Title: New limits for large N matrix and tensor models

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Abstract: In this talk, I will describe the framework of large D matrix models, which provides new limits for matrix models where the sum over planar graphs simplifies when D is large. The basic degrees of freedom are a set of D real matrices of size NxN which is invariant under O(D). These matrices can be naturally interpreted as a real tensor of rank three, making a compelling connection with tensor models. Furthermore, they have a natural interpretation in terms of D-brane constructions in string theory. I will present a way to define a large D scaling of the coupling constants such that the sum over Feynman graphs of fixed genus in matrix models admits a well-defined large D expansion. In particular, in the large D limit, the sum over planar graphs truncates to a tractable, yet non-trivial, sum over generalized melonic graphs. This family of graphs has been shown to display very interesting properties, especially in the case of quantum mechanical models such as the SYK model and SYK-like tensor models. If time allows, I will also explain how one can use the large D limit of matrix models to simplify the sum over all genera, which is notoriously divergent.

New limits for large N matrix and tensor models

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Outline

1. Context and motivation
2. Large D matrix models
   - Preliminaries
   - Feynman rules
   - Large N and large D expansions
3. Double scaling limit
4. Summary and outlook

Context (1)

Random **matrix** models provide a description of random discretized **surfaces**

- In perturbation theory, Feynman graphs correspond to ribbon graphs, dual to discretized surfaces
- Free energy admits a well-defined $1/N$ expansion governed by the genus of the Feynman graphs

$$F = \sum_g N^{2-2g} F_g$$

- When $N \to \infty$, **planar graphs** ($g = 0$) dominate the perturbative expansion

Matrix models appear in many areas of physics, in particular in string theory, holography and **quantum gravity**

Context (2)

Random **tensor** models generalize random matrix models $\rightarrow$ random discretized geometries in **higher dimensions**

- They allow for a larger class of interactions and they admit different interesting large N expansions
- When $N \rightarrow \infty$, leading Feynman graphs are often called 'melonic' in a broad sense

From point of view of random geometries, tensor models are somewhat disappointing at the moment $\rightarrow$ **branched polymer** phases

However, their usefulness may also lie somewhere else $\rightarrow$ recent **connection** with SYK physics and holography

Context (3)

The SYK model caught a lot of attention in the recent years

- N fermions coupled between each other with all-to-all **random** couplings: $H_{\text{int}} \sim J_{ijkl}\psi_i\psi_j\psi_k\psi_l$
- **Solvable** in the large N limit (melonic graphs), including at strong coupling
- Emergent **conformal** symmetry in the IR, which is spontaneously and explicitly broken $\rightarrow$ $NAdS_2/NCFT_1$
- **Quasi-normal behavior**, **maximally chaotic**, etc.

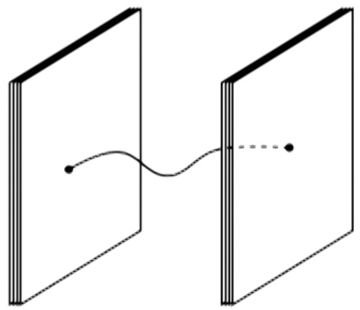
Connection with random tensor models $\rightarrow$ they are both **melonic theories**

In particular, one can construct **genuine** QM models with random tensors $\rightarrow$ SYK-like tensor models

Motivation

Both SYK model and SYK-like tensor models are rather **exotic** from point of view of holography

$\rightarrow$ Matrix d.o.f. are often **natural**



$$(A_\alpha)_{ab}, \quad 0 \leq \alpha \leq p$$

$$(X_\mu)_{ab}, \quad 0 \leq \mu \leq D = d - p - 1$$

Motivation

Melonic limit can be achieved with O(D)-**invariant** matrix models, with symmetry $U(N)^2 \times O(D)$ and d.o.f.

$$(X_\mu)_{a_1 a_2} = X_{a_1 a_2 \mu}\,, \quad 1 \le a_1, a_2 \le N\,, \quad 1 \le \mu \le D$$

More general framework → **matrix-tensor models** with symmetry $O(N)^2 \times O(D)^r$ and d.o.f.

$$(X_{\mu_1 \cdots \mu_r})_{a_1 a_2} = X_{a_1 a_2 \mu_1 \cdots \mu_r}\,, \quad 1 \le a_1, a_2 \le N\,, \quad 1 \le \mu_i \le D$$

→ Set of D^r matrices / tensor of rank $R = r + 2$

→ Additional parameter D allows to consider the **large D limit**

In this talk, focus on matrix-tensor models with $r = 1$

Large D matrix models

Preliminaries

Basic variable:

$$(X_\mu)_{a_1 a_2} = X_{a_1 a_2 \mu}\,, \quad 1 \leq a_1, a_2 \leq N\,, \quad 1 \leq \mu \leq D$$

transforming in the fundamental representation of $O(N)^2 \times O(D)$, with transformation law

$$X_{a_1 a_2 \mu} \to X'_{a_1 a_2 \mu} = O^{(1)}_{a_1 a'_1} O^{(2)}_{a_2 a'_2} O^{(3)}_{\mu\mu'} X_{a'_1 a'_2 \mu'}$$

Invariant **action** with single-trace interactions

$$S = ND\left(\frac{1}{2}\,\mathrm{tr}(X_\mu X_\mu^T) + \sum_a \tau_a \mathcal{I}_{\mathcal{B}_a}(X)\right)$$

where

$$\mathcal{I}_{\mathcal{B}_a}(X) = \mathrm{tr}(X_\mu X_\nu^T \cdots X_\rho X_\sigma^T)$$

Feynman rules

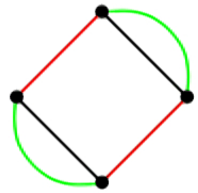
Interested in the perturbative expansion of the free energy onto vacuum connected **Feynman graphs**

→ Two representations for the vertex associated with each interaction term $\mathcal{I}_{\mathcal{B}_a}$

- **Stranded** representation: ribbon graph vertices with additional lines associated with $O(D)$ symmetry
- **Colored** representation: 3-regular edge-colored graphs, called interaction bubbles and denoted as $\mathcal{B}_a$

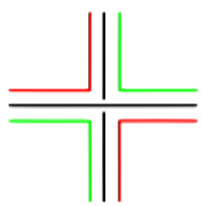
Examples of order four

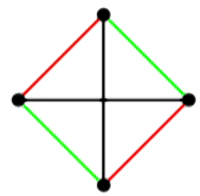
$$\mathrm{tr}(X_\mu X_\mu^T X_\nu X_\nu^T) = X_{a_1 a_2 \mu} X_{b_1 a_2 \mu} X_{b_1 b_2 \nu} X_{a_1 b_2 \nu}$$



$$g(\mathcal{B}_a) = 0$$

$$\mathrm{tr}(X_\mu X_\nu^T X_\mu X_\nu^T) = X_{a_1 a_2 \mu} X_{b_1 a_2 \nu} X_{b_1 b_2 \mu} X_{a_1 b_2 \nu}$$





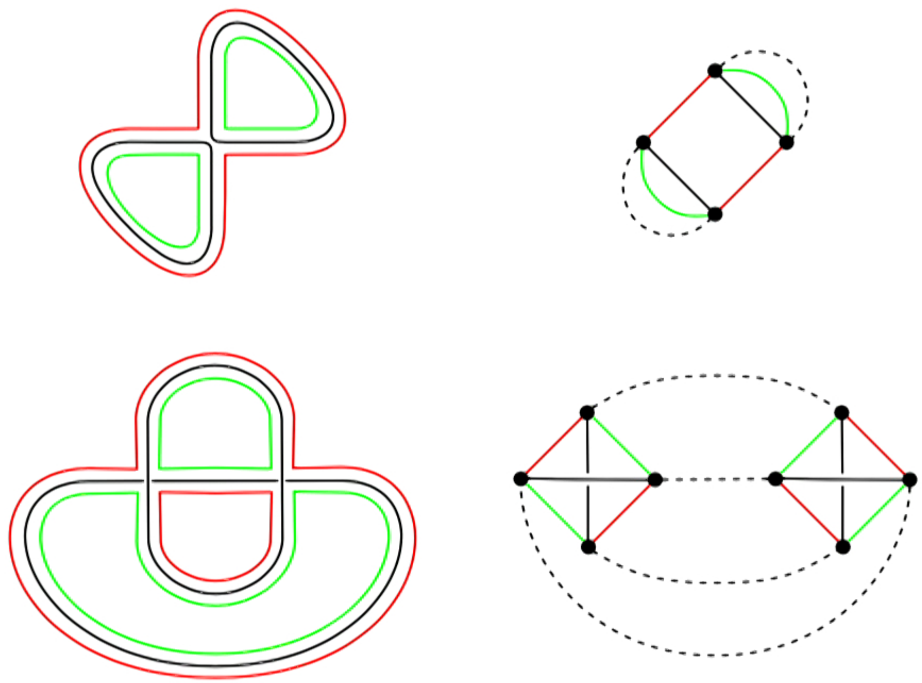
$$g(\mathcal{B}_a) = \frac{1}{2}$$

Feynman graphs

Also two representations:

- **Stranded** representation: interaction vertices connected by propagators, which are ribbon edges with an internal line
- **Colored** representation: vertices in interaction bubbles connected by a new set of edges of color 0

Examples



Standard scaling

Need to specify how the coupling constants **scale** as $N, D \to \infty$

$$S = ND\left(\frac{1}{2}\operatorname{tr}(X_\mu X_\mu^T) + \sum_a \tau_a \mathcal{I}_{\mathcal{B}_a}(X)\right)$$

- Large N 't Hooft scaling: τ_a fixed as $N \to \infty$
- Large D standard scaling: τ_a fixed as $D \to \infty$

$\rightarrow$ Familiar scalings for matrix and vector models

Result: free energy admits **well-defined** large N and large D expansions:

$$F = \sum_{g \in \frac{1}{2}\mathbb{N}} \sum_{L \in \mathbb{N}} N^{2-2g} D^{1-L} F_{g,L}$$

Expansions and leading sector

Expansions:

$$F = \sum_{g \in \frac{1}{2}\mathbb{N}} \sum_{L \in \mathbb{N}} N^{2-2g} D^{1-L} F_{g,L}$$

→ Large N expansion governed by genus g, large D expansion governed by the **Gurau degree** L (related to the number of 'loops')

→ Expansion parameter: $1/D$

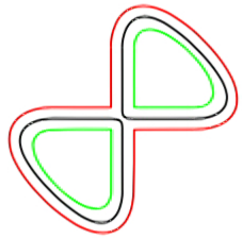
→ Large N and large D limits 'commute'

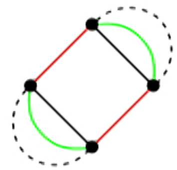
Leading sector:

- As $N \to \infty$, **planar** graphs ($g = 0$) dominate the $1/N$ expansion
- As $D \to \infty$, the sum over planar graphs truncates to a sum over **melonic** graphs ($g = L = 0$)

Melonic graphs

Example:





- Melons can be constructed in a **recursive** way and they can be enumerated
- In the continuum limit ($N = D$), they give rise to the universality class of **branched polymers**
- In quantum field theories in $d > 0$, they correspond to **tadpoles**

Enhanced scaling

Large D **enhanced scaling**:

$$\tau_a = D^{g(\mathcal{B}_a)} \lambda_a$$

with λ_a fixed when $D \to \infty$

Result: free energy F still admits **well-defined** large N and large D expansions

→ F is first expanded at large N

$$F = \sum_{g \in \frac{1}{2}\mathbb{N}} N^{2-2g} F_g$$

→ Then, each F_g is expanded at large D

$$F_g = \sum_{\ell \in \mathbb{N}} D^{1+g-\frac{\ell}{2}} F_{g,\ell}$$

Expansions and leading sector

Expansions:

$$F = \sum_{g \in \frac{1}{2}\mathbb{N}} N^{2-2g} \sum_{\ell \in \mathbb{N}} D^{1+g-\frac{\ell}{2}} F_{g,\ell}$$

→ Large D expansion governed by new parameter ℓ (related to the **index**)

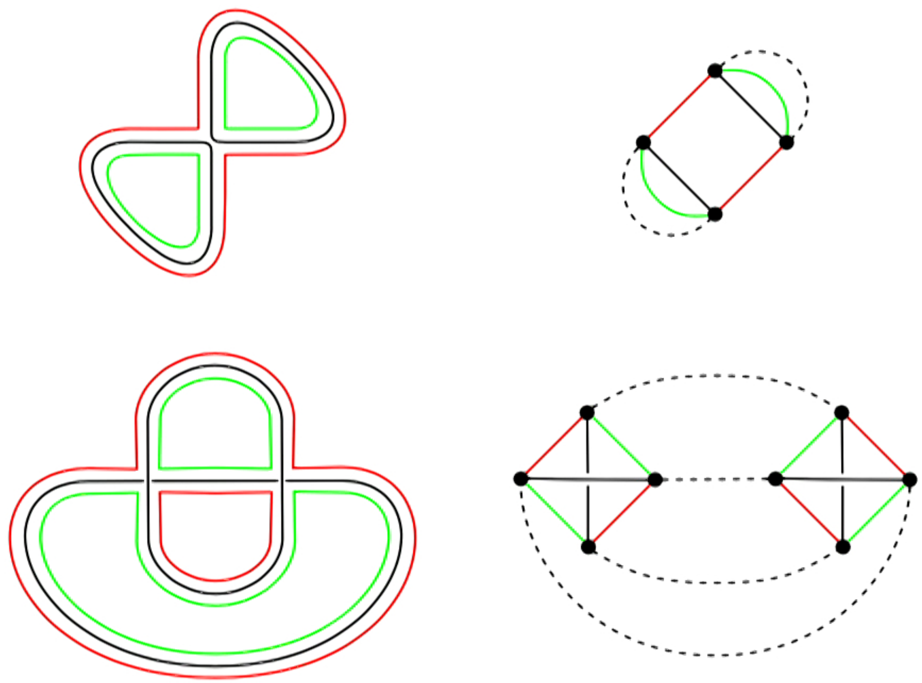
→ Expansion parameter: $1/\sqrt{D}$

→ The two limits do **not** commute: $N \to \infty$ first, $D \to \infty$ second

Leading sector:

- As $N \to \infty$, **planar** graphs ($g = 0$) still dominate
- As $D \to \infty$, the sum over planar graphs now truncates to a sum over **generalized melonic** graphs ($g = \ell = 0$)

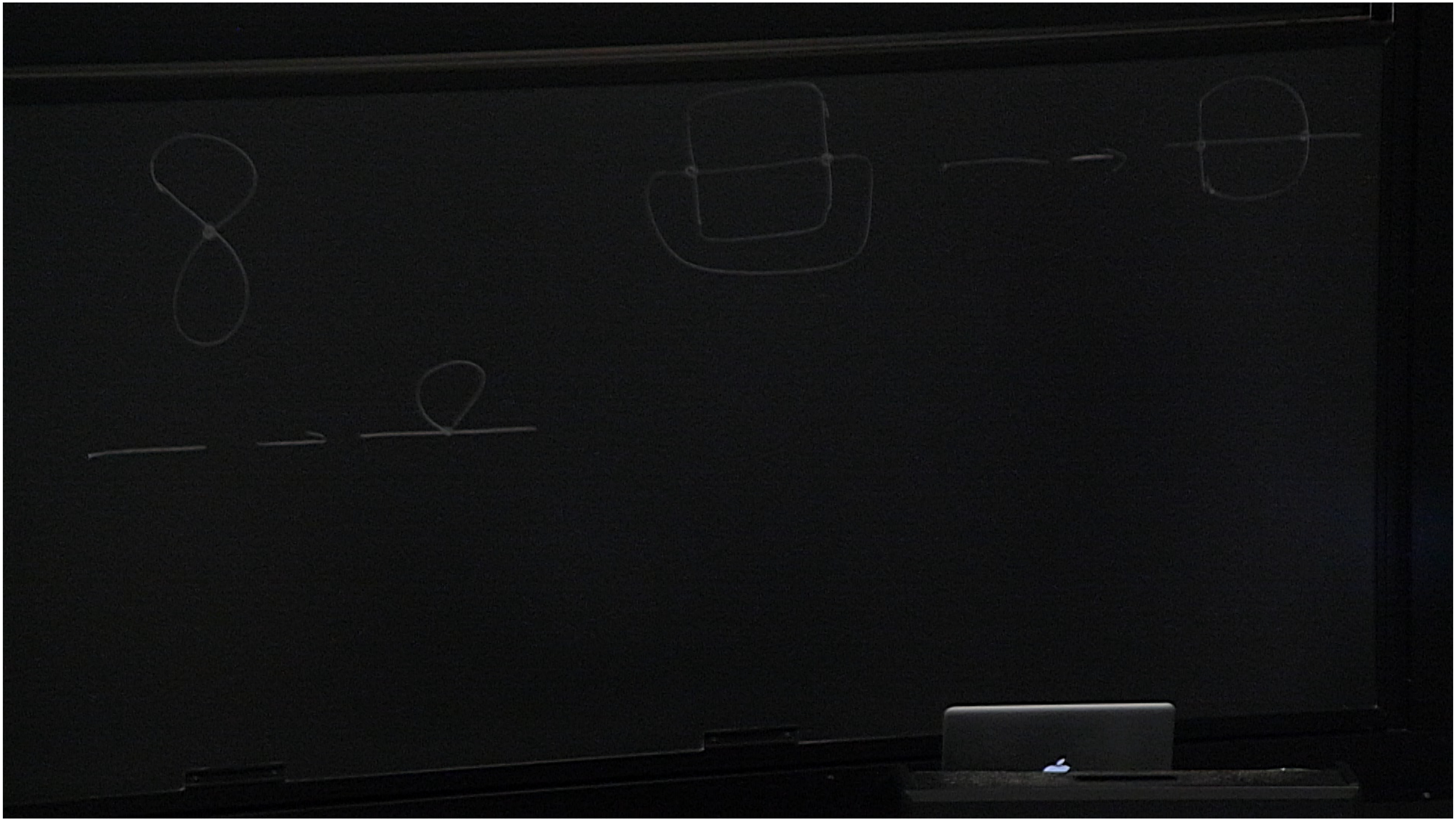
Generalized melonic graphs



Generalized melonic graphs

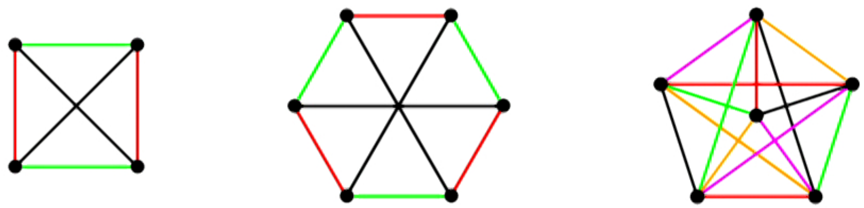
Properties:

- Generalized melons form a strictly **larger** family of graphs than melons
- No general classification of generalized melons, only known to contain **particular** interactions
  - → melonic, maximal single-trace (MST), ...
- There exist a **recursive**, tree-like structure in generalized melons
- Critical behavior in the continuum limit not known in general, but still **branched polymers** for known cases
- Enhanced scaling crucial for building tensor models with **SYK-like physics**



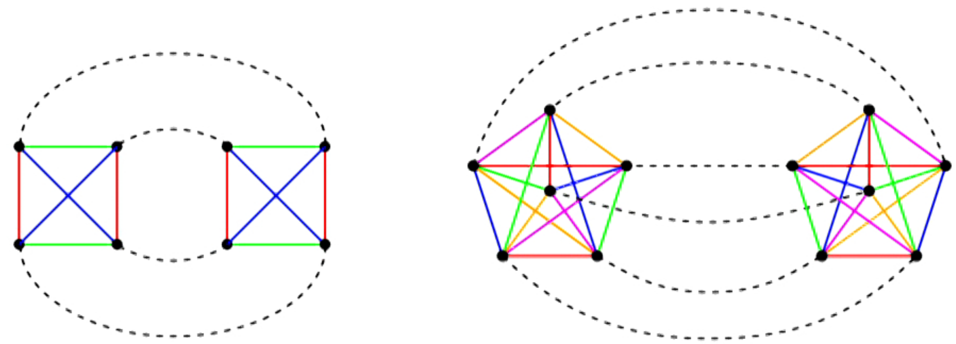
MST interactions

MST interaction: $F_{ij} = 1$ for all pairs of colors (i, j)



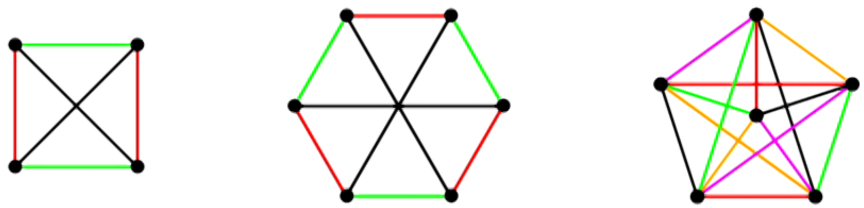
MST interactions

Result: leading sector of models with MST interactions always contain **mirror melons**



MST interactions

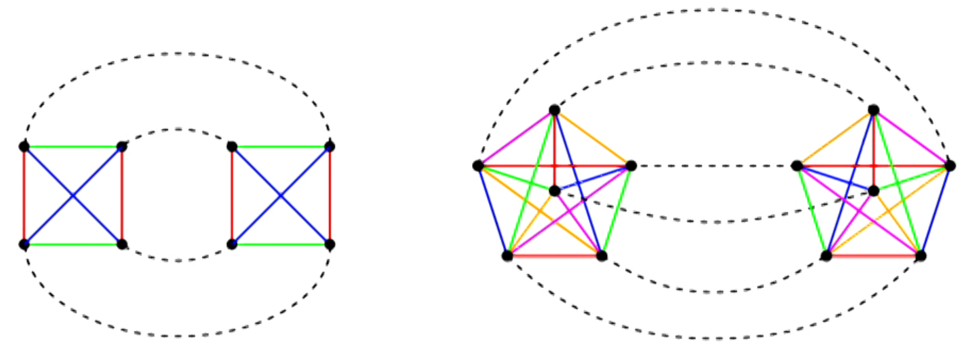
MST interaction: $F_{ij} = 1$ for all pairs of colors (i, j)



→ **Result**: leading sector of models with MST interactions always contain **mirror melons**

MST interactions

Result: leading sector of models with MST interactions always contain **mirror melons**



→ These are the Feynman graphs that yield physics similar to the SYK model

→ They also play an important in large N QFTs in $d \geq 2$

Double scaling limit

Double scaling limit

Ongoing work with Dario Benedetti, Sylvain Carrozza and Reiko Toriumi $\rightarrow$ towards a truncation of the **sum over all genera**

Focus on a model with tetrahedric interaction with coupling λ

$$F(\lambda) = \sum_{g \in \frac{1}{2}\mathbb{N}} N^{2-2g} \sum_{\ell \in \mathbb{N}} D^{1+g-\frac{\ell}{2}} F_{g,\ell}(\lambda)$$

$$= \sum_{g \in \frac{1}{2}\mathbb{N}} \left(\frac{N}{\sqrt{D}} \right)^{2-2g} \sum_{\ell \in \mathbb{N}} D^{2-\frac{\ell}{2}} F_{g,\ell}(\lambda)$$

$\rightarrow$ This suggests to take the limits $N, D \rightarrow \infty$ while keeping $M = N/\sqrt{D}$ **fixed**

$$\lim_{\substack{D\to\infty \\ M \text{ fixed}}} \frac{F(\lambda)}{D^2} = \sum_{g \in \frac{1}{2}\mathbb{N}} M^{2-2g} F_{g,0}(\lambda)$$

Summary

Summary

- New large D limit for matrix models $\rightarrow$ **truncation** of the sum over planar graphs to a sum over generalized melons
- Similar results hold for general matrix-tensors $(X_{a_1 a_2})_{\mu_1 \cdots \mu_r}$ and also for tensors ($N = D$)
  - $\rightarrow$ New class of **solvable** large N QFTs
- **Larger** class of leading order Feynman graphs than for standard scaling; includes MST interactions
  - $\rightarrow$ Able to capture the SYK physics
  - $\rightarrow$ Play important role in tensor field theories

Outlook

Outlook

- Fully **characterize** generalized melonic graphs
- Prove that the double scaled sum over all genera has a **finite** radius of convergence at large D
- Study large N and D expansions for **Hermitian** models
  - $\rightarrow$ Symmetry **reduced** to $U(N) \times O(D)$
  - $\rightarrow$ Matrices are required to be **traceless** in addition
- Understand the melonic limit with **loop equations** (Scwhinger-Dyson equations)