Title: Effective field theory near and far from equilibrium

Speakers: Paolo Glorioso

Series: Condensed Matter

Date: December 06, 2019 - 1:30 PM

URL: http://pirsa.org/19120016

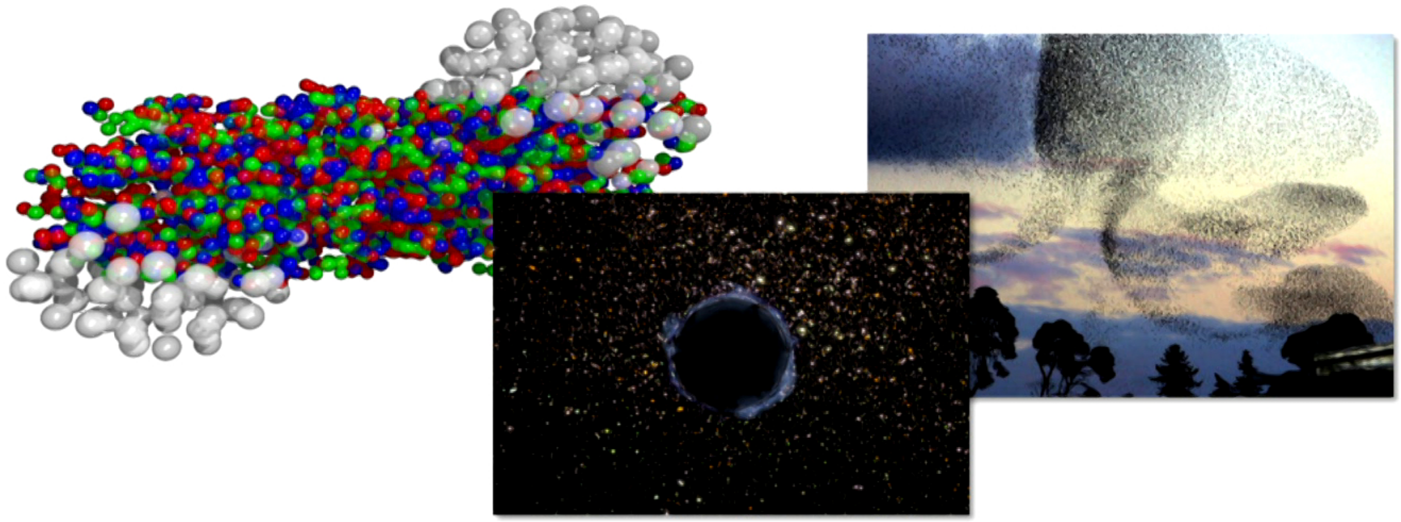
Abstract: In this talk I will discuss effective field theories for two classes of non-equilibrium systems, one far and one near equilibrium. The backbone of the approach is the Schwinger-Keldysh formalism, which is the natural starting point for doing field theory in non-equilibrium situations. In the first part of the talk I will present an effective response for topological driven (Floquet) systems, which are inherently far from equilibrium. As an example, I will discuss a chiral Floquet drive coupled to a background $U(1)$ field, which gives rise to a theta term in the response action, and show that this is independent of smooth deformations of the underlying system. In the second part, I will discuss an ongoing project using effective field theories for hydrodynamics. I will show that chiral diffusion for interacting systems in 1+1 dimensions, which may be relevant to edge transport in quantum Hall systems, has an infrared instability. I will then discuss the fate of this instability.

Effective field theory near and far from equilibrium

Paolo Glorioso

December 6, 2019

with
Hong Liu [1805.09331]
Andrey Gromov and Shinsei Ryu [1908.03217]
Luca Delacretaz, *in progress*



Non-equilibrium physics displays a huge variety of phenomena in nature. These range from heavy ion collisions to black holes dynamics, from driven systems to non-equilibrium steady states, etc.

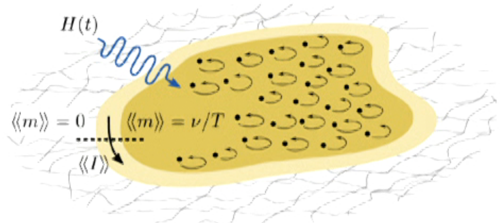
Many open questions: thermalization, information paradox, turbulence, ...

What is a suitable framework which captures systematically these phenomena?

Goal: encode low-energy description of non-equilibrium systems into effective field theories, independent of microscopic details.

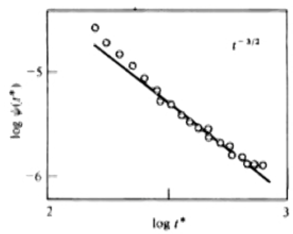
Direct applications include:

- Topological response of periodically driven systems, which are inherently far from equilibrium.



[Nathan et al., '16]

- Systematic computation of hydrodynamic fluctuations. E.g. long-time tails, renormalization of transport. Current methods (e.g. stochastic hydro) are not systematic.



[Boon, "Molecular hydrodynamics," '91]

Outline

1. Far from equilibrium: Floquet topological response
   - General construction
   - Chiral Floquet drive
2. Near equilibrium: infrared instability of chiral diffusion
   - EFT for hydrodynamics
   - Loop corrections
3. Conclusions

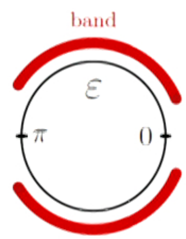
Far from equilibrium:
non-equilibrium topology and Floquet systems

Non-equilibrium topology and Floquet systems

Floquet systems have time-dependent periodic Hamiltonian

$$H(t+T) = H(t), \quad U(t) = \mathcal{T} e^{-i \int_0^t H(s) ds}$$

- There is no strict notion of energy.
- Can define quasi-energies $\varepsilon_n \sim \varepsilon_n + \frac{2\pi}{T}$. Energy analog of Bloch theory.

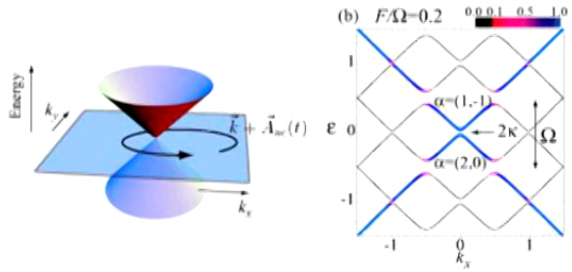


[Fruchart, '15]

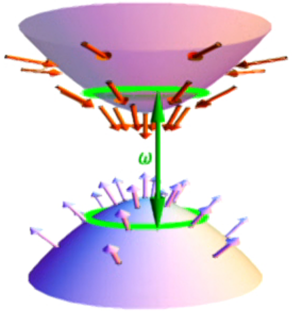
- Numerous recent theoretical works
  Reviews: [Harper,Roy,Rudner,Sondhi '19; Rudner,Lindner '19]

Dynamical generation of topology

- Circularly polarized light opens a gap [Oka,Aoki '09]



- Time periodic magnetic field [Lindner,Refael,Galitski '11]

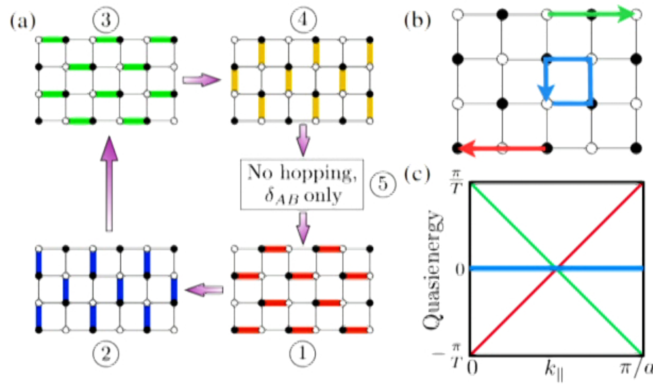


Experiments
[Wang,Steinberg,Jarillo-Herrero,Gedik '13]
[Rechtsman,Zeuner,Plotnik,Lumer,Nolte,Segev,Szameit '13]
[Jotzu,Messer,Desbuquois,Lebrat,Uehlinger,Greif,Esslinger '14]

A canonical model: chiral Floquet drive

[Rudner, Lindner, Berg, Levin '12]

$$H(t) = -J \sum_{r \in A} (c^\dagger_{r+d(t)} c_r + c^\dagger_r c_{r+d(t)}), \qquad d(t) = \uparrow \rightarrow \downarrow \leftarrow$$



[Rudner et al., '12]

- Edge states [Fidkowski, Po, Potter, Vishwanath '16; Roy, Harper '16; Po et al. '16; von Keyserlingk, Sondhi '16]
- Quantized topological invariants [Rudner et al. '12; Iadecola, Hsieh '17]
- Quantized response: magnetization [Nathan et al. '16; Nathan et al. '19]

Recall: Static topological phases

For time-independent Hamiltonians, $H(t) = H_0$, a successful approach to many-body topological systems is that of topological field theory.

- Detect topological phases by coupling the system to background gauge fields.
- Example: integrate out fermions in $(2+1)$-dimensions

$$Z[A] = \int \mathcal{D}\psi \mathcal{D}\bar{\psi} e^{-S[\psi,\bar{\psi},A]} = e^{-S_{\text{eff}}[A]}$$

- Response action S_{eff} is local, imaginary, and topological

$$S_{\text{eff}}[A] = i\frac{\nu}{4\pi}\int d^3x \varepsilon^{\mu\nu\rho} A_\mu \partial_\nu A_\rho, \qquad \nu = \text{integer}$$

- Powerful to diagnose and predict new topological phases.
- Works only when notion of ground state and gap are well-defined.

Aim: Reproduce the success of time-independent approach to Floquet systems.

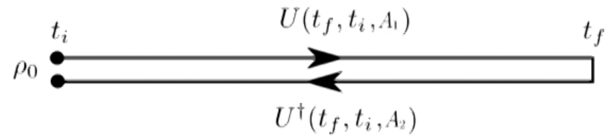
- Effective field theory approach.
- Diagnostic tool of topological order.

General setup – Schwinger-Keldysh approach

For systems out of equilibrium, the natural starting point is the Schwinger-Keldysh trace

$$e^{iW[A_1,A_2]} = \mathrm{Tr}\left[U(t_i,t_f;A_1)\rho_0 U^\dagger(t_i,t_f;A_2)\right]$$

- Analog of $Z[A_\mu]$ for time-independent Hamiltonians.
- ρ_0 is an initial state
- $U(t_i,t_f;A)$: unitary coupled to an external gauge field
  - Two unitaries for forward and backward evolutions
  - Two gauge fields A_1, A_2 for forward and backward evolutions



General setup

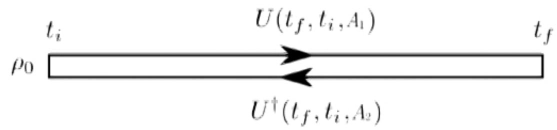
$$e^{iW[A_1,A_2]} = \mathrm{Tr}\left[U(t_i,t_f;A_1)\rho_0 U^\dagger(t_i,t_f;A_2)\right]$$

SK for Floquet topological systems:

- Initial state: Infinite temperature Gibbs ensemble

$$\rho_0 = \frac{e^{\alpha Q}}{\mathrm{Tr}e^{\alpha Q}}, \qquad \mathcal{Q} = \sum_r (n_r - \tfrac{1}{2})$$

- Real time contour: Integer multiple of Floquet period T



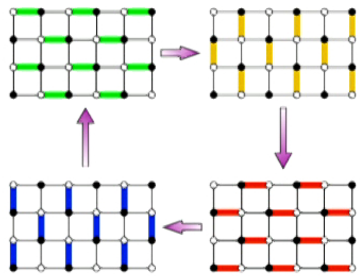
$$t_f - t_i = \kappa T, \quad \kappa = \text{integer}$$

- Background: Static background

$$A_0 = 0, \qquad \vec{A}(t,\vec{r}) = \vec{A}(\vec{r})$$

up to gauge fixing.
Note: gauge invariance under $A_{1,2} \to A_{1,2} + \partial\lambda_{1,2}$ with $\lambda_1 = \lambda_2$ at $t = t_i, t_f$.

Topological response: chiral Floquet drive



On a closed manifold:

$$e^{iW[A_1,A_2]} = \frac{1}{2\cosh(\frac{\alpha}{2})^N} \prod_r \left[e^{-\frac{\alpha}{2}} + e^{\frac{\alpha}{2}} e^{i\int \frac{dt}{T}(B_{1r} - B_{2r})} \right]$$

where

- $\int \frac{dt}{T}$ =integer
- N total number of lattice sites
- $B_r = A_x(r) + A_y(r + b_1) - A_x(r + b_1 + b_2) - A_y(r + b_2)$ flux collected by a particle starting at r

Topological response: chiral Floquet drive

For slowly varying background, leads to a spatial theta term:

$$e^{iW[A_1,A_2]} = e^{i\frac{\Theta(\alpha)}{2\pi}\int \frac{dt}{T}\int d^2r[B_1(r)-B_2(r)]}, \qquad B(r) = d\vec{A}(r)$$

where

$$\Theta(\alpha) = \theta + f(\alpha), \quad \theta = \Theta(\alpha = 0), \quad f(\alpha) = -f(-\alpha)$$

- θ is quantized due to large gauge invariance of each A_1, A_2 and particle-hole symmetry.
- From explicit evaluation:

$$\theta = \pi, \qquad f(\alpha) = -\pi \tanh\frac{\alpha}{2}$$

- Independent of metric of the spatial manifold $\Rightarrow$ topological term.

Topological response: chiral Floquet drive

$$e^{iW[A_1,A_2]} = e^{i\frac{\Theta(\alpha)}{2\pi}\int \frac{dt}{T}\int d^2r[B_1(r)-B_2(r)]}$$

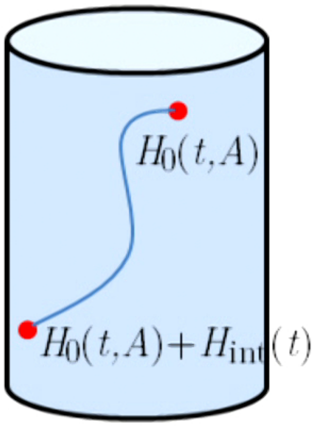
$\Theta(\alpha)$ independent of continuous deformations:

$$H(t,A) = H_0(t,A) + \lambda H_{\text{int}}(t)$$

H_0 : chiral Floquet drive, H_{int} : many-body interaction

Independent of λ as far as response remains local.

Sketch of the proof:



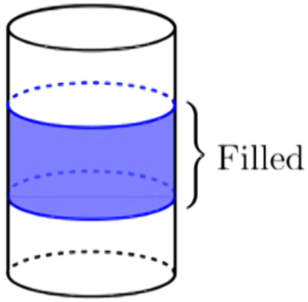
1. $\frac{\delta W}{\delta A_{1i}(r)} = -i\int dt\,\mathrm{Tr}[\rho_0 J^i(r,t)] \equiv -i\langle J^i(r)\rangle$
2. $\langle J^i(r)\rangle = \mathrm{Tr}[\rho\frac{\delta H_0}{\delta A_i}] = 0$
3. EFT:
   $W = \frac{1}{2\pi}\int \frac{dt}{T}\int d^2r\,\Theta(\alpha,r)(B_1(r) - B_2(r))$

$$\Rightarrow \quad \frac{\delta W}{\delta A_{1i}} \propto \int dt\,\varepsilon^{ij}\partial_j\Theta(\alpha,r) = 0$$

Topological response: chiral Floquet drive

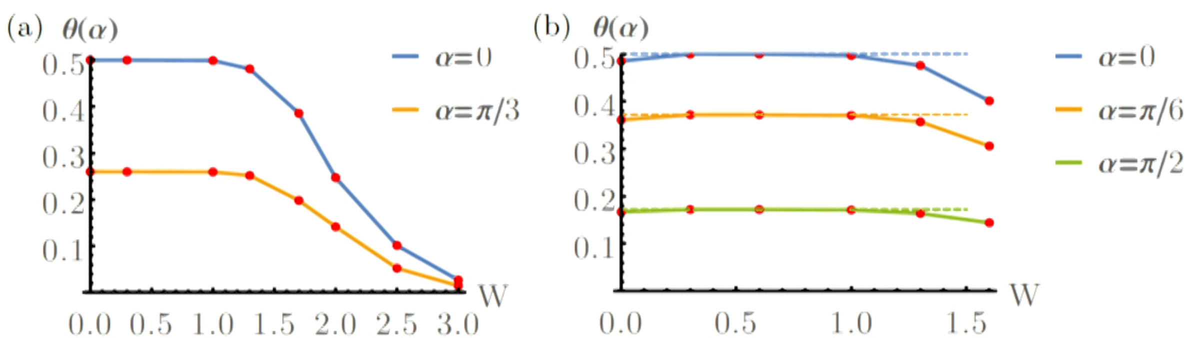
Numerical test of topological stability.

Open system:



$$H = H_0(t, A) + \sum_r w_r c_r^\dagger c_r + V_0 \sum_r (-1)^{\eta_r} c_r^\dagger c_r$$

$w_r \in (-W, W)$ random, uniformly distributed



Plot of $\Theta(\alpha)$ as a function of disorder strength W, for various values of α when $\lambda = 0$ (a) and $\lambda = 0.1(b)$.

Remarks

1. Theta term can be related to quantized magnetization [Nathan et al. '16; Nathan et al. '19]
2. Relation to chiral unitary index [Po et al. '16]
3. Higher dimensions
4. Geometric response

Local thermal equilibrium:
infrared instability of low-dimensional chiral diffusion

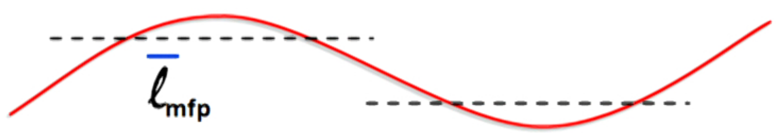
Chiral diffusion in 1+1

Low-energy theory of a conserved $U(1)$ current with chiral anomaly near thermal equilibrium

$$\partial_\mu J^\mu = c\varepsilon^{\mu\nu} F_{\mu\nu}$$

$c = \frac{\nu}{4\pi}$: anomaly coefficient, $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$.

Hydrodynamic approach:



- Neglect energy-momentum conservation
- Local equilibrium: $\rho = e^{-\frac{1}{T}(H-\mu(t,x)Q)}$

Chiral diffusion in 1+1

$$\partial_\mu J^\mu = c\varepsilon^{\mu\nu} F_{\mu\nu}$$

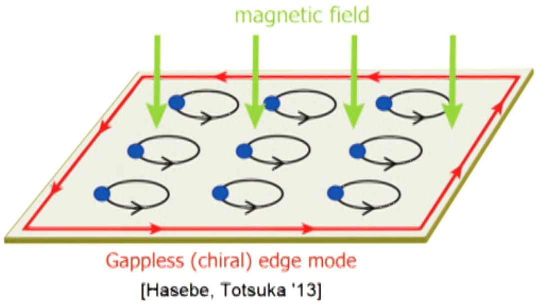
$$J^t = n(\mu) = \chi\mu + \frac{1}{2}\chi'\mu^2 + \cdots, \quad J^x = -4c\mu - \sigma\partial_x\mu$$

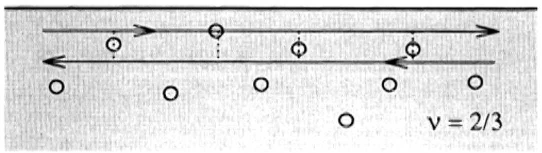
- $-4c\mu$ required by second law [Son,Surowka '09].
- Chiral diffusion:

$$\chi\partial_t\mu - 4c\partial_x\mu - \sigma\partial_x^2\mu + \frac{1}{2}\chi'\partial_t\mu^2 = 0$$

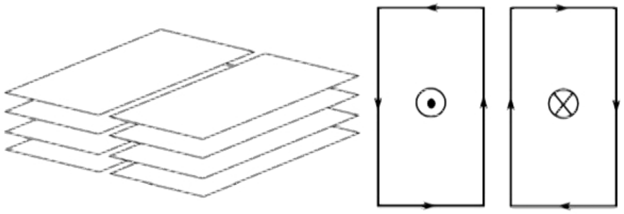
Motivations

- Edge of quantum Hall systems [Kane,Fisher '95; Ma,Feldman '19]





- Surface chiral metals [Balents,Fisher '95; Sur,Lee '13]



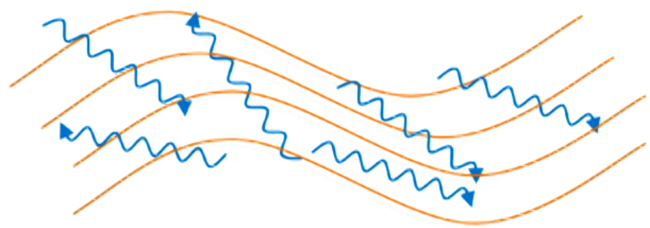
- Chiral magnetic effect [Vilenkin '80; Son,Spivak '13; Yamamoto '15]

$$\vec{J} \propto \mu \vec{B}$$

Motivations

- Local thermal equilibrium $\Rightarrow$ need to include thermal fluctuations
- Result of the interplay between thermal fluctuations and interactions of collective modes

$$\partial_\mu J^\mu = \eta, \qquad \langle \eta \rangle = 0, \qquad \langle \eta(t,x)\eta(0)\rangle = -2T\sigma\delta(t)\delta''(x)$$



- These induce renormalization of transport and change qualitatively the far IR picture.
- For dramatic IR effects, typically need momentum conservation. [Kovtun '12;Chen-Lin,Delacretaz,Hartnoll '18]

I will show:

- Chiral diffusion breaks down in the IR
- It persists even without momentum conservation!
- It furnishes a novel mechanism to flow to a non-trivial IR fixed point.

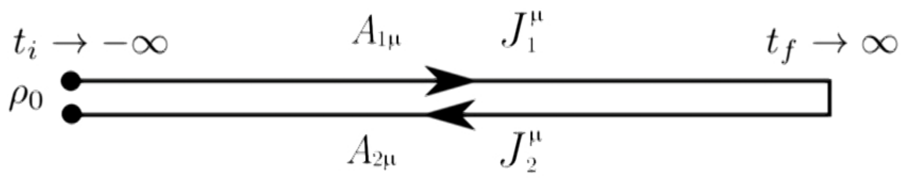
EFT of chiral diffusion

- Consider a quantum system in a thermal state $\rho_0 = e^{-\beta H}/\mathrm{Tr}(e^{-\beta H})$ with

$$\partial_\mu J^\mu = c\varepsilon^{\mu\nu} F_{\mu\nu}$$

- Background sources: $A_{1\mu}, A_{2\mu}$

$$e^{iW[A_1,A_2]} = \mathrm{Tr}\left[U(A_1)\rho_0 U^\dagger(A_2)\right] = \int_{\rho_0} D\psi_1 D\psi_2 e^{iS[\psi_1,A_1]-iS[\psi_2,A_2]}$$



- Anomalous conservation of J_1^μ and J_2^μ implies the Ward identity

$$W[A_{1\mu}+\partial_\mu\lambda_1, A_{2\mu}+\partial_\mu\lambda_2] = W[A_{1\mu}, A_{2\mu}] + c\int \lambda_1 F_{1\mu\nu} - c\int \lambda_2 F_{2\mu\nu}$$

EFT of chiral diffusion

$$W[A_{1\mu}+\partial_\mu\lambda_1, A_{2\mu}+\partial_\mu\lambda_2] = W[A_{1\mu}, A_{2\mu}] + c\int \lambda_1 F_{1\mu\nu} - c\int \lambda_2 F_{2\mu\nu}$$

- W is non-local due to long-living modes associated to $\partial_\mu J_1^\mu = 0$ and $\partial_\mu J_2^\mu = 0$.
- "Unintegrate" long-living modes [Haehl,Loganayagam,Rangamani '15; Crossley, PG, Liu '15; Jensen, Pinzani-Fokeeva, Yarom '17;. . .]

$$e^{iW[A_1,A_2]} = \int D\varphi_1 D\varphi_2\, e^{iS_{\text{hydro}}[A_1,\varphi_1;A_2,\varphi_2]}$$

$$\varphi_1, \varphi_2 : \text{ long living modes}$$

- S_{hydro} local, satisfies several symmetries. Precisely recovers diffusion in the saddle-point limit.

IR INSTABILITY

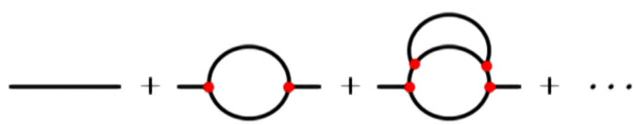
$$\partial_\mu J^\mu = \chi \partial_t \mu - 4c\partial_x \mu - \sigma \partial_x^2 \mu + \frac{1}{2}\chi' \partial_t \mu^2 = \eta$$

It is convenient to change coordinates to a frame co-moving with the chiral front: $x \to x + \frac{4a}{\chi}t$. Upon rescaling various quantities:

$$\partial_t \mu - \partial_x^2 \mu + \lambda \partial_x(\mu^2) = \eta$$

Scaling $\partial_t \sim \partial_x^2$, the interaction λ is relevant! This has dramatic consequences:

$$\langle J^i(\omega) J^i(-\omega) \rangle_{\text{ret}} \sim \sigma i\omega + \lambda^2 (i\omega)^{-\frac{1}{2}} + \lambda^4 (i\omega)^{-1} + \cdots$$



Correlation function grows with time!

Fate in the IR

What is the fate of chiral diffusion in the IR?
To get a sense, consider higher-dimensional generalization:

$$J^x = -4c\mu - \sigma\partial_x\mu, \qquad J^\perp = -\sigma_\perp\nabla_\perp\mu$$

Rescaled coupling λ is marginal in $2+1$ and irrelevant in $3+1$.

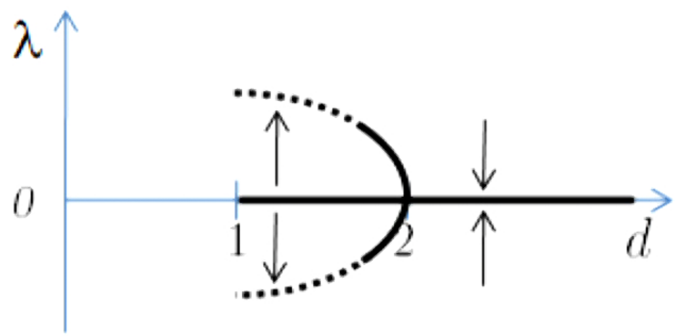
- $(2+1)-d$: surface chiral metals
- $(3+1)-d$: chiral magnetic effect with large background magnetic field.

Fate in the IR

Integrate out momentum shell $e^{-l}\Lambda < |k| < \Lambda$:

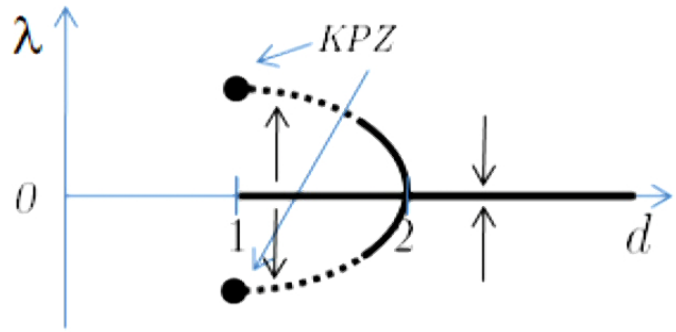
$$\frac{\partial\lambda}{\partial l} = \frac{1}{2}\varepsilon\lambda - \frac{\lambda^3}{2\pi}, \qquad \varepsilon = 2 - d$$

The theory is marginally irrelevant in $d = 2$ and has a non-trivial fixed point at $\varepsilon = 2 - d > 0$!



Fate in the IR

In $1+1$ dimensions, the theory is equivalent to KPZ (Kardar-Parisi-Zhang) universality class.



- Diffusive fluctuations around the chiral front at $x + \frac{4c}{\chi} t$ are in the KPZ universality class.
- Chiral diffusion flows to $\omega = k + k^z$, $z = \frac{3}{2}$, leading to the exact scaling

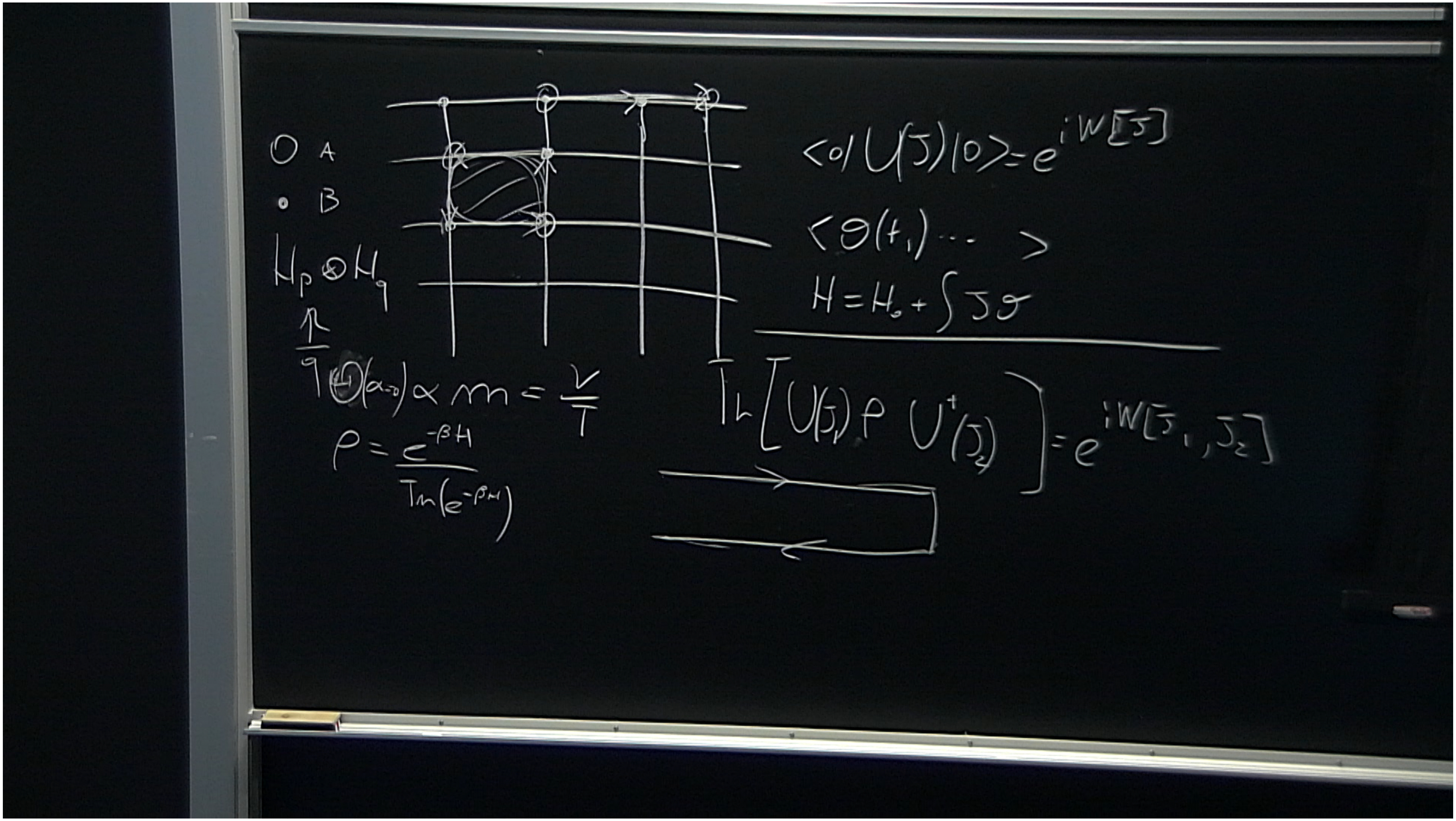
$$\sigma(\omega) = \langle J^i(\omega) J^i(-\omega) \rangle_{\text{sym}} \sim \frac{1}{\omega^{1/3}}$$

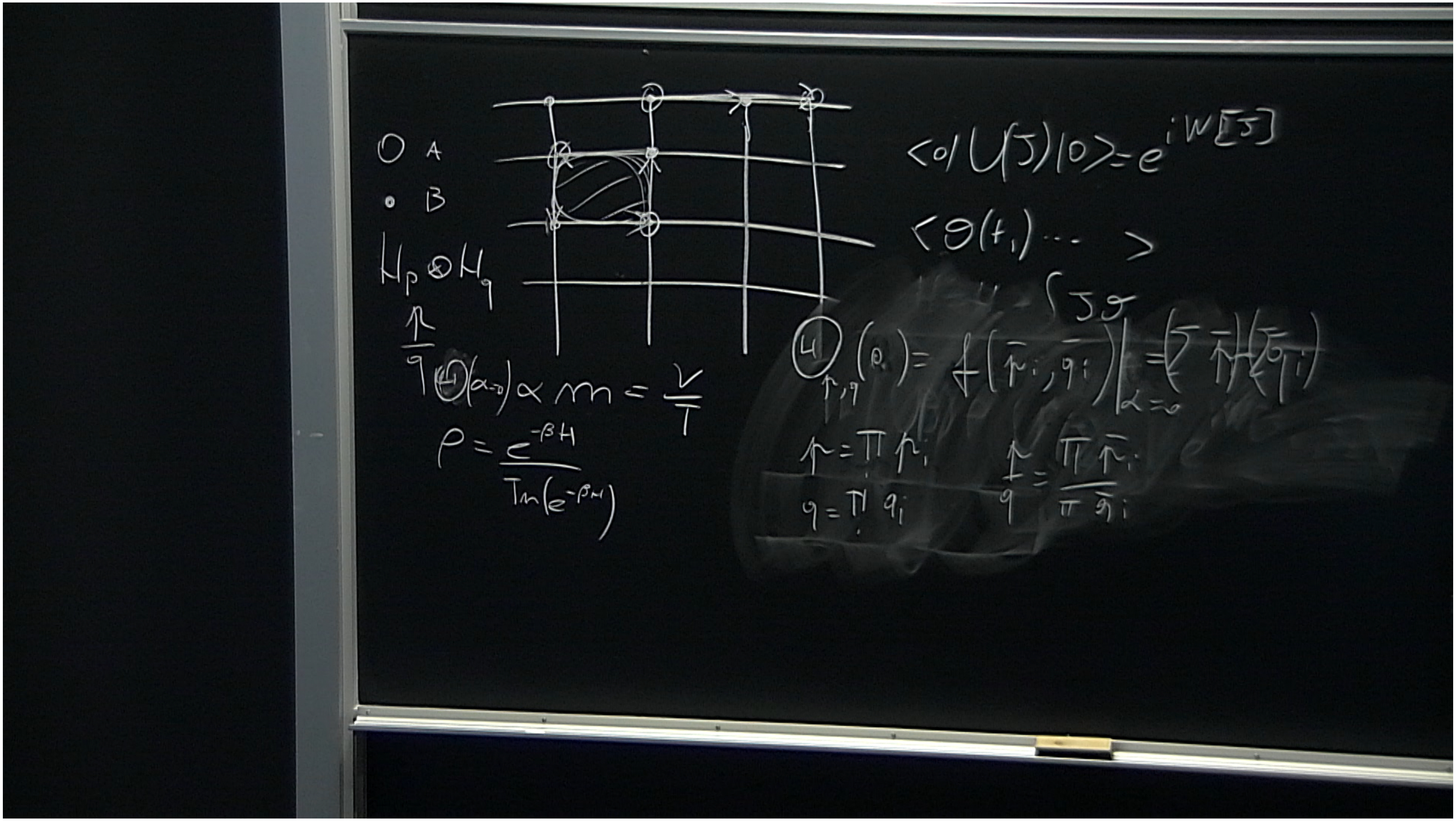
Remarks

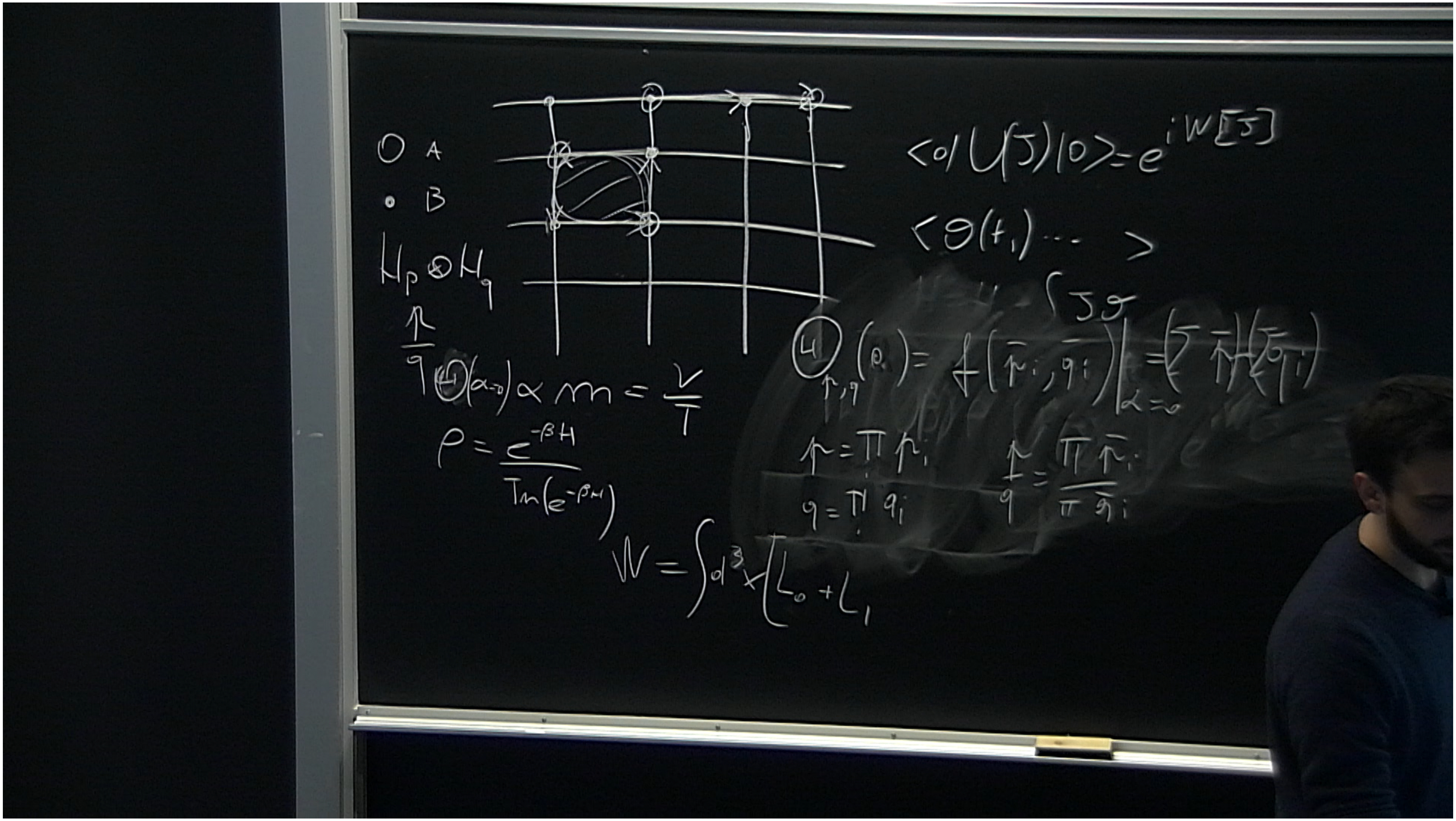
- Infrared instability of chiral diffusion
- Persists without momentum conservation
- Relevant to edge physics

Future directions

1. Chiral diffusion: include energy conservation; connection to other known systems?
2. Floquet: geometric response; constraints on $\Theta(\alpha)$? time-ordering sensitive topological response? Non-topological properties?
3. Other directions: open systems and novel constraints


A
B
H_p ⊗ H_q
<0| U(J) |0> = e^{iW[J]}
H = H_0 + ∫ Jθ
ρ = e^{-βH} / Tr(e^{-βH})
Tr[U(J_1) ρ U†(J_2)] = e^{iW[J_1, J_2]}


A
B
Hp ⊗ Hq
<0|U(J)|0> = e^{iW[J]}
ρ = e^{-βH} / Tr(e^{-βH})


A
B
$H_p \otimes H_q$
$\langle 0 | U(J) | 0 \rangle = e^{iW[J]}$
$\langle \mathcal{O}(t_1) \cdots \rangle$
$\rho = \frac{e^{-\beta H}}{\mathrm{Tr}(e^{-\beta H})}$
$W = \int d^3x \, [L_0 + L_1$