Title: Quantum mechanics and the covariance of physical laws in quantum reference frames

Speakers: Flaminia Giacomini

Collection: Indefinite Causal Structure

Date: December 09, 2019 - 4:00 PM

URL: http://pirsa.org/19120021

Abstract: In physics, every observation is made with respect to a frame of reference. Although reference frames are usually not considered as degrees of freedom, in all practical situations it is a physical system which constitutes a reference frame. Can a quantum system be considered as a reference frame and, if so, which description would it give of the world? Here, we introduce a general method to quantise reference frame transformations, which generalises the usual reference frame transformation to a â€œsuperposition of coordinate transformationsâ€•. We describe states, measurement, and dynamical evolution in different quantum reference frames, without appealing to an external, absolute reference frame, and find that entanglement and superposition are frame-dependent features. The transformation also leads to a generalisation of the notion of covariance of dynamical physical laws, to an extension of the weak equivalence principle, and to the possibility of defining the rest frame of a quantum system.

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Quantum mechanics and the covariance of physical laws in quantum reference frames

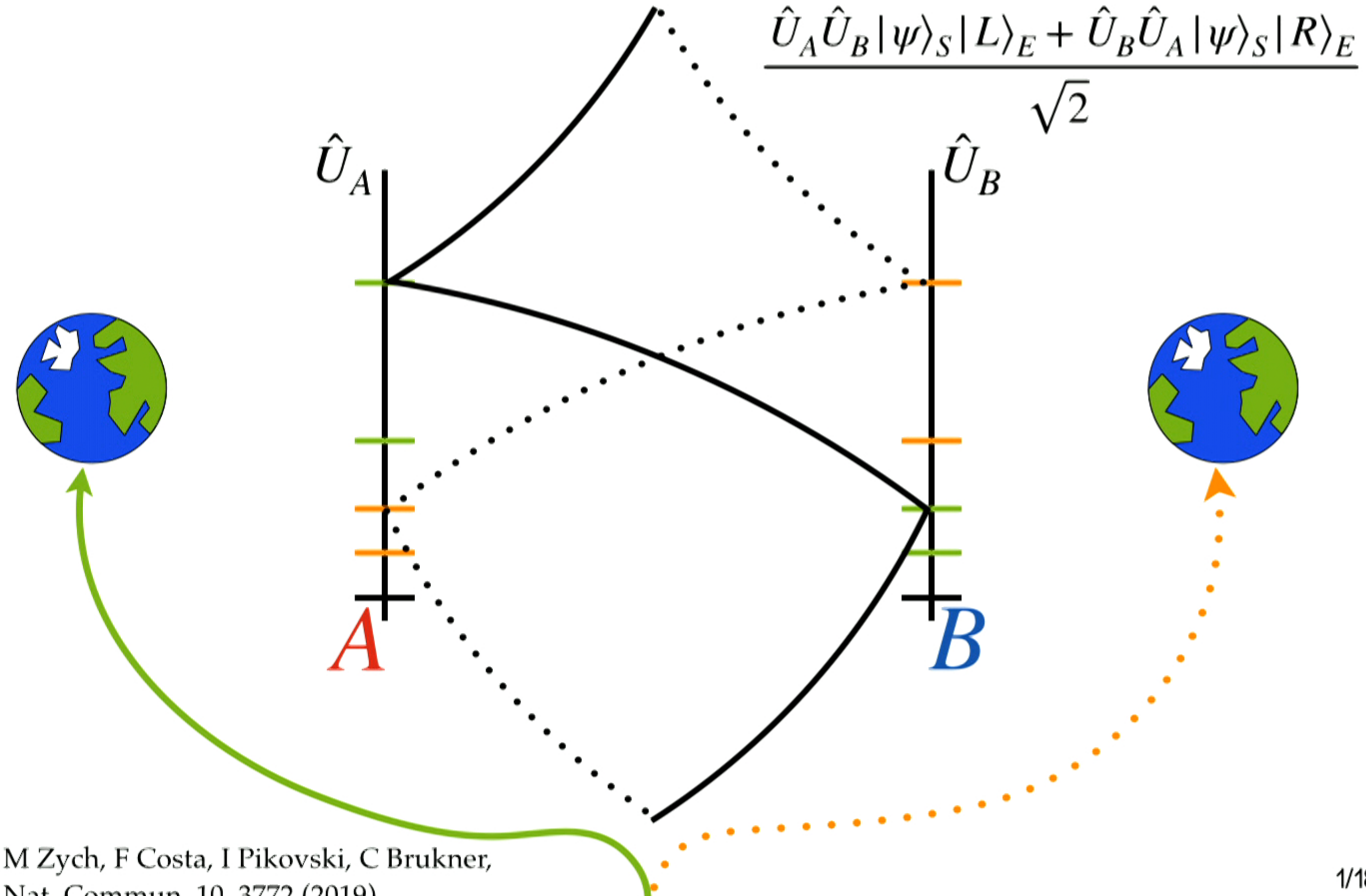
Flaminia Giacomini, Esteban Castro Ruiz, Časlav Brukner,

Ref: Nat. Commun. 10, 494 (2019)

CoQuS
ComplexQuantumSystems

Indefinite Causal Structure Workshop
Waterloo, 9 December 2019

The gravitational switch



M Zych, F Costa, I Pikovski, C Brukner,
Nat. Commun. 10, 3772 (2019)

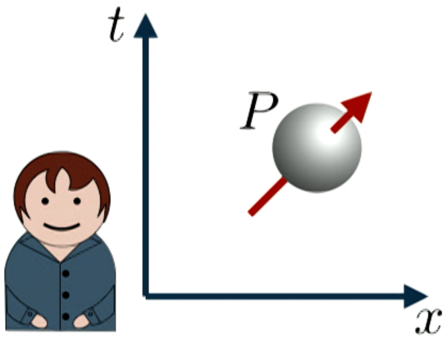
1

INTRODUCTION

What is a reference frame?

Reference frames are abstract entities, used to fix the point of view from which observations are carried out.

The laws of physics are the same regardless of the choice of the reference frame. (**Principle of covariance**).



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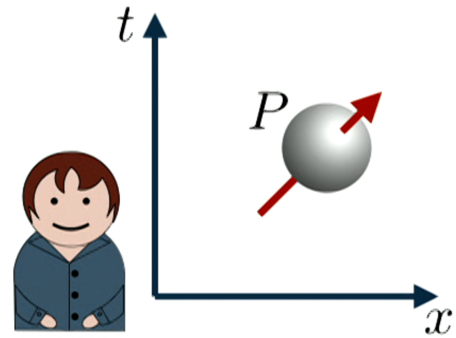

Translation	$\hat{U}_T = e^{\frac{i}{\hbar} X_0 \hat{p}}$	
Galilean boost	$\hat{U}_B = e^{\frac{i}{\hbar} v \hat{G}}$	$\hat{G} = \hat{p}t - m\hat{x}$
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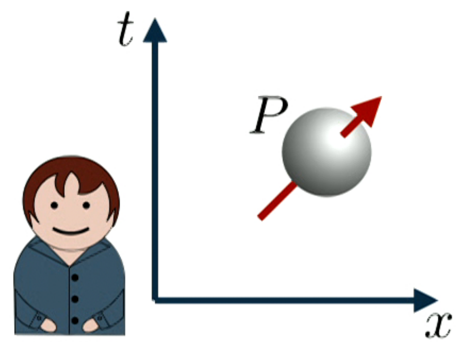

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The reference frame enters the transformation as a **parameter**.

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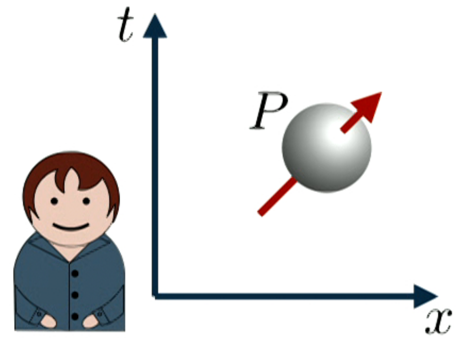

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Covariance of physical laws

$$\hat{H}' = \hat{U}\hat{H}\hat{U}^\dagger + i\hbar \frac{d\hat{U}}{dt}\hat{U}^\dagger$$

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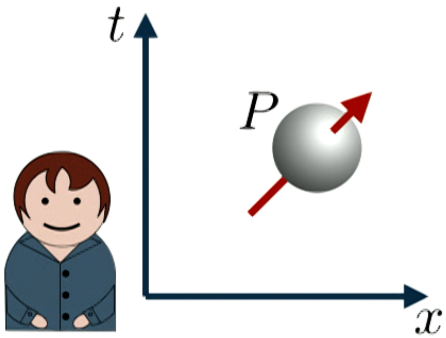

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Symmetry

$$\hat{H}' = \hat{H}$$

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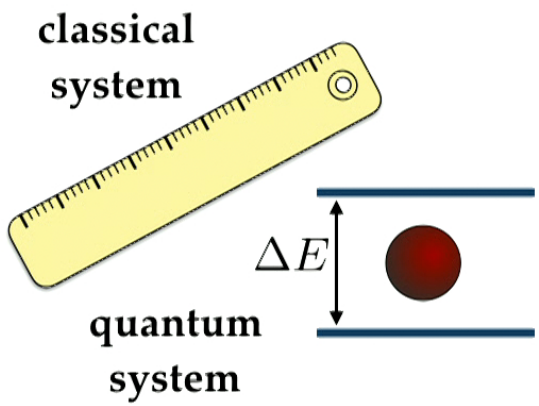

Every observation is carried out by means of a physical system…

… and taking as reference a physical object.

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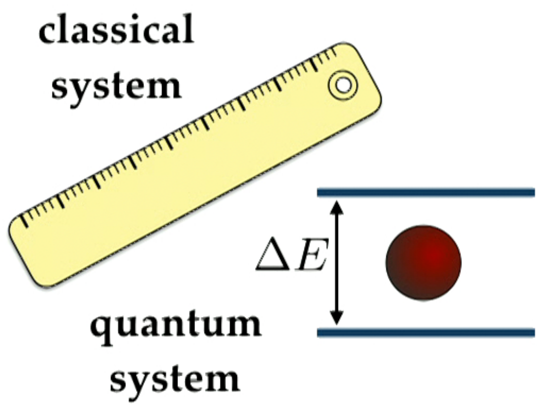

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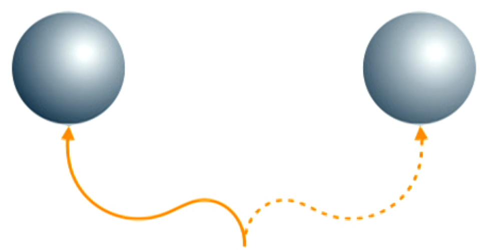
Quantum systems can be in superposition, or entangled



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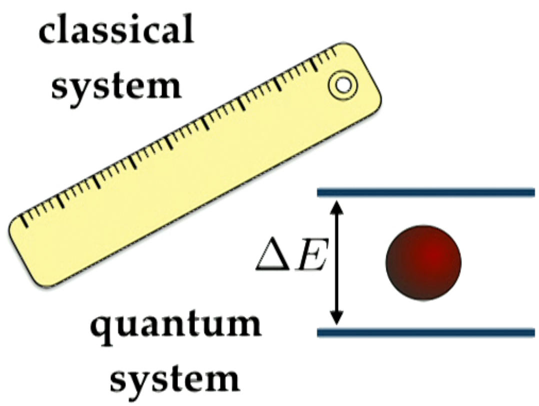

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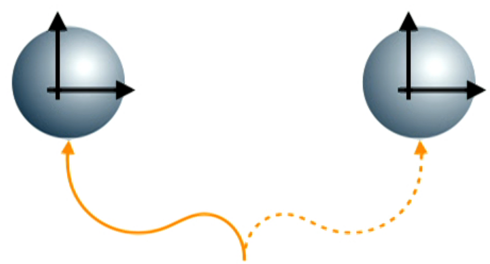
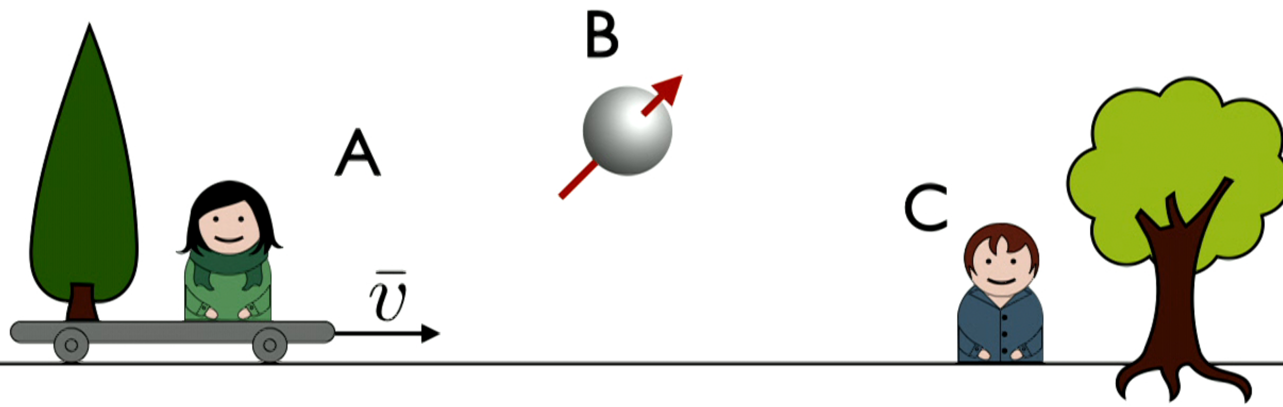


Can we associate a quantum reference frame to a quantum particle?

What is a reference frame?



A quantum reference frame can be "attached" to any physical system.

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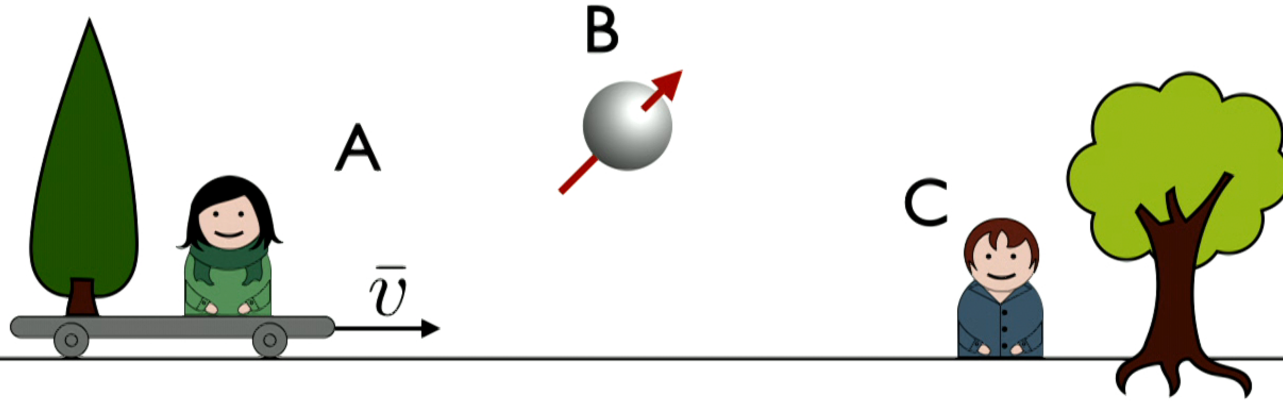
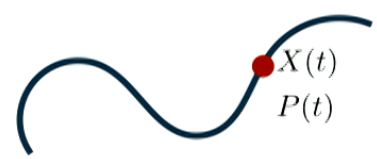


A quantum reference frame can be "attached" to any physical system.

We need to consider the **dynamical degrees of freedom** of the reference frame.

$$\begin{cases} X(t) \\ P(t) \end{cases}$$



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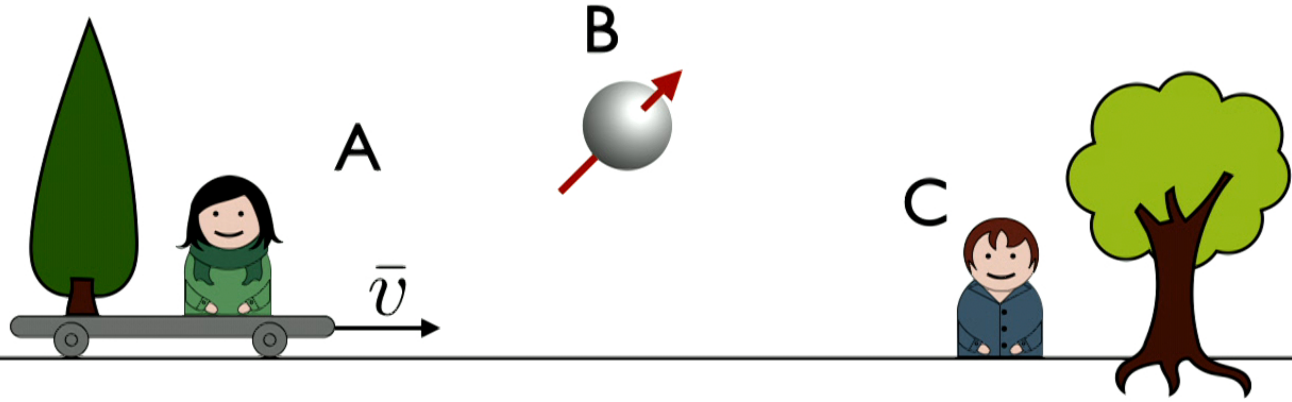

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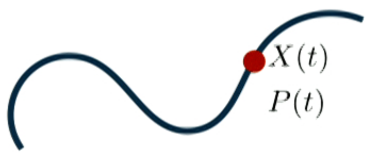


The system can behave according to either **classical** or **quantum** mechanics.

Covariance in QRFs

The goal is the **generalisation** of the **principle of covariance.**

⇓

Covariance in quantum reference frames

The laws of physics are the same irrespective of the choice of the quantum RF.

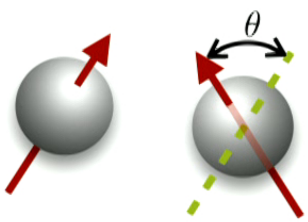
Outline

- The formalism for quantum reference frames: transformation, notion of relative state, dynamics

Examples of transformations

- Superposition of spatial translations
- Superposition of Galilean boosts
- Weak equivalence principle in quantum reference frames

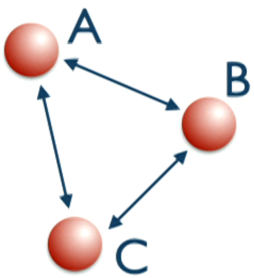
Quantum reference frames

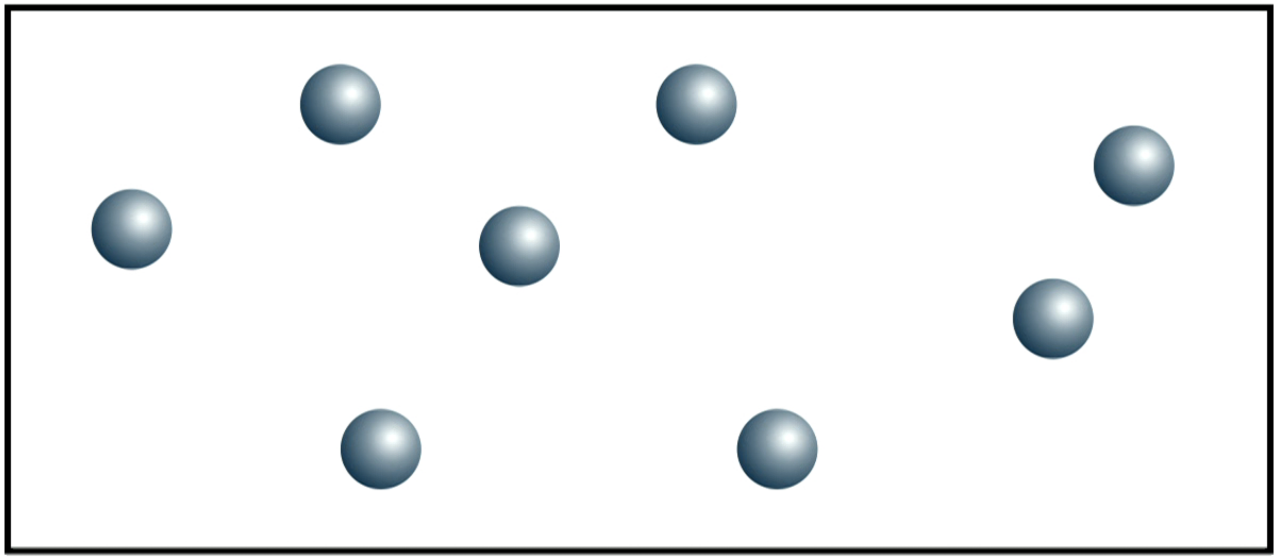


Elements of the formalism

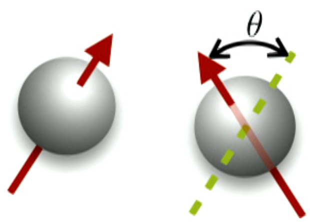
Relational approach: only **relative** quantities are considered.

No need of an absolute reference frame.





Quantum reference frames



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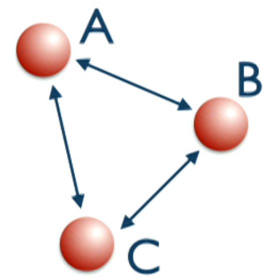

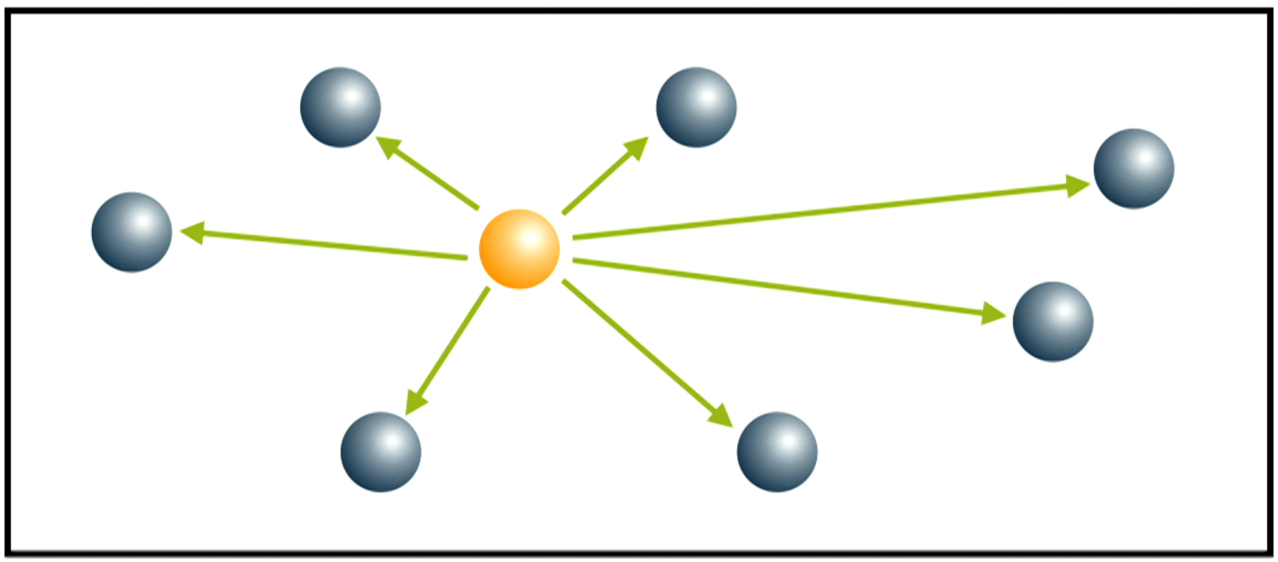

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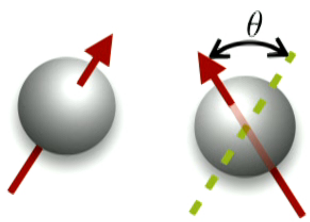
Quantum reference frames



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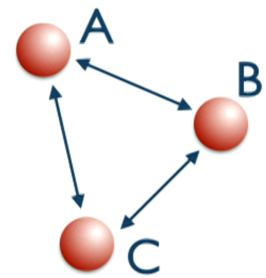

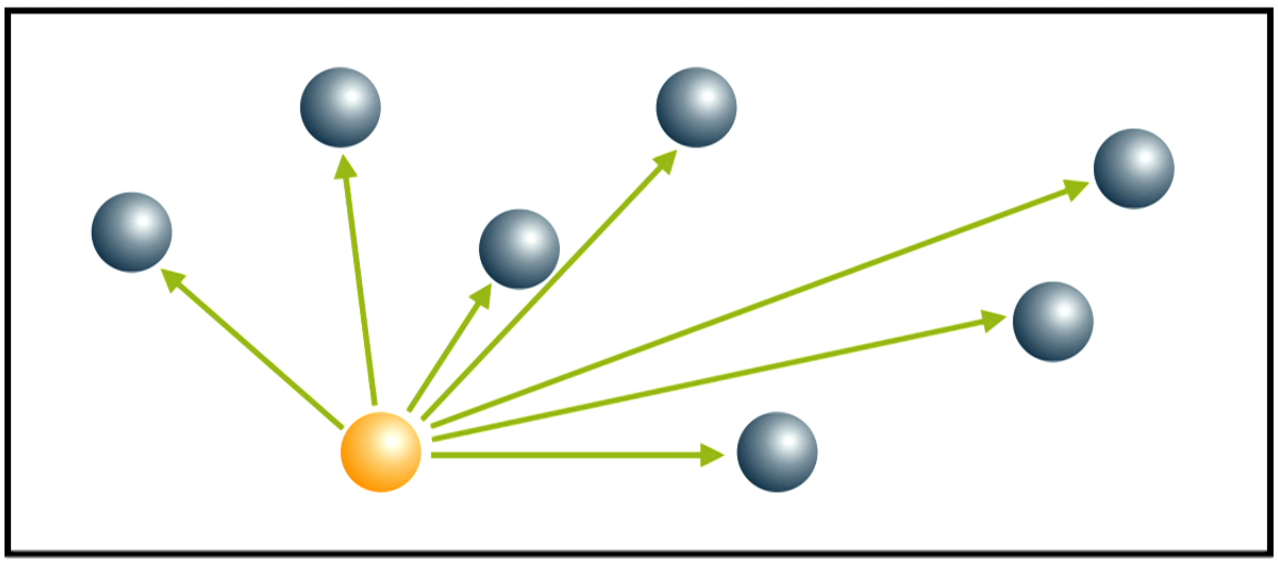

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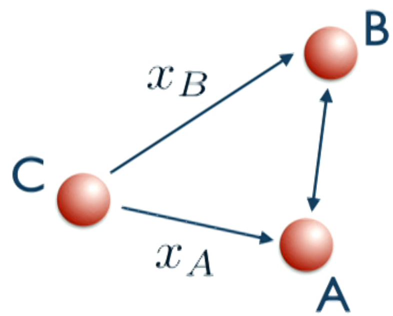
Transformation of the state

Simplest case: transformation to relative coordinates

$$q_B = x_B - x_A$$

$$q_C = -x_A$$



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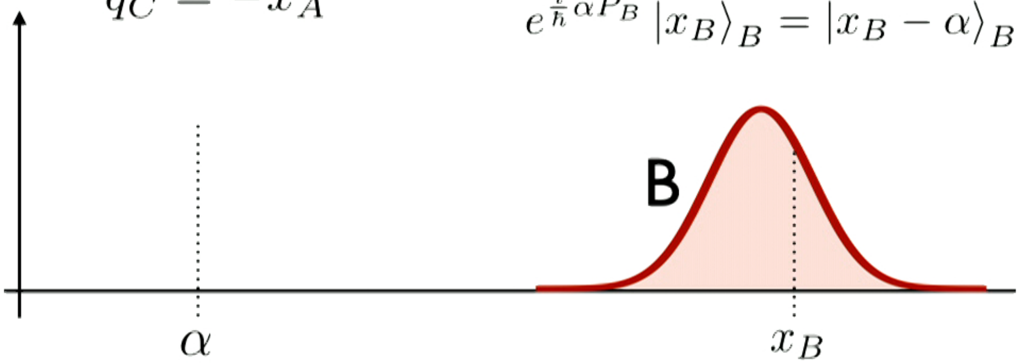

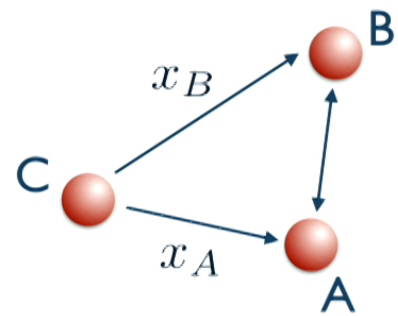

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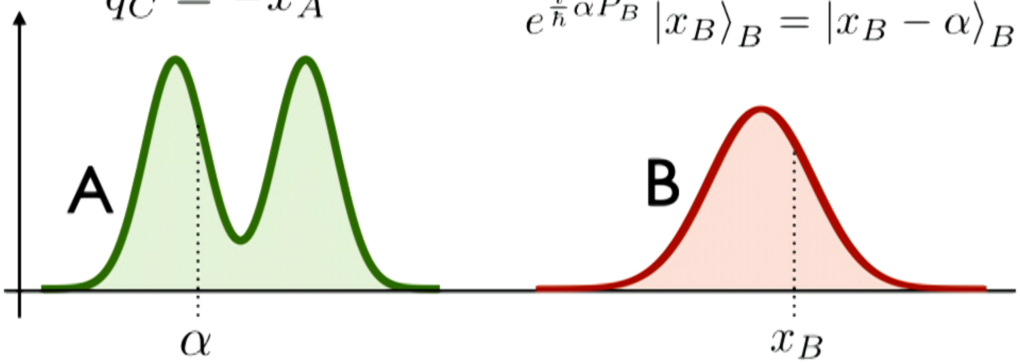

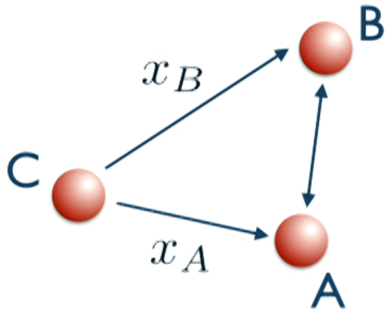

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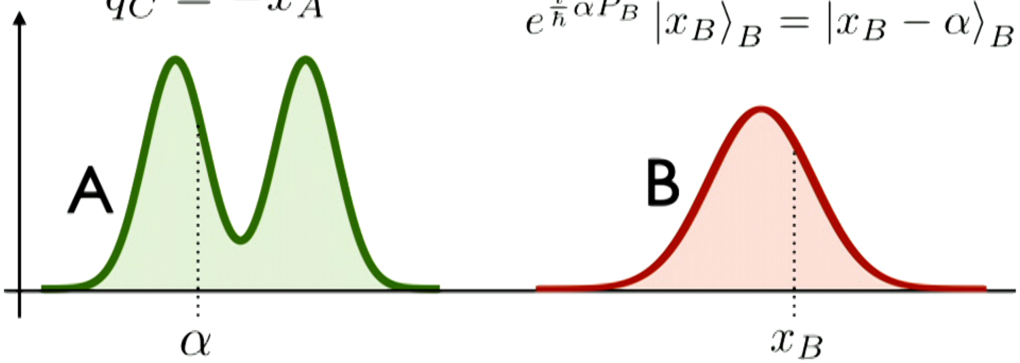

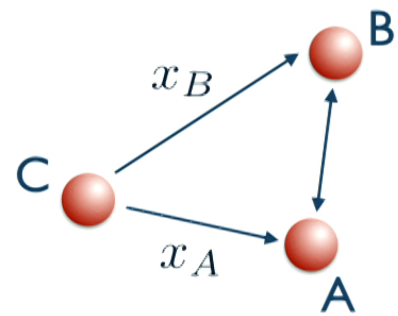

For the whole state:

$$e^{\frac{i}{\hbar}\hat{X}_A \hat{P}_B} |\psi\rangle_A |\phi\rangle_B$$

usually a parameter of the group!

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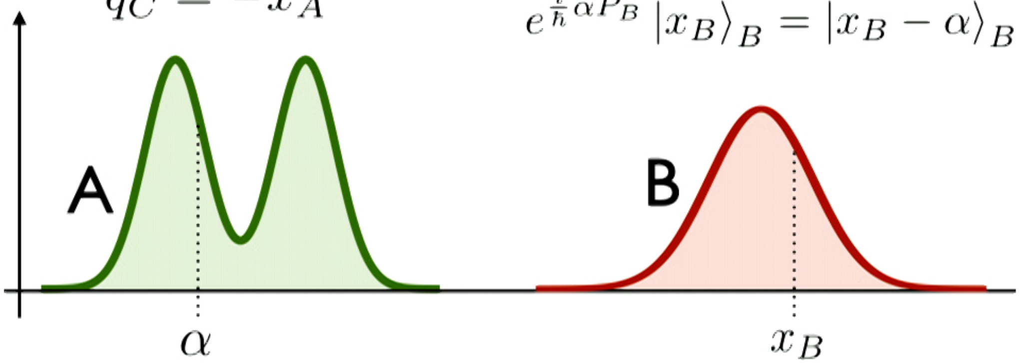

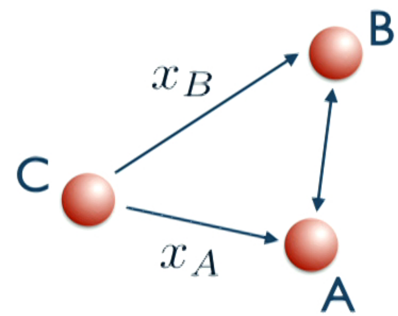

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- Wavepackets instead of sharp position/velocities
- Quantum superposition, entanglement

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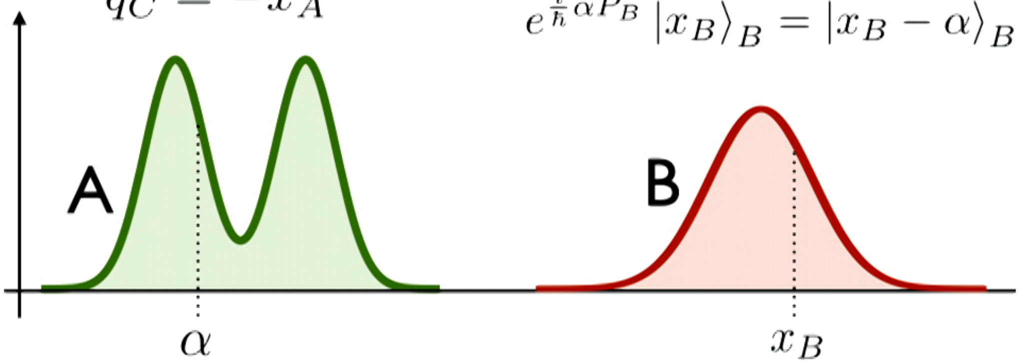

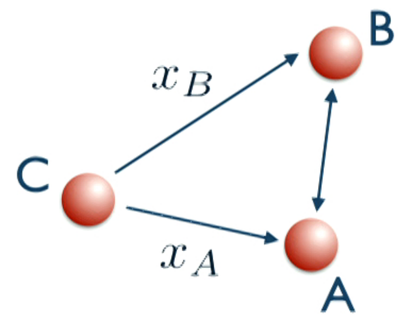

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$$\boxed{\hat{S}_x = \hat{\mathcal{P}}_{AC} e^{\frac{i}{\hbar}\hat{x}_A \hat{p}_B}}$$

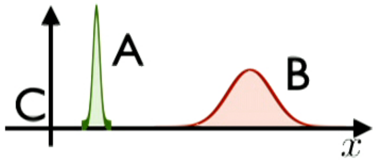
$$\rho_{BC}^{(A)} = \hat{S}_x \rho_{AB}^{(C)} \hat{S}_x^\dagger$$

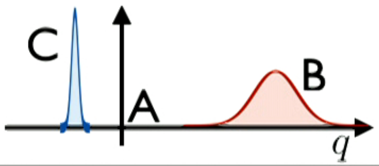
$\hat{\mathcal{P}}_{AC}$: parity operator + swap between A and C.

Example: Relative states

$$\hat{S}_x = \hat{\mathcal{P}}_{AC} e^{\frac{i}{\hbar}\hat{x}_A \hat{p}_B}$$

Localised state of A





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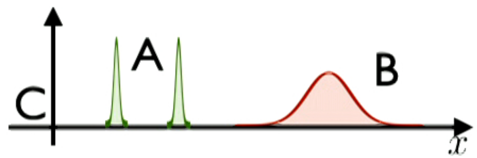
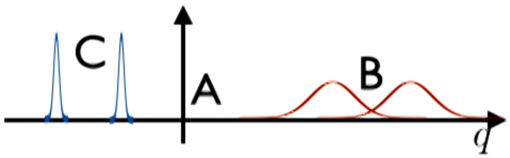
Product state and spatial superposition





A: new reference frame; B: quantum system; C: old reference frame

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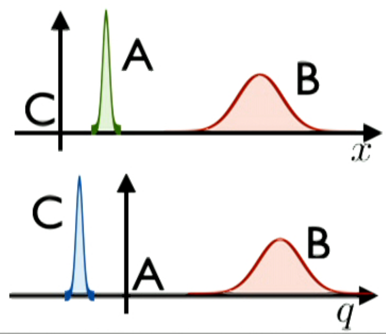

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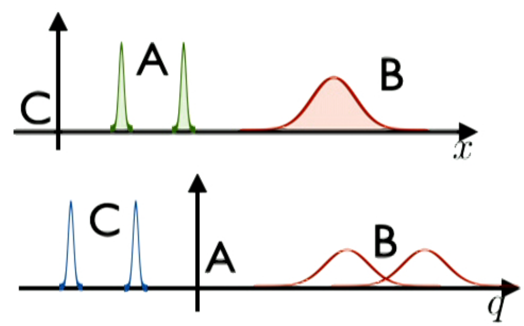
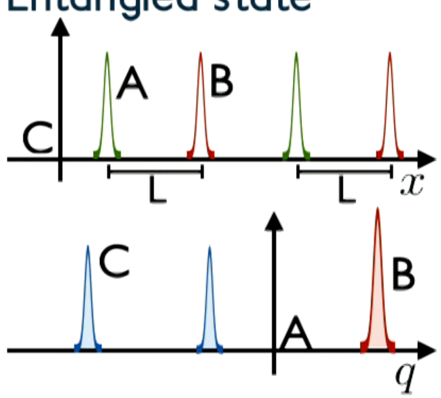
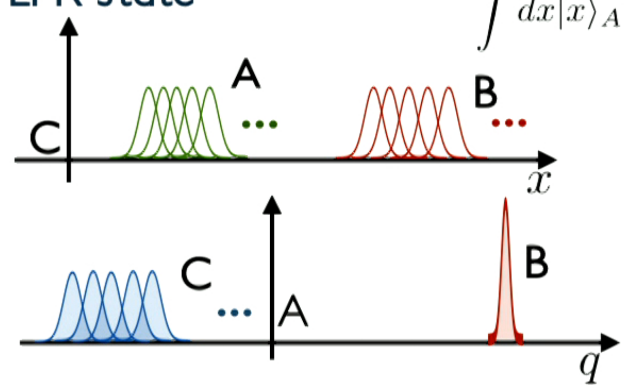


Entangled state



EPR state

$$\int dx |x\rangle_A |x + X\rangle_B$$



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3

TEMPORAL EVOLUTION: DYNAMICS

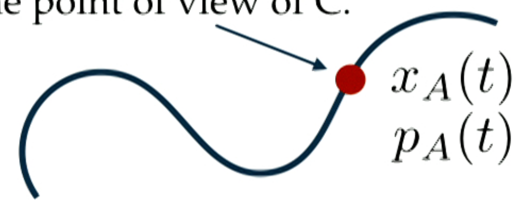
Dynamical reference frames

Reference frames are not anymore parameters in a transformation, but are associated to a hamiltonian.

C describes A and B: $H_{AB}^{(C)}$

A describes B and C: $H_{BC}^{(A)}$

A obeys Hamilton's equations of motion from the point of view of C.



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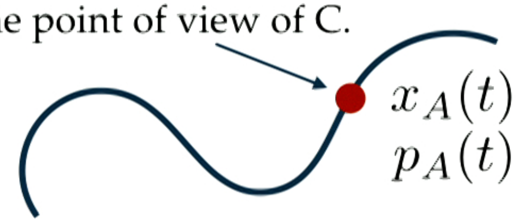

The hamiltonian changes according to a canonical transformation, which is the same in the classical and quantum case:

$$H'(q_i, \pi_i) = H(x_i(q_i, \pi_i), p_i(q_i, \pi_i)) + \frac{\partial F}{\partial t}$$

new hamiltonian

old hamiltonian in new variables

generator of the canonical transformation

The Schrödinger equation

Schrödinger equation in C's reference frame

$$i\hbar \frac{d\rho_{AB}^{(C)}}{dt} = \left[H_{AB}^{(C)}, \rho_{AB}^{(C)}(t)\right]$$

A: new reference frame
B: quantum system
C: old reference frame

To change to the frame of A we apply the transformation $\hat{S}$

$$i\hbar \frac{d\rho_{BC}^{(A)}}{dt} = \left[H_{BC}^{(A)}, \rho_{BC}^{(A)}(t)\right]$$

$$\hat{H}_{BC}^{(A)} = \hat{S}\hat{H}_{AB}^{(C)}\hat{S}^\dagger + i\hbar \frac{d\hat{S}}{dt}\hat{S}^\dagger$$

$$\hat{\rho}_{BC}^{(A)} = \hat{S}\hat{\rho}_{AB}^{(C)}\hat{S}^\dagger$$

The evolution in the new reference frame is unitary.

The Schrödinger equation

Schrödinger equation in C's reference frame

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A: new reference frame
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$$\hat{H}_{BC}^{(A)} = \hat{S}\hat{H}_{AB}^{(C)}\hat{S}^{\dagger} + i\hbar\frac{d\hat{S}}{dt}\hat{S}^{\dagger}$$

$$\hat{\rho}_{BC}^{(A)} = \hat{S}\hat{\rho}_{AB}^{(C)}\hat{S}^{\dagger}$$

The evolution in the new reference frame is unitary.

We define a symmetry transformation as:

$$\hat{S}\hat{H}\left(\{m_i, \hat{x}_i, \hat{p}_i\}_{i=A,B}\right)\hat{S}^{\dagger} + i\hbar\frac{d\hat{S}}{dt}\hat{S}^{\dagger} = \hat{H}\left(\{m_i, \hat{x}_i, \hat{p}_i\}_{i=B,C}\right)$$

General transformation

$$\hat{S} = e^{-\frac{i}{\hbar}\hat{H}_C t}\hat{\mathcal{P}}^{(i)}_{AC}\Pi_n e^{\frac{i}{\hbar}\hat{f}^n_A(t)\hat{O}^n_B} e^{\frac{i}{\hbar}\hat{H}_A t}$$

General transformation

$$\hat{S} = e^{-\frac{i}{\hbar}\hat{H}_C t}\hat{\mathcal{P}}^{(i)}_{AC}\Pi_n e^{\frac{i}{\hbar}\hat{f}^n_A(t)\hat{O}^n_B} e^{\frac{i}{\hbar}\hat{H}_A t}$$

generalisation of the standard change of reference frame

$$\hat{U}_i = \Pi_n e^{\frac{i}{\hbar}f^n(t)\hat{O}^n_B}$$

General transformation

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generalisation of the standard change of reference frame

$$\hat{U}_i = \Pi_n e^{\frac{i}{\hbar}f^n(t)\hat{O}^n_B}$$

function describing the relationship between old and new reference frame

General transformation

Hamiltonian of the new reference frame A as seen from C

$$\hat{S} = e^{-\frac{i}{\hbar}\hat{H}_C t}\hat{\mathcal{P}}^{(i)}_{AC}\Pi_n e^{\frac{i}{\hbar}\hat{f}^n_A(t)\hat{O}^n_B} e^{\frac{i}{\hbar}\hat{H}_A t}$$

generalised parity operator (switches the equations of motion of A and C)

generalisation of the standard change of reference frame

$$\hat{U}_i = \Pi_n e^{\frac{i}{\hbar}f^n(t)\hat{O}^n_B}$$

function describing the relationship between old and new reference frame

General transformation

$$\hat{S} = e^{-\frac{i}{\hbar}\hat{H}_C t}\hat{\mathcal{P}}^{(i)}_{AC}\Pi_n e^{\frac{i}{\hbar}\hat{f}^n_A(t)\hat{O}^n_B} e^{\frac{i}{\hbar}\hat{H}_A t}$$

Hamiltonian of the new reference frame A as seen from C

generalised parity operator (switches the equations of motion of A and C)

generalisation of the standard change of reference frame $\hat{U}_i = \Pi_n e^{\frac{i}{\hbar}f^n(t)\hat{O}^n_B}$

function describing the relationship between old and new reference frame

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General transformation

Hamiltonian of the old reference frame C as seen from A

Hamiltonian of the new reference frame A as seen from C

$$\hat{S} = e^{-\frac{i}{\hbar}\hat{H}_C t}\hat{\mathcal{P}}^{(i)}_{AC}\Pi_n e^{\frac{i}{\hbar}\hat{f}^n_A(t)\hat{O}^n_B} e^{\frac{i}{\hbar}\hat{H}_A t}$$

generalised parity operator (switches the equations of motion of A and C)

generalisation of the standard change of reference frame $\hat{U}_i = \Pi_n e^{\frac{i}{\hbar}f^n(t)\hat{O}^n_B}$

function describing the relationship between old and new reference frame

Translations in QRFs

The new QRF is described by system A at time 0.

$$|\Psi_0\rangle_{AB} = \frac{1}{\sqrt{2}}\left(|x_1\rangle_A + |x_2\rangle_A\right)|\phi_0\rangle_B \qquad \hat{H}^{(C)}_{AB} = \frac{\hat{p}_A^2}{2m_A} + \frac{\hat{p}_B^2}{2m_B}$$

We want to jump to the QRF of A

$$\hat{S}_T = \exp\left(-\frac{i}{\hbar}\frac{\hat{\pi}_C^2}{2m_C}t\right)\hat{\mathcal{P}}^{(x)}_{AC}\exp\left(\frac{i}{\hbar}\hat{x}_A\hat{p}_B\right)\exp\left(\frac{i}{\hbar}\frac{\hat{p}_A^2}{2m_A}t\right)$$

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The new QRF is described by system A at time 0.

$$|\Psi_0\rangle_{AB} = \frac{1}{\sqrt{2}}\left(|x_1\rangle_A + |x_2\rangle_A\right)|\phi_0\rangle_B$$

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We want to jump to the QRF of A

$$\hat{S}_T = \exp\left(-\frac{i}{\hbar}\frac{\hat{\pi}_C^2}{2m_C}t\right)\hat{\mathscr{P}}_{AC}^{(x)}\exp\left(\frac{i}{\hbar}\hat{x}_A\hat{p}_B\right)\exp\left(\frac{i}{\hbar}\frac{\hat{p}_A^2}{2m_A}t\right)$$

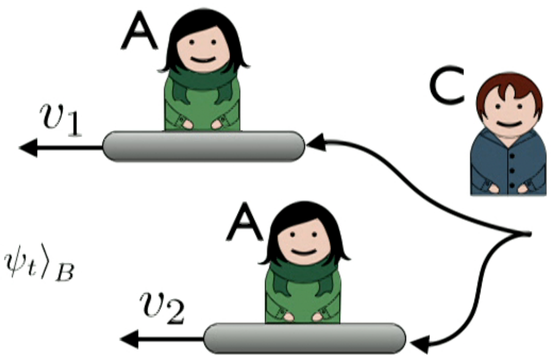
$\hat{S}_x$ translation to a reference frame which is frozen in time.

$$\hat{H}_{BC}^{(A)} = \frac{\hat{\pi}_B^2}{2m_B} + \frac{\hat{\pi}_C^2}{2m_C}$$

The free hamiltonian is symmetric under generalised translations.

The boost in QRFs

We want to boost to a QRF A moving in a superposition of velocities from the point of view of C.



$$\hat{H}_{AB}^{(C)} = \frac{\hat{p}_A^2}{2m_A} + \frac{\hat{p}_B^2}{2m_B}$$

QRF of A

$$|\Psi_t\rangle_{AB} = \frac{1}{\sqrt{2}} \left(|m_A v_1\rangle_A + |m_A v_2\rangle_A \right) |\psi_t\rangle_B$$

$$\hat{S}_b = \exp\left(-\frac{i}{\hbar} \frac{\hat{\pi}_C^2}{2m_C} t \right) \hat{\mathcal{P}}_{AC}^{(v)} \exp\left(\frac{i}{\hbar} \frac{\hat{p}_A}{m_A} \hat{G}_B \right) \exp\left(\frac{i}{\hbar} \frac{\hat{p}_A^2}{2m_A} t \right)$$

- parity and swap
- sets velocity of C to the opposite of velocity of A

$$\pi_C = -\frac{m_C}{m_A} p_A$$

Operator replacing v!

Galilean boost on B controlled by the velocity of A

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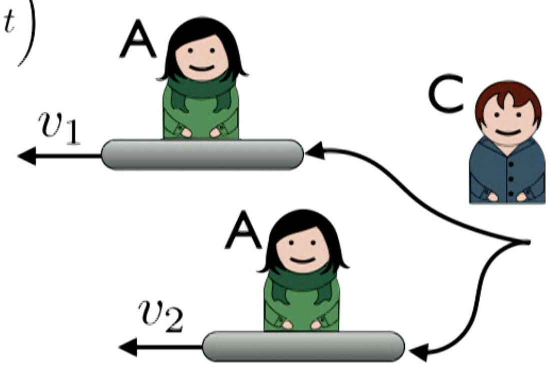
The boost in QRFs

$$\hat{S}_b = \exp\left(-\frac{i}{\hbar}\frac{\hat{\pi}_C^2}{2m_C}t\right)\hat{\mathcal{P}}_{AC}^{(v)}\exp\left(\frac{i}{\hbar}\frac{\hat{p}_A}{m_A}\hat{G}_B\right)\exp\left(\frac{i}{\hbar}\frac{\hat{p}_A^2}{2m_A}t\right)$$



From the QRF of C

$$\hat{H}_{AB}^{(C)} = \frac{\hat{p}_A^2}{2m_A} + \frac{\hat{p}_B^2}{2m_B}$$

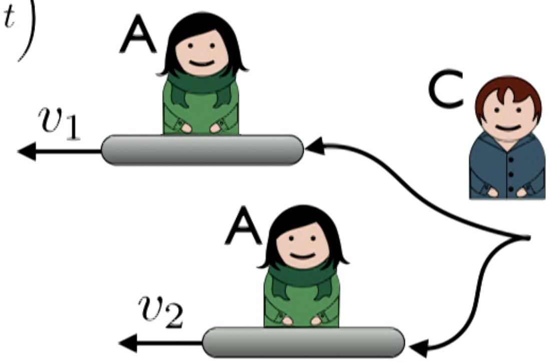
$$|\Psi_t\rangle_{AB} = \frac{1}{\sqrt{2}}\left(|m_A v_1\rangle_A + |m_A v_2\rangle_A\right)|\psi_t\rangle_B$$

From the QRF of A

$$\hat{H}_{BC}^{(A)} = \frac{\hat{\pi}_B^2}{2m_B} + \frac{\hat{\pi}_C^2}{2m_C}$$

The boost in QRFs

$$\hat{S}_b = \exp\left(-\frac{i}{\hbar}\frac{\hat{\pi}_C^2}{2m_C}t\right)\hat{\mathcal{P}}_{AC}^{(v)}\exp\left(\frac{i}{\hbar}\frac{\hat{p}_A}{m_A}\hat{G}_B\right)\exp\left(\frac{i}{\hbar}\frac{\hat{p}_A^2}{2m_A}t\right)$$



From the QRF of C

$$\hat{H}_{AB}^{(C)} = \frac{\hat{p}_A^2}{2m_A} + \frac{\hat{p}_B^2}{2m_B}$$

$$|\Psi_t\rangle_{AB} = \frac{1}{\sqrt{2}}\left(|m_A v_1\rangle_A + |m_A v_2\rangle_A\right)|\psi_t\rangle_B$$

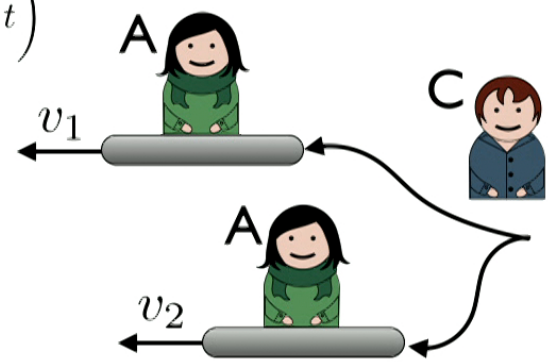
From the QRF of A

$$\hat{H}_{BC}^{(A)} = \frac{\hat{\pi}_B^2}{2m_B} + \frac{\hat{\pi}_C^2}{2m_C}$$

The free hamiltonian is symmetric under superposition of Galilean boosts.

The boost in QRFs

$$\hat{S}_b = \exp\left(-\frac{i}{\hbar}\frac{\hat{\pi}_C^2}{2m_C}t\right)\hat{\mathcal{P}}_{AC}^{(v)}\exp\left(\frac{i}{\hbar}\frac{\hat{p}_A}{m_A}\hat{G}_B\right)\exp\left(\frac{i}{\hbar}\frac{\hat{p}_A^2}{2m_A}t\right)$$



From the QRF of C

$$\hat{H}_{AB}^{(C)} = \frac{\hat{p}_A^2}{2m_A} + \frac{\hat{p}_B^2}{2m_B}$$

$$|\Psi_t\rangle_{AB} = \frac{1}{\sqrt{2}}\left(|m_A v_1\rangle_A + |m_A v_2\rangle_A\right)|\psi_t\rangle_B$$

From the QRF of A

$$\hat{H}_{BC}^{(A)} = \frac{\hat{\pi}_B^2}{2m_B} + \frac{\hat{\pi}_C^2}{2m_C}$$

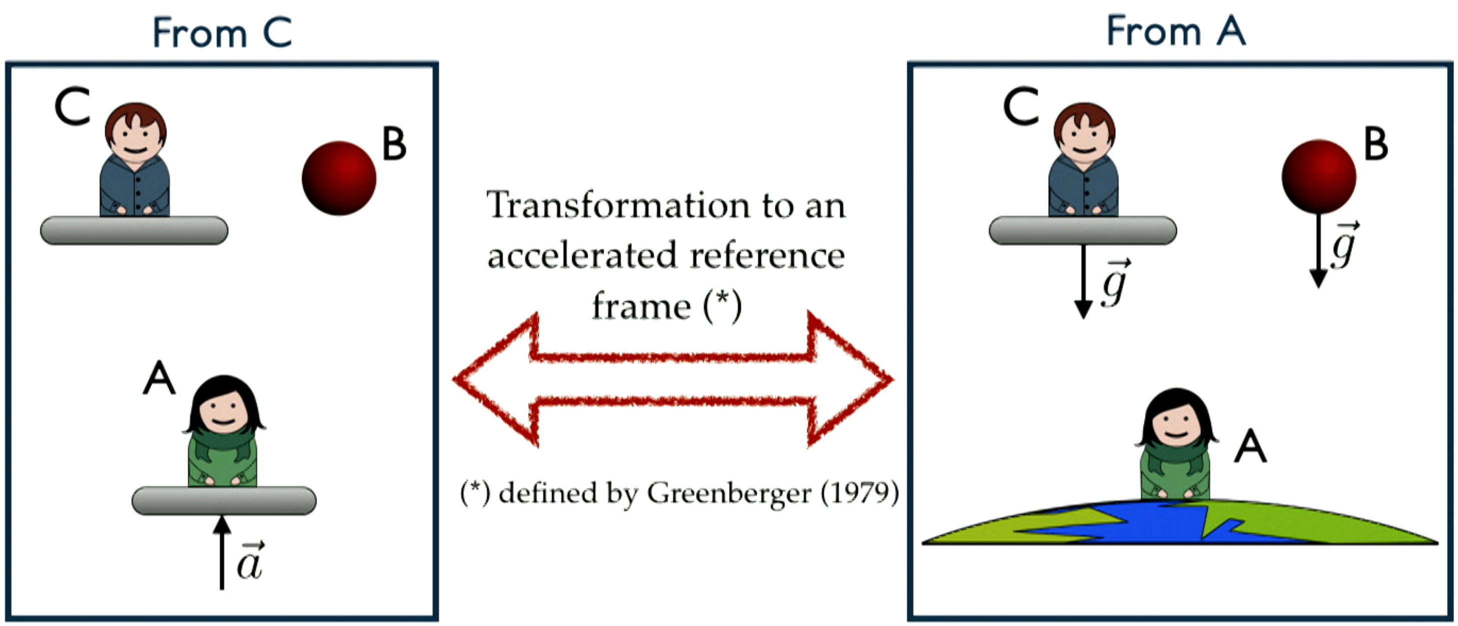
The free hamiltonian is symmetric under superposition of Galilean boosts.

$$|\Psi'_t\rangle_{BC} = \frac{1}{\sqrt{2}}\left(e^{\frac{i}{\hbar}v_1\hat{G}_B}|\psi_t\rangle_B|-m_C v_1\rangle_C + e^{\frac{i}{\hbar}v_2\hat{G}_B}|\psi_t\rangle_B|-m_C v_2\rangle_C\right)$$

Weak equivalence principle

The physical effects as seen from a reference frame moving with constant and uniform acceleration are indistinguishable from those as seen in a uniform gravitational field.

$$m_I \vec{a} = m_g \vec{g} \quad \rightsquigarrow \quad \vec{a} = \vec{g}$$



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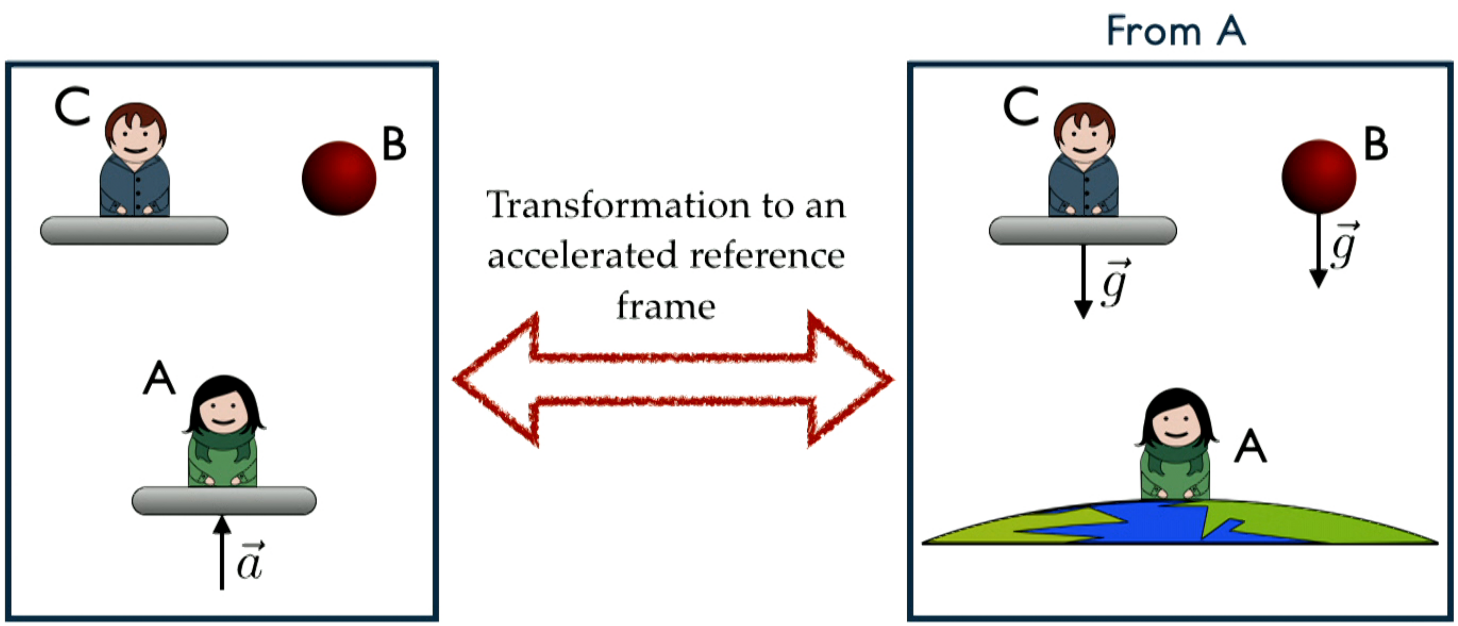
Weak equivalence principle

The physical effects as seen from a reference frame moving ~~with~~ in a superposition of constant and uniform accelerations are indistinguishable from those as seen in a superposition of uniform gravitational fields.

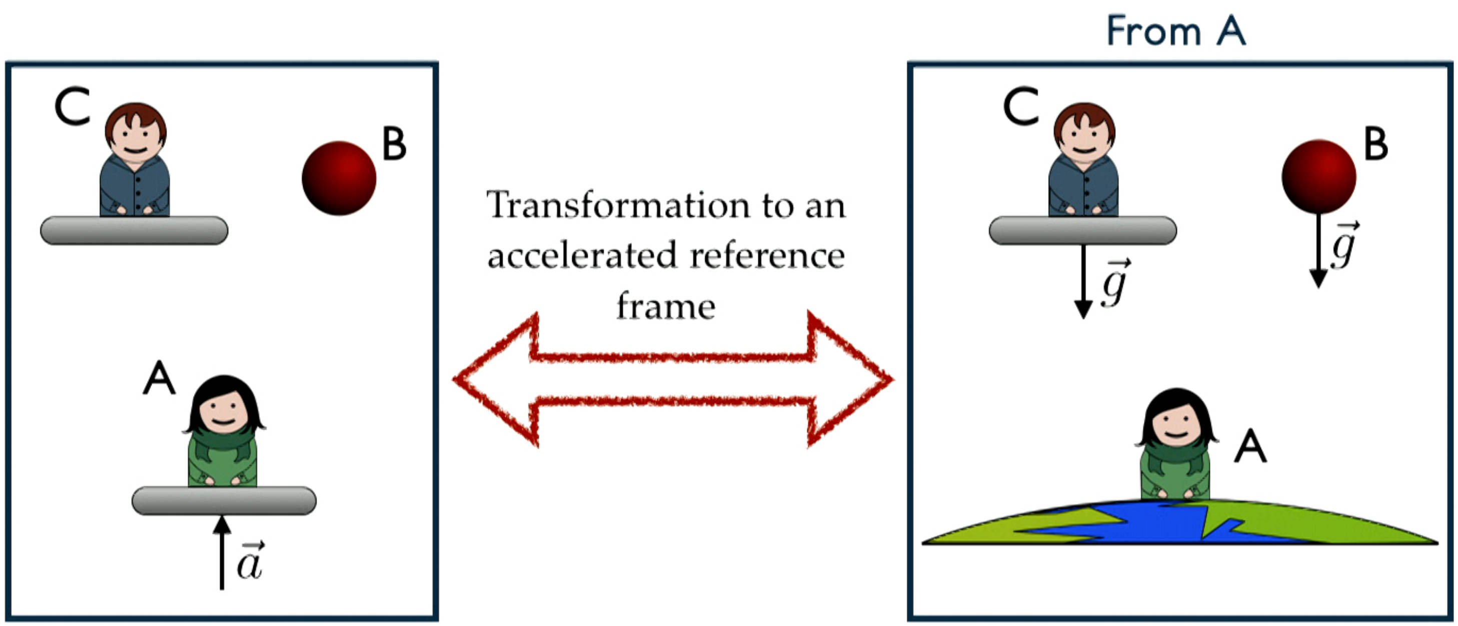
$$m_I\vec{a} = m_g\vec{g} \quad \rightsquigarrow \quad \vec{a} = \vec{g}$$



Weak equivalence principle

The physical effects as seen from a reference frame moving ~~with~~ **in a superposition of** constant and uniform acceleration**s** are indistinguishable from those as seen in a **superposition of** uniform gravitational field**s**.

$m_I \not{\vec{a}}\,\hat{a} = m_g \not{\vec{g}}\,\hat{g}$ → ~~$\vec{a} = \vec{g}$~~ $\hat{a} = \hat{g}$



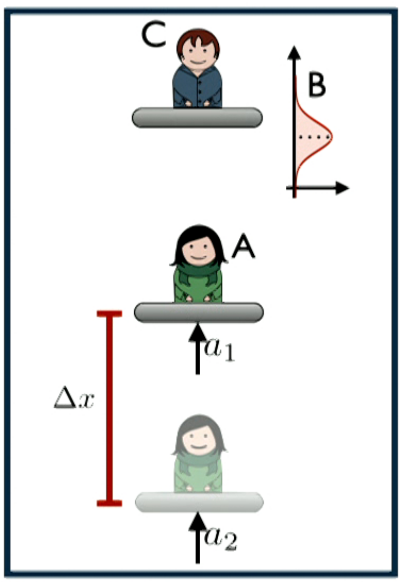
Weak equivalence principle

The physical effects as seen from a reference frame moving ~~with~~ **in a superposition of** constant and uniform acceleration**s** are indistinguishable from those as seen in a **superposition of** uniform gravitational field**s**.

$$m_I \hat{a} = m_g \hat{g} \quad (\text{replacing } m_I \vec{a} = m_g \vec{g})$$

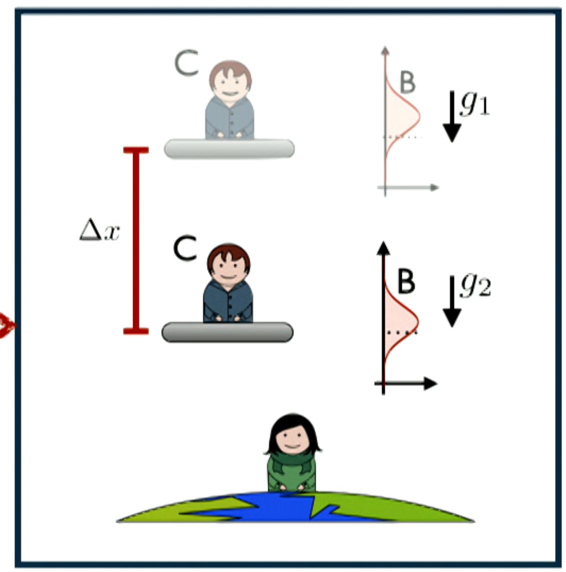
~~$\vec{a} = \vec{g}$~~ $\hat{a} = \hat{g}$

From C



Transformation to a quantum reference frame in superposition of accelerations

From A



WEP for QRFs

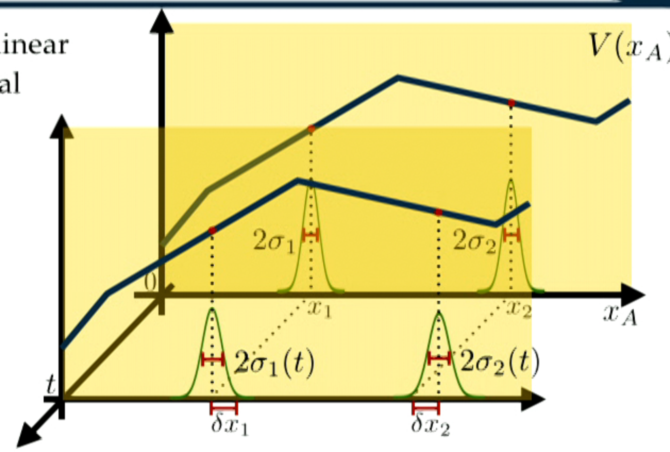
$$\hat{H}_{AB}^{(C)} = \frac{\hat{p}_A^2}{2m_A} + \frac{\hat{p}_B^2}{2m_B} + V(\hat{x}_A)$$

Piecewise linear potential

State of A at time t:

$$|\psi_0(t)\rangle_A = \frac{1}{\sqrt{2}}\left(|\psi_1(t)\rangle_A + |\psi_2(t)\rangle_A\right)$$

$$a_1 = -\frac{1}{m_A}\frac{dV(\hat{x}_A)}{d\hat{x}_A}\Big|_{x_1(t)} \qquad a_2 = -\frac{1}{m_A}\frac{dV(\hat{x}_A)}{d\hat{x}_A}\Big|_{x_2(t)}$$



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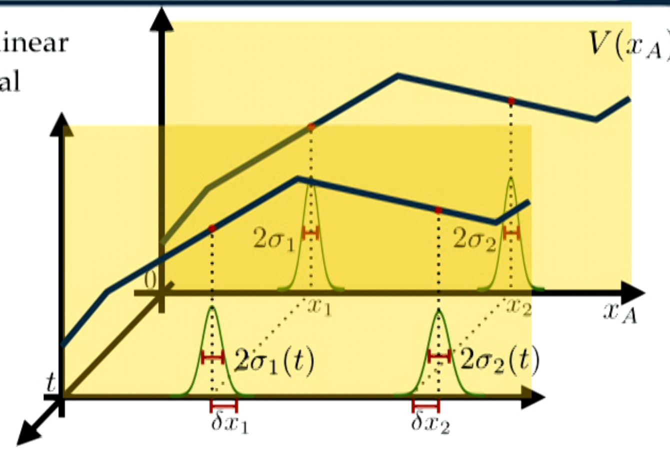
$$\hat{H}_{AB}^{(C)} = \frac{\hat{p}_A^2}{2m_A} + \frac{\hat{p}_B^2}{2m_B} + V(\hat{x}_A)$$

Piecewise linear potential



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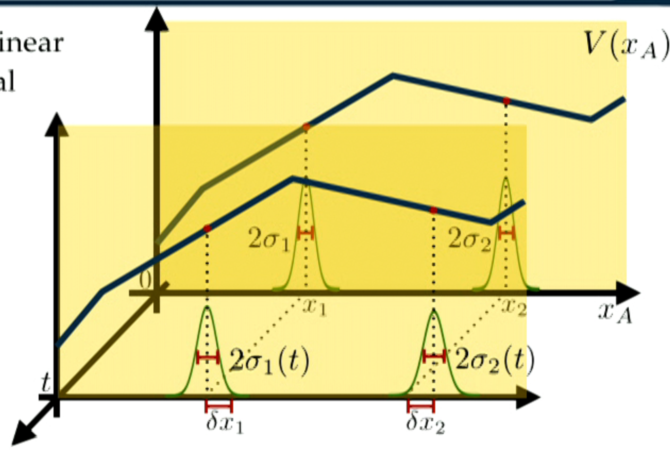
$$\hat{S}_{EP} = e^{-\frac{i}{\hbar}\left(\frac{\hat{\pi}_C^2}{2m_C}+\frac{m_C}{m_A}V(-\hat{q}_C)\right)t}\hat{\mathcal{P}}_{AC}^{(v)}\hat{Q}_t e^{\frac{i}{\hbar}\left(\frac{\hat{p}_A^2}{2m_A}+V(\hat{x}_A)\right)t}$$

WEP for QRFs

$$\hat{H}_{AB}^{(C)} = \frac{\hat{p}_A^2}{2m_A} + \frac{\hat{p}_B^2}{2m_B} + V(\hat{x}_A)$$

Piecewise linear potential



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$$\hat{S}_{EP} = e^{-\frac{i}{\hbar}\left(\frac{\hat{\pi}_C^2}{2m_C} + \frac{m_C}{m_A}V(-\hat{q}_C)\right)t}\hat{\mathcal{P}}_{AC}^{(v)}\hat{Q}_t e^{\frac{i}{\hbar}\left(\frac{\hat{p}_A^2}{2m_A} + V(\hat{x}_A)\right)t}$$

Generalisation of the Greenberger operator

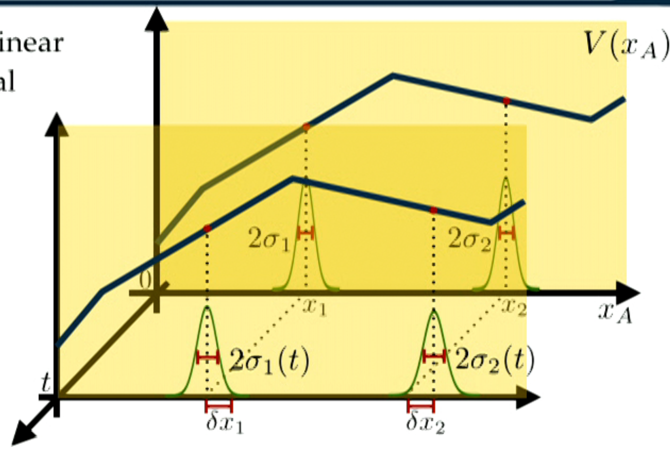
In the reference frame of A

$$\hat{H}_{BC}^{(A)} = \frac{\hat{\pi}_B^2}{2m_B} + \frac{\hat{\pi}_C^2}{2m_C} + \frac{m_C}{m_A}V(-\hat{q}_C) - \frac{m_B}{m_A}\frac{dV}{dx_A}\Big|_{-\hat{q}_C}\hat{q}_B$$

WEP for QRFs

$$\hat{H}_{AB}^{(C)} = \frac{\hat{p}_A^2}{2m_A} + \frac{\hat{p}_B^2}{2m_B} + V(\hat{x}_A)$$

Piecewise linear potential



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$$\hat{S}_{EP} = e^{-\frac{i}{\hbar}\left(\frac{\hat{\pi}_C^2}{2m_C} + \frac{m_C}{m_A}V(-\hat{q}_C)\right)t}\hat{\mathcal{P}}_{AC}^{(v)}\hat{Q}_t e^{\frac{i}{\hbar}\left(\frac{\hat{p}_A^2}{2m_A} + V(\hat{x}_A)\right)t}$$

Generalisation of the Greenberger operator

In the reference frame of A

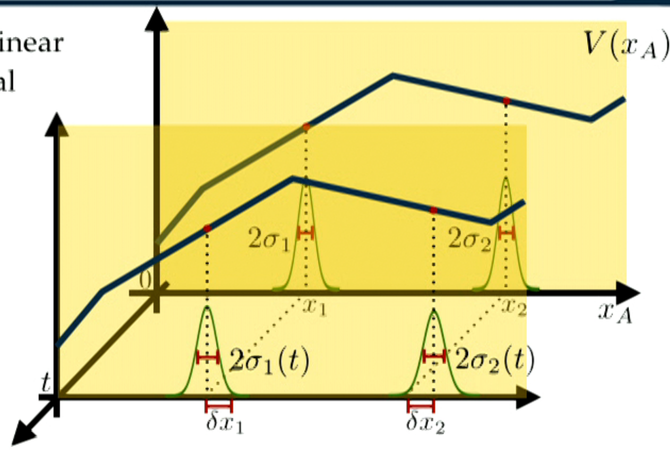
$$\hat{H}_{BC}^{(A)} = \frac{\hat{\pi}_B^2}{2m_B} + \frac{\hat{\pi}_C^2}{2m_C} + \frac{m_C}{m_A}V(-\hat{q}_C) - \frac{m_B}{m_A}\frac{dV}{dx_A}\Big|_{-\hat{q}_C}\hat{q}_B$$

$$|\psi'(t)\rangle_{BC} = \frac{1}{\sqrt{2}}\left(|\alpha_1'(t)\rangle_C Q_t^1|\phi(t)\rangle_B + |\alpha_2'(t)\rangle_C Q_t^2|\phi(t)\rangle_B\right)$$

WEP for QRFs

$$\hat{H}^{(C)}_{AB} = \frac{\hat{p}_A^2}{2m_A} + \frac{\hat{p}_B^2}{2m_B} + V(\hat{x}_A)$$

Piecewise linear potential



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$$\hat{S}_{EP} = e^{-\frac{i}{\hbar}\left(\frac{\hat{\pi}_C^2}{2m_C} + \frac{m_C}{m_A}V(-\hat{q}_C)\right)t}\hat{\mathcal{P}}^{(v)}_{AC}\hat{Q}_t e^{\frac{i}{\hbar}\left(\frac{\hat{p}_A^2}{2m_A} + V(\hat{x}_A)\right)t}$$

Generalisation of the Greenberger operator

In the reference frame of A

$$\hat{H}^{(A)}_{BC} = \frac{\hat{\pi}_B^2}{2m_B} + \frac{\hat{\pi}_C^2}{2m_C} + \frac{m_C}{m_A}V(-\hat{q}_C) - \frac{m_B}{m_A}\frac{dV}{dx_A}\Big|_{-\hat{q}_C}\hat{q}_B$$

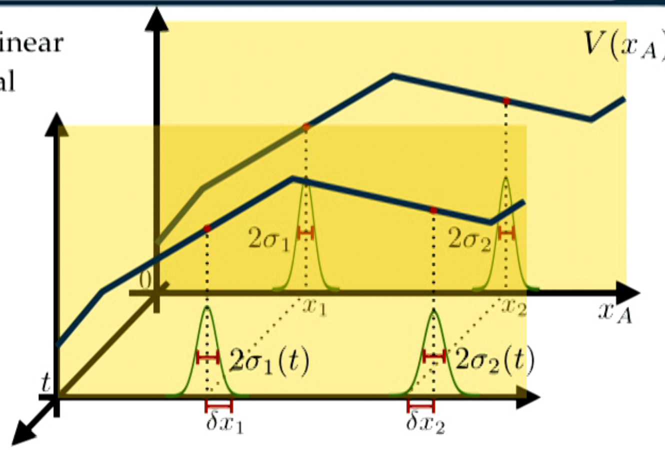
Standard Greenberger operators for the accelerations 1 and 2

$$|\psi'(t)\rangle_{BC} = \frac{1}{\sqrt{2}}\left(|\alpha_1'(t)\rangle_C Q_t^1|\phi(t)\rangle_B + |\alpha_2'(t)\rangle_C Q_t^2|\phi(t)\rangle_B\right)$$

WEP for QRFs

$$\hat{H}_{AB}^{(C)} = \frac{\hat{p}_A^2}{2m_A} + \frac{\hat{p}_B^2}{2m_B} + V(\hat{x}_A)$$

Piecewise linear potential



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$$a_1 = -\frac{1}{m_A}\frac{dV(\hat{x}_A)}{d\hat{x}_A}\bigg|_{x_1(t)} \qquad a_2 = -\frac{1}{m_A}\frac{dV(\hat{x}_A)}{d\hat{x}_A}\bigg|_{x_2(t)}$$

$$\hat{S}_{EP} = e^{-\frac{i}{\hbar}\left(\frac{\hat{\pi}_C^2}{2m_C} + \frac{m_C}{m_A}V(-\hat{q}_C)\right)t}\hat{\mathcal{P}}_{AC}^{(v)}\hat{Q}_t e^{\frac{i}{\hbar}\left(\frac{\hat{p}_A^2}{2m_A} + V(\hat{x}_A)\right)t}$$

Generalisation of the Greenberger operator

In the reference frame of A

$$\hat{H}_{BC}^{(A)} = \frac{\hat{\pi}_B^2}{2m_B} + \frac{\hat{\pi}_C^2}{2m_C} + \frac{m_C}{m_A}V(-\hat{q}_C) - \frac{m_B}{m_A}\frac{dV}{dx_A}\bigg|_{-\hat{q}_C}\hat{q}_B$$

B moves as if it were in a superposition of gravitational fields!

Standard Greenberger operators for the accelerations 1 and 2

$$|\psi'(t)\rangle_{BC} = \frac{1}{\sqrt{2}}\left(|\alpha_1'(t)\rangle_C Q_t^1|\phi(t)\rangle_B + |\alpha_2'(t)\rangle_C Q_t^2|\phi(t)\rangle_B\right)$$

Summary

Operational and relational formalism for quantum reference frames.

Question

How can we describe the world from the point of view of a non-idealised reference frame, i.e. associated to a quantum state and to a dynamical equation of motion?

Need to find a more general law to change the reference frame.

This leads to a **generalisation of the notion of covariance**, which has been explored in the two cases of the **superposition of spatial translations** and the **superposition of Galilean boosts**.

The **weak equivalence principle** can also be extended to quantum reference frames.