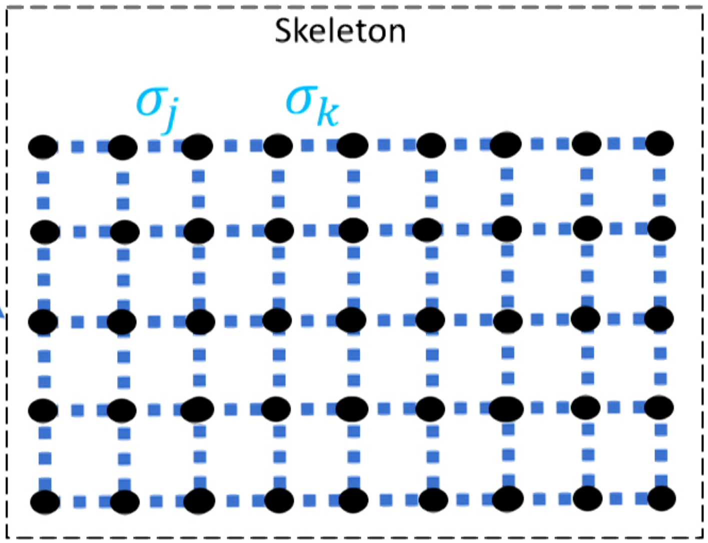
Lost ☹

Manifold

$$\int Dg_{ab} = \prod_{z \epsilon M} \int dg_{ab}(z)$$

$$\int D\sigma = \prod_{(x,y)\epsilon M\times M} \int d\sigma(x,y)$$

Kept ☺



Choice IV

Gravitational amplitude

Task & Strategy

Choice I: focus on g

Choice II: path integral

$$A = \sum_{g} A_{QG}[g] \sum_{m} A_M[g, m]$$

Choice III: $g = \sigma$

$$A = \sum_{\sigma} A_{QG}[\sigma] \sum_{m} A_M[\sigma, m]$$

Choice IV: gravitational amplitude

$$A_{QG}[\sigma] = ?$$

$$A_{QG}[g_{ab}] = e^{iS_{EH}[g_{ab}]}$$

$$S_{EH}[g_{ab}] = \int R\sqrt{-g}dx^4$$

1. Introduce $\Delta[\sigma]$ as a measure of curvature
2. Parker's magic relates $A_{QG}[g_{ab}]$ to $\Delta[\sigma]$
3. Write $A_{QG}[\sigma]$ in terms of $\Delta[\sigma]$

1. $\Delta[\sigma]$ as a measure of curvature

$$\boxed{\Delta = C\exp\{-\int \theta ds'\}/s^{-d}}$$

1. Meaning: Measures curvature by geodesic (de)focusing
(Visser 1993)

curved spacetime geodesic density / Flat spacetime geodesic density

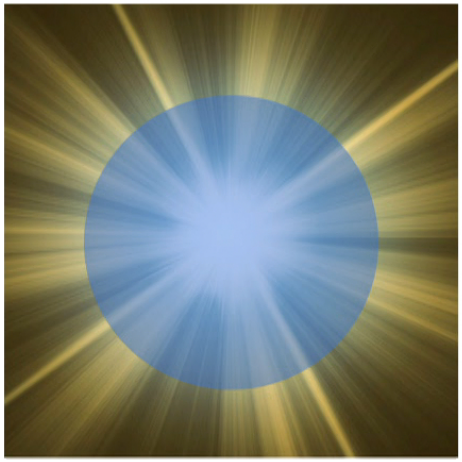
2. In terms of σ

geodesic distance $s = |2\sigma|^{1/2}$

expansion $\theta(x) = \frac{\sigma^a{}_a(x,y)-1}{s}$

3. Wilson line $\mathcal{P}\exp\{\int A_a dx^a\}$

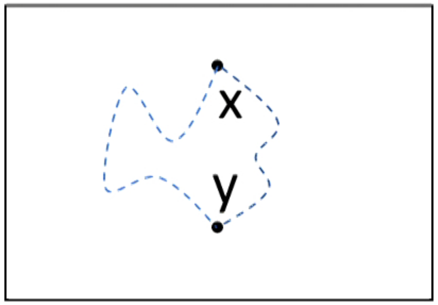
Δ - Van Vleck-Morette determinant



2. Parker's magic

(Parker 1979, Bekenstein & Parker 1981)

$$\exp\{i(\frac{\sigma}{2l} + clR)\} \xleftrightarrow{\sum_{\text{path}}} (\Delta[\sigma])^{3c} \exp\{i\frac{\sigma}{2l}\}$$



3. Write $A_{QG}[\sigma]$ in terms of $\Delta[\sigma]$

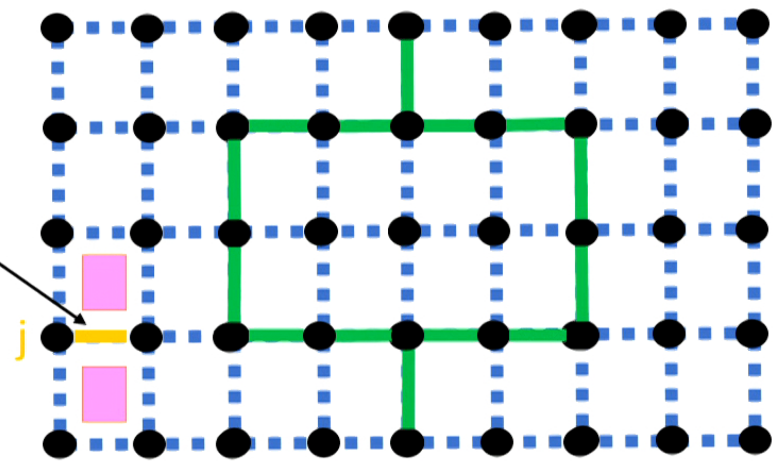
1 Infinitesimal Einstein-Hilbert

$$\exp\left\{i\bar{\alpha}d^4x_j\sqrt{-g_j}R_j\right\}$$

2

$$\exp\left\{i\alpha_j s_j \Delta_j^{-1} R_j\right\}$$

$$\alpha_j = \alpha \sum_{\{k,m,n\}} s_k s_m s_n$$



3

$$c_j \to \alpha_j \Delta_j^{-1}$$

$$l_j \to s_j$$

4

$$\exp\left\{i\left(\frac{\sigma}{2l} + clR\right)\right\} \xleftrightarrow{\sum_{\text{path}}} (\Delta[\sigma])^{3c} \exp\left\{i\frac{\sigma}{2l}\right\}$$

5 Combine with matter to obtain $A_{QG}[\sigma]$ later

Choice V

Matter amplitude

Matter amplitude: strategy

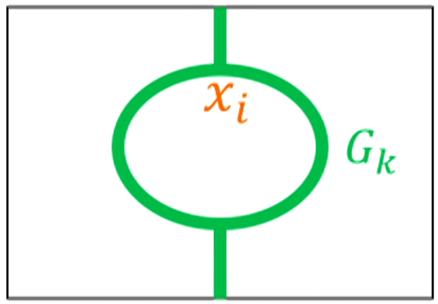
1. Feynman diagrammatic matter QFT

2. Localized Feynman diagrams - Worldline formalism

3. Put on skeleton

1. Feynman diagrammatic matter QFT

$$(\Box + m^2 + \xi R)\phi(x) = 0$$



Γ Feynman diagrams

Vertex factor

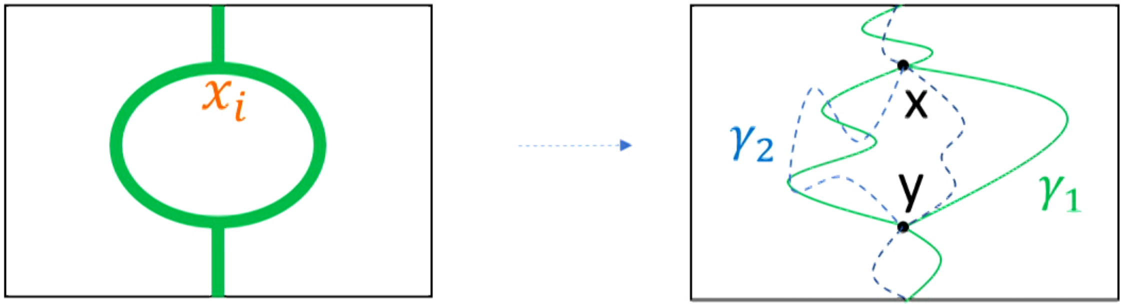
$$\sum_\Gamma \prod_i \int \sqrt{-g}dx_i \frac{V[\Gamma]}{N[\Gamma]} \prod_{k\in\Gamma} G_k$$

Propagators

Vertex integrals

Symmetry factor

2. Localized Feynman diagrams - Worldline formalism



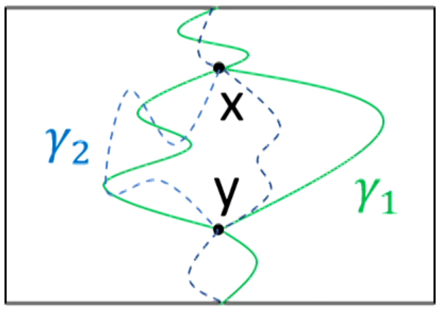
γ correlation diagrams

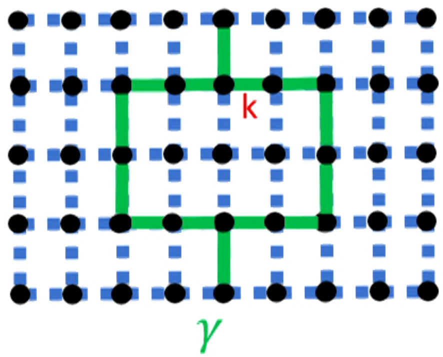
$$G(x,y) = i\int_0^\infty \langle x, l|y, 0\rangle\, e^{-im^2 l} dl$$

$$\langle x, l|y,0\rangle = \int d[x(l')]\exp\left\{ i\int_0^l dl'[\tfrac{1}{4}g_{ab}\frac{dx^a}{dl'}\frac{dx^b}{dl'} - (\xi - \tfrac{1}{3})R(l')]\right\}$$

Scalar field is not just for scalar field!
External, spacetime dof by worldline curve;
Internal dof by Grassman variables etc.

3. Put on skeleton





$$A_M[k \in \gamma, g] = \int \frac{dl_k}{(4\pi i l_k)^2} \exp\left\{ i\frac{\sigma_k}{2l_k} - i(\xi - \frac{1}{3})R_k l_k - im^2 l_k \right\}$$

$$A_M[\gamma, g] = \prod_{k\in\gamma} A_M[k \in \gamma, g] V[\gamma]$$

Summary I

- Choice I: focus on g
- Choice II: path integral
  -Indefinite causal structure
- Choice III: $g = \sigma$
  -Skeleton (for relational local variable)
- Choice IV: gravitational amplitude – Parker's magic
  -Different from all other approaches
- Choice V: matter amplitude – Worldline formalism

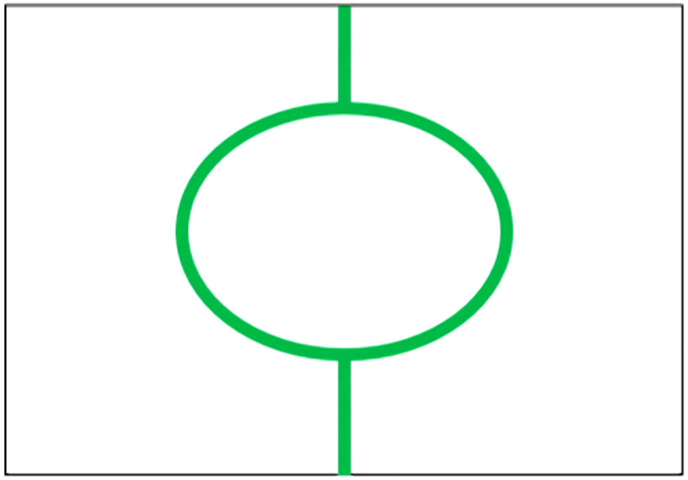
Motivations

1. Practical
   $\sigma(x,y) = \sigma(x',y')$
2. Matter-friendly
   $A_M \to e^{i\sigma/2l - im^2 l}$
3. Causal structure manifest
   σ =,<,> 0

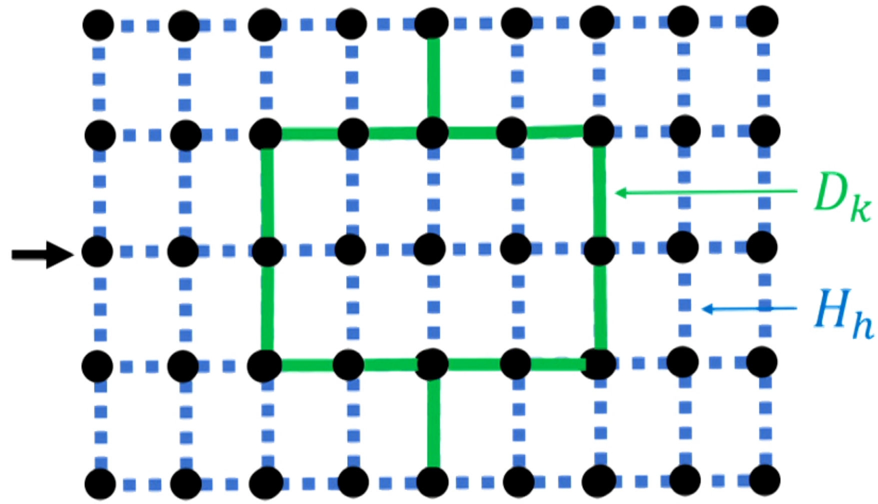
Quantitative analysis - 1
Identifying useful observables - 2
Including matter - 2
Signature of spacetime -3

Summary II: Main result

matter
QFT



$$\sum_\Gamma \prod_i \int dx_i \frac{V[\Gamma]}{N[\Gamma]} \prod_{k\in\Gamma} G_k$$



WQG

Non-perturbative, background independent

$$\sum_\Gamma \sum_\sigma \frac{V[\gamma_\Gamma]}{N[\gamma_\Gamma,\sigma]} \prod_{h\notin\gamma_\Gamma} \underbrace{\Delta_h^{3\alpha_h\Delta_h^{-1}}}_{H_h} \prod_{k\in\gamma_\Gamma} \underbrace{\int \frac{dl_k}{(4\pi i l_k)^2}\Delta_k^{3C_k}\exp\left\{i\frac{\sigma_k}{2l_k} - i m^2 l_k\right\}}_{D_k}$$

Why single out causal structure for study in QG?

- Quantum causal structure generically present among QG approaches
- Why order an ice-cream with milk flavor?
- Why single out causal structure in studying QG?
- Answer: upper bound in causal uncertainty
- Potential fixed point under change of scale
- Potential insights into the UV structure of matter and gravity

Thank you!