Title: PSI 2019/2020 - Condensed Matter (Wang) - Lecture 13

Speakers: Chong Wang

Collection: PSI 2019/2020 - Condensed Matter (Wang)

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Today:

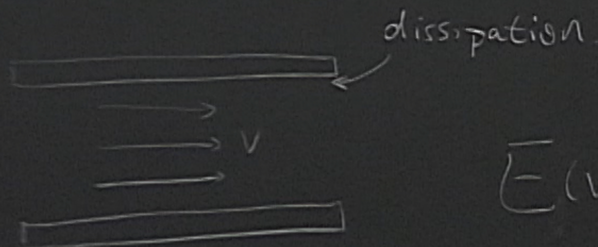
- Coments on. ① Persistent flow
  ② Anderson-Higgs

- Abrikosov vortex lattice

Superfluidity (Landau)

Assume Galilean (He-3/4, metals with small Fermi surfaces)

$\varepsilon(k) \sim \frac{k^2}{2m}$

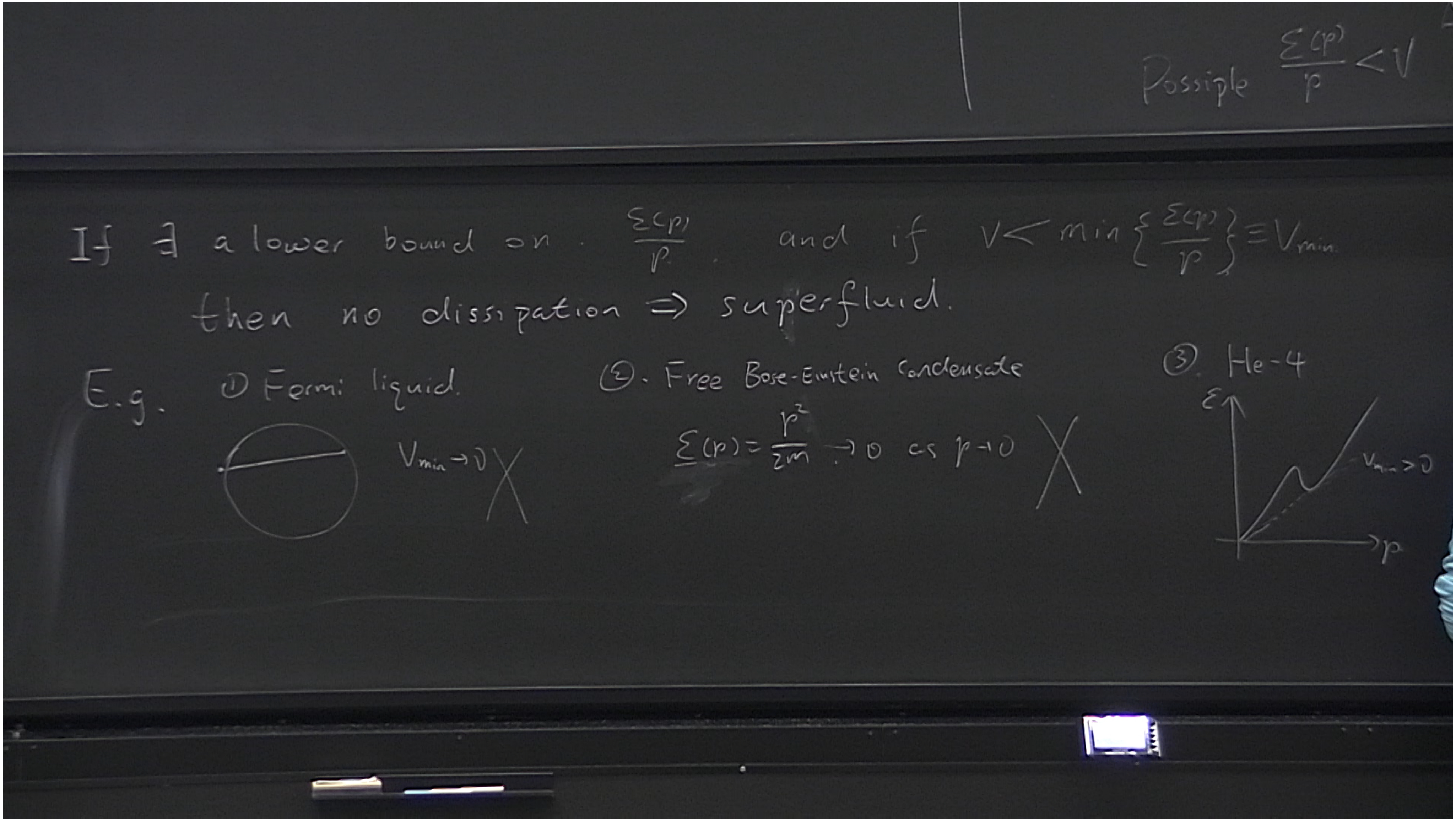


$$E(v,P) = E(v=0,p) + \vec{p}\cdot\vec{v} + \frac{1}{2}Mv^2$$

Suppose a quasi-particle carries momentum $\vec{p}$ and energy $\varepsilon(p)>0$ at rest.

$$\Delta E = \varepsilon(p) + \vec{p}\cdot\vec{v} \quad \text{favored if} < 0$$

Possible $\frac{\varepsilon(p)}{p} < V$


Possible $\frac{\varepsilon(p)}{p} < V$
If $\exists$ a lower bound on $\frac{\varepsilon(p)}{p}$ and if $V < \min\left\{\frac{\varepsilon(p)}{p}\right\} \equiv V_{min}$
then no dissipation $\Rightarrow$ superfluid.
E.g.
① Fermi liquid.
$V_{min} \to 0$ X
②. Free Bose-Einstein Condensate
$\varepsilon(p) = \frac{p^2}{2m} \to 0$ as $p \to 0$ X
③. He-4
ε
$V_{min} > 0$
p

$\Delta E = \varepsilon(p) + p \cdot v$ favored if < 0

Possible $\frac{\varepsilon(p)}{p} < V$

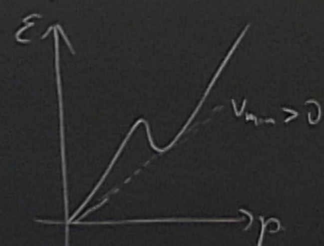
d if $v < \min\left\{\frac{\varepsilon(p)}{p}\right\} \equiv V_{min}$

-fluid.

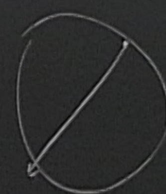
nstein condensate

$\to 0$ as $p \to 0$ X

③ He-4



④ Paired Fermi Surface.



$\frac{\Delta}{2k_F} \approx V_{min}$

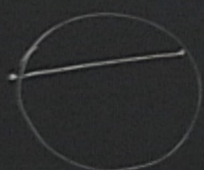
(He-3)

Possible $\frac{\varepsilon(p)}{p} < V$

If $\exists$ a lower bound on $\frac{\varepsilon(p)}{p}$ and if $V < \min\left\{\frac{\varepsilon(p)}{p}\right\} \equiv V_{min}$

then no dissipation $\Rightarrow$ superfluid.

E.g. ① Fermi liquid



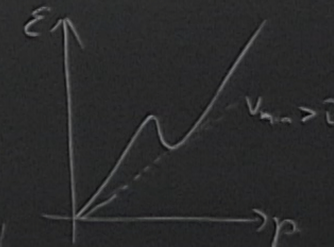
$V_{min} \to 0$ ✗

②. Free Bose-Einstein Condensate

$\varepsilon(p) = \frac{p^2}{2m} \to 0$ as $p \to 0$ ✗

Not SF

③ He-4



$\Delta E = \varepsilon(p) + p \cdot v$

Possible $\frac{\varepsilon(p)}{p} < v$

and if $v < \min\left\{\frac{\varepsilon(p)}{p}\right\} \equiv v_{min}$

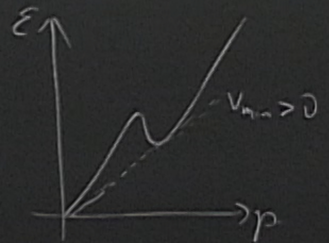
uperfluid.

Bose-Einstein Condensate

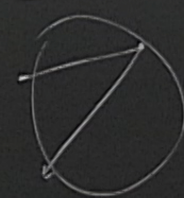
$\varepsilon = \frac{p^2}{2m} \to 0$ as $p \to 0$ X

SF

③ He-4

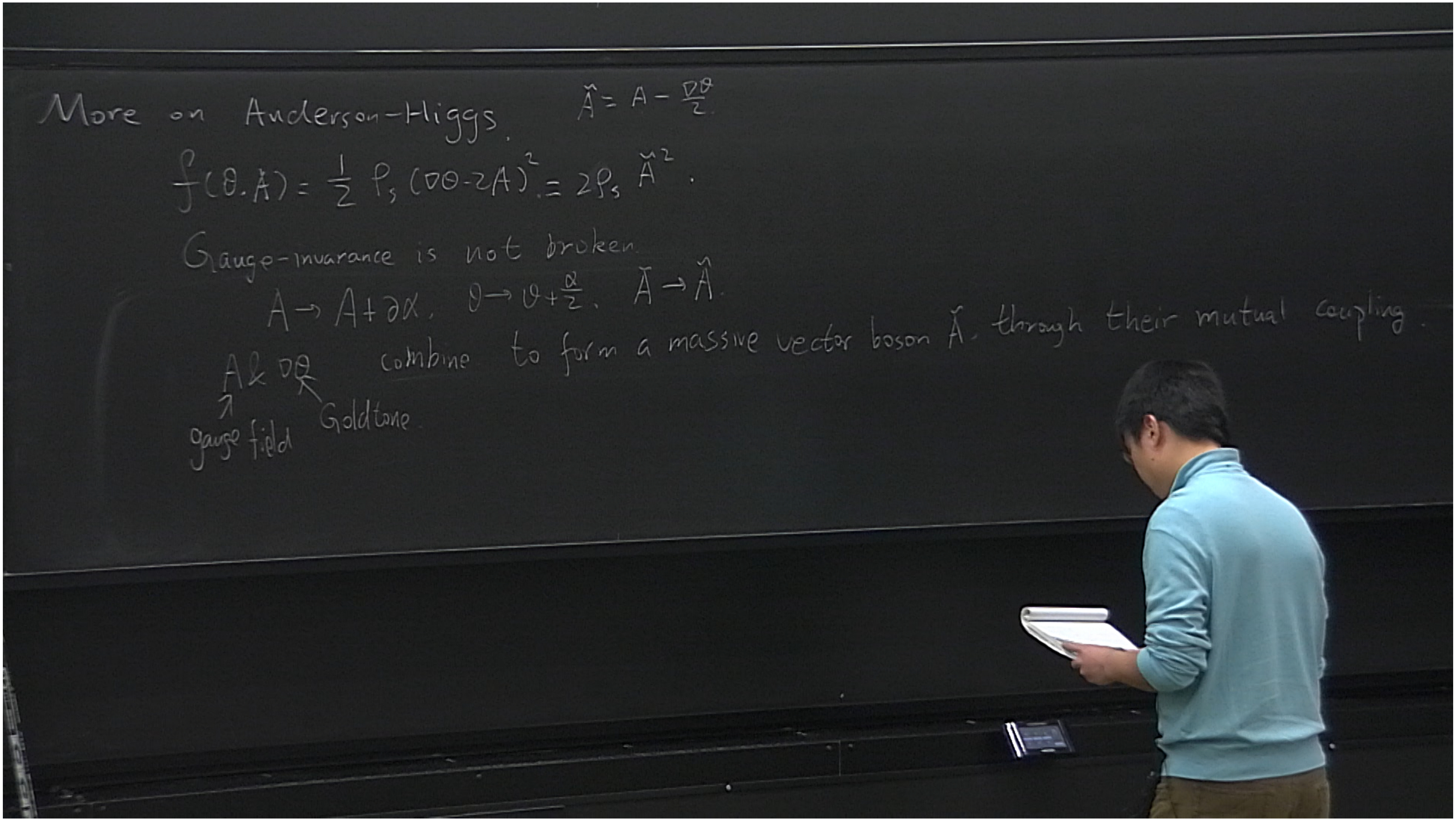


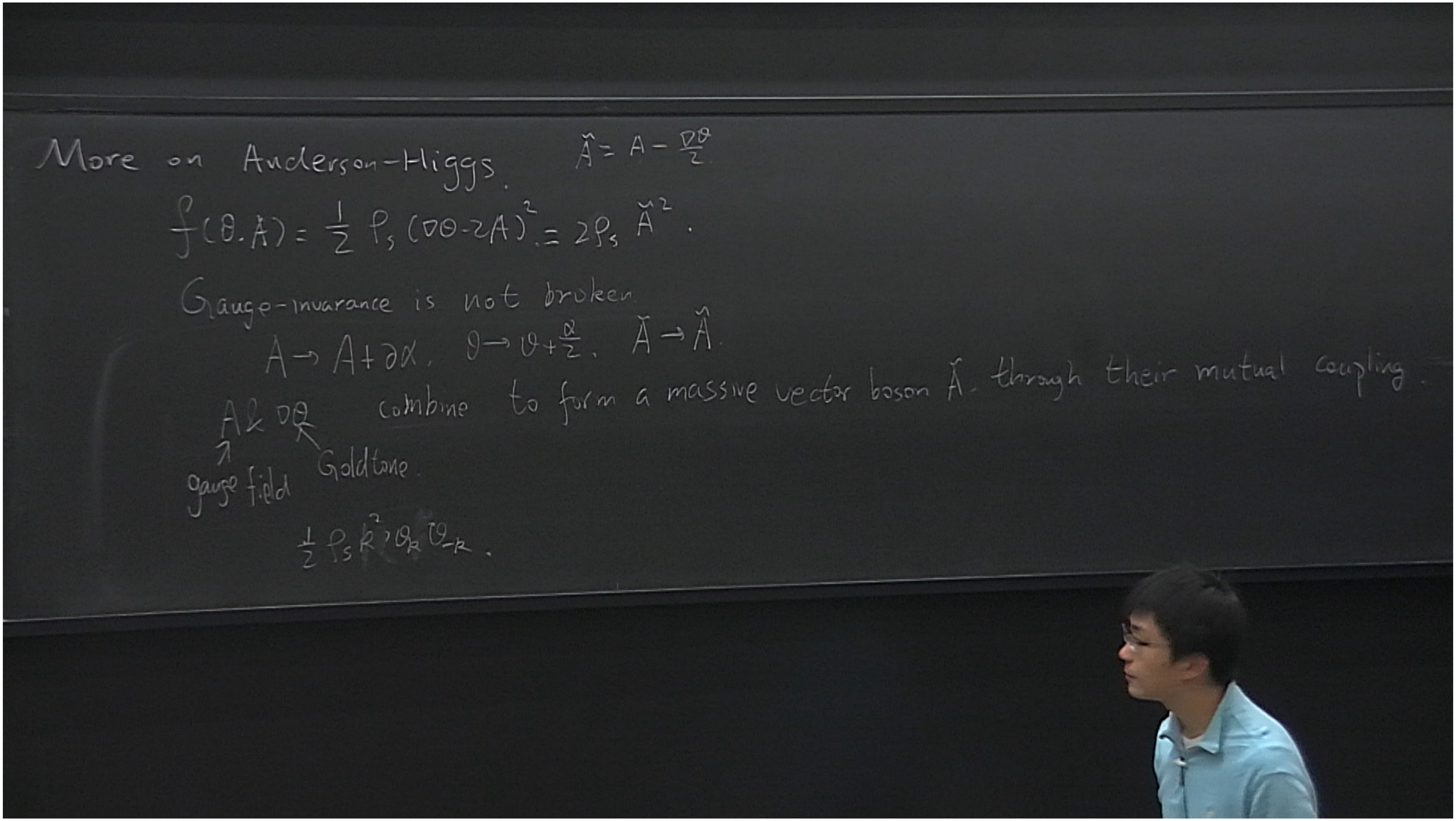
④ Paired Fermi Surface



$\frac{\Delta}{2k_F} \approx v_{min}$

SF


More on Anderson-Higgs. $\tilde{A} = A - \frac{\nabla\theta}{2}$.
$f(\theta, A) = \frac{1}{2}\rho_s(\nabla\theta - 2A)^2 = 2\rho_s \tilde{A}^2$.
Gauge-invarance is not broken.
$A \to A + \partial\alpha$, $\theta \to \theta + \frac{\alpha}{2}$, $\tilde{A} \to \tilde{A}$.
A & $\nabla\theta$ combine to form a massive vector boson $\tilde{A}$ through their mutual coupling.
gauge field
Goldtone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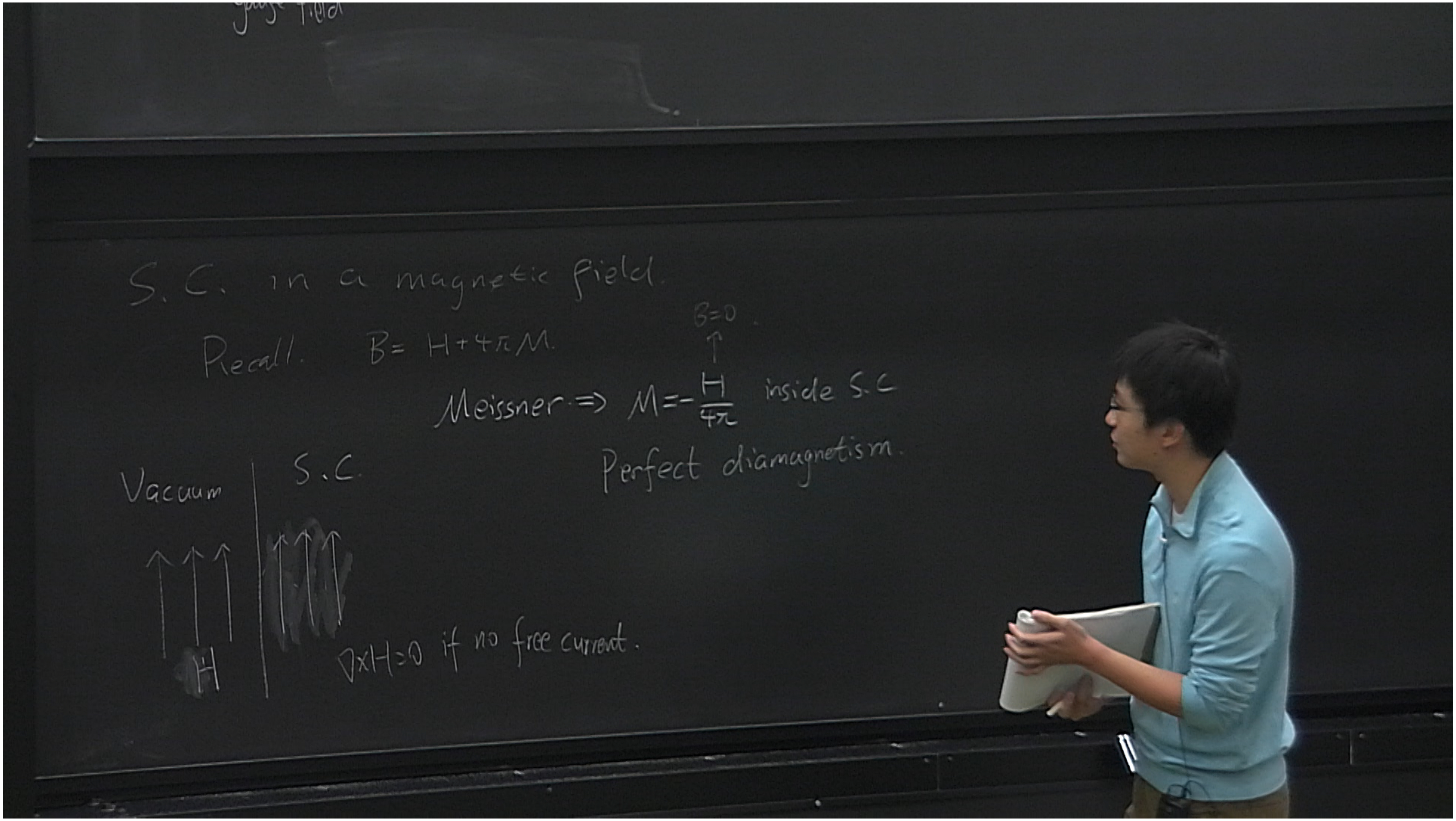
gauge field
Goldstone.
$\frac{1}{2}\rho_s k^2 \theta_k \theta_{-k}$.


S.C. in a magnetic field.
Recall. B= H+4πM.
B=0
Meissner ⇒ M=-H/4π inside S.C
Perfect diamagnetism.
Vacuum
S.C.
H
∇×H=0 if no free current.

$$f(B) = \frac{1}{2}\left|(\nabla - i2A)\Phi\right|^2 - \frac{1}{\xi^2}|\Phi|^2 + b|\Phi|^4 + \frac{1}{8\pi}B^2$$

$B=0$

$\uparrow$

$= -\frac{H}{4\pi}$ inside S.C.

fect diamagnetism.

$$g(H) = f(B) - \frac{BH}{4\pi}$$

Rule: fix H, minimize g w.r.t. B.

S.C. : $B=0$. $g(H) = f_{GL}(B=0) = f_{S.C}(0)$

Normal metal: $g_{nm}(H) \simeq f_{nm}(0) + \frac{B^2}{8\pi} - \frac{BH}{4\pi} = f_{nm}(0) - \frac{H^2}{8\pi}$

minimize w.r.t. B

At $T=0$ $f_{S.C}(0) - f_{nm}(0) \simeq -n_e \cdot \frac{\Delta}{E_F} \Delta$

$\Rightarrow g_{sc} = g_{nm}$ at $\frac{H_c^2}{8\pi} = \frac{n_0 \Delta_0^2}{E_F} = \left(\frac{\Phi_0}{\lambda_L \xi}\right)^2$ $\Phi_0 = \frac{hc}{e} = 2\pi$

S.C. : $B=0$. $g(H) = f_{GL}(B=0) = f_{S.C}(0)$

Normal metal: $g_{nm}(H) \simeq \underbrace{f_{nm}(0) + \frac{B^2}{8\pi} - \frac{BH}{4\pi}}_{} = f_{nm}(0) - \frac{H^2}{8\pi}$

minimize w.r.t. B

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$\Phi_0 = \frac{hc}{e} = 2\pi$

H_c: thermodynamic critical f

S.C. : $B=0$. $g(H) = f_{GL}(B=0) = f_{S.C}(0)$

Superconductor

Normal metal: $g_{nm}(H) \simeq \underbrace{f_{nm}(0) + \frac{B^2}{8\pi} - \frac{BH}{4\pi}}_{\text{minimize w.r.t. } B} = f_{nm}(0) - \frac{H^2}{8\pi}$

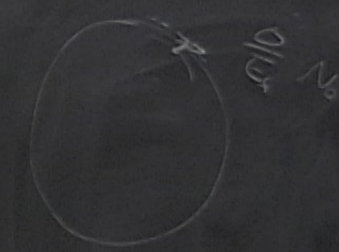
(nm)

$\boxed{\text{At } T=0}$, $\boxed{f_{S.C}(0) - f_{nm}(0) \simeq -n_0 \frac{\Delta}{E_F}\Delta}$

$\Rightarrow g_{sc} = g_{nm}$ at $\boxed{\frac{H_c^2}{8\pi} = \frac{n_0 \Delta_0^2}{E_F} = \left(\frac{\Phi_0}{\lambda_L \xi}\right)^2}$

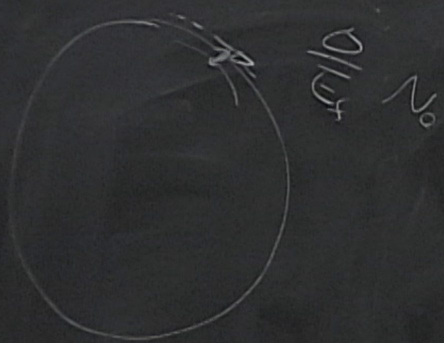
$\Phi_0 = \frac{hc}{e} = 2\pi$

H_c: thermodynamic critical field.



$H < H_c$, $g_{sc} < g_{nm} \Rightarrow$ S.C

$H > H_c$, $g_{nm} < g_{cs} \Rightarrow$ N.M.



odynamic critical field.

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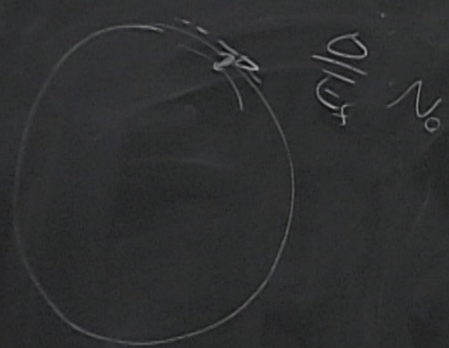
Type I



...dynamic critical field.

$\Rightarrow g_{sc} = f_{nm}$ at $\frac{H_c^2}{8\pi} = \frac{\quad}{E_F} = \left(\frac{\quad}{\lambda_L \xi}\right)$

H_c: thermodynamic

$H = H_c$

Type 1: $\xi \gg \lambda_L$ (Typical pure metal S.C)

(in fact only need $\xi > \sqrt{2}\lambda_L$, no proof)

Surface tension of boundary > 0 ? < 0 ?

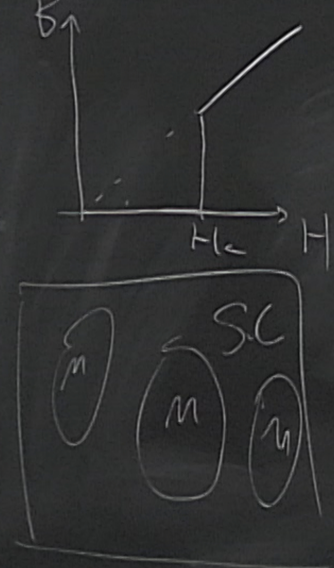
H Normal Metal | S.C

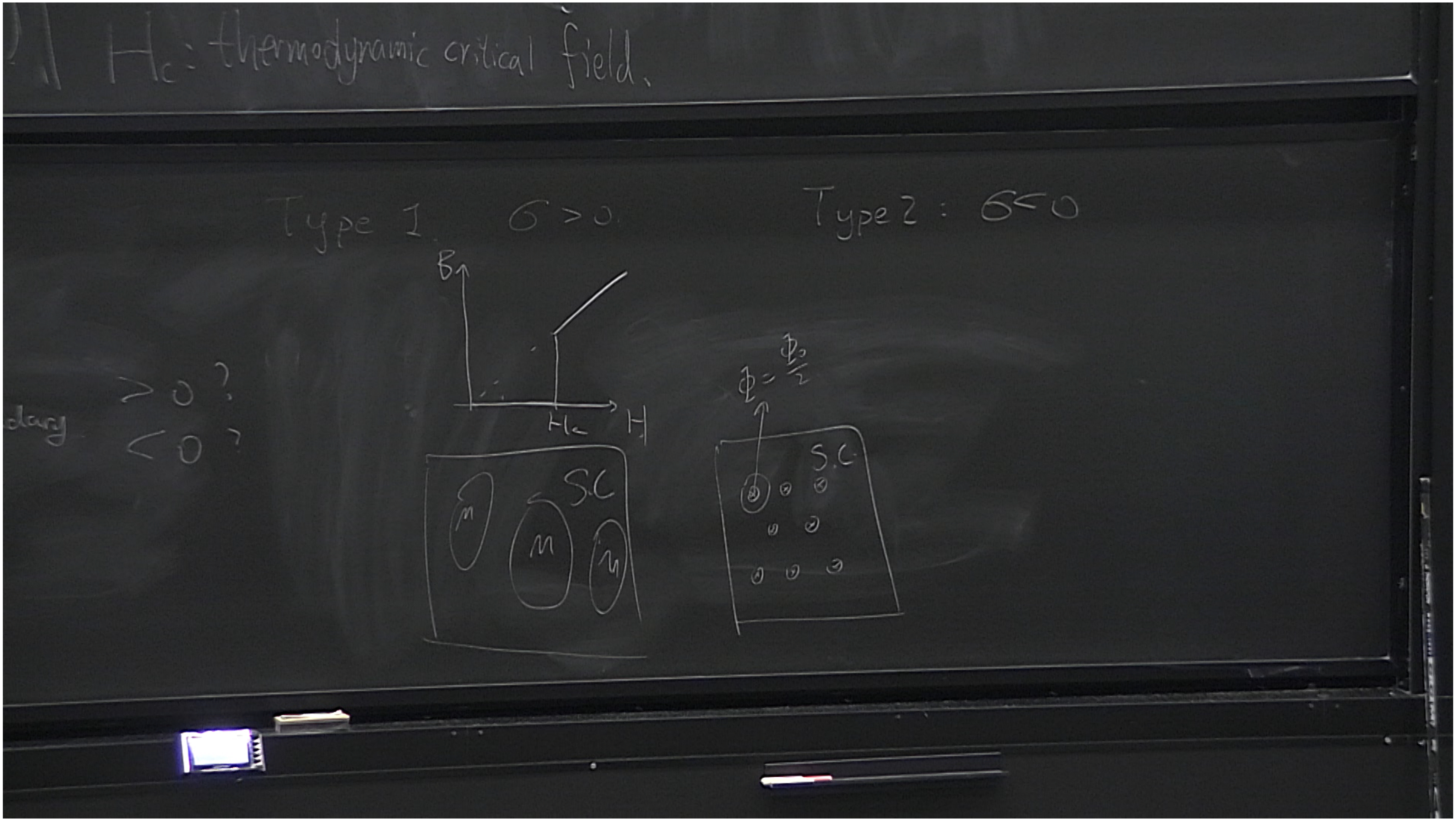
H_c : thermodynamic critical field.

Type I $\sigma > 0$.

dary > 0 ?

< 0 ?



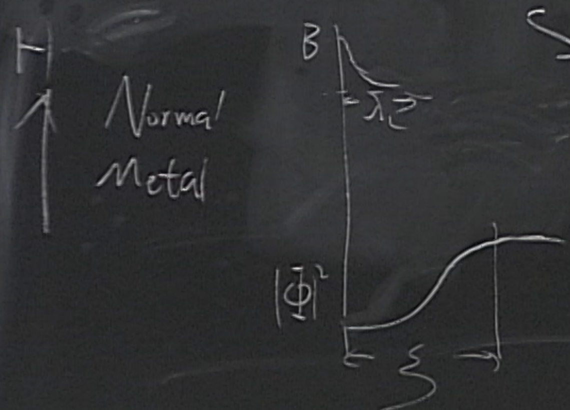


H_c: thermodynamic critical field.
Type 1. $\sigma > 0$.
Type 2 : $\sigma < 0$
B
H_c
H
S.C
S.C
$\Phi = \frac{\Phi_0}{2}$
> 0 ?
< 0 ?

$H = H_c$.

Type 1: $\xi \gg \lambda_L$ (Typical pure metal S.C)

(in fact only need $\xi > \sqrt{2}\lambda_L$, no proof)



S.C

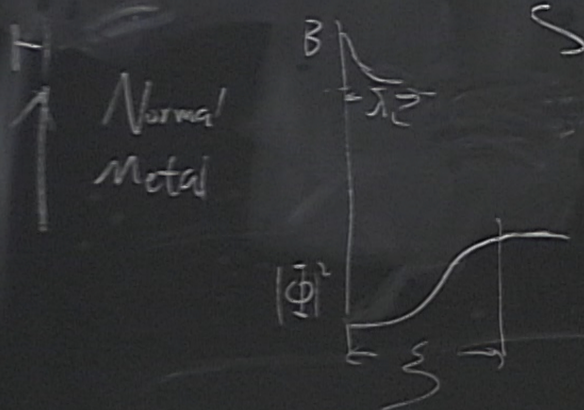
Surface tension of boundary

σ.

$H = H_c$

Type 1: $\xi >> \lambda_L$ (Typical pure metal S.C)

(in fact only need $\xi > \sqrt{2}\lambda_L$, no proof)



Surface tension of boundary >0 ? <0 ?

σ.

$$\sigma = \xi (f_{nm} - f_{sc}) \sim \xi \frac{H_c^2}{8\pi} > 0.$$

Type 2, $\xi \ll \lambda_L$ $(\xi < \sqrt{2}\lambda_L) \rightarrow$ Alloy S.C., High-T_c Cuprate.

$$B = B_0 e^{-\frac{x}{\lambda_L}}, \qquad g = \underbrace{\frac{1}{2}\rho_s(\nabla\theta - 2A)^2}_{\frac{1}{8\rho_s} j_0^2} + \frac{B^2}{2\pi} - \frac{BH}{4\pi}$$

$$j \sim \rho_s(\nabla\theta - 2A) \sim \frac{1}{4\pi}\partial_x B = \frac{1}{4\pi\lambda_L} B$$

Type 2, $\xi \ll \lambda_L$ $(\xi < \sqrt{2}\lambda_L) \rightarrow$ Alloy S.C., High-T_c Cuprate.

$$B = B_0 e^{-\frac{x}{\lambda_L}},$$

$$g = \underbrace{\frac{1}{2}\rho_s(\nabla\theta - 2A)^2}_{\frac{1}{8\rho_s} j_0^2 = \frac{B^2}{8\pi}} + \frac{B^2}{8\pi} - \frac{BH}{4\pi} = \frac{B^2}{4\pi} - \frac{BH}{4\pi}$$

$$j \sim \rho_s(\nabla\theta - 2A) \sim \frac{1}{4\pi}\partial_x B = \frac{1}{4\pi\lambda_L}B$$

$$\frac{1}{\lambda_L^2} = 16\pi\rho_s$$

Alloy S.C., High-T_c Cuprate.

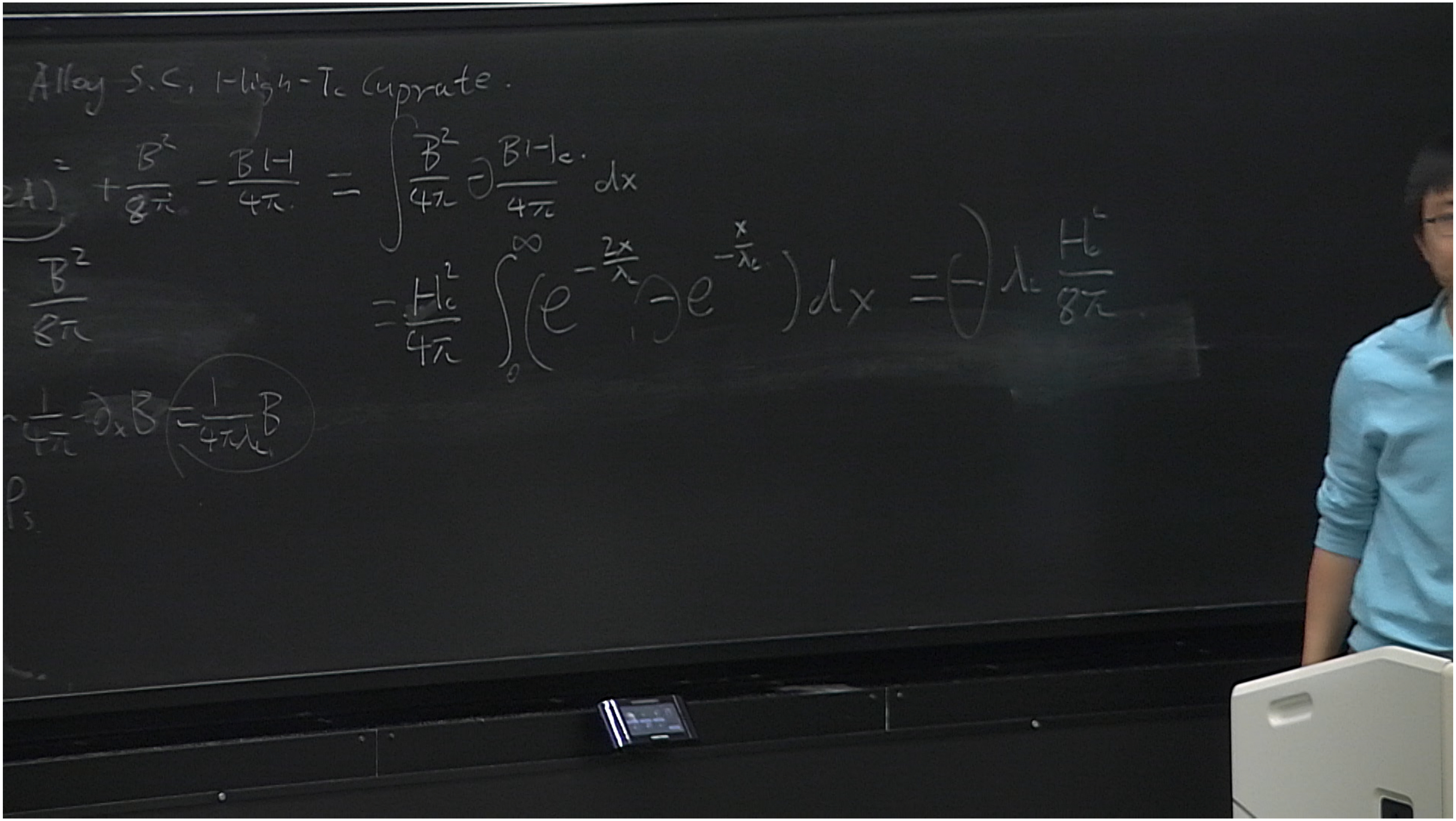
$$A)^2 + \frac{B^2}{8\pi} - \frac{BH}{4\pi} = \int \frac{B^2}{4\pi} - \frac{BH_c}{4\pi}\, dx$$

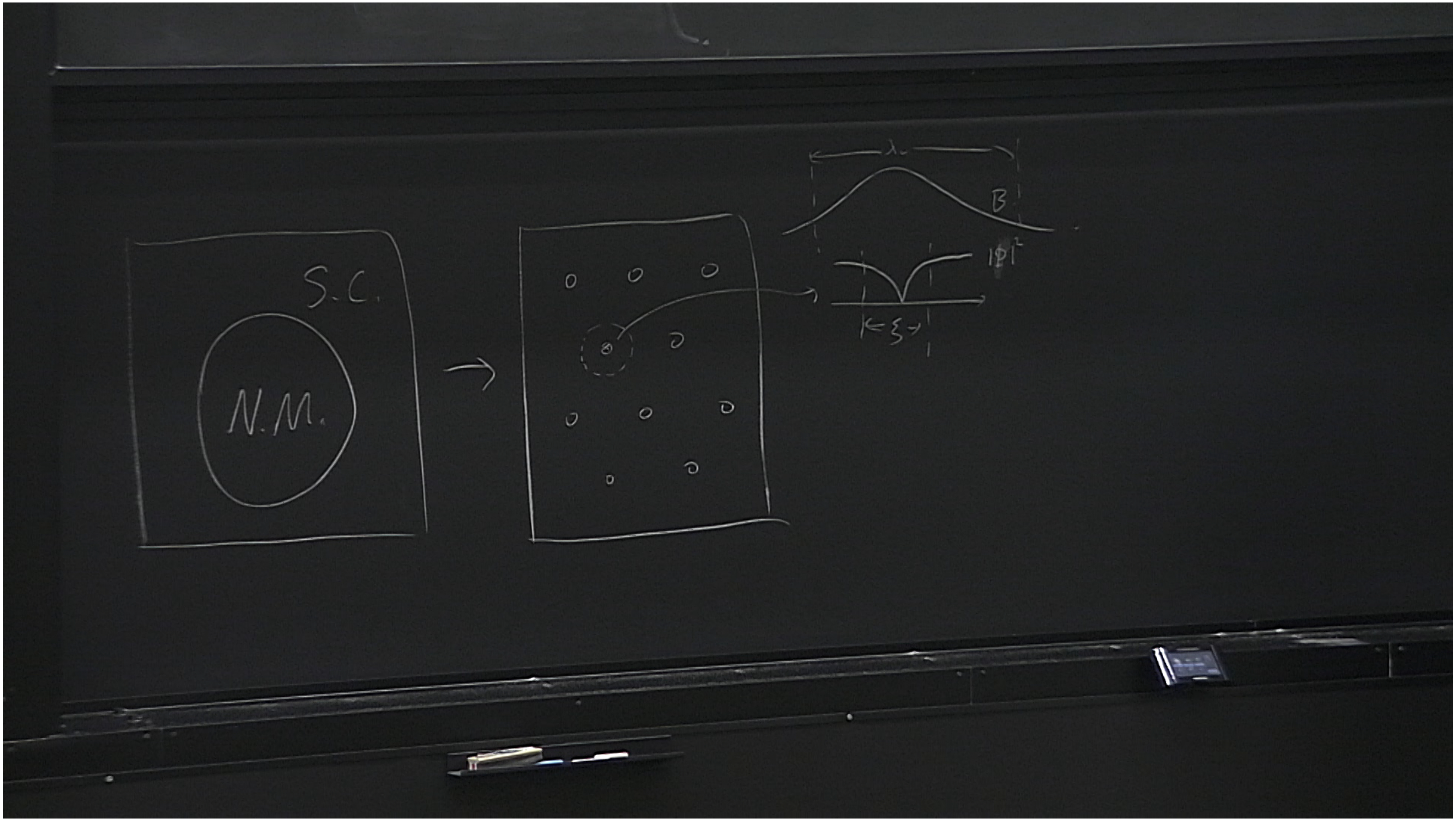
$$\frac{B^2}{8\pi}$$

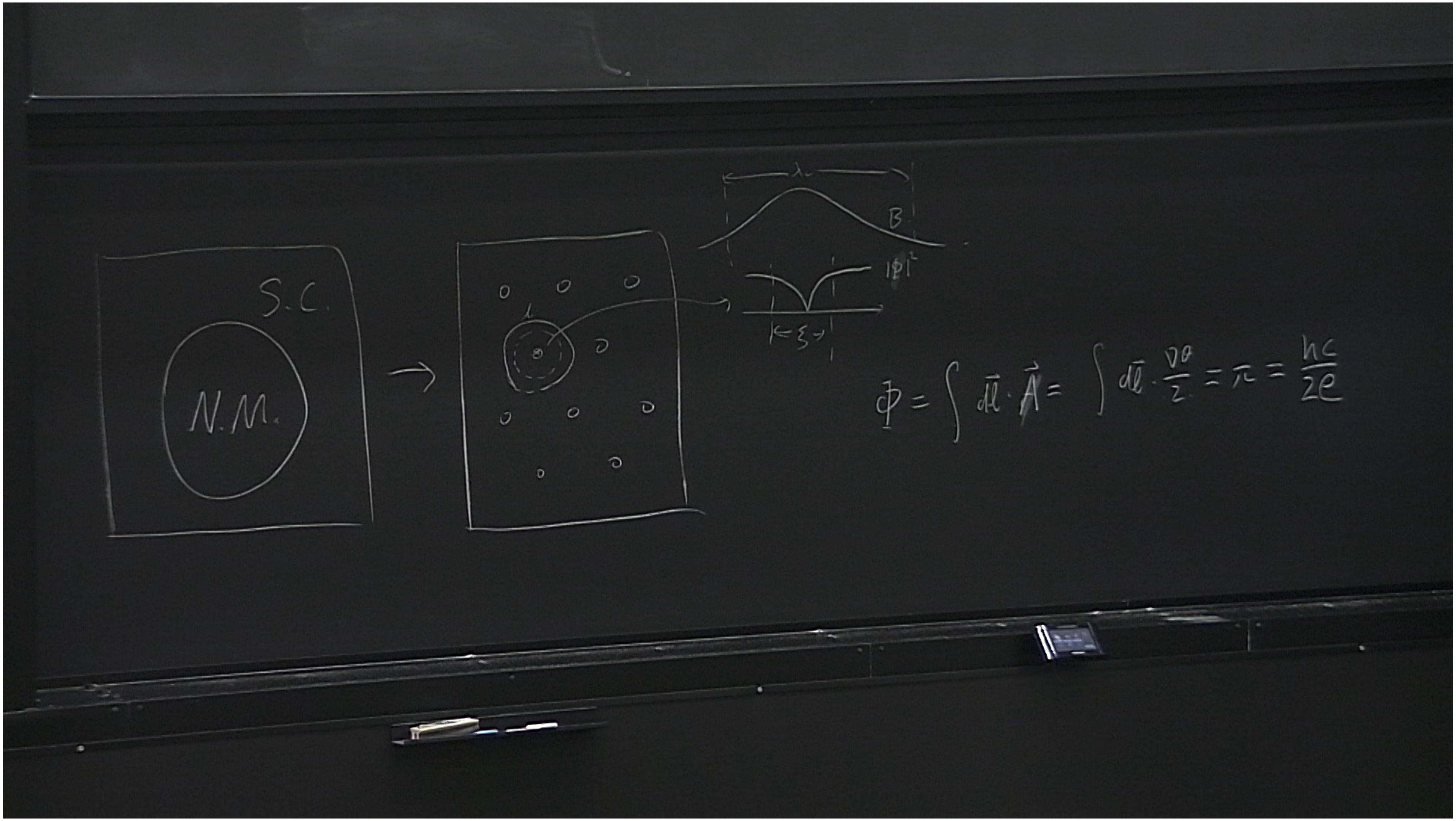
$$= \frac{H_c^2}{4\pi}\int \left(e^{-\frac{2x}{\lambda}} - e^{-\frac{x}{\lambda_c}}\right) dx = -\lambda_c \frac{H_c^2}{8\pi}$$

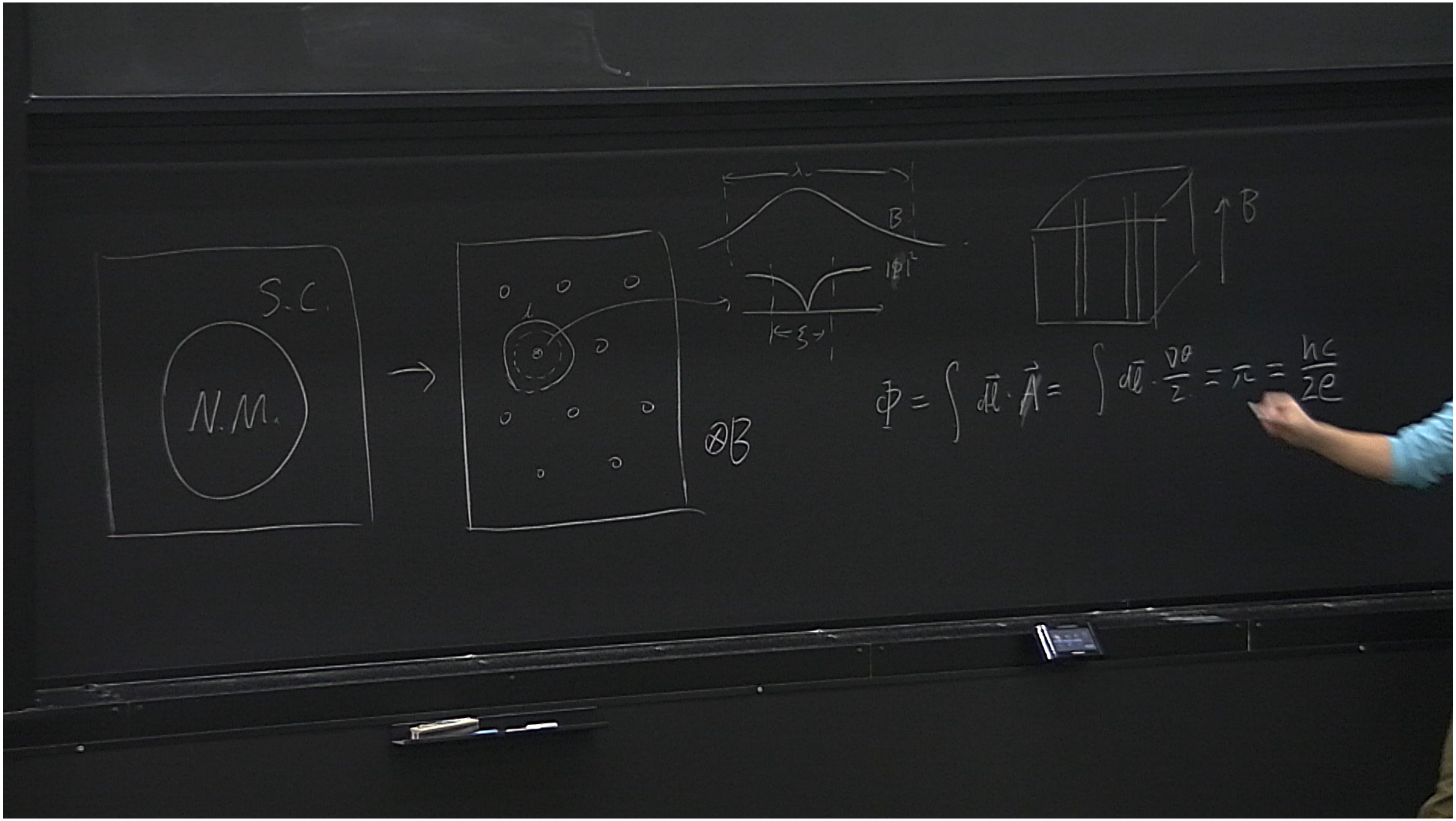
$$\frac{1}{4\pi}\partial_x B = -\frac{1}{4\pi\lambda_c} B$$

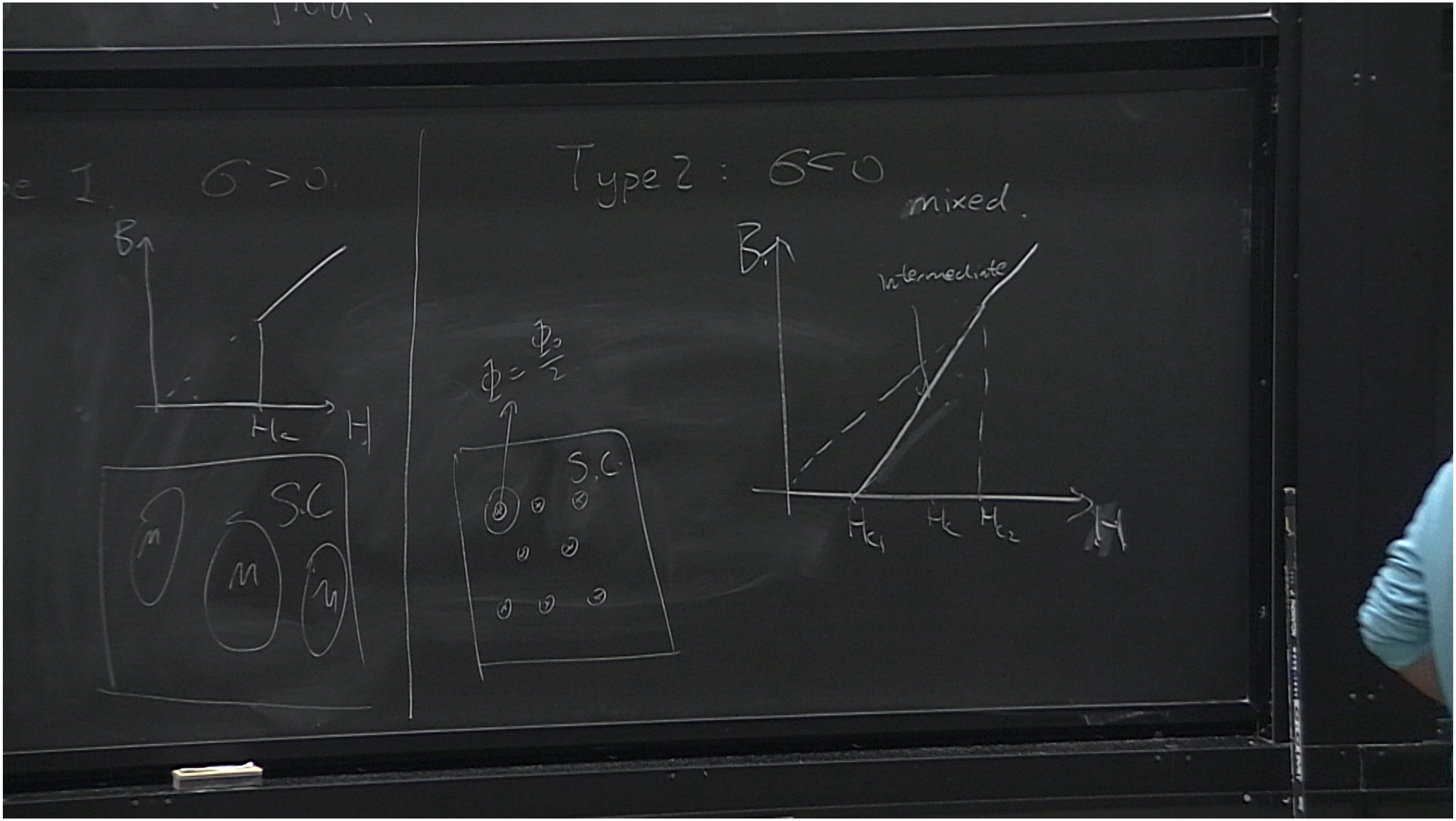
ρ_s

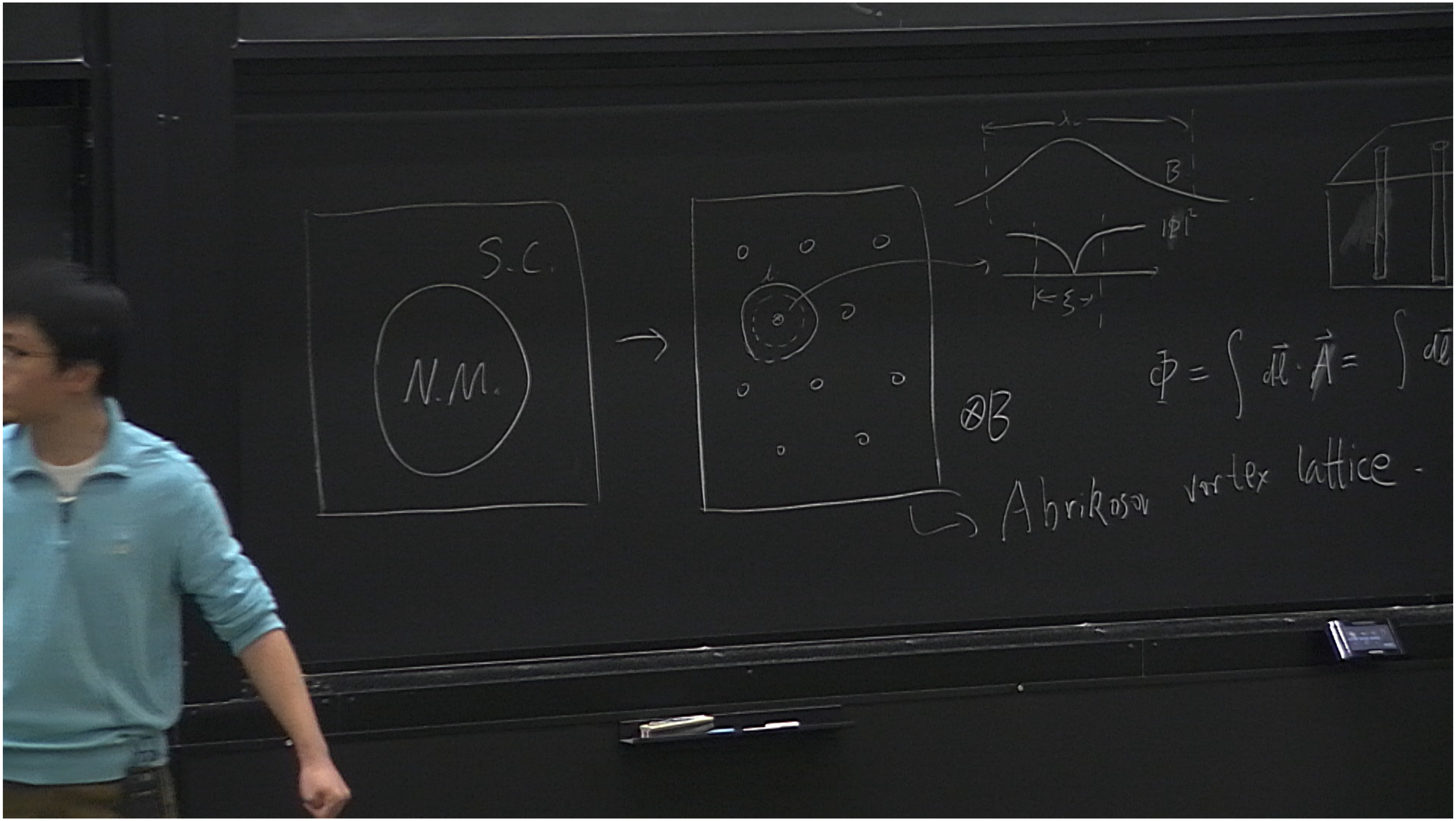

Alloy S.C, High-Tc Cuprate.


S.C.
N.M.
λ
B
$|\Phi|^2$
ξ


S.C.
N.M.
λ
B
|φ|²
ξ
Φ = ∫ dl · A = ∫ dl · ∇θ/2 = π = hc/2e


S.C.
N.M.
⊗B
λ
B
$|\psi|^2$
ξ
B
$\Phi = \int d\vec{l} \cdot \vec{A} = \int d\vec{l} \cdot \frac{\nabla\theta}{2} = \pi = \frac{hc}{2e}$


Type 2 : σ<0
mixed
intermediate
S.C
S.C.


S.C.
N.M.
B
$|\Phi|^2$
ξ
$\otimes B$
$\Phi = \int d\vec{l} \cdot \vec{A} = \int d\vec{l}$
Abrikosov vortex lattice.