Title: PSI 2019/2020 - Condensed Matter (Wang) - Lecture 15

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Collection: PSI 2019/2020 - Condensed Matter (Wang)

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Today: 2d topological superconductor. non-abelian anyons

Refs. Ivanov. PRL 2001.

Read & Green. PRB 2000

Kitaev. arXiv:0506438

Length & Completeness.

elian anyons

Consider a BCS-like state in 2d spinless fermion.

$$\text{BdG}: H = \sum_k \left[\xi(k) c_k^\dagger c_k + \frac{\Delta}{2}\left(e^{i\theta} c_{-k} c_k + e^{-i\theta} c_k^\dagger c_{-k}^\dagger \right) \right].$$

$$\xi(k) = \frac{k^2}{2m} - E_f \,, \quad \Delta \ll E_f \,. \quad e^{i\theta} = \frac{k_x + i k_y}{|k|} \quad (p_x - ip_y \text{ pairing})$$

No material know to have p-ip pairing for electron.

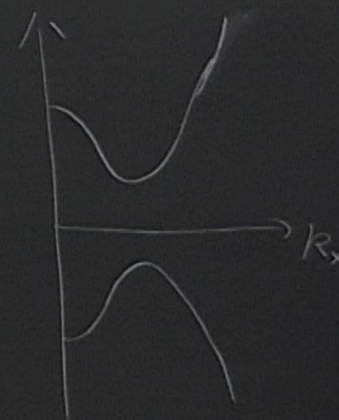
"Fractional quantum Hall" at $\frac{n_e}{B} = 5/2$ is belived to have p-ip pairing for "Composite fermion".

$$H = \sum_{\substack{\infty < k_y < \infty \\ 0 < k_x < \infty}} \left(c_k^{+} \;\; c_{-k} \right) \begin{pmatrix} \xi(k) & \Delta e^{-i\theta} \\ \Delta e^{i\theta} & -\xi(k) \end{pmatrix} \begin{pmatrix} c_k \\ c_{-k}^{+} \end{pmatrix}$$

D

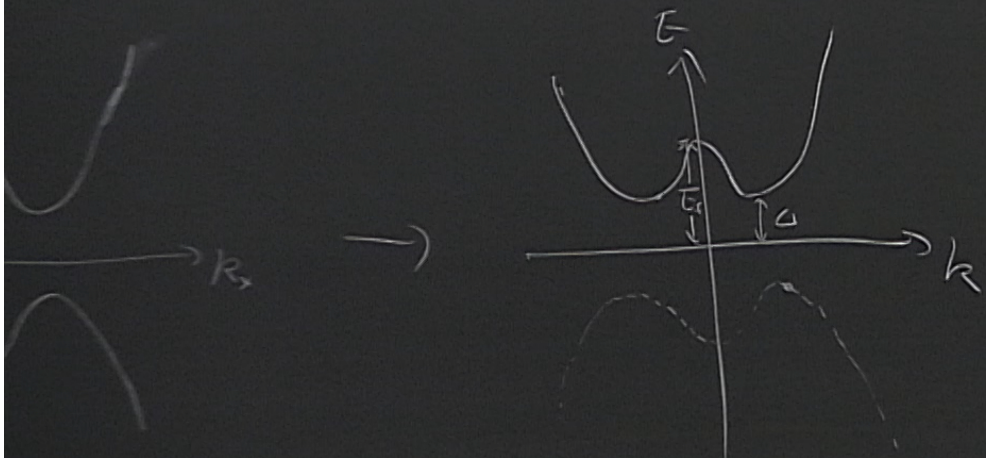
$$\text{Eigenvalue} : \pm\sqrt{\xi^2 + \Delta^2} = \pm E(k)$$

$$H = \sum_{0 < k_x} \left(\gamma^{+}_{k,+} , \; \gamma^{+}_{k,-} \right) \begin{pmatrix} E(k) & \\ & -E(k) \end{pmatrix} \begin{pmatrix} \gamma_{k,+} \\ \gamma_{k,-} \end{pmatrix}$$



Define $\gamma_k = \begin{cases} \gamma_{k,+} & \text{if } k_x > 0 \\ \gamma^{+}_{-k,-} & \text{if } k_x < 0. \end{cases}$

$$\Rightarrow H = \sum_k E(k)\, \gamma_k^{+} \gamma_k$$



Use a redundant formulation

Introduce a two-component Ψ_k, $k \in \mathbb{R}^2$.

$$H = \sum_k \Psi_k^\dagger \begin{pmatrix} \xi(k) & \Delta e^{-i\theta} \\ \Delta e^{i\theta} & \xi(-k) \end{pmatrix} \Psi_k = \sum_k \Psi_k^\dagger H_k \Psi_k$$

only half of Ψ_k is physical.

$$\Psi_k = \underset{\uparrow}{U} \, \Psi_{-k}^{*} \xrightarrow{\text{Some algebra}} \text{requires } \underline{U^\dagger H_k U = -H_{-k}^T} \Rightarrow U = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

2x2 Unitary matrix.

Use a redundant formulation

Introduce a two-component Ψ_k, $k \in \mathbb{R}^2$

$$H = \sum_k \Psi_k^\dagger \begin{pmatrix} \xi(k) & \Delta e^{-i\theta} \\ \Delta e^{i\theta} & \xi(-k) \end{pmatrix} \Psi_{-k} = \sum_k \Psi_{-k}^\dagger H_k \Psi_k$$

only half of Ψ_k is physical.

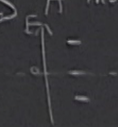
$$\underset{\uparrow}{\Psi_k = U} \, \Psi_{-k}^{*} \xrightarrow{\text{Some algebra}} \text{requires } \underbrace{U^\dagger H_k U = -H_{-k}^T}$$

2×2 Unitary matrix.

→ a.k.a. "particle-hole symmetry of S.C."

→ Redundancy, not a Symm

$$\Rightarrow U = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \Rightarrow$$

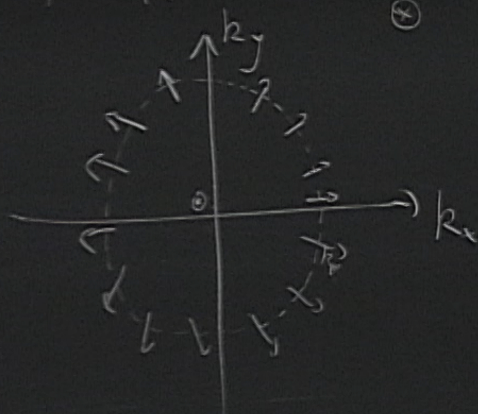


A mode with $E>0$ → Anoter mode with $-E$.

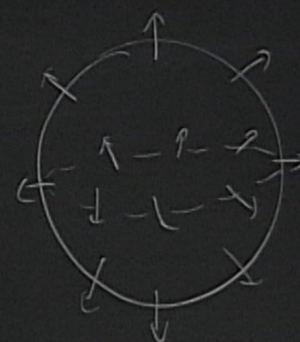
If a single mode at $E=0$ ⇒ Stable!

$$H_k = \xi(k)\,\sigma_z + \Delta\cos\theta\,\sigma_x + \Delta\sin\theta\,\sigma_y = \vec{d}(k)\cdot\vec{\sigma}.$$

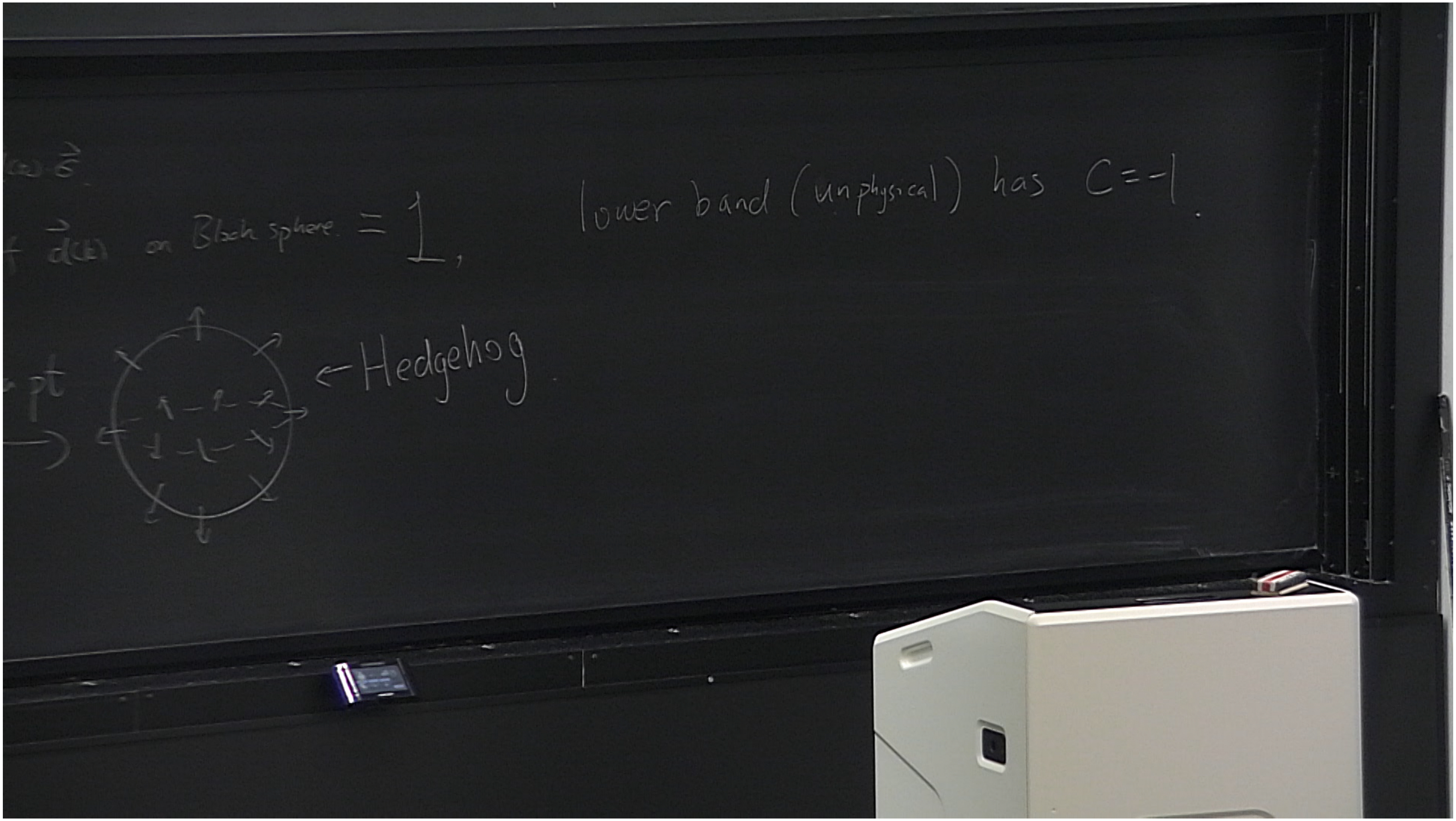
Chern # of upper band = winding number of $\vec{d}(k)$ on Bloch sphere $= 1$, low



identify ∞ as a pt $\rightsquigarrow$



$\leftarrow$ Hedgehog.


(k)·σ
f d(k) on Bloch sphere = 1,
lower band (unphysical) has C = -1.
pt
← Hedgehog

$\xi(k) = \frac{k^2}{2m} - E_f$, $\Delta \ll E_f$. $e^{i\theta} = \frac{k_x + i k_y}{|k|}$ ($p_x - i p_y$ pairing)

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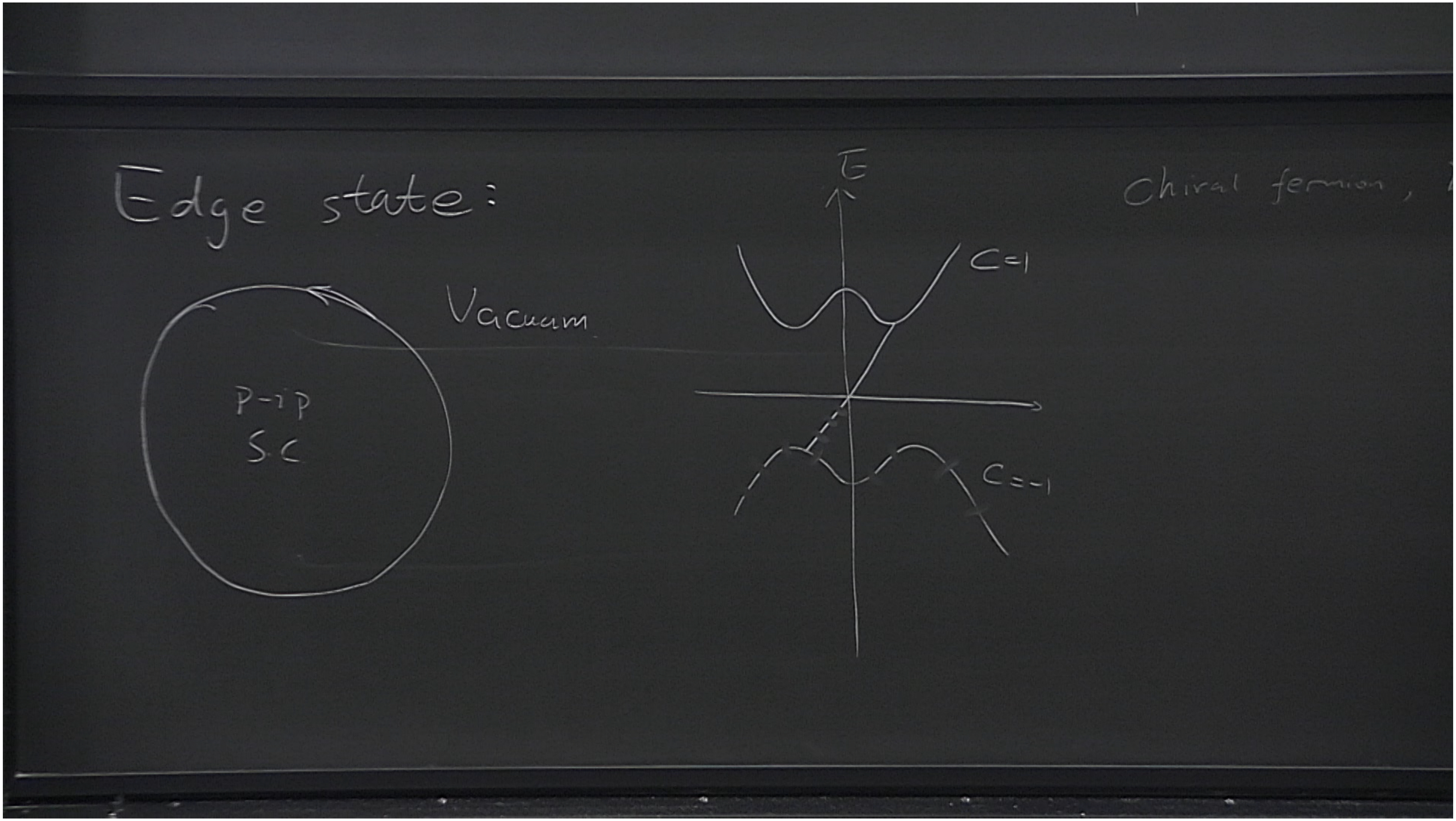
γ_{k+} if $k_x > 0$

$\gamma^{\dagger}_{-k-}$ if $k_x < 0$.

$\sum_k E(k)\, \gamma_k^{\dagger} \gamma_k$

Dispersion $E(k)$ ∼ Same as s-wave.

2d band → Chern number



Edge state:
Vacuum
p-ip
S.C
E
C=1
C=-1
Chiral fermion,

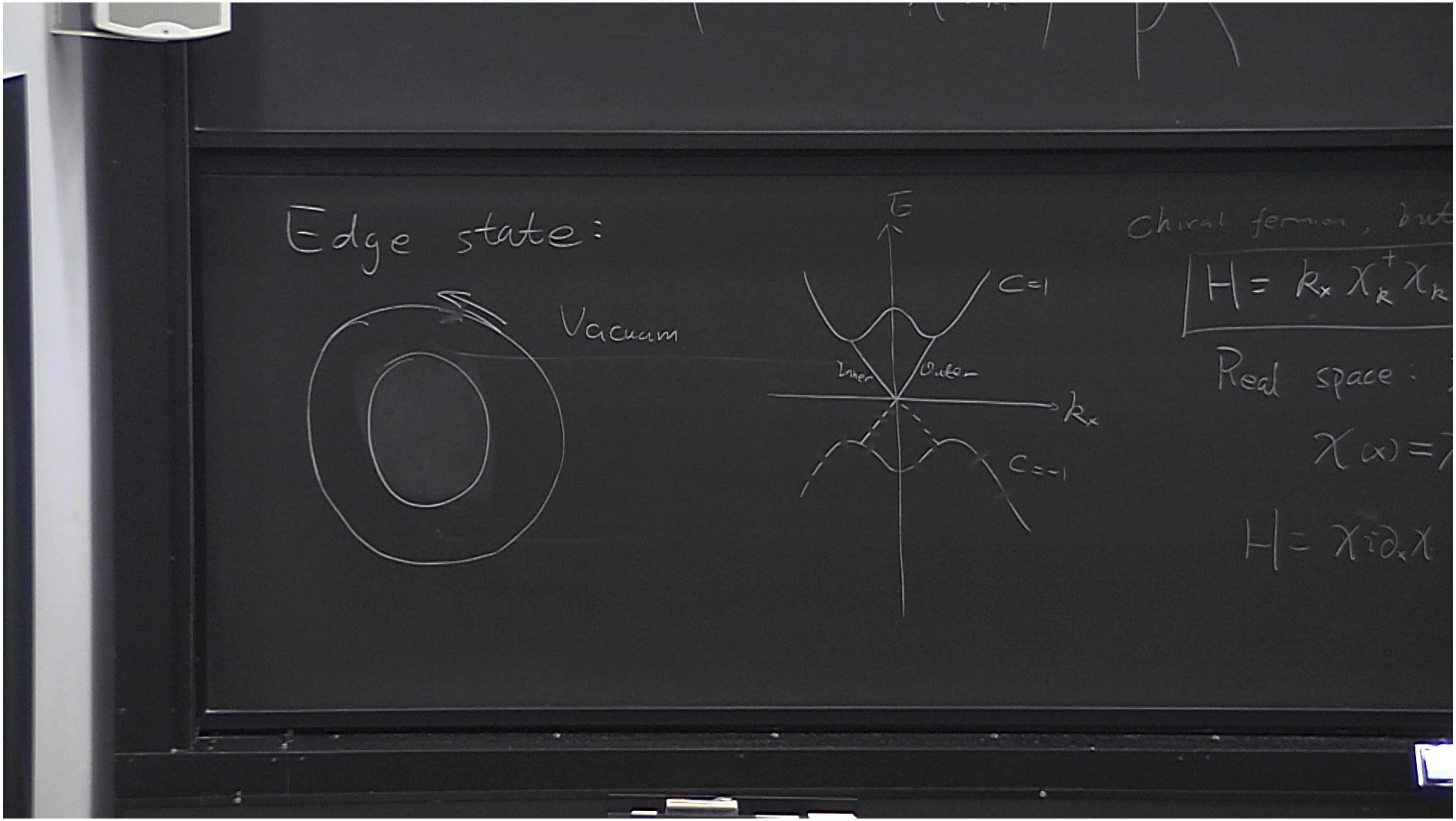
Chiral fermion, but only $E>0$ modes physical.

$$\boxed{H = k_x \chi_k^\dagger \chi_k \quad . \quad \chi_{-k} = \chi_k^\dagger}$$ Chiral Majorana fermion.

Real space: $$\chi(x) = \int_{-\infty}^{\infty} \frac{dk}{2\pi} e^{ikx} \chi(k) = \int_0^{\infty} \frac{dk}{2\pi} \left[e^{ikx}\chi_k + e^{-ikx}\chi_k^\dagger \right]$$: Hermitian.

$\chi(x) = \chi^\dagger(x) \leftarrow$ Majorana fermion
(Real)

$$H = \chi i \partial_x \chi$$


Edge state:
Vacuum
E
C=1
Inner
Outer
k_x
C=-1
Chiral fermion, but
$H = k_x \chi_k^\dagger \chi_k$
Real space:
$\chi(x) =$
$H = \chi i\partial_x \chi$

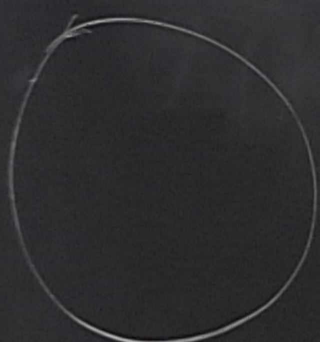
$H = \chi i \partial_x \chi$

wha

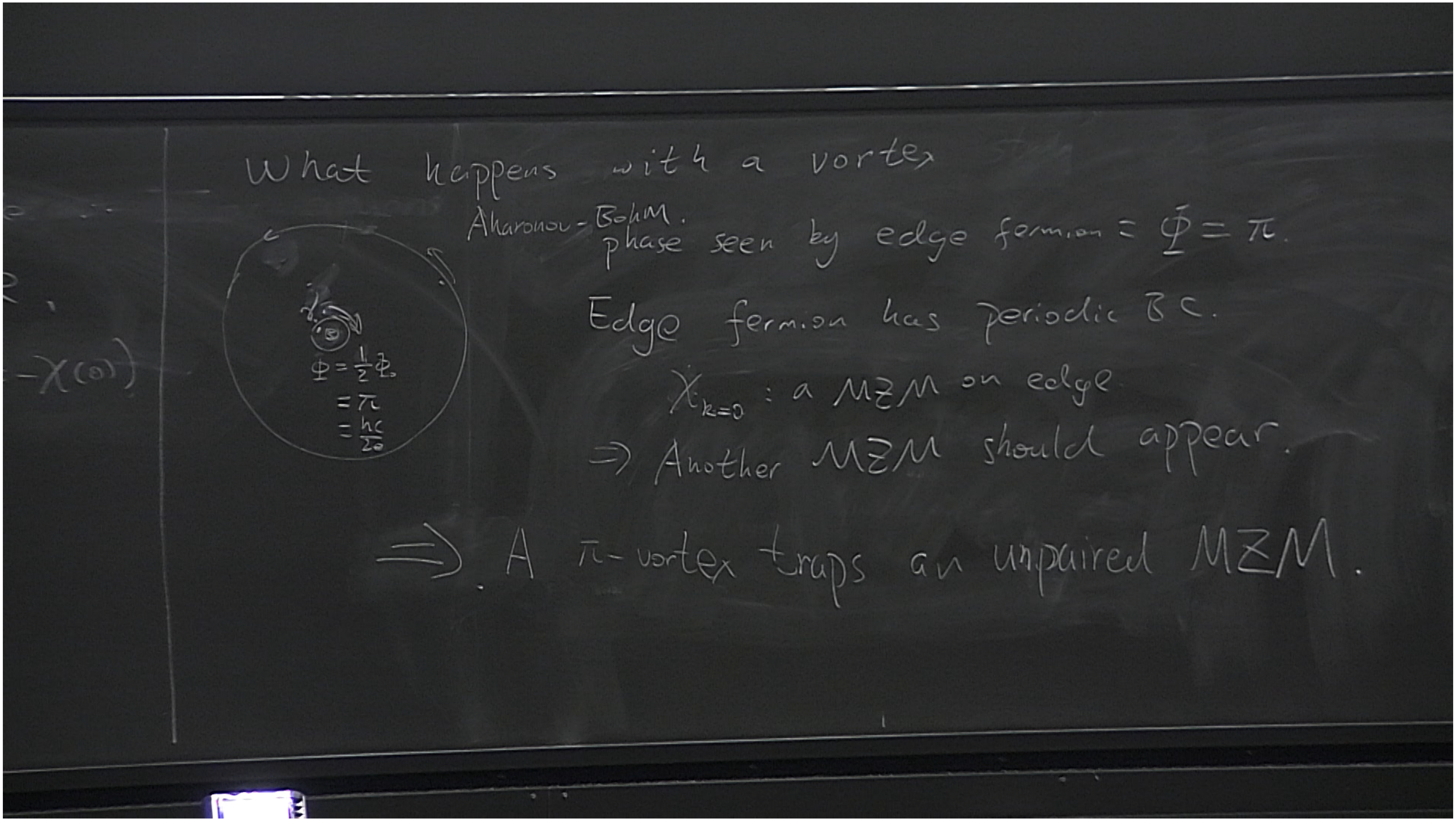
$k_x = 0$?? If this mode exist $\Rightarrow$ Majorana zero mode

$\boxed{\chi_0 = \chi_0^+}$ should not appear alone.

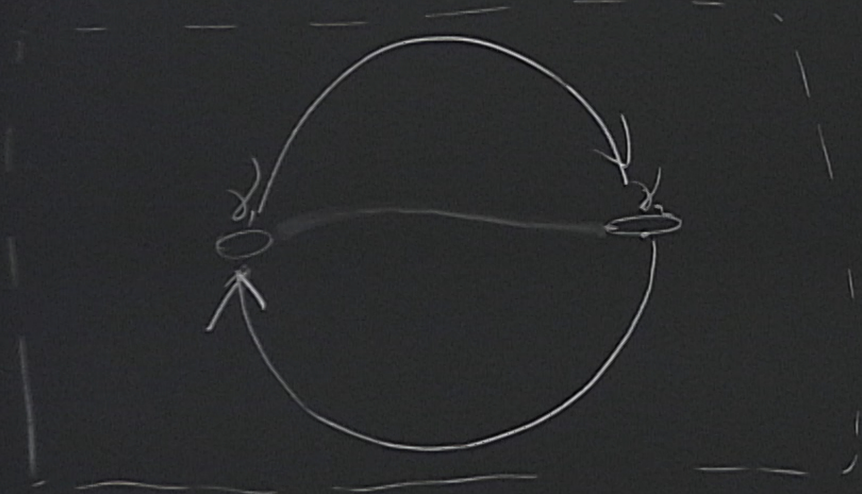
Edge has anti-periodic b.c ($\chi(L) = -\chi(0)$)



$$k_x = \frac{(2n+1)\pi}{L}$$

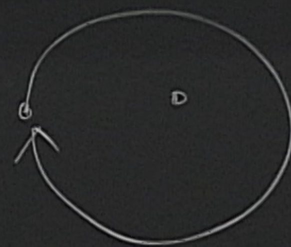

What happens with a vortex
Aharonov-Bohm.
phase seen by edge fermion = $\Phi = \pi$.
Edge fermion has periodic BC.
$\chi_{k=0}$: a MZM on edge
$\Rightarrow$ Another MZM should appear.
$\Rightarrow$ A π-vortex traps an unpaired MZM.
$\Phi = \frac{1}{2}\Phi_0$
$= \pi$
$= \frac{hc}{2e}$
$-\chi(0))$

Consider 2 vortices (far away from each other)



2-dim $\mathcal{H} = (|\Omega\rangle, \hat{c}^{\dagger}|\Omega\rangle)$

$\hat{c} = \frac{\gamma_1 + i\gamma_2}{2}$, $\hat{c}|\Omega\rangle = 0$

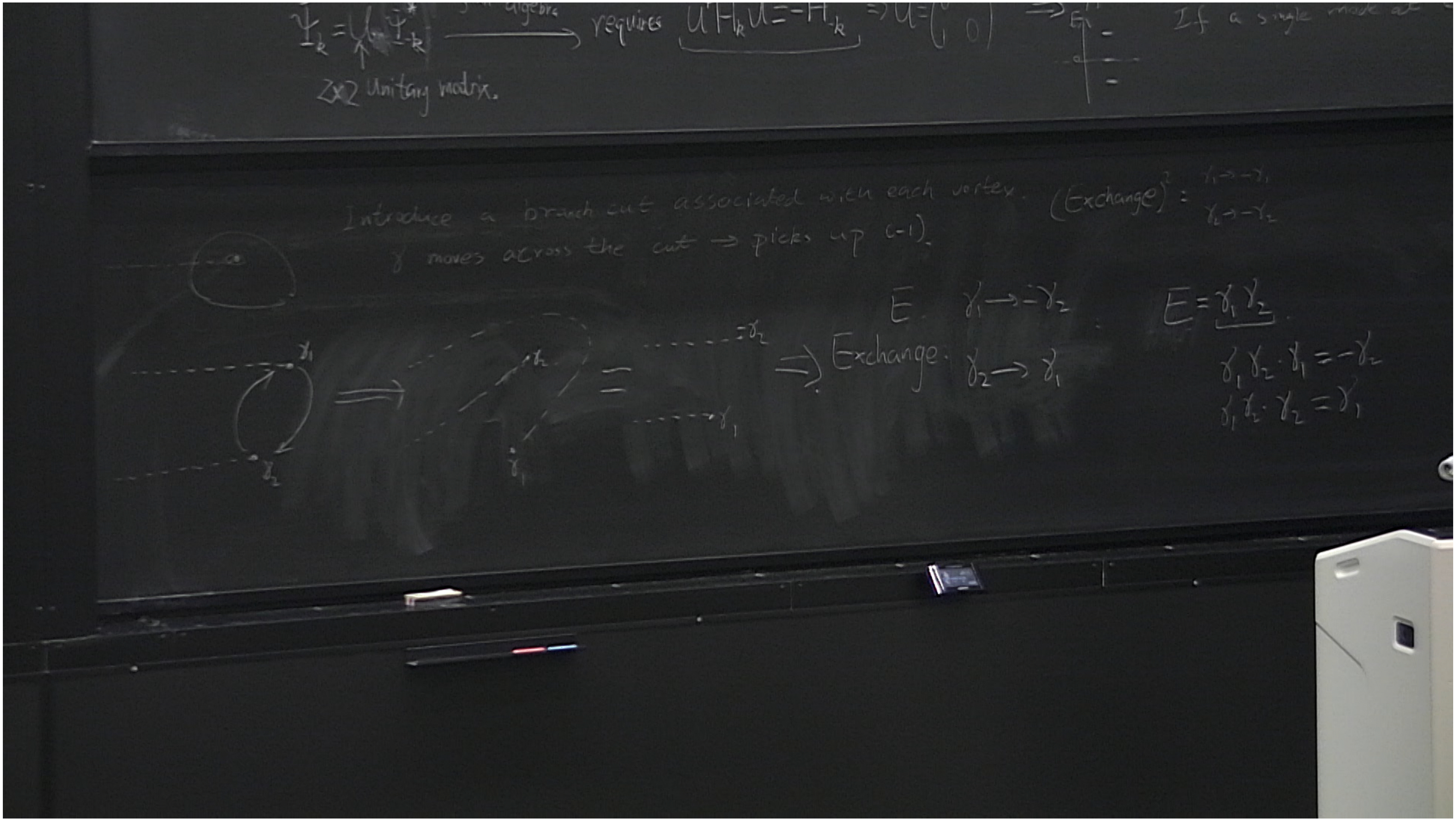
Exchange twice $\simeq$  3d: has to be trivial.

2d: non-contractable.

$\Rightarrow$ Exchange doesn't have to give ± 1.

If exchange: $e^{i\theta} \neq \pm 1 \Rightarrow$ Abelian anyon

If degeneracy: Matrix $\Rightarrow$ non-Abelian anyon.


2x2 Unitary matrix.
requires $U^\dagger H_k U = -H_{-k}$
Introduce a branch cut associated with each vortex.
γ moves across the cut $\Rightarrow$ picks up (-1).
$(\text{Exchange})^2$: $\gamma_1 \to -\gamma_1$, $\gamma_2 \to -\gamma_2$
E: $\gamma_1 \to -\gamma_2$
$\Rightarrow$ Exchange: $\gamma_2 \to \gamma_1$
$E = \gamma_1 \gamma_2$
$\gamma_1 \gamma_2 \cdot \gamma_1 = -\gamma_2$
$\gamma_1 \gamma_2 \cdot \gamma_2 = \gamma_1$

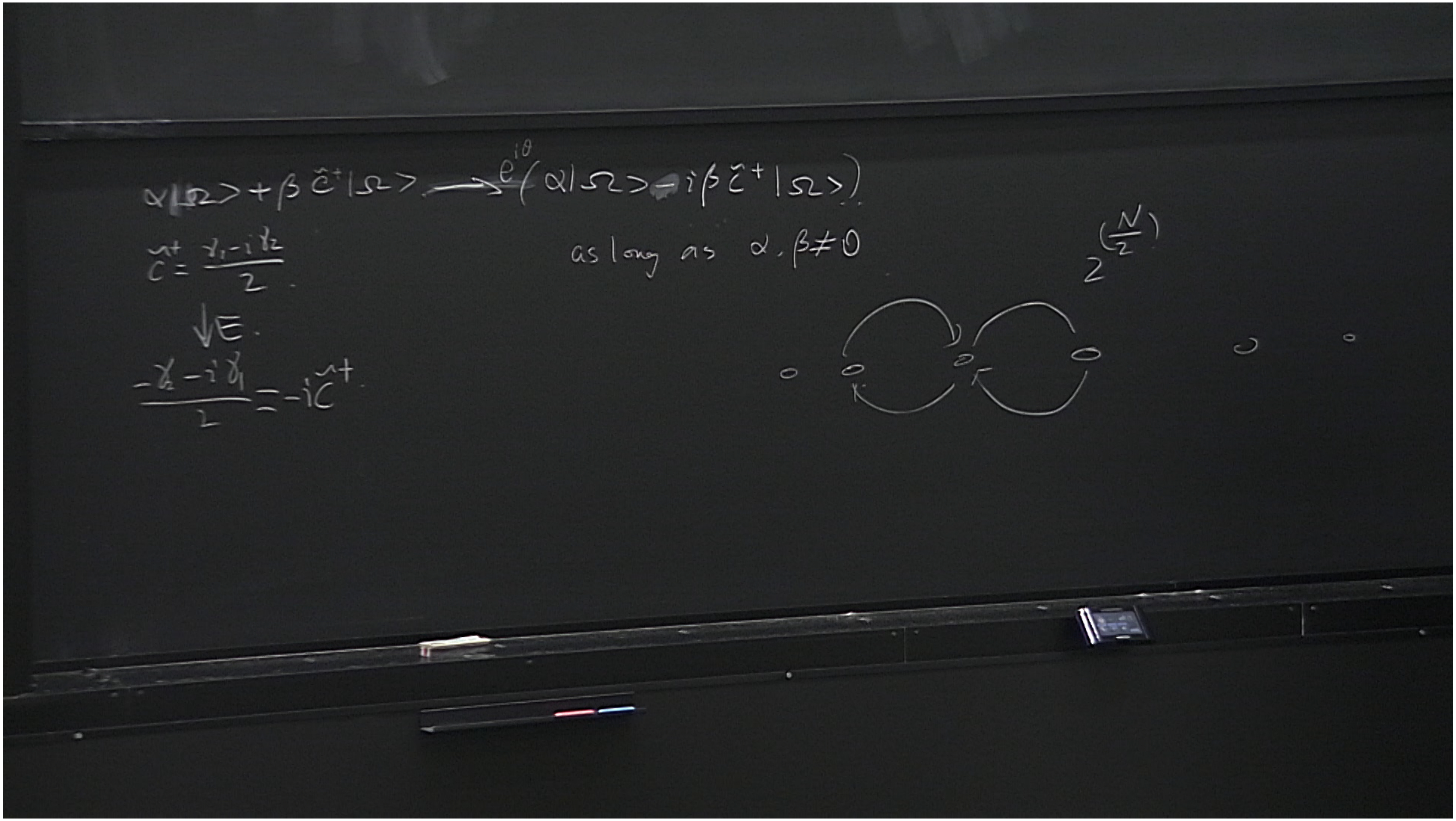
$$\alpha|\Omega\rangle + \beta\,\tilde{c}^{\dagger}|\Omega\rangle \longrightarrow e^{i\theta}\left(\alpha|\Omega\rangle - i\beta\,\tilde{c}^{\dagger}|\Omega\rangle\right)$$

$$\tilde{c}^{\dagger} = \frac{\gamma_1 - i\gamma_2}{2}$$

as long as $\alpha, \beta \neq 0$

$$\downarrow E$$

$$\frac{-\gamma_2 - i\gamma_1}{2} = -i\tilde{c}^{\dagger}$$


$\alpha|\Omega\rangle + \beta \tilde{c}^{\dagger}|\Omega\rangle \longrightarrow e^{i\theta}(\alpha|\Omega\rangle - i\beta \tilde{c}^{\dagger}|\Omega\rangle)$
$\tilde{c}^{\dagger} = \frac{\gamma_1 - i\gamma_2}{2}$
$\downarrow E$.
$\frac{-\gamma_2 - i\gamma_1}{2} = -i\tilde{c}^{\dagger}$
as long as $\alpha, \beta \neq 0$
$2^{(\frac{N}{2})}$