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Collection: Indefinite Causal Structure

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Enhancing Entanglement with a Quantum Temporal Switch

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Work in progress in collaboration with: Alessio Belenchia, Esteban Castro-Ruiz, Costantino Budroni, Magdalena Zych, Caslav Brukner, and Robert B. Mann

Indefinte Causal Structure; Dec 9th, 2019.

UNIVERSITY OF WATERLOO | IQC Institute for Quantum Computing

Indefinite Causal Order

- Quantum physics may admit a causal structure where the order of events is indefinite[1,2].
- This provides advantages in computation[3] and communication efficiency[4,5].
- It is not known how ICO will affect entanglement of quantum fields.

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[2] G. Chiribella, G. M. D'Ariano, P. Perinotti, and B. Valiron, Phys. Rev. A 88, 022318 (2013)
[3] M. Araújo, F. Costa, and Č. Brukner, Phys. Rev. Lett. 113, 250402 (2014)
[4] A. Feix, M. Araújo, and Č. Brukner, Physical Review A 92, 052326 (2015)
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Entanglement Harvesitng

- Qubit-field interactions can be non-destructively probed[6].
- Can be used to probe the entanglement structure of quantum fields in various spacetimes (such as near a black hole[7]).
- Entanglement harvesting can be exploited for
  - Seismology[8].
  - Rangefinding[9].

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[6] P. Corona-Ugalde, M. Onuma-Kalu, R. B. Mann, Phys. Rev. A 96 (2017)

[7] L. J. Henderson, R. A. Hennigar, R. B. Mann, A. R. H. Smith, and J. Zhang, Class. Quant. Grav. 35, 21LT02 (2018)

[8] E. G. Brown, W. Donnelly, A. Kempf, R. B. Mann, E. Martín-Martínez, N. C. Menicucci, New J. Phys. 16 (2014)

[9] G. Salton, R. B. Mann, N. C. Menicucci, New J. Phys. 17 035001 (2015)

The entanglement harvesting protocol

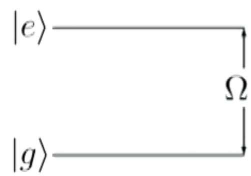
1. Begin with two *uncorrelated* particle detectors (atoms, qubits, harmonic oscillators, etc.).
2. Allow them to locally interact with a free quantum field (taken here to be a massless scalar field in the vacuum state) **for some time**.
3. The detectors can become *entangled*, even if they remained spacelike separated![10,11]

   This entanglement has been swapped from the field to the detectors.

[10] A. Valentini, Physics Letters A 153, 321 (1991).
[11] B. Reznik, Foundations of Physics 33, 167 (2003), ISSN 1572-9516

The Unruh-DeWitt detector model[12]

- This model is a good approximation for the light-matter interaction when no angular momentum is exchanged.



- Unruh-DeWitt detectors are two level systems separated by an energy gap Ω.
  Particle detection = Transition from $|g\rangle$ to $|e\rangle$.
- We cannot measure a field directly; we always measure the responce of something that couples to the field.

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[12] B. L. Hu, S-Y. Lin and J. Louko, Class. Quant. Grav. 29 224005 (2012).

THE INTERACTION HAMILTONIAN

Two detectors A and B couple to a massless scalar field $\hat{\phi}(\boldsymbol{x}, t)$ through

$$\hat{H}_I(t) = \lambda \sum_{D \in \{A,B\}} \frac{d\tau_D}{dt} \chi_D[\tau_D(t)] \hat{\mu}_D[\tau_D(t)] \int d^3\boldsymbol{x}\, F_D\big(\boldsymbol{x} - \boldsymbol{x}_D[\tau_D(t)]\big) \hat{\phi}(\boldsymbol{x}, t)$$

where,

- λ is the coupling strength (small).
- $\chi_D(\tau_D)$ is the switching function of each detector.
- $\hat{\mu}_D(\tau_D)$ is the monopole moment of each detector.

$$\hat{\mu}_D(\tau_D) = \hat{\sigma}_D^+ \mathrm{e}^{\mathrm{i}\Omega_D \tau_D} + \hat{\sigma}_D^- \mathrm{e}^{-\mathrm{i}\Omega_D \tau_D}$$

- $F_D(\boldsymbol{x})$ is the spacial profile of each detector.
- $\boldsymbol{x}_D(\tau_D)$ is the spacetime position of each detector.
- τ_D is the proper time of each detector.

Evolution of the system

1. The detector-field system is initially in the state

$$\hat{\rho}_0 = |0\rangle_\phi \langle 0| \otimes |g\rangle_A \langle g| \otimes |g\rangle_B \langle g| \, .$$

2. It evolves under

$$\hat{U} = -\mathcal{T} \exp\left(-\mathrm{i} \int_{-\infty}^{\infty} dt \, \hat{H}_I(t)\right) = \mathbb{1} + \lambda \hat{U}^{(1)} + \lambda^2 \hat{U}^{(2)} + \mathcal{O}\left(\lambda^3\right)$$

to the final state

$$\hat{\rho} = \hat{U} \hat{\rho}_0 \hat{U}^\dagger .$$

3. Trace out the field to get the final state of the detectors

$$\hat{\rho}_{AB} = \mathrm{Tr}_\phi\left[\hat{\rho}\right] = \begin{pmatrix} \rho_{11} & 0 & 0 & \rho_{14} \\ 0 & \rho_{22} & \rho_{23} & 0 \\ 0 & \rho_{23}^* & \rho_{33} & 0 \\ \rho_{14}^* & 0 & 0 & \rho_{44} \end{pmatrix} + \mathcal{O}(\lambda^4).$$

Quantifying entanglement

- The entanglement of formation is a measure of entanglement of ρ_{AB}, which has the interpretation as the number of Bell pairs required to prepare ρ_{AB} using LOCC.

$$E_f(\hat{\rho}_{AB}) := h\left(\frac{1+\sqrt{1-\mathcal{C}(\hat{\rho}_{AB})^2}}{2}\right)$$

where $h(x) := -x\log(x) - (1-x)\log(1-x)$ and $\mathcal{C}(\hat{\rho}_{AB})$ is the concurrence.

- The concurrence is also an entanglement monotone

$$\mathcal{C}(\hat{\rho}_{AB}) := \max\Big\{0, \big[(\sqrt{\rho_{11}\rho_{44}} + |\rho_{14}) - (\sqrt{\rho_{22}\rho_{33}} + |\rho_{23}|) \\ - (\sqrt{\rho_{11}\rho_{44}} - |\rho_{14}) - (\sqrt{\rho_{22}\rho_{33}} - |\rho_{23}|)\big]\Big\}$$

Causal ordering of the detectors

- We have the freedom to choose when the detectors turn on and off.
- Why not condition the switchings on a control qubit $|i\rangle_C$?

$$\chi_{A,i}(t) = \chi(t - T_{A,i}), \qquad \chi_{B,i}(t) = \chi(t - T_{B,i})$$

- Now the initial state of the detector-field-control system is

$$\hat{\rho}_0 = |0\rangle_A \langle 0| \otimes |0\rangle_B \langle 0| \otimes |0\rangle_F \langle 0| \otimes |+\rangle_C \langle +| \,.$$

- And the new interaction Hamiltonian is

$$\hat{H}_I(t) = \lambda \sum_{i\in\{0,1\}} \sum_{D\in\{A,B\}} \int d^3\boldsymbol{x}\, F_D\big(\boldsymbol{x} - \boldsymbol{x}_D(t)\big)\, \chi_{D,i}(t)\hat{\mu}_D(t) \otimes \hat{\phi}(\boldsymbol{x}, t) \otimes |i\rangle_C \langle i|$$

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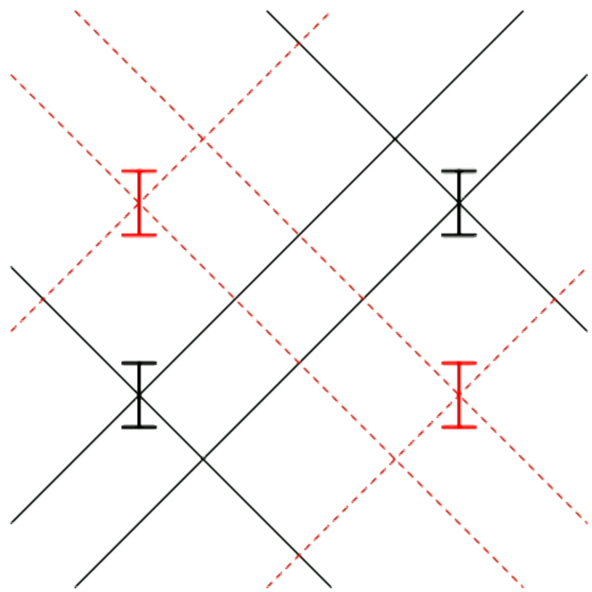
Some terminology

We consider 2 scenarios:

1. **Cause-effect** (CE): Not always spacelike separated detectors are in a superposition of $|A \text{ before } B\rangle$ and $|B \text{ before } A\rangle$ ($T_{A,0} = T_{B,1}$ and $T_{B,0} = T_{A,1}$)

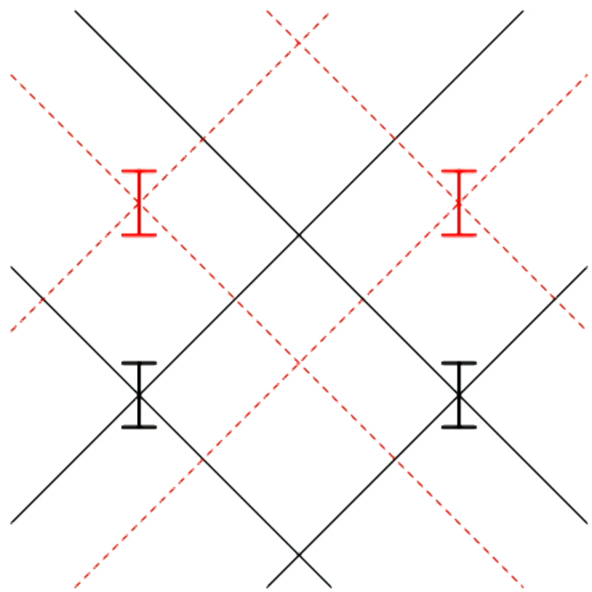


13/35

Some terminology

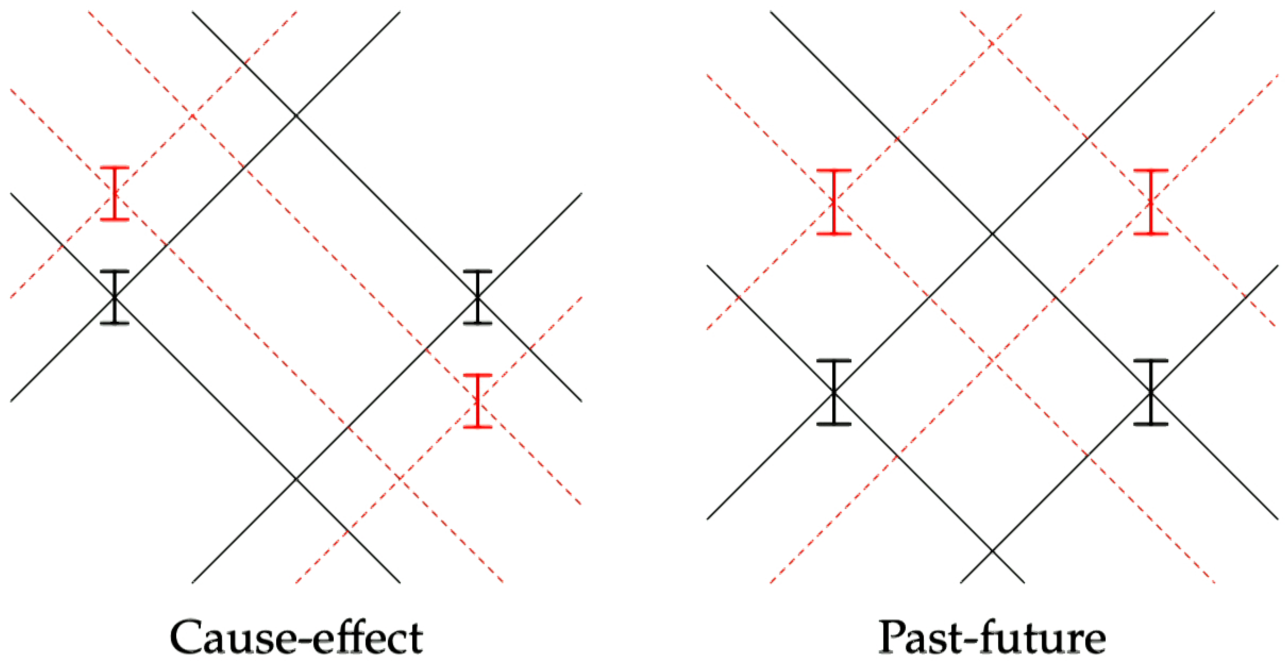
We consider 2 scenarios:

2. **Past-future** (PF): Spacelike separated detectors are in a superposition of $|\text{early}\rangle$ and $|\text{late}\rangle$ ($T_{A,0} = T_{B,0}$ and $T_{A,1} = T_{B,1}$)



These are distinct scenarios

- You can transform *one* branch of the CE superposition to look like *one* branch of the PF superposition (or vice-versa), the *other* branch will never look the same.



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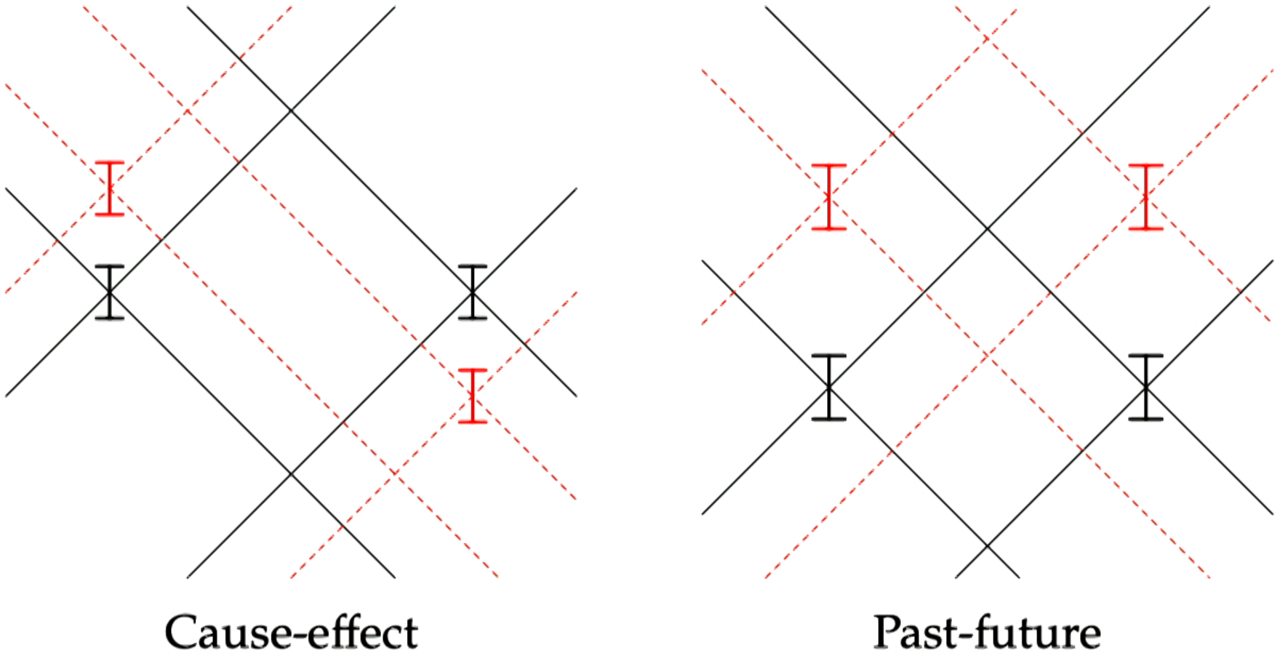

Final State

The final state of the detector-control system (to lowest order in the interaction strength) is

$$\hat{\rho}_{ABC} = \sum_{i,j\in\{0,1\}} \begin{pmatrix} 1 + Y_{ii} + Y_{jj}^* & 0 & 0 & \mathcal{M}_{jj}^* \\ 0 & P_{B,ij} & \mathcal{L}_{AB,ji}^* & 0 \\ 0 & \mathcal{L}_{AB,ij} & P_{A,ij} & 0 \\ \mathcal{M}_{ii} & 0 & 0 & 0 \end{pmatrix} \otimes |i\rangle_C \langle j| \,.$$

Measurements of the control qubit

The control qubit can be:

1. measured in the computation basis
2. traced out (thrown away)
3. measured in the $\pm$ basis

(1) Measure in the computation basis

If the control qubit is measured in the computational basis with result $|i\rangle$, the resulting AB subsystem is described by

$$\hat{\rho}_{AB}^{(i)} = \begin{pmatrix} 1-(P_{A,ii}+P_{B,ii}) & 0 & 0 & \mathcal{M}_{ii}^* \\ 0 & P_{B,ii} & \mathcal{L}_{AB,ii}^* & 0 \\ 0 & \mathcal{L}_{AB,ii} & P_{A,ii} & 0 \\ \mathcal{M}_{ii} & 0 & 0 & 0 \end{pmatrix} + \mathcal{O}(\lambda^4)$$

where $i \in \{0,1\}$.
This is "standard" entanglement harvesting.

(2) Trace out the control bit

If the control qubit traced out the resulting reduced density matrix is

$$\hat{\rho}_{AB}^{(\mathrm{Tr})} = \begin{pmatrix} 1 - \left(P_A^{(\mathrm{Tr})} + P_B^{(\mathrm{Tr})}\right) & 0 & 0 & \mathcal{M}^{(\mathrm{Tr})*} \\ 0 & P_B^{(\mathrm{Tr})} & \mathcal{L}_{AB}^{(\mathrm{Tr})*} & 0 \\ 0 & \mathcal{L}_{AB}^{(\mathrm{Tr})} & P_A^{(\mathrm{Tr})} & 0 \\ \mathcal{M}^{(\mathrm{Tr})} & 0 & 0 & 0 \end{pmatrix} + \mathcal{O}(\lambda^4)$$

where

$$P_D^{(\mathrm{Tr})} = \frac{1}{2}\left(P_{D,00} + P_{D,11}\right)$$

$$\mathcal{L}_{AB}^{(\mathrm{Tr})} = \frac{1}{2}\left(\mathcal{L}_{AB,00} + \mathcal{L}_{AB,11}\right)$$

$$\mathcal{M}^{(\mathrm{Tr})} = \frac{1}{2}\left(\mathcal{M}_{00} + \mathcal{M}_{11}\right)$$

(3) Measure in the computation basis

If the control qubit is measured in the computational basis with result $|+\rangle$, the resulting AB subsystem is described by

$$\hat{\rho}_{AB}^{(+)} = \begin{pmatrix} 1 - \left(P_A^{(+)} + P_B^{(+)}\right) & 0 & 0 & \mathcal{M}^{(\mathrm{Tr})^*} \\ 0 & P_B^{(+)} & \mathcal{L}_{AB}^{(+)^*} & 0 \\ 0 & \mathcal{L}_{AB}^{(+)} & P_A^{(+)} & 0 \\ \mathcal{M}^{(\mathrm{Tr})} & 0 & 0 & 0 \end{pmatrix} + \mathcal{O}(\lambda^4)$$

where

$$P_D^{(+)} = \frac{1}{4}\left(P_{D,00} + P_{D,01} + P_{D,10} + P_{D,11}\right)$$

$$\mathcal{L}_{AB}^{(+)} = \frac{1}{4}\left(\mathcal{L}_{AB,00} + \mathcal{L}_{AB,01} + \mathcal{L}_{AB,10} + \mathcal{L}_{AB,11}\right)$$

$$\mathcal{M}^{(\mathrm{Tr})} = \frac{1}{2}\left(\mathcal{M}_{00} + \mathcal{M}_{11}\right)$$

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The general reduced density matrix

In all three cases, the reduced density matrix has the same form.

$$\hat{\rho}_{AB}^{(\cdot)} = \begin{pmatrix} 1-\left(P_A^{(\cdot)}+P_B^{(\cdot)}\right) & 0 & 0 & \mathcal{M}^{(\cdot)*} \\ 0 & P_B^{(\cdot)} & \mathcal{L}_{AB}^{(\cdot)\,*} & 0 \\ 0 & \mathcal{L}_{AB}^{(\cdot)} & P_A^{(\cdot)} & 0 \\ \mathcal{M}^{(\cdot)} & 0 & 0 & 0 \end{pmatrix} + \mathcal{O}(\lambda^4)$$

- $P_A^{(\cdot)}$ and $P_B^{(\cdot)}$ are the transition probability of detector A and B respectively.
- $\mathcal{M}^{(\cdot)}$ encodes the nonlocal correlations.
- $\mathcal{L}_{AB}^{(\cdot)}$ encodes the total correlations.

CONCURRENCE

To lowest order in the interaction strength, the concurrence is

$$\mathcal{C}\left(\hat{\rho}_{AB}\right) = 2\max\left[0, |\mathcal{M}| - \sqrt{P_A P_B}\right] + \mathcal{O}(\lambda^4)$$

This expression has a nice interpretation:
The detectors are entangled when the non-local correlations dominate the local noise.

Setup

We take our detectors to:

- be point-like. $F_D(\boldsymbol{x}) = \delta^{(3)}(\boldsymbol{x})$
- have cosine switching.

$$\chi(t) = \begin{cases} \cos\left(\sqrt{\frac{2}{\pi}}\frac{t}{\sigma}\right), & -\sqrt{2\pi^3} \leq t \leq \sqrt{2\pi^3} \\ 0, & \text{otherwise} \end{cases}$$

- Have identical energy gaps. $\Omega_A\sigma = \Omega_B\sigma = 1$

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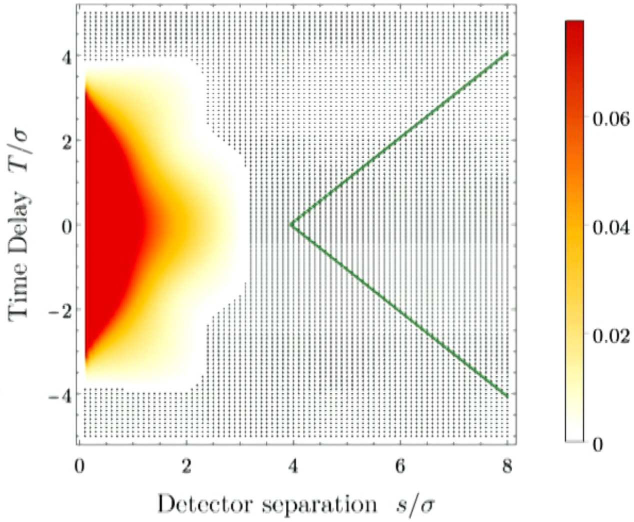
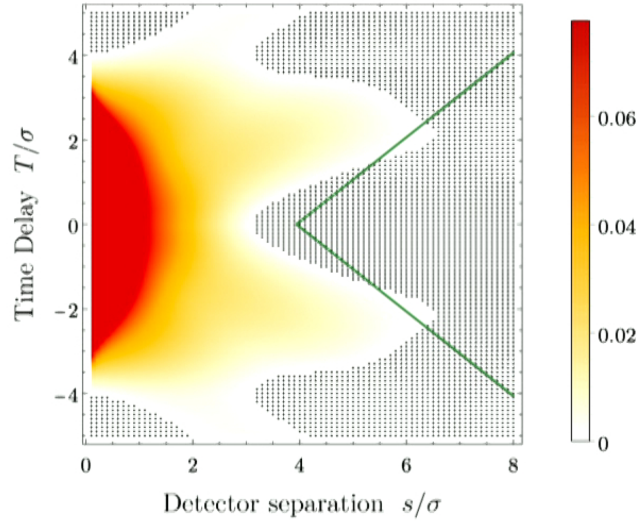
Entanglement enhancement



$\mathcal{C}\left(\hat{\rho}_{AB}^{(\mathrm{Tr})}\right)/\lambda^2$



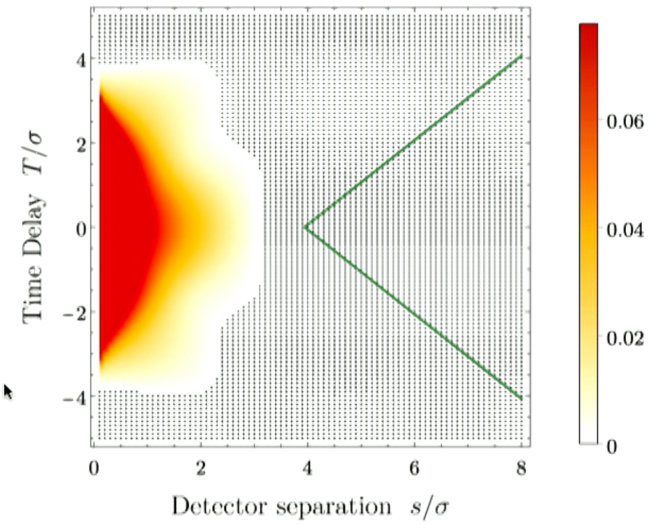
$\mathcal{C}\left(\hat{\rho}_{AB}^{(\mathrm{CE})}\right)/\lambda^2$

The green lines marks the line of spacelike separation

- There is significant enhancement in the regions where entanglement harvesting is possible
- EH is now possible in regions of spacelike separation

25/35

Entanglement enhancement



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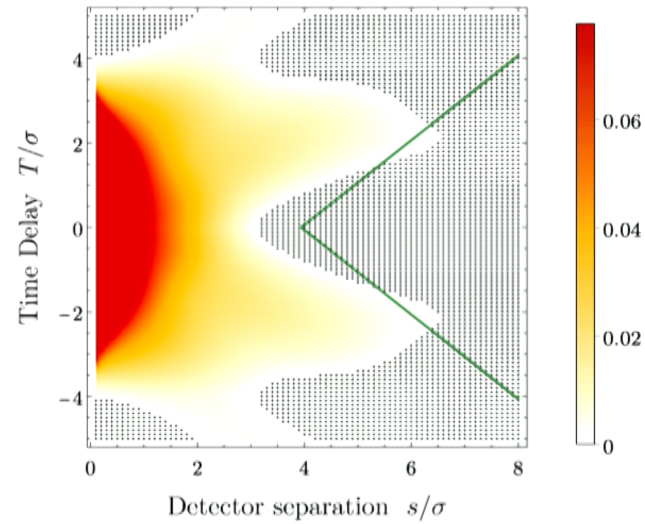

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What is going on here?

Recall:

- $\mathcal{C}(\hat{\rho}_{AB}) = 2\max\left[0, |\mathcal{M}| - \sqrt{P_A P_B}\right]$
- $\mathcal{M}^{(+)} = \mathcal{M}^{(\mathrm{Tr})}$

It turns out $P_D^{(\mathrm{Tr})} \leq P_D^{(+)}$ (perturbativly) regardless of the spacial profile or the switching function.

The cross terms $P_{D,01}$ and $P_{D,10}$ lower the transition probability of the detector

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What about double switching?

- It is natural to ask how these results compare with a "double-switching" function

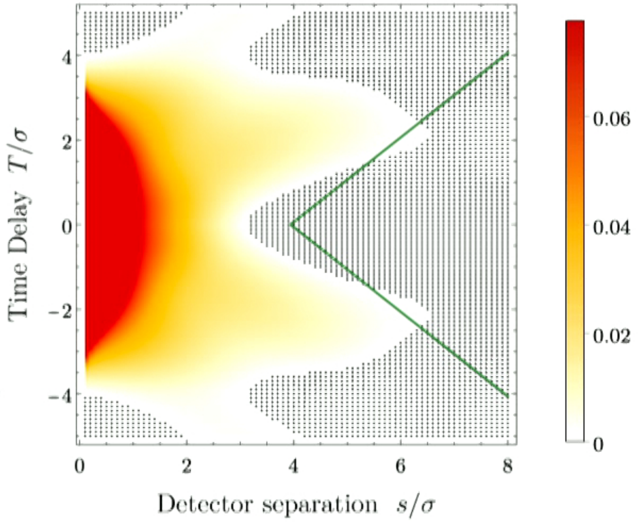
$$\chi_D^{(\mathrm{DS})}(t) = \frac{1}{2}\left[\chi_{D,0}(t) + \chi_{D,1}(t)\right]$$

$$\hat{\rho}_{AB}^{(\mathrm{DS})} = \begin{pmatrix} 1 - \left(P_A^{(+)} + P_B^{(+)}\right) & 0 & 0 & \mathcal{M}^{(\mathrm{DS})*} \\ 0 & P_B^{(+)} & \mathcal{L}_{AB}^{(+)*} & 0 \\ 0 & \mathcal{L}_{AB}^{(+)} & P_A^{(+)} & 0 \\ \mathcal{M}^{(\mathrm{DS})} & 0 & 0 & 0 \end{pmatrix} + \mathcal{O}(\lambda^4)$$

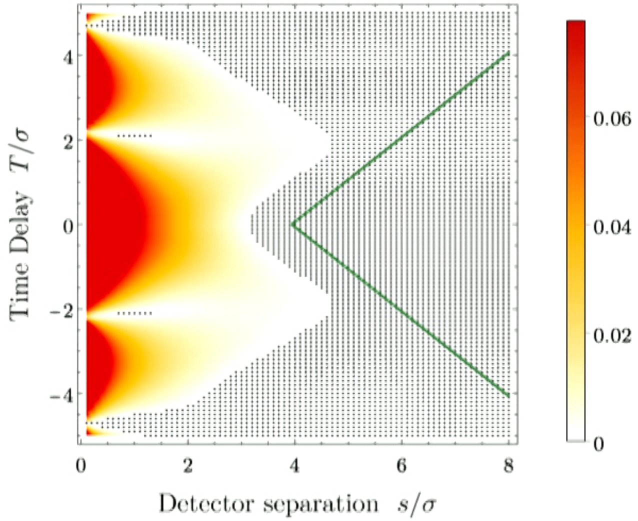
where

$$\begin{aligned} \mathcal{M}^{(\mathrm{DS})} &= \frac{1}{4}\left(\mathcal{M}_{00} + \mathcal{M}_{01} + \mathcal{M}_{10} + \mathcal{M}_{11}\right) \\ &= \frac{1}{2}\left(\mathcal{M}^{(\mathrm{CE})} + \mathcal{M}^{(\mathrm{PF})}\right) \end{aligned}$$

WHAT ABOUT DOUBLE SWITCHING



$\mathcal{C}\left(\hat{\rho}_{AB}^{(CE)}\right)/\lambda^2$



$\mathcal{C}\left(\hat{\rho}_{AB}^{(DS)}\right)/\lambda^2$

The green lines marks the line of spacelike separation

- The resulting entanglement of the detectors can distinguish between indefinite temporal order and a double switch.
- Double switching does not have spacelike EH

δ-SWITCHING

If we take the switching function to be

$$\chi_{D,i}(t) = \eta\delta(t - T_{D,i}) \qquad\qquad T_B \geq T_A$$

the final state of the detector(s) can be known exactly!

$$\hat{\rho}_{ABC} = \sum_{i,j\in\{0,1\}} \begin{pmatrix} \rho_{11} & 0 & 0 & \rho_{14} \\ 0 & \rho_{22} & \rho_{23} & 0 \\ 0 & \rho_{23}^{*} & \rho_{33} & 0 \\ \rho_{14}^{*} & 0 & 0 & \rho_{44} \end{pmatrix} \otimes |i\rangle_C \langle j|$$

We need to give them a spacial profile

$$F_D(\boldsymbol{x}) = \exp\left(-\frac{(\boldsymbol{x} - \boldsymbol{x}_D)^2}{a^2}\right)$$

The no-go theorm [13]

Two UDW detectors with arbitrary spacial profiles, coupling strengths and delta switching functions **cannot harvest entanglement** from a coherent state of the scalar field.

But we can cheat.

[13] P Simidzija and E. Martín-Martínez Phys. Rev. D 96, 065008

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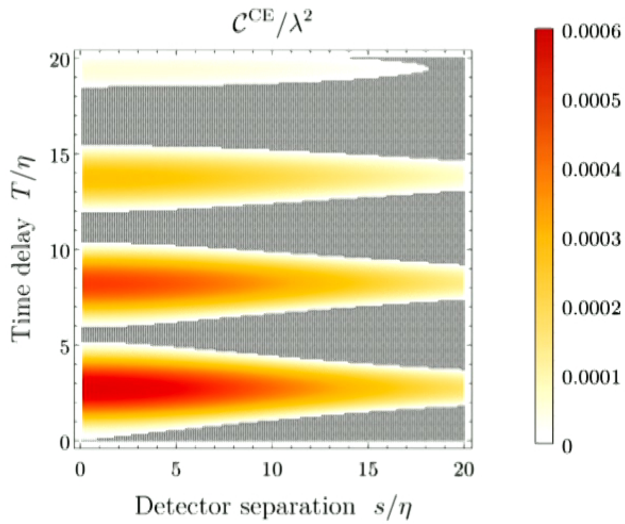
Cheating the no-go theorm



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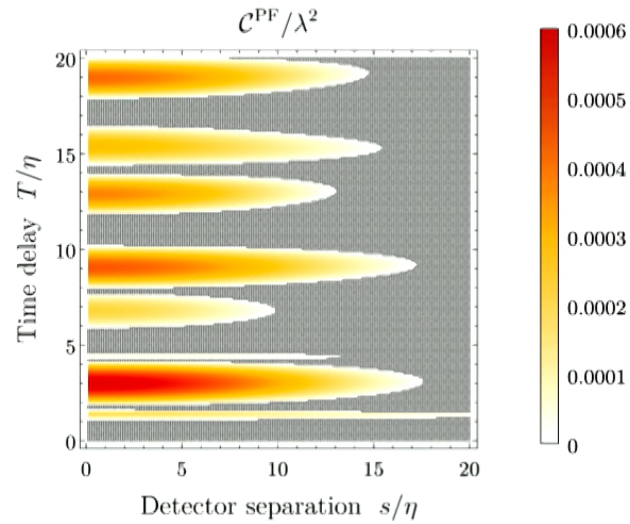


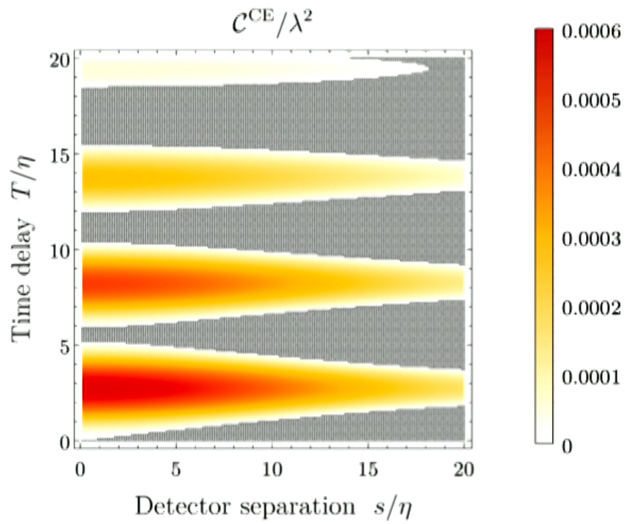
$\mathcal{C}\left(\hat{\rho}_{AB}^{(\mathrm{PF})}\right)/\lambda^2$

The detectors are timelike separated in these figures

($a = 9\eta$ and $\Omega_A\eta = \Omega_B\eta = 1$)

- The detectors are on **only once** in each branch of the superposition
- But we still see entanglement harvesting!

Cheating the no-go theorm



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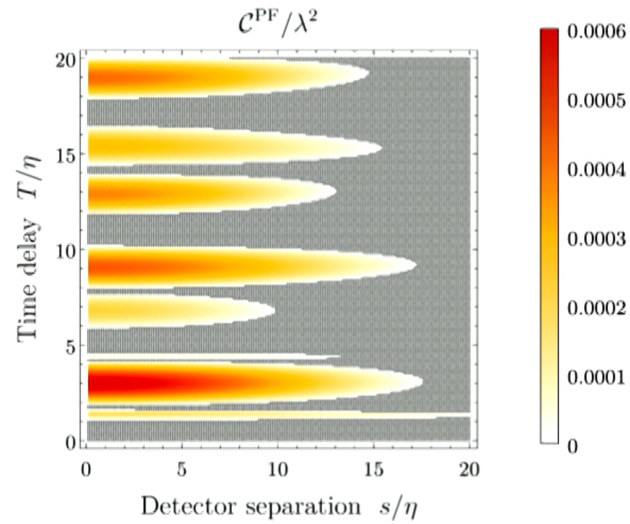

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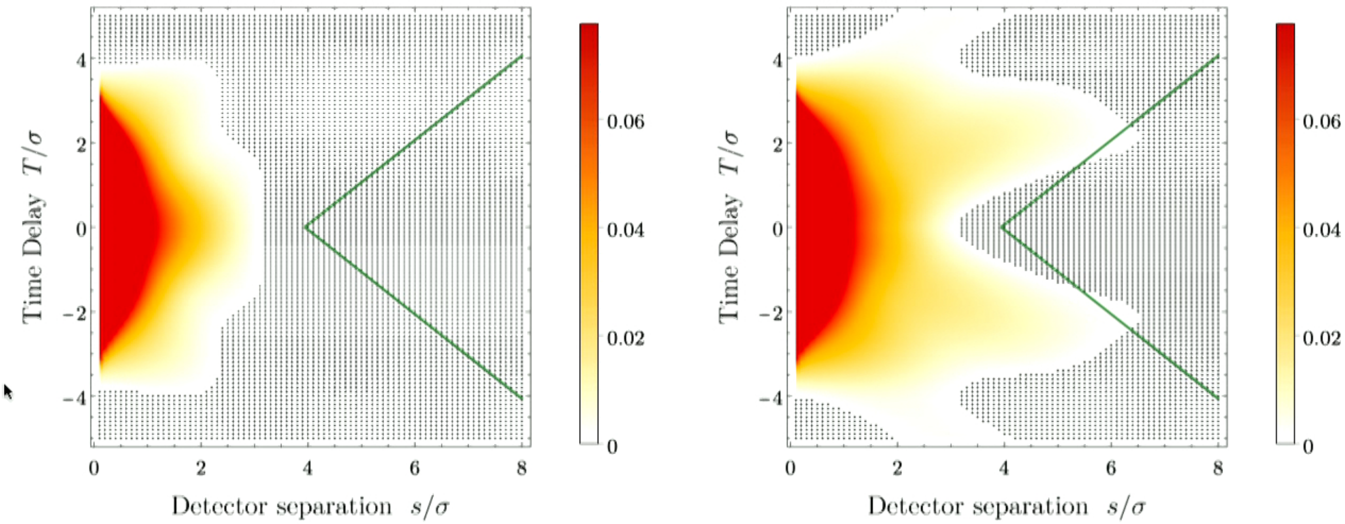
Conclusions

Indefinte temoral order:

- Enhances entanglement harvesting, particularly in spacelike separated regions.
- Reduces detector transition probability.
- Is distinct from double switching.
- Allows us to cheat the no-go theorem and harvest entanglement with delta switching.

Entanglement enhancement



$\mathcal{C}\left(\hat{\rho}_{AB}^{(\mathrm{Tr})}\right)/\lambda^2$ $\qquad$ $\mathcal{C}\left(\hat{\rho}_{AB}^{(\mathrm{CE})}\right)/\lambda^2$

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- There is significant enhancement in the regions where entanglement harvesting is possible
- EH is now possible in regions of spacelike separation