

Title: PSI 2019/2020 - Quantum Field Theory II (David) - Lecture 16

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Collection: PSI 2019/2020 - Quantum Field Theory II (David)

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Renormalization of Gauge Theories: A first look

QED $U(1)$: A_μ gauge potential
 phase

$\alpha = 1 \dots 4$ Dirac

$\mu = 1 \dots 4$ Lorentz indices

$\Psi, \bar{\Psi}^\alpha$ Dirac Field.

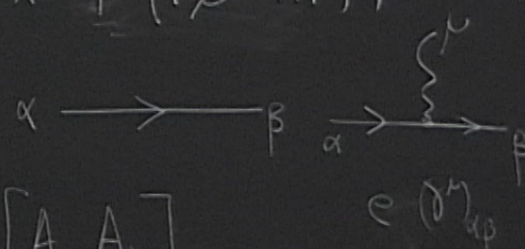
charge e

$$-\frac{1}{4} \int d^4x F_{\mu\nu} F^{\mu\nu}$$



$$\int d^4x \bar{\Psi} (i \not{D} - m) \Psi$$

$$\not{D} = (\not{\partial} + ie \not{A})$$



Gauge

$SU(2)$ or $SU(3)$

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu - ig [A_\mu, A_\nu]$$

charge g

A_μ^a

$a = 1, 3$

$= 1, 8$

$\Psi^{\alpha,i}$

$i = 1, 2$ or $1, 3$

Renormalization of Gauge Theories: A first look

QED $U(1)$: A_μ gauge potential

$$-\frac{1}{4} \int d^4x F_{\mu\nu} F^{\mu\nu} \quad \mu \sim \nu$$

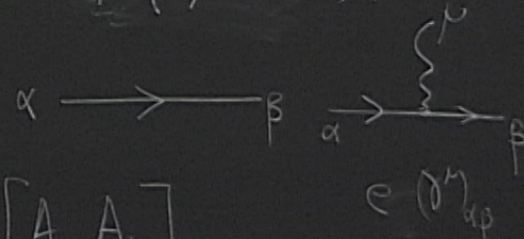
$\alpha = 1 \dots 4$ Dirac

$\mu = 1 \dots 4$ Lorentz indices

$\psi, \bar{\psi}$ Dirac Field

charge e

$$\int d^4x \bar{\psi} (i \not{D} - m) \psi \quad \not{D} = \not{\partial} - ie \not{A}$$



Gauge $SU(2)$ or $SU(3)$

charge g A_μ^a $a = 1, 3$ or $1, 8$

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu - ig [A_\mu, A_\nu]$$

$\psi^{\alpha,i}$

$i = 1, 2$ or $1, 3$

+ Ghosts $c, \bar{c}$

1st look

$$\psi \quad \mu \quad \nu$$

$$(-m)\psi \quad \not{D} = (\not{D} - ie\not{A})$$

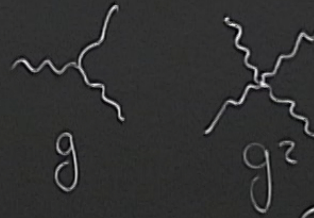
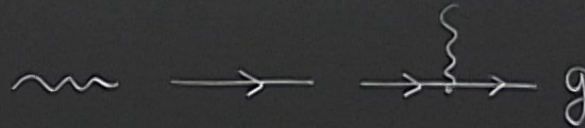
$$\beta \xrightarrow{\alpha} \beta \quad e \gamma_{\mu}$$

ghosts $c, \bar{c}$

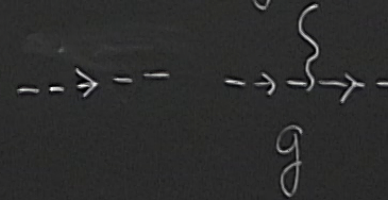
QED



Gauge + Dirac



+ Ghosts



QED

Gauge + Dirac

g

g^2

+ Ghosts

g

1 loop diagram

e^2

e^2

e^3

charge g

A_μ

$a = 1, 3$
 $= 1, 8$

$\psi^{a,i}$

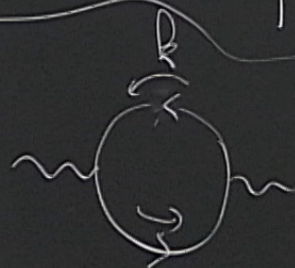
$i = 1, 2$ or $1, 3$

$p \rightarrow$ wavy line

$$\frac{1}{p^2}$$

$\rightarrow$ straight line

$$\frac{1}{p}$$

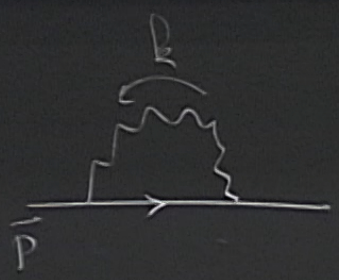


internal momentum

$$\int d^4 k \frac{1}{k^2} \rightarrow \text{diverge } \Lambda^2 + p^2 \log \Lambda$$

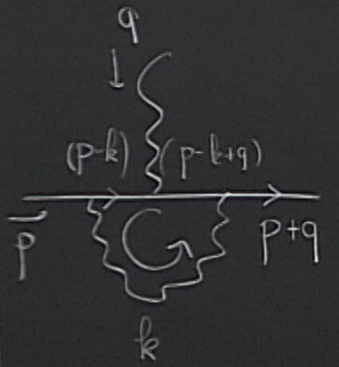
$$|k| < \Lambda$$

$\alpha_{1,3}$ + Ghosts $C, \bar{C}$



$$\int d^4 k \frac{1}{k^2 \cdot k} \sim \Lambda + \cancel{\log} \Lambda$$

UV $\leftarrow$ con
 $\Lambda^2 \cdot A_p^2$



$$\int d^4 k \frac{1}{k^2} \frac{1}{k} \frac{1}{k} \sim \log \Lambda$$

$$\frac{1}{k^2} \frac{1}{p-k} \frac{1}{p+q-k}$$

$p^2 \log \Lambda$

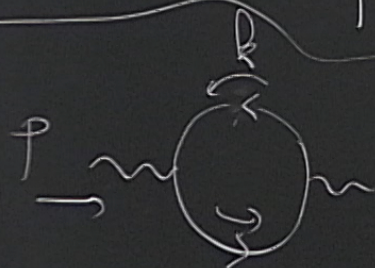
charge g A_μ $a=1,3$
 $=1,8$ $\psi^{a,i}$ $i=1,2$ or $1,3$ + Ghost

$p \rightarrow$ wavy line

$$\frac{1}{p^2}$$

$\rightarrow$ straight line

$$\frac{1}{p}$$



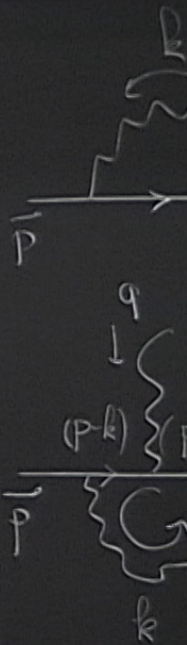
internal momentum

large k

$$\int d^4k \frac{1}{k^2} \rightarrow \text{diverge}$$

$$|k| < \Lambda$$

$$\Lambda^2 + p^2 \log \Lambda$$



$$S_R = \int d^4x \frac{A}{2} (\partial_\mu \phi)^2 + \frac{B}{2} \phi^2 + \frac{G}{4!} \phi^4$$

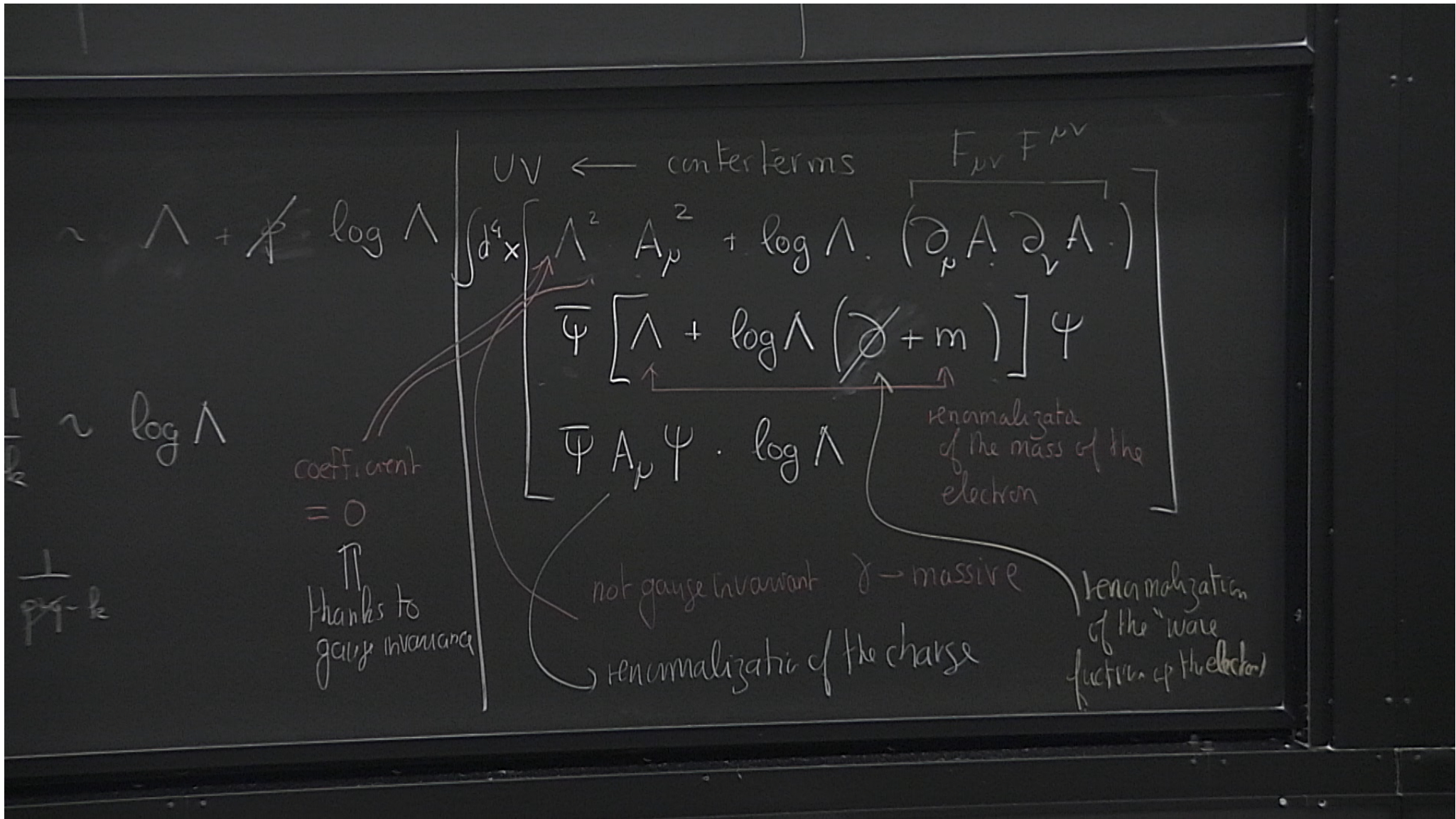
$\uparrow$ field $\uparrow$ mass $\uparrow$ renormalization of the coupling g

$$\sqrt{A} \phi = \Phi$$

QED

$$S_R = \int A \bar{\Psi} \not{\partial} \Psi + B \bar{\Psi} m \Psi + C \bar{\Psi} A \Psi + D F_{\mu\nu}^2$$

$\uparrow$ Field Ψ $\uparrow$ mass $\uparrow$ charge e $\downarrow$ Field A_μ



UV ← counter terms

$F_{\mu\nu} F^{\mu\nu}$

$\sim \Lambda + \cancel{\log \Lambda}$

$\int d^4x \left[\Lambda^2 A_\mu^2 + \log \Lambda \cdot (\partial_\mu A \partial_\nu A) \right]$

$\bar{\Psi} \left[\Lambda + \log \Lambda (\cancel{\not{\partial}} + m) \right] \Psi$

$\bar{\Psi} A_\mu \Psi \cdot \log \Lambda$

renormalization of the mass of the electron

not gauge invariant $\rightarrow$ massive

renormalization of the charge

renormalization of the "wave function of the electron"

coefficient = 0
 $\Uparrow$
thanks to gauge invariance

$\frac{1}{k} \sim \log \Lambda$

$\frac{1}{p^2 - k}$

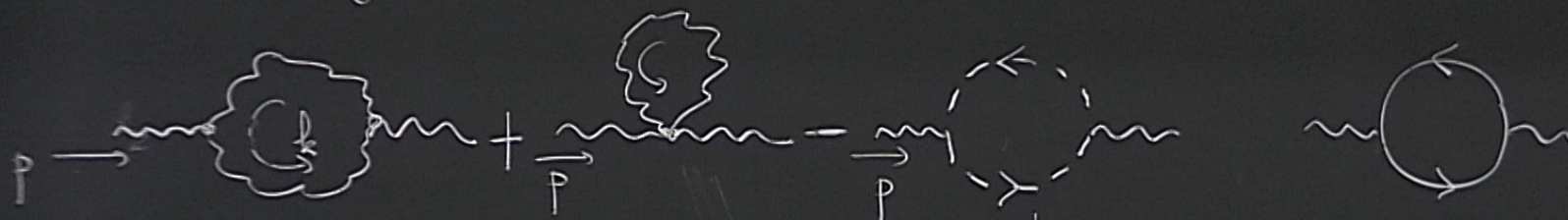
$$|k| \sim \Lambda$$

$SU(2)$ gauge theory

Gauge + Fermions

: identical

Renormal



$$\int d^4k \frac{1}{k^2} \frac{1}{k^2} (k \text{ or } p)(k \text{ or } p)$$

CT for $(\partial_\nu A_\nu - \partial_\nu A_\nu)^2$

$$\boxed{\Lambda^2} + \boxed{p^2 \log \Lambda}$$

no linear term by parity

$$\boxed{\Lambda^2}$$

$$\boxed{\Lambda^2} + \boxed{p^2 \log \Lambda}$$

here too

$$\Rightarrow \boxed{\circ}$$

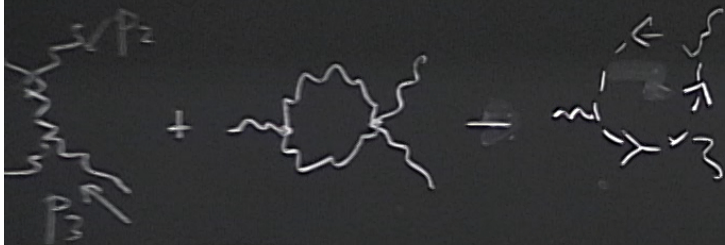
thanks to the ghosts

100

renormalization of the charge

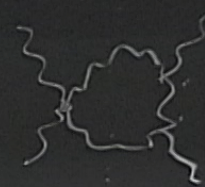
function of the electron

Gauge Theory G. 't Hooft 72



$$+ p_{\mu 3} \log \Lambda \rightarrow \text{coeff} = 0$$

$$A_\mu A_\nu \rightarrow \partial_\mu A^\nu A^\mu A_\nu$$



$$\int d^4 k \left(\frac{1}{k^2} \right)^4 k^\mu, p^\mu$$

$$\Rightarrow \log \Lambda \boxed{A A A A}$$

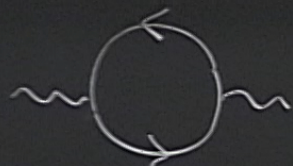
$$\Lambda^4 p^4 + \Lambda^2 p^2 + \log \Lambda$$

$$\Lambda^3 p^3 + \Lambda p$$

gauge invariance

hence

identical



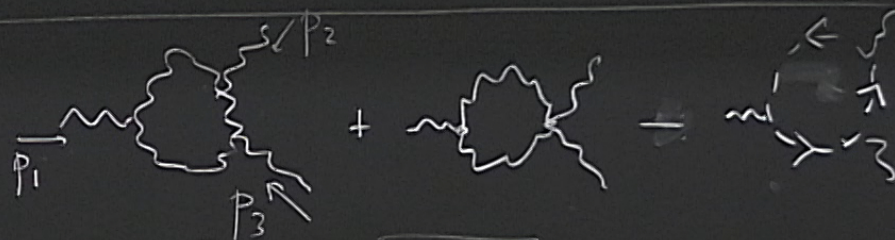
CT for $(\partial_\mu A_\nu - \partial_\nu A_\mu)^2$

$$\log \Lambda$$



thanks to the ghosts

Renormalization of Gauge Theory G. 't Hooft 72



$$\Lambda$$

$$\Lambda \cdot A_\mu A_\mu A_\mu$$

$$+ p_{1,2,3} \log \Lambda$$

coeff = 0

$$\partial_\mu A^\nu A^\mu A_\nu$$

g

$$\int d^4 k \left(\frac{1}{k^2} \right)^4 k^4$$

$$\Lambda^4 p^4 + \Lambda^2 p^2$$

$$\Lambda^3 p^3 +$$

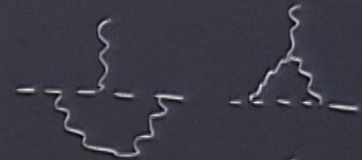
PollEv.com/dan.wohns.778

screened by Higgs Mechanism

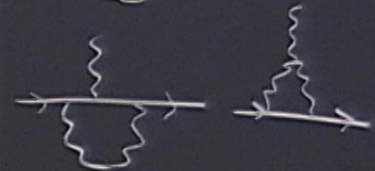
$$SU(3) \otimes \underbrace{SU(2) \otimes U(1)}_{U(1)}$$

u	d	s	c	t	b	
e, ν_e	μ, ν_μ	τ, ν_τ				BSM
$\otimes$						GV
W_+, W_-, Z_0, γ, H						

charge ghost



charge Dirac



charge of the gauge field



Dimensional Renormalization $\Rightarrow$ Minimal Subtraction Scheme

dimension $d \neq 4$

MS, $\overline{\text{MS}}$

respects gauge + Lorentz

$\log \Lambda \rightarrow$ poles $\frac{1}{d-4}$ UV divergences

renormalization = subtract the poles

$[A^\mu, \tilde{A}]$

BRST Symmetry Z6

Gauge Theory Gauge Fix it

$$(A_\mu)$$



$$(A_\mu) + (c, \bar{c})$$

$$S_{\text{YM}}[A_\mu]$$

huge local
gauge symmetry

$$S_{\text{G Fixed}}[A_\mu, c, \bar{c}]$$

not gauge invariant
any more

Is there a remnant of Gauge Invariance

ϵ do not depend on X

Yes, there is global symmetry (rigid)

involves the gauge group

BUT: the generators are anticommuting fields ϵ

$$A_\mu \rightarrow A_\mu + D_\mu \alpha \quad \Rightarrow \quad A_\mu = A_\mu + \epsilon D_\mu C$$

"anti-ghost" "ghost"

Supersymmetry mixes $A, C, \bar{C}$

ϵ do not depend on X

BRST Symmetry Z6

Gauge Theory Gauge Fix it

$$(A_\mu)$$



$$(A_\mu) + (c, \bar{c})$$

$$S_{\text{YM}}[A_\mu]$$

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Is there a remnant of Gauge Invariance?
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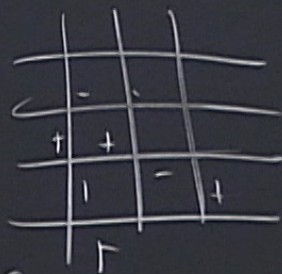
Supersymmetry mixes $A, c, \bar{c}$

Condensed Matter

$$E = - \sum_{\langle ij \rangle} S_i \cdot S_j J_{ij}$$

$$S_i = \pm 1$$

J_{ij} also are ± 1 : Fixed



Spin glass

Quenched coupling



BRST too

ϵ do not depend on X $\Rightarrow$ currents and charge

(rigid)

gauge fields ϵ

- Gauge
- Strings
- CFT
- Quantum Gravity
- etc...

$A_\mu + \epsilon D_\mu C$
ghost ghost

ϵ do not depend on X

internal momentum

$$|k| < \Lambda$$

Asymptotic Freedom

charge g is renormalized

$$\alpha = g^2 \text{ coupling}$$

$E \rightarrow$ energy $\Rightarrow$ Effective coupling $\alpha(E)$

For ϕ^4 coupling g , effective c., $g(E)$

gauge invariance

renormaliz

Result at 1 loop

$$E \frac{d}{dE} \alpha(E) = \beta(\alpha(E)) = -b_2 \cdot \alpha(E)^2 + \dots$$

$\uparrow$ negative

b_2

gauge invariance

renormaliz

Result at 1 loop

$$E \frac{d}{dE} \alpha(E) = \beta(\alpha(E)) = -b_2 \cdot \alpha(E)^2 + \dots$$

↑ negative

$$-b_2 = -\frac{2}{(4\pi)^2} \left(-\frac{11}{3} C_2(G) + \frac{4}{3} N_F C(r) \right)$$

$C_2(G)$

$f^{acd} f^{bcd}$

$= \delta^{ab} C_2(G)$

↑ representation of the Fermion

↑ # families (Fermions)

$SU(2) \quad C_2 = 2$

$SU(N) \quad C_2 = N$

$\frac{1}{k^2} \quad \frac{1}{p-k} \quad \frac{1}{p-q-k}$
 (Gross-Wilczek) + (Politzer) $\Rightarrow$ QCD
 SU(3)

$\Uparrow$
 Thanks
 gauge !!

Result at 1 loop

$$E \frac{d}{dE} \alpha(E) = \beta(\alpha(E)) = -b_2 \cdot \alpha(E)^2 + \dots$$

$\uparrow$ negative

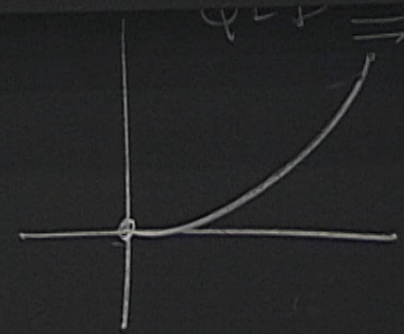
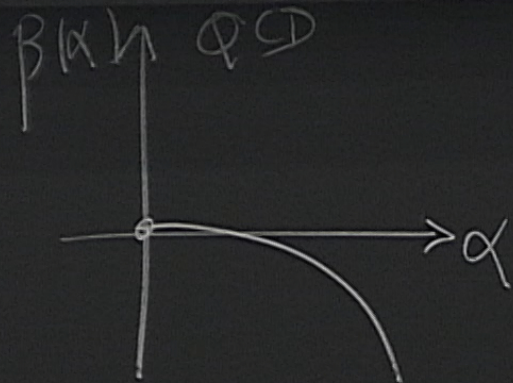
$$-b_2 = -\frac{2}{(4\pi)^2} \left(-\frac{11}{3} C_2(G) + \frac{4}{3} N_F C(r) \right)$$

$C_2(G)$

$$f^{acd} f^{bcd} = \delta^{ab} C_2(G)$$

$\uparrow$
 representation
 of the Fermion

$$N_F = 0$$



$$\alpha(E) \rightarrow 0 \quad \text{when } E \nearrow \infty$$

Weak at high energy

Strong at low energy

Confinement

Asymptotic Freedom (at high energy)

$\in$ do not depend on X $\Rightarrow$ currents and charge

- Gauge
- Strings
- CFT
- Quantum Gravity
- etc...