Title: Aspects of non-perturbative unitarity in Quantum Field Theory

Speakers: Alessia Platania

Series: Quantum Gravity

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Abstract: According to the Asymptotic Safety conjecture, a (non-perturbatively) renormalizable quantum field theory of gravity could be constructed based on the existence of a non-trivial fixed point of the renormalization group flow.
The existence of this fixed point can be established, e.g., via the non-perturbative methods of the functional renormalization group (FRG). In practice, the use of the FRG methods requires to work within truncations of the gravitational action, and higher-derivative operators naturally lead to the presence of several poles in the propagator. The question is whether these poles represent a real problem for the unitarity of the theory.

Using QED as a working example, in this talk I will discuss some aspects of non-perturbative unitarity in Quantum Field Theory. I will show that the inclusion of quantum effects at all scales is of crucial importance to assess unitarity of field theories. In particular, poles appearing in truncations of the action could correspond to fake degrees of freedom of the theory. Possible conditions to determine, within truncations, whether a pole represents a fake or a genuine degree of freedom of the theory will also be discussed.

Motivation

- **Einstein-Hilbert gravity**: unitary, but *perturbatively non-renormalizable*
- **Quadratic gravity** is a *renormalizable* theory, but has one *massive spin-2 ghost*

$$S = -\int d^4x \sqrt{-g}\,\{\gamma R + \alpha R_{\mu\nu} R^{\mu\nu} - \beta R^2\}$$

Stelle, PRD 16 (1977) 953-969

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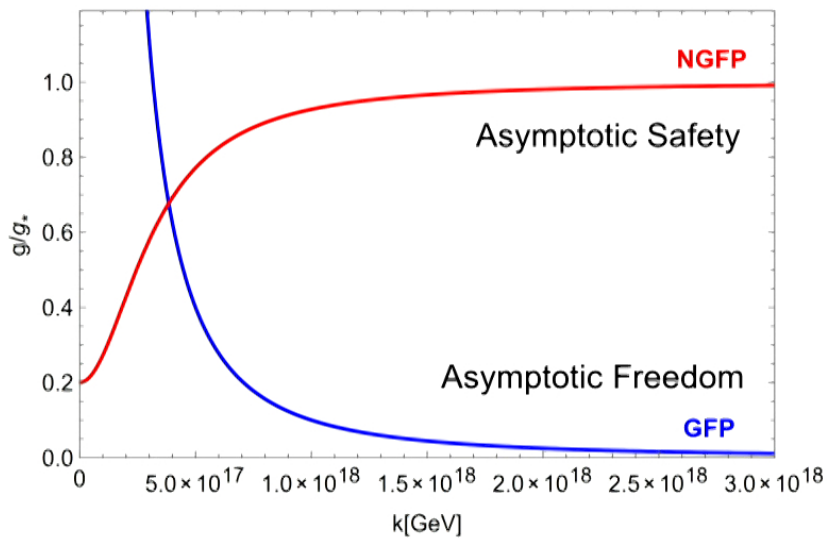

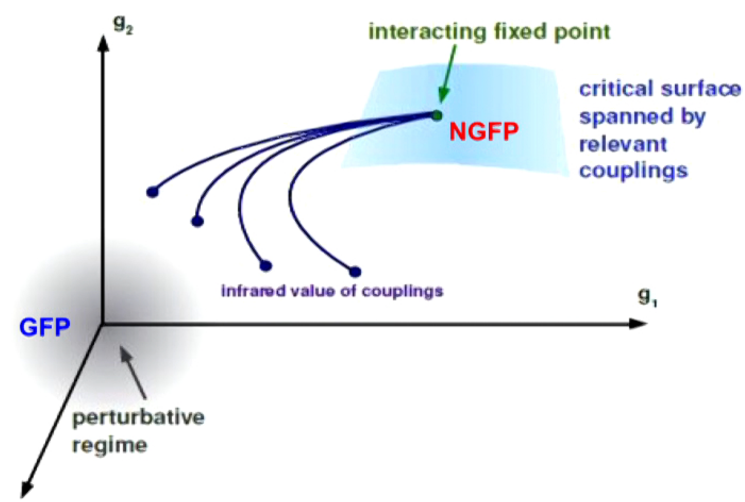

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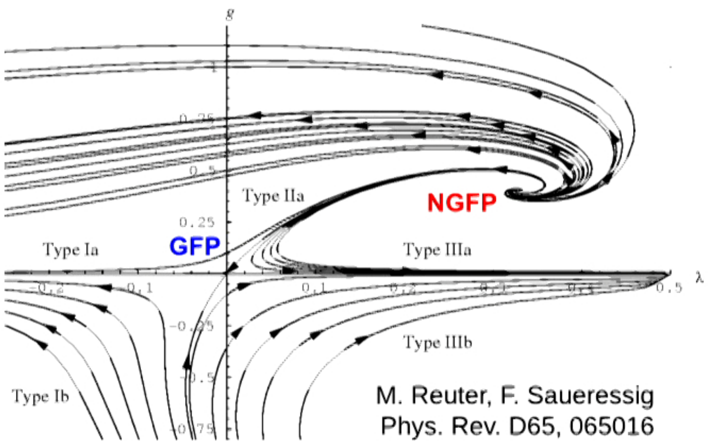

M. Reuter, F. Saueressig
Phys. Rev. D65, 065016

Functional (non-perturbative) RG

→ **Fixed points** of the RG flow:

- **GFP** → saddle point
- **NGFP** → UV-attractor

→ Extended truncations:

- **3 relevant directions**

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Unitarity?

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Non-perturbative effects could be important for the understanding of fundamental properties of quantum field theories, such as **renormalizability and unitarity**

Non-perturbative unitarity: Although quadratic gravity has a ghost, *quantum effects* could make the ghost unstable, thus restoring unitarity

Salam, Strathdee (1978)
E. S. Fradkin, A. A. Tseytlin (1981)

Unitarity

- **Unitarity condition**

$$S^\dagger S = \mathbb{I} \qquad S = \mathbb{I} + iT$$

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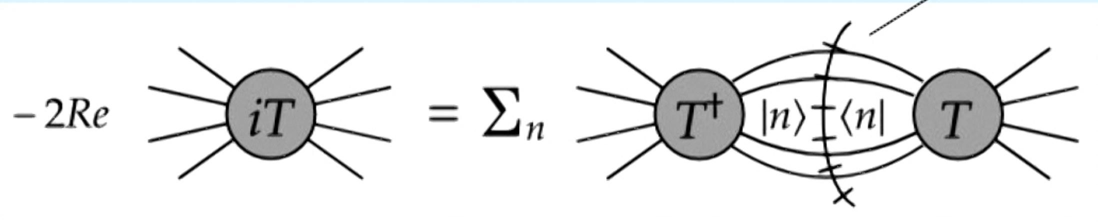
Cutting rules
T'Hooft, Veltman (1973)



$$|n\rangle \in \mathcal{F}$$

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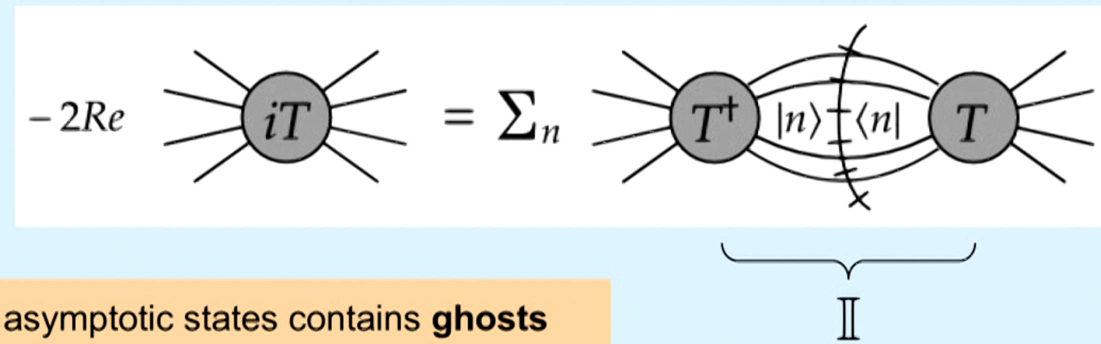

$|n\rangle \in \mathcal{F}$

If the space of asymptotic states contains **ghosts**

$\langle m|n\rangle = (-1)^{\alpha_n}\delta_{mn} \qquad$ "$\mathbb{I}$" $= \sum_n (-1)^{\alpha_n}|n\rangle\langle n|$

⇒ Loss of physical unitarity

Spectral representation and unitarity

- **Spectral representation**

$$\Delta(q^2) = i\int_0^\infty d\mu^2 \, \frac{\rho(\mu^2)}{q^2 - \mu^2 + i\varepsilon}$$

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For a **stable particle**, the spectral density is a **Dirac delta**

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- **Propagator**

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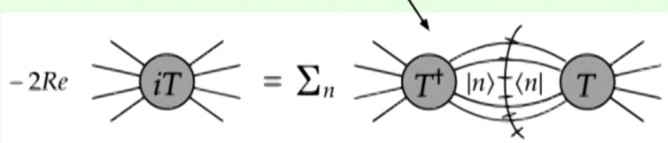

Salam, Strathdee (1978), Fradkin, Tseytlin (1981)
Donoghue, Menezes (2019)

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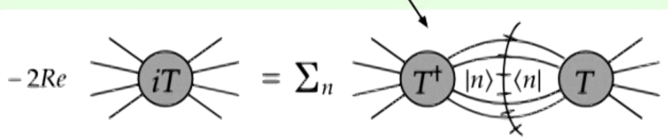

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$$\Delta(q^2) = \frac{i}{q^2-m_0^2} \longrightarrow \Delta(q^2) = \frac{i}{q^2-m_0^2-\Sigma(q^2)}$$

Bare propagator

Dressed propagator

self-energy

Functional Renormalization Group

Solving the **quantum theory** is equivalent to solve the functional **renormalization group equation**

$$k\partial_k \Gamma_k = \frac{1}{2}\mathrm{STr}\left\{ \left(\Gamma_k^{(2)} + \mathcal{R}_k \right)^{-1} k\partial_k \mathcal{R}_k \right\}$$

C. Wetterich. *Phys. Lett. B* 301:90 (1993)
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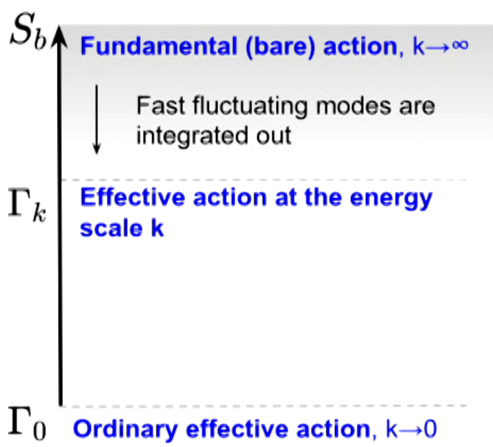

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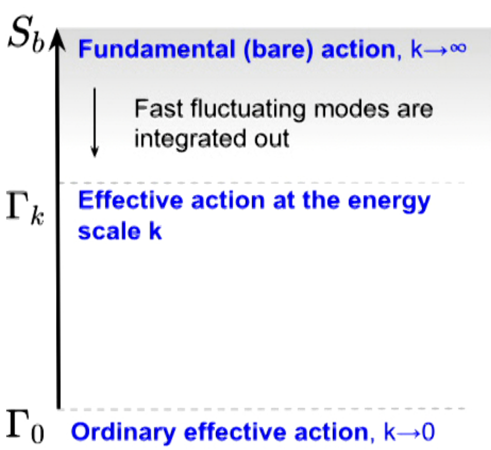

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S_b **Fundamental (bare) action**, k→∞

Fast fluctuating modes are integrated out

Γ_k **Effective action at the energy scale k** → All terms compatible with symmetry and field content of the theory are generated

Γ_0 **Ordinary effective action**, k→0 → *All quantum fluctuations are integrated out → non-locality*
Incorporates all quantum effects → fully-dressed quantities
This is the object to use to check **unitarity**!

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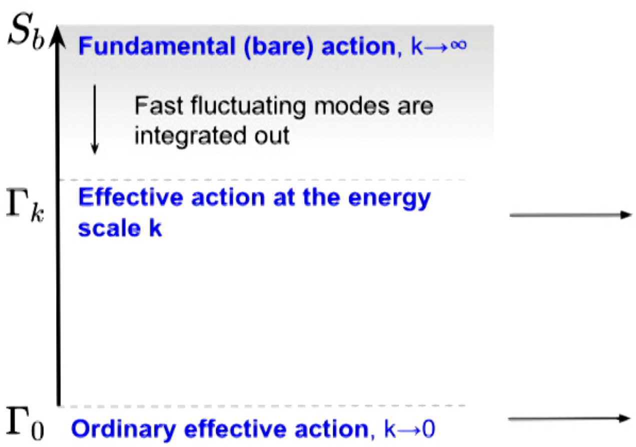
$$k\partial_k\Gamma_k = \frac{1}{2}\mathrm{STr}\left\{\left(\Gamma_k^{(2)} + \mathcal{R}_k\right)^{-1} k\partial_k\mathcal{R}_k\right\}$$

Problem: Need to work within truncation ⇒ higher-derivatives ⇒ Poles

Questions:
- What is the nature of these poles?
- Are these poles removed by quantum effects?
- Connection between poles in finite truncation and poles in the effective action?
- How do we understand, within truncation, if these poles are dangerous for unitarity?



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Unitarity in QED

Take the **one-loop effective action** as a **toy model for the full effective action**

$$\Gamma_{QED} = -\frac{1}{4}\int d^4x \, \{F_{\mu\nu} P(\Box) F^{\mu\nu}\} \qquad P(q^2) = 1 - \frac{\alpha}{3\pi}\log\left(\frac{m_{th}^2 - q^2}{m_{th}^2}\right) - \frac{q^2}{M^2} \qquad m_{th} = 2m_f$$

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In this case the propagator has one massless pole and one massive ghost pole

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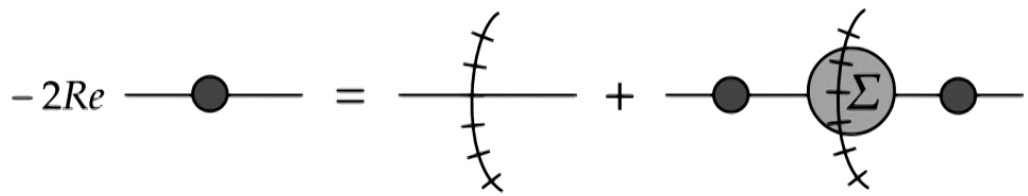
Absorptive part of the propagator



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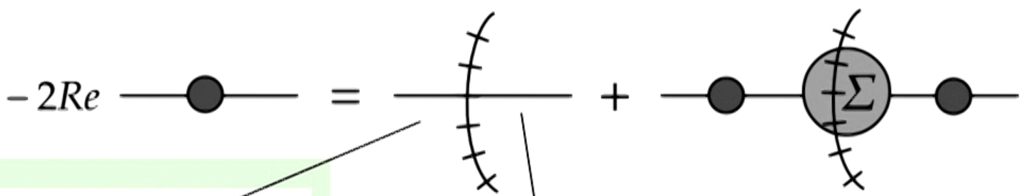

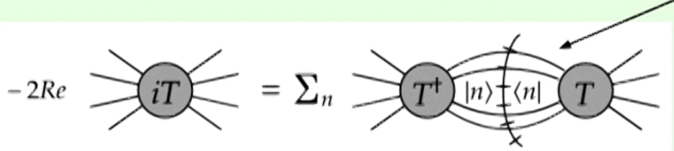

$$\rho \sim \delta(q^2)$$

What happens within truncations?

$$\Gamma_k = -\frac{1}{4}\int d^4x\{F_{\mu\nu}P_k F^{\mu\nu}\}$$

$$P_k(z) = \sum_{n=1}^{\infty} a_n(k)\, z^n \qquad z = q^2/m_{th}^2$$

$$k = 0 \qquad P(z) = 1 - z + \frac{\alpha}{3\pi}\sum_{n=1}^{\infty}\frac{z^n}{n} \equiv 1 - z - \frac{\alpha}{3\pi}\log(1-z)$$

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Finite truncation of the action (e.g., derivative expansion of the action)

$$P^N(z) = 1 - z + \frac{\alpha}{3\pi}\sum_{n=1}^{N}\frac{z^n}{n}$$

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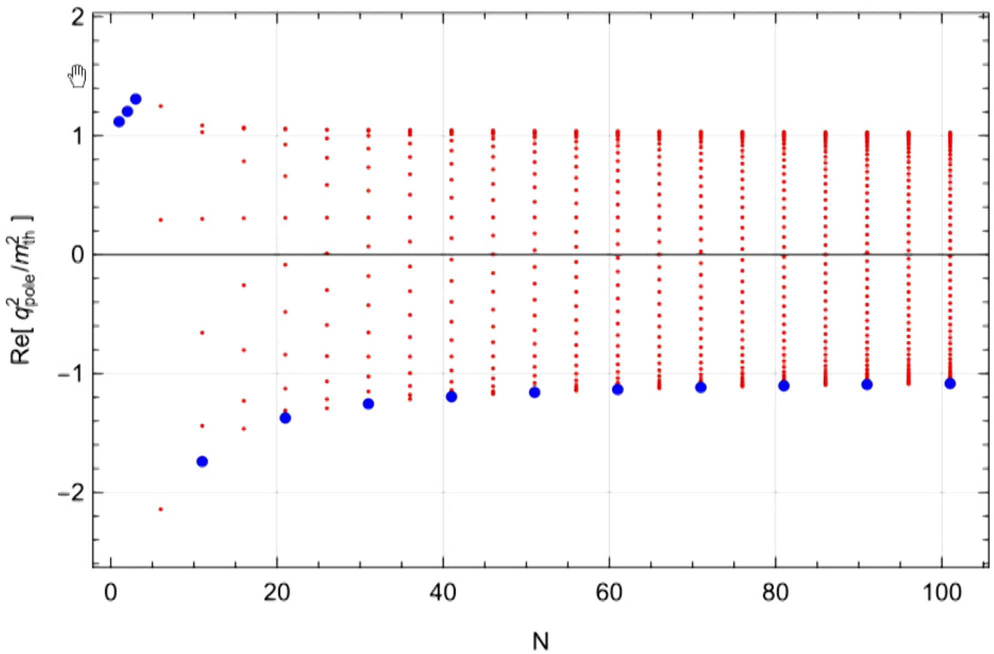

- Real poles
- Complex poles

Persistent ghost pole at

$$q^2 \sim -m_{th}^2$$

It is a pole for N odd

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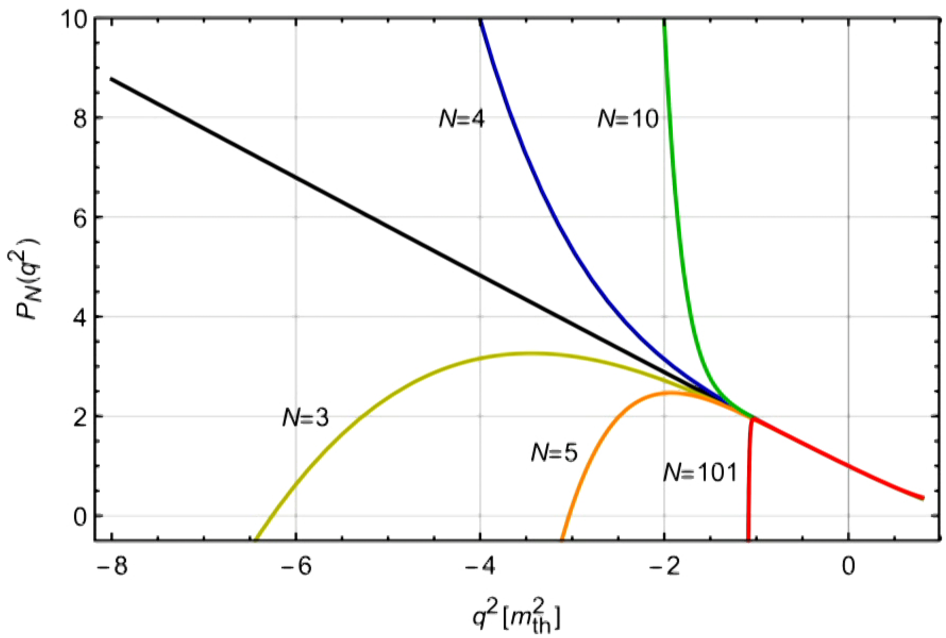

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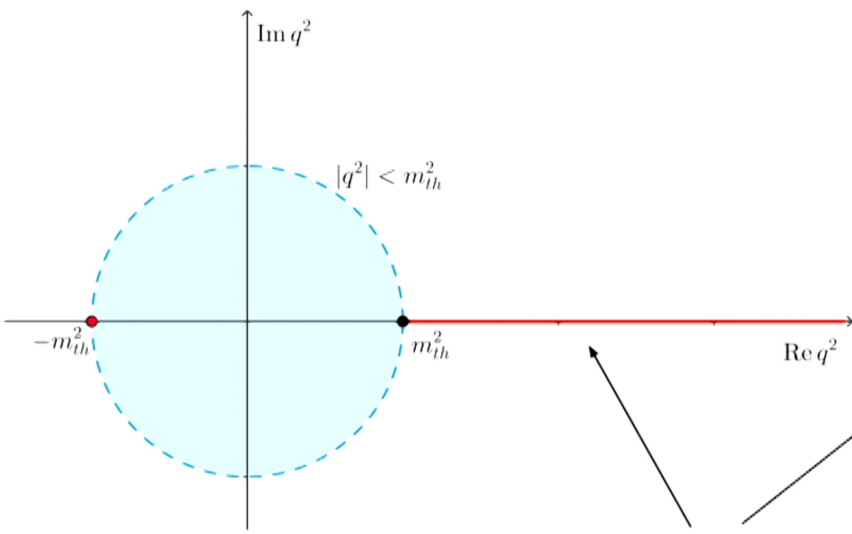

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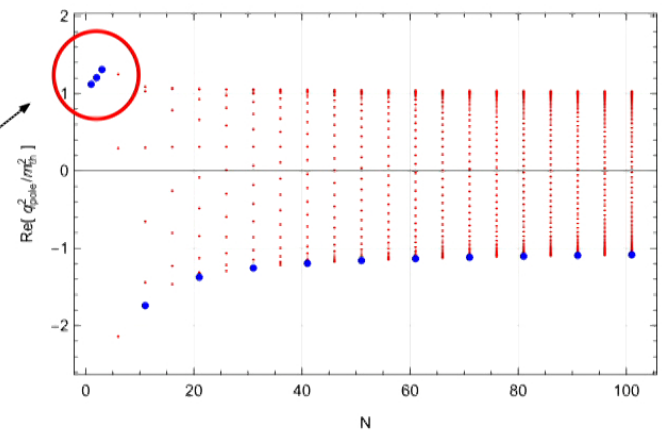

Unstable ghost lives in the branch cut (cannot be seen in any perturbative expansion)

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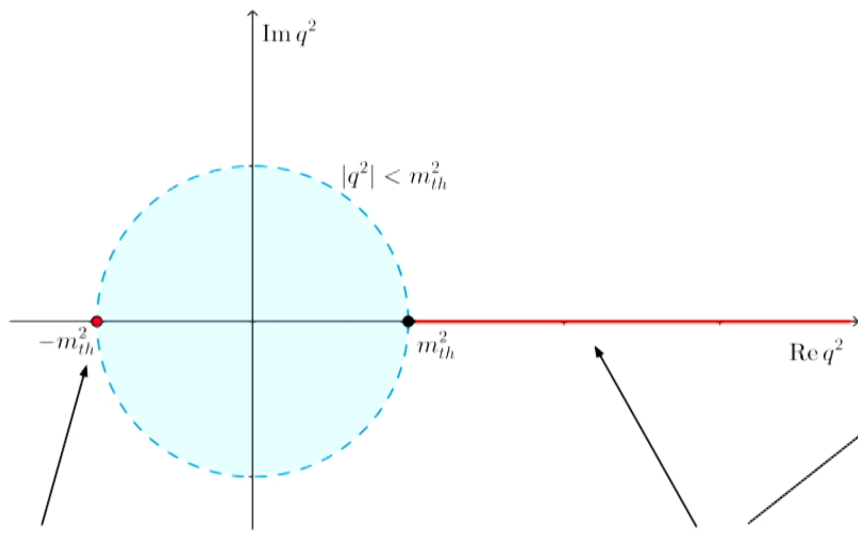
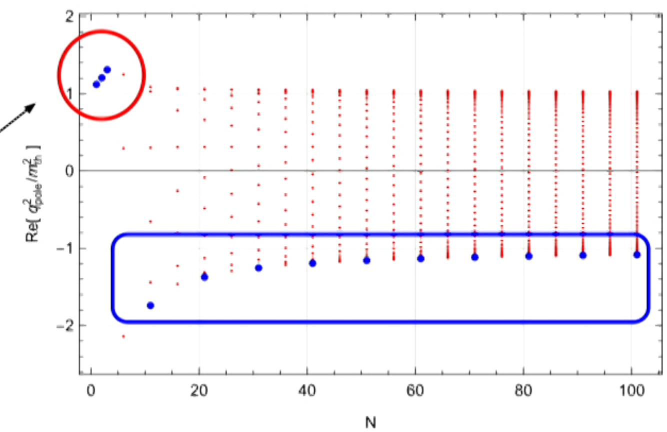


The apparent ghost pole is generated by the convergence properties of the function P(z)

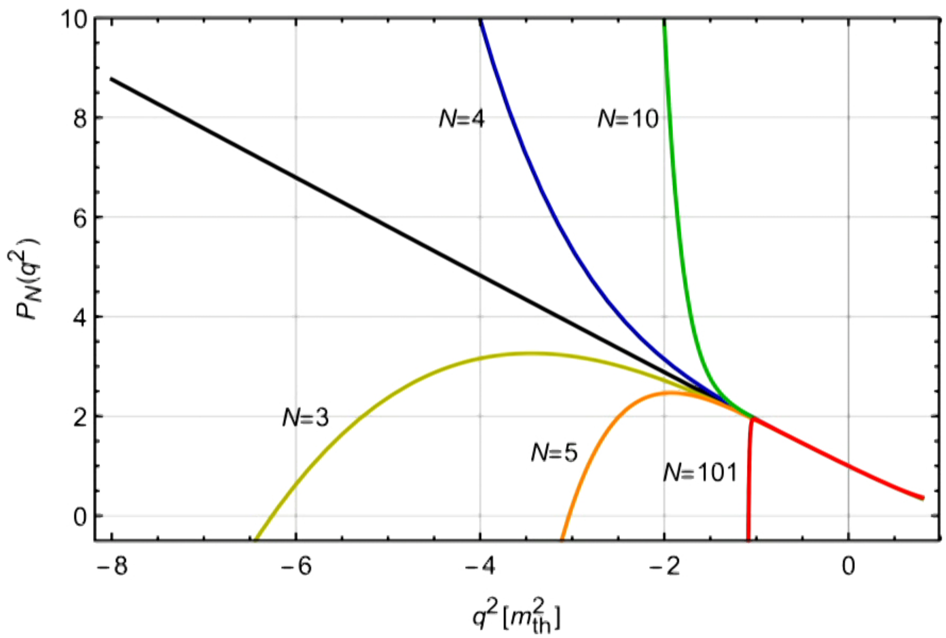


Fake ghost living in the principal branch of the Log (not appearing in the full theory)

Unstable ghost lives in the branch cut (cannot be seen in any perturbative expansion)

What happens within truncations?

$$P^N(z) = 1 - z + \frac{\alpha}{3\pi}\sum_{n=1}^{N}\frac{z^n}{n} \qquad z = q^2/m_{th}^2 \qquad \alpha = 1 \qquad \Delta(q^2) \sim \frac{i}{q^2 P^N(q^2)}$$



The apparent ghost pole is generated by the convergence properties of the function P(z)

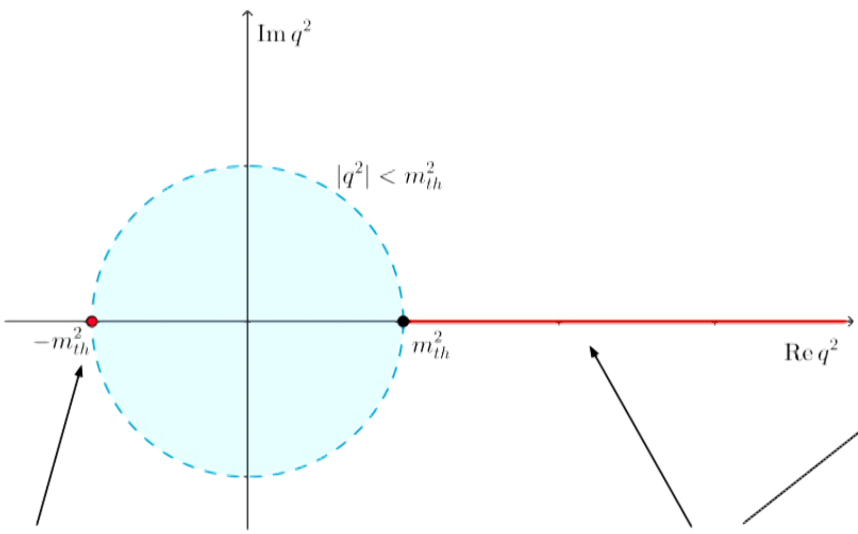
Persistent ghost pole at

$$q^2 \sim -m_{th}^2$$

It is a pole for N odd

What happens within truncations?

$$P^N(z) = 1 - z + \frac{\alpha}{3\pi}\sum_{n=1}^{N}\frac{z^n}{n} \qquad z = q^2/m_{th}^2 \qquad \alpha = 1 \qquad \boxed{\Delta(q^2) \sim \frac{i}{q^2 P^N(q^2)}}$$



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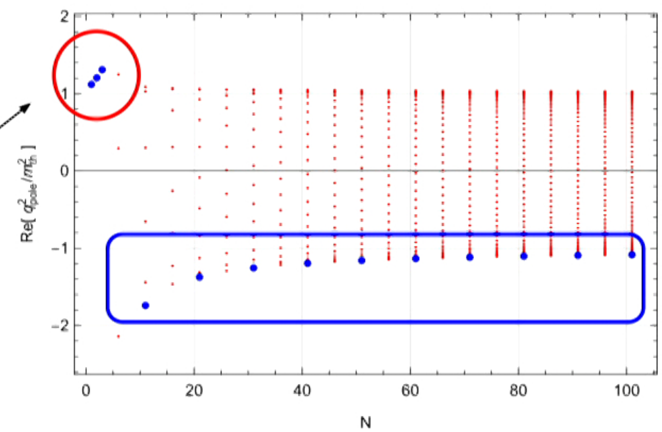

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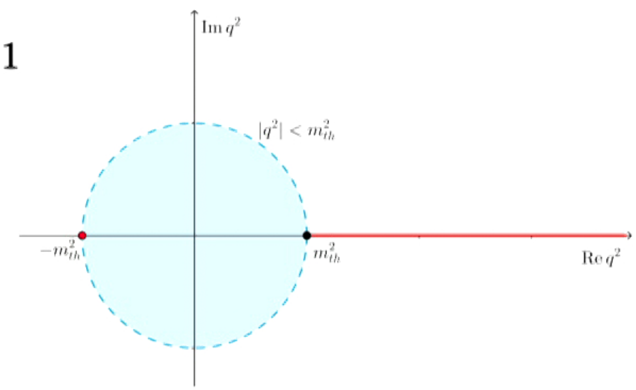
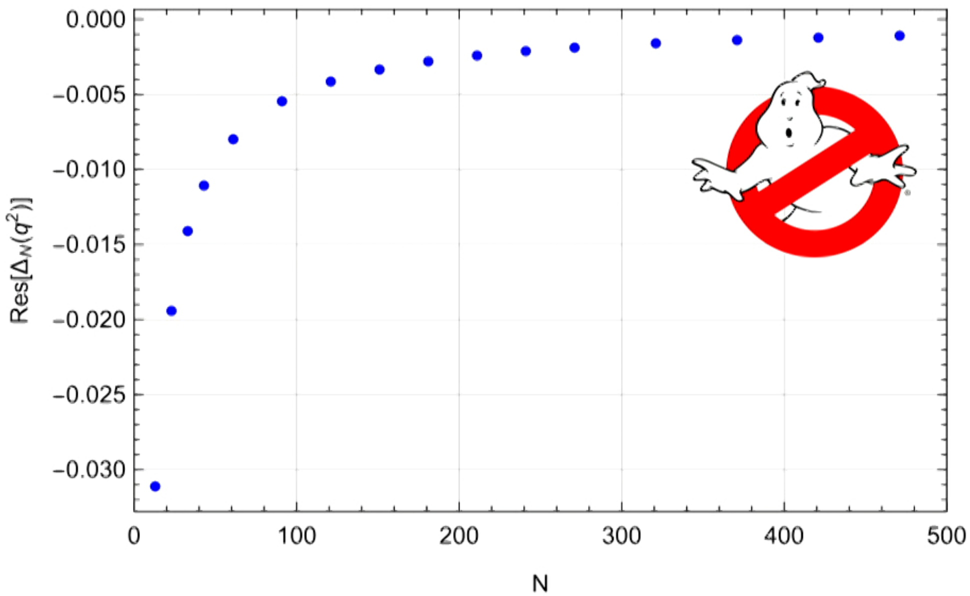
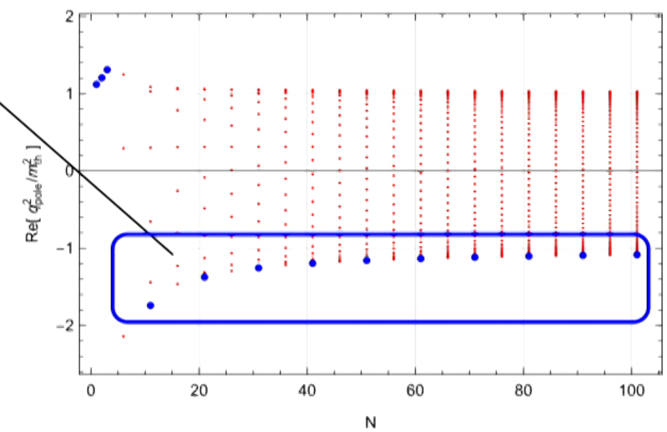


The answer lies in the **residue**!

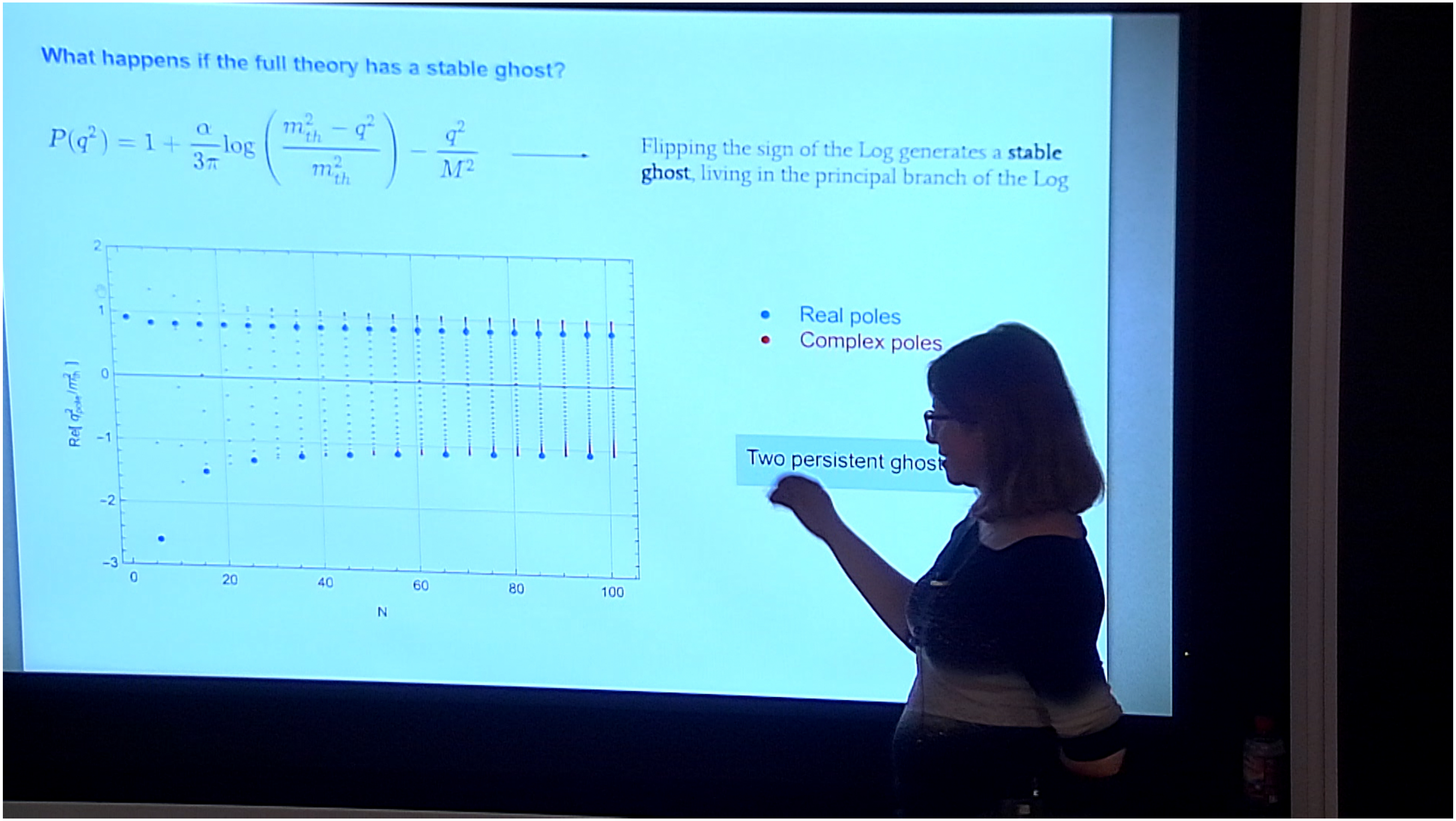




What happens if the full theory has a stable ghost?

$$P(q^2) = 1 + \frac{\alpha}{3\pi}\log\left(\frac{m_{th}^2 - q^2}{m_{th}^2}\right) - \frac{q^2}{M^2} \longrightarrow$$

Flipping the sign of the Log generates a **stable ghost**, living in the principal branch of the Log


What happens if the full theory has a stable ghost?
$P(q^2) = 1 + \frac{\alpha}{3\pi}\log\left(\frac{m_{th}^2 - q^2}{m_{th}^2}\right) - \frac{q^2}{M^2}$
Flipping the sign of the Log generates a **stable ghost**, living in the principal branch of the Log
Real poles
Complex poles
Two persistent ghost
$\mathrm{Re}[\,q^2_{pole}/m^2_{th}\,]$
N
0
20
40
60
80
100
2
1
0
-1
-2
-3

What happens if the full theory has a stable ghost?

$$P(q^2) = 1 + \frac{\alpha}{3\pi}\log\left(\frac{m_{th}^2 - q^2}{m_{th}^2}\right) - \frac{q^2}{M^2}$$

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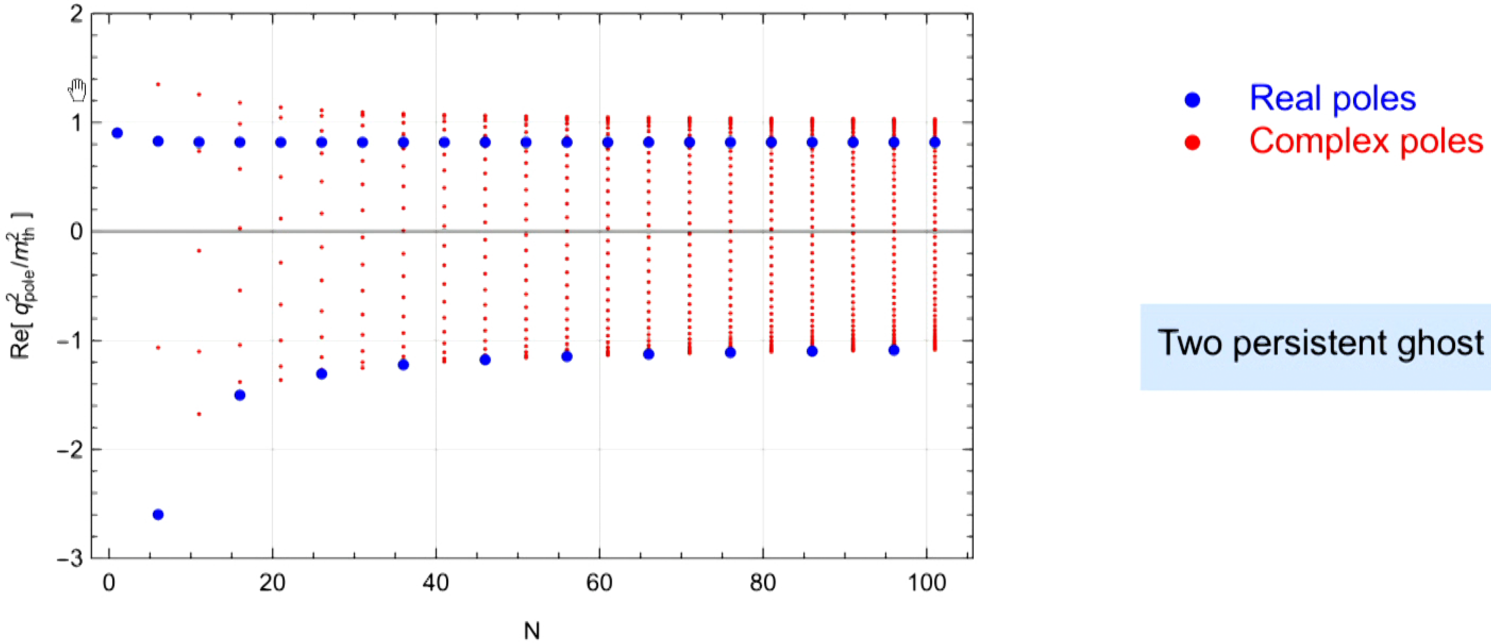

Two persistent ghost poles!

What happens if the full theory has a stable ghost?

$$P(q^2) = 1 + \frac{\alpha}{3\pi}\log\left(\frac{m_{th}^2 - q^2}{m_{th}^2}\right) - \frac{q^2}{M^2} \longrightarrow$$

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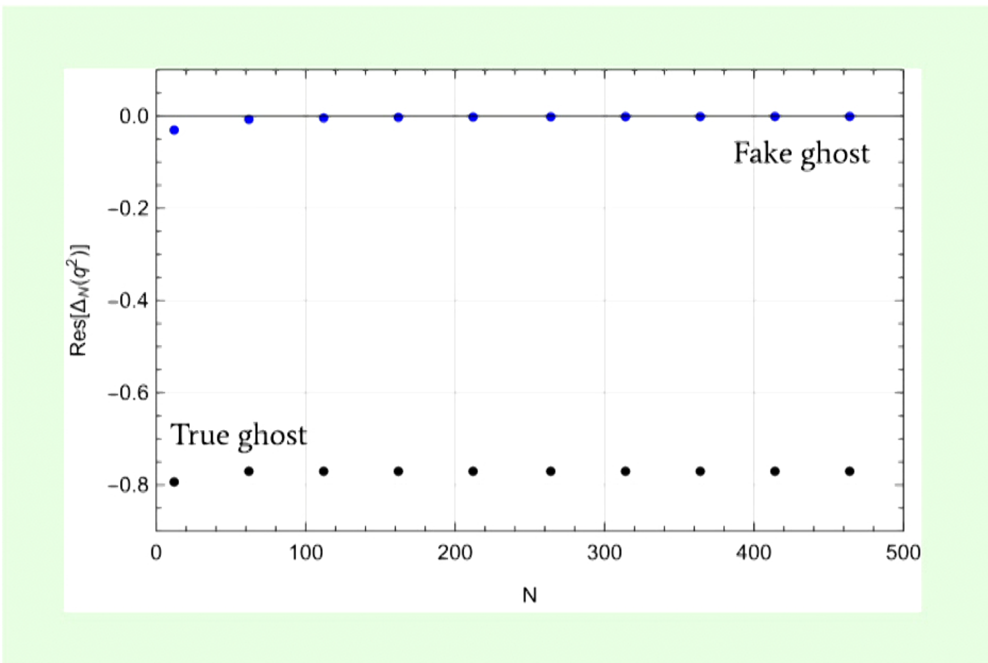

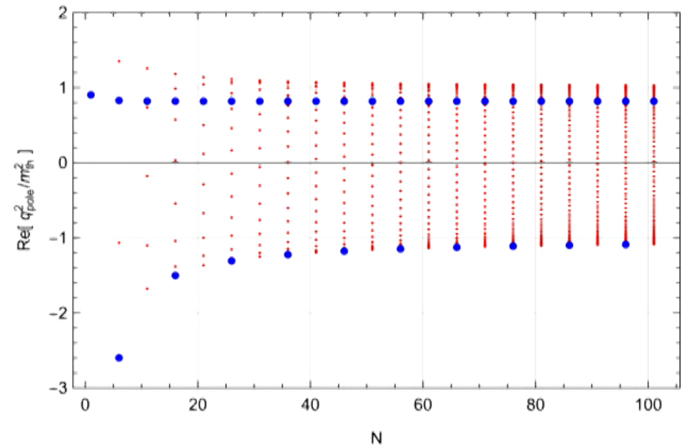

Stable ghost in the full theory
→ persistent **negative residue**

Fake ghost (generated by convergence properties of P(z))
→ **residue approaches zero**

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Similar results are obtained from other toy models for the fully-quantum effective action

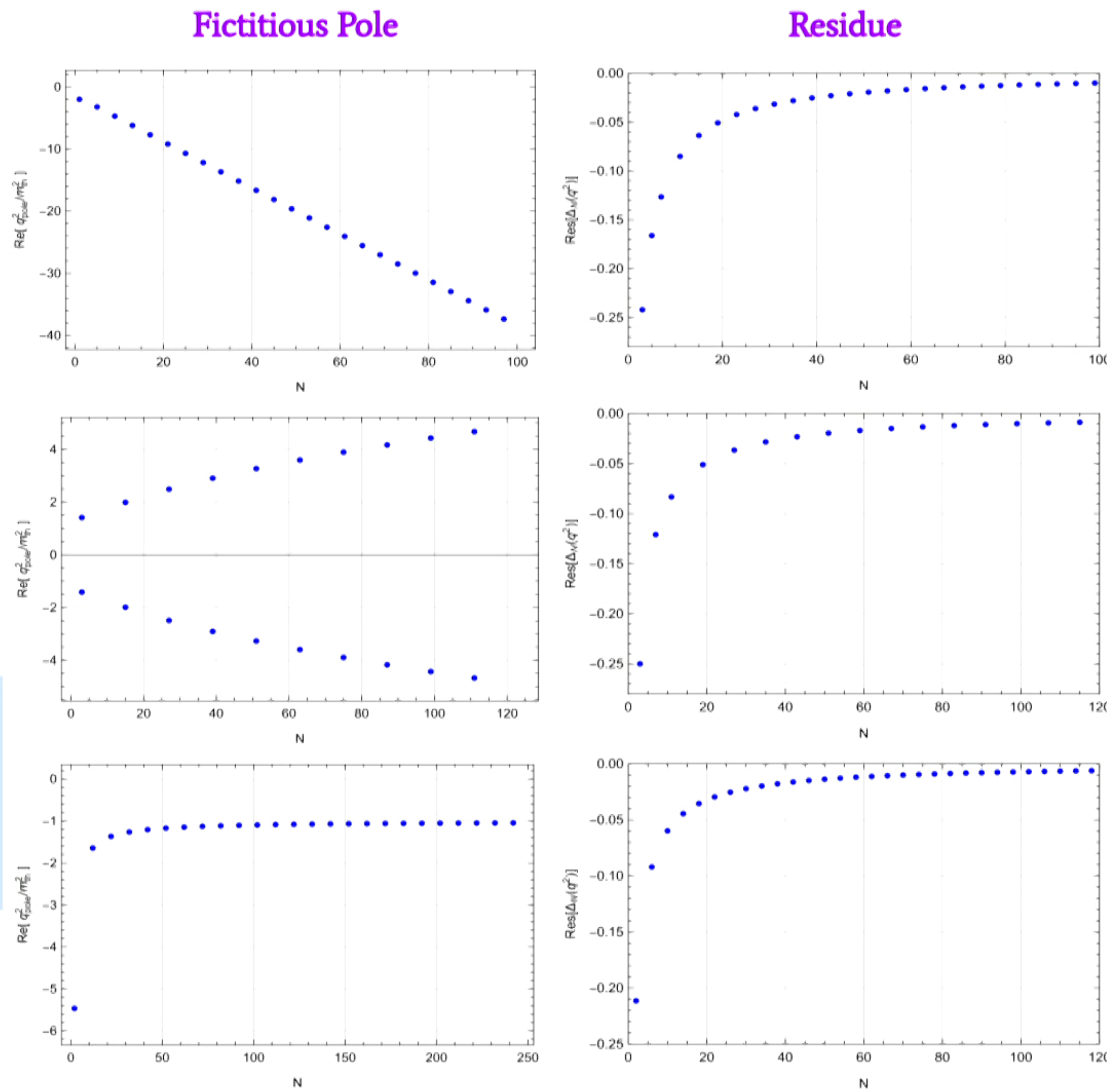
$$P(q^2) = 1 + \exp(q^2/M^2)$$

Ghost-free full theory
(P(z): entire function)
Truncation: fake ghosts move to infinity, residue approaches zero

$$P(q^2) = 1 + \exp(-q^4/M^4)$$

Full theory with unstable ghosts
(P has finite radius of convergence)
Truncation: fake ghost moves to boundary of domain of converg., residue approaches zero

$$P(q^2) = (1 - q^2/M^2)^{1/2}$$



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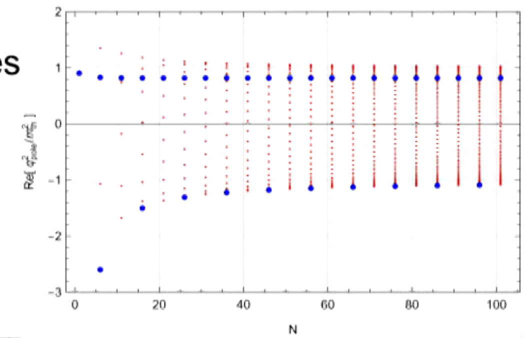
Summary & Conclusions

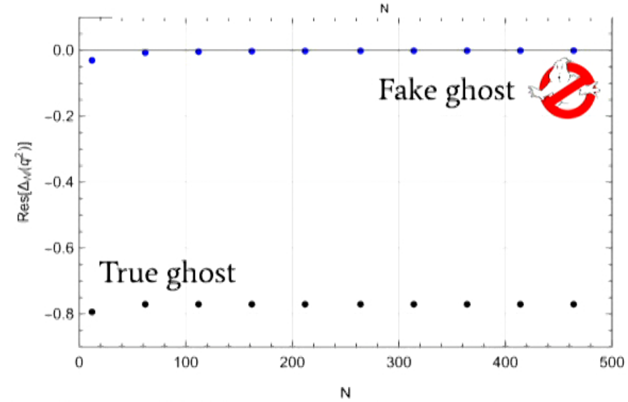
- We discussed **unitarity** from the point of view of the Functional Renormalization Group



- Including ***all quantum fluctuations*** is crucial for unitarity: it determines which states appear in the sum over states in the **optical theorem**

- **Truncations** / derivative expansion of the action ⇒ **fictitious poles**

- The fictitious pole is a "***fake ghost***": its **residue approaches zero** when a sufficiently large number of terms in the action are included.
  Ghosts in the full theory are instead characterized by a **persistent negative residue**.



- Still no proof of the "**residue conjecture**" (work in progress), but **no countexample** found so far

- **Evolution of residue with N**: **criterium to determine nature of poles at finite truncation order**

- Most reliable instrument to check *unitarity in full glory*: **fully-quantum effective action**

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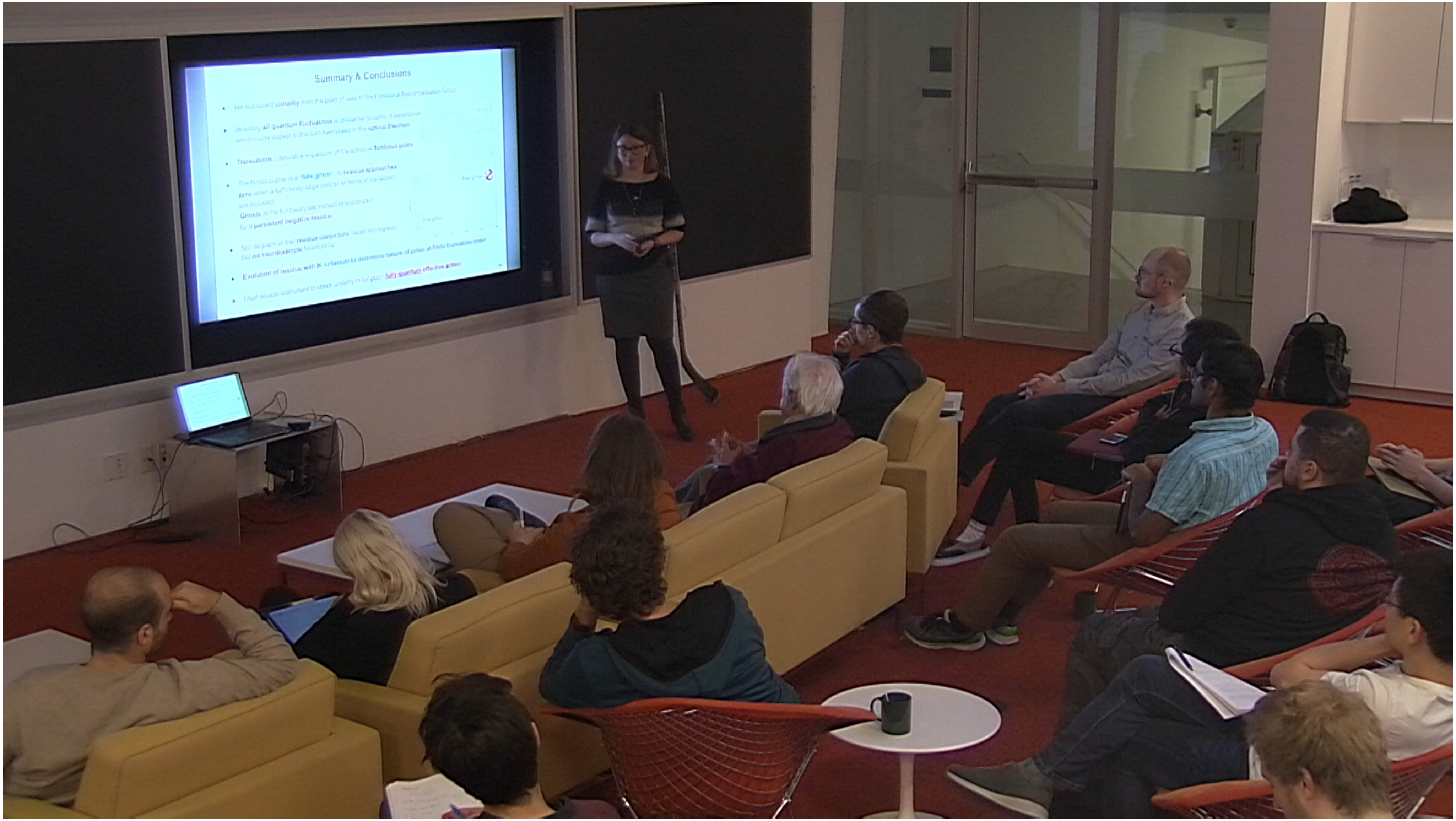
Summary & Conclusions