Title: PSI 2019/2020 - Quantum Field Theory II (David) - Lecture 15

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# Higgs Mechanism

* Symmetry Breaking Goldstone Boscns
* Chaiged Condensates
* Higgs Mechanism in QFT

U(1) Symmetry

Scalar Field $\vec{\Phi} = (\Phi_1, \Phi_2)$ $\Phi = \Phi_1 + i\Phi_2$

2 spin 0 : real ↑ complex

1 complex Field $U(1)$ symmetry $\bar{\Phi} = \Phi_1 - i\Phi_2$

Action $S[\vec{\Phi}] = \int d^d x \, \frac{1}{2}(\partial_\mu \vec{\Phi} \partial^\mu \vec{\Phi}) + \frac{t}{2}(\vec{\Phi}\cdot\vec{\Phi}) + \frac{g}{8}(\vec{\Phi}\cdot\vec{\Phi})^2$

$g > 0$ $S[\Phi] = \int d^q x \, \frac{1}{2}(\partial_\mu \Phi \, \partial^\mu \bar{\Phi}) + \frac{t}{2}(\Phi\cdot\bar{\Phi}) + \frac{g}{8}(\Phi\cdot\bar{\Phi})^2$

2 bosons charged $b_+, b_-$ $m$ = mass

Charge conjugation $\bar{b}_+ = b_-$,

$t = m^2$

Scalar Field $\vec{\Phi} = (\Phi_1, \Phi_2)$ $\Phi = \Phi_1 + i\Phi_2$

2 spin 0 : real

1 complex Field $U(1)$ symmetry

complex $\bar{\Phi} = \Phi_1 - i\Phi_2$

$t = m^2 > 0$

$t < 0$

Action $S_E[\vec{\Phi}] = \int d^d x \; \frac{1}{2}(\partial_\nu \vec{\Phi}\, \partial^\nu \vec{\Phi}) + \frac{t}{2}(\vec{\Phi}\cdot\vec{\Phi}) + \frac{g}{8}(\vec{\Phi}\cdot\vec{\Phi})^2$

$g > 0$ $S_E[\Phi] = \int d^d x \; \frac{1}{2}(\partial_\nu \Phi\, \partial^\nu \bar{\Phi}) + \underbrace{\frac{t}{2}(\Phi\cdot\bar{\Phi}) + \frac{g}{8}(\Phi\cdot\bar{\Phi})^2}_{V(\Phi)}$

2 bosons charged $b_+, b_-$ $m$ = mass

Charge conjugation $\bar{b}_+ = b_-$,

Euclidean action

$\phi = \phi_1 + i\,\phi_2$

complex

$\bar{\phi} = \phi_1 - i\,\phi_2$

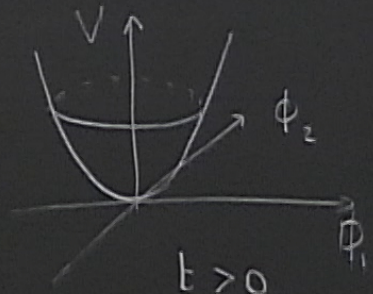
$+\frac{t}{2}(\vec{\phi}\,\vec{\phi}) + \frac{g}{8}(\vec{\phi}\cdot\vec{\phi})^2$

$+\frac{t}{2}(\phi\cdot\bar{\phi}) + \frac{g}{8}(\phi\cdot\bar{\phi})^2$

$V(\phi)$

Euclidean action

$t = m^2 > 0$ minimum of $V$ is $\phi = 0$



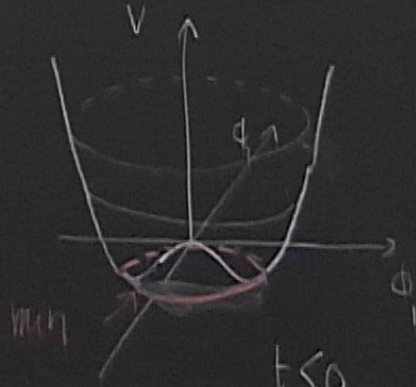

$t < 0$ minima degenerate

$t > 0$

$\phi = \phi_{min} e^{i\theta}$ $\quad \theta \in [0, 2\pi)$

$\vec{\phi} = \phi_{min}(\cos\theta, \sin\theta)$

$\phi_{min} = \sqrt{\frac{-2t}{g}}$




$t < 0$

$b_+, b_-$ $m$ : mass $V(\phi)$

Charge conjugation $\bar{b}_+ = b_-$, Euclidean action

$$Q = \int d\vec{x}\, J_0(x)$$

$$J_\mu = \frac{1}{2}\left(\phi\, \partial_\mu \bar{\phi} - \bar{\phi}\, \partial_\mu \phi\right)$$

total charge $J_0$. $\vec{J}$

$J$ d.o. charge

$\phi_{min} e^{i\theta}$

Charge conjugation

$t > 0$ $|\Omega\rangle$ symmetric $U(1)$ $Q = \int d\vec{x}$

$Q|\Omega\rangle = 0$ $Q$ charge operator

$t < 0$ $|\Omega(\theta)\rangle$ $\langle\Omega(\theta)|\phi|\Omega(\theta)\rangle = \phi_{min} e^{i\theta}$

$Q|\Omega(\theta)\rangle \neq 0$

$U(1)$ symmetry is spontaneously broken

$m$ = mass $\quad V(\phi)$

Charge conjugation $\bar{b}_+ = b_-$, $\quad$ Euclidean action

$$Q = \int d\vec{x}\, J_0(x)$$

$$J_\mu = \frac{i}{2}\left(\phi\, \partial_\mu \bar{\phi} - \bar{\phi}\, \partial_\mu \phi\right)$$

total charge $J_0$. $\quad \vec{J}$

$J$

d.o. charge

$$[Q, H] = 0$$

$= \phi_{min} e^{i\theta}$

$\partial^\mu A^a_\mu(x) = 0$

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$x = (t, \vec{x})$

$a = 1, 2, 3$

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d dim of the lattice
n dim of the group

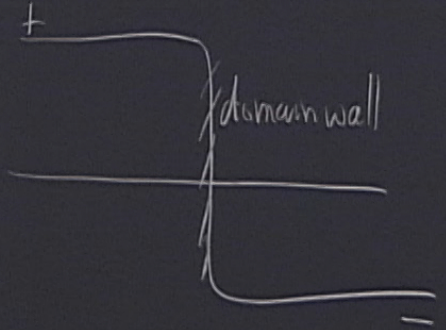
N site



N × d × n d.o.f gauge Field

N × n d.o.f gauge transf.

N × (d-1) × n  n = 1



$\phi_1 + i\,\phi_2$

ex

$\phi_1 - i\,\phi_2$

$\vec{\Phi}) + \frac{g}{8}(\vec{\phi}\cdot\vec{\Phi})^2$

$\cdot\bar{\Phi}) + \frac{g}{8}(\phi\cdot\bar{\Phi})^2$

$V(\phi)$

n action

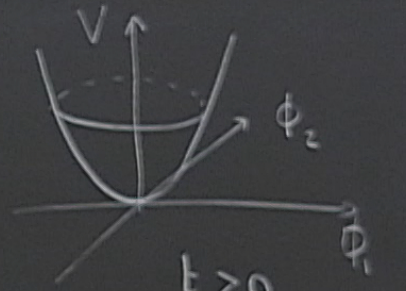
$t = m^2 > 0$ minimum of $V$ is $\phi = 0$

$t < 0$ minima degenerate

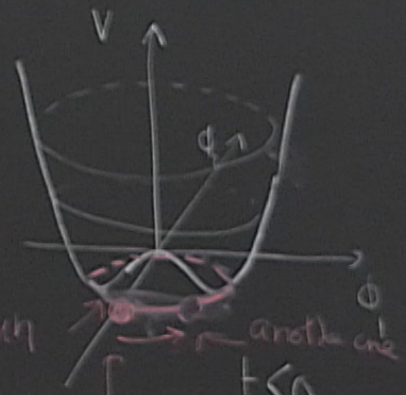
$\phi = \phi_{min} e^{i\theta}$ $\quad \theta \in [0, 2\pi)$

$\vec{\phi} = \phi_{min}(\cos\theta, \sin\theta)$

$$\phi_{min} = \sqrt{\frac{-2t}{g}}$$




$t > 0$




$\partial_\nu\bar{\Phi} - \bar{\Phi}\cdot\partial_\nu\phi)$

Charge conjugation $b_+ = b_-$,

Euclidean action

$$Q = \int d\vec{x}\, J_0(x)$$

$$J_\mu = \frac{i}{2}\left(\phi\, \partial_\mu \bar{\phi} - \bar{\phi}\, \partial_\mu \phi\right)$$

$t \leq 0$

total charge $J_0$. $\vec{J}$

$J$

d.o. charge

$$[Q, H] = 0$$

$= \phi_{mn} e^{i\theta}$

$$S[\vec{\Phi}] = \int d^4x \; \frac{1}{2}\left(\partial_\mu H \partial^\mu H + (-2+)H^2\right) + \frac{1}{2}\left(\partial_\mu \varphi \partial^\mu \varphi\right) + \text{interaction terms}$$

$H$, $\sigma$ are real fields

$H$ massive boson with mass $m_H^2 = (-2+) > 0$ — curvature of $V$ in radial direction

$\varphi$ massless boson $m_\varphi^2 = 0$ — no curvature, potential is flat in transverse direction

S.S.B. $\Longrightarrow$ massless boson : Nambu-Goldstone boson (Coleman)

$U(1) \longrightarrow I$ pattern of S.B

$O(3) \longrightarrow O(2)$ 2 Goldstone bosons $\vec{\Phi} = (\Phi_1, \Phi_2, \Phi_3)$ $\vec{\Phi}_0 = \Phi_{min.}\,\vec{n}$, $\vec{n} \in S_2$ unit sphere in 3 dim.

$$[\phi] = \int d^4x \; \frac{1}{2}(\partial_\mu \phi \cdot \partial^\mu \bar{\phi}) + \frac{t}{2}(\phi \cdot \bar{\phi}) + \frac{g}{8}(\phi \bar{\phi})^2$$

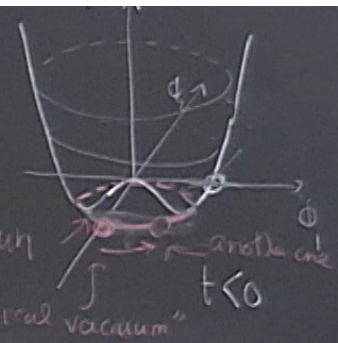
$b_+, b_-$ $\quad m$ = mass

$\bar{b}_+ = b_-$,

$V(\phi)$

Euclidean action

$$\phi_{min} = \sqrt{\frac{-2t}{g}}$$




$$\int d\vec{x}\, \bar{J}_0(x)$$

$$J_\mu = \frac{i}{2}\left(\phi \cdot \partial_\mu \bar{\Phi} - \bar{\Phi} \cdot \partial_\mu \Phi\right)$$

total Charge $J_0$. $\vec{J}$

$J$ d.o. charge

$H] = 0$

$t \leq 0$ what is the mass ?

$$\vec{\Phi}_0 = \Phi_m(1, 0) \qquad \phi_{min} = \sqrt{-2t/g}$$

$$\vec{\Phi}(x) = \bar{\Phi}_0 + \hat{\vec{\Phi}} = (\phi_{min} + H(x), \varphi(x))$$

small fluctuations — radial — transverse

$$V(\vec{\Phi}) = -\frac{t^2}{2g} + (-t)H^2 + \boxed{\cdot H^3 + \cdot H\varphi^2 + H^4 + \cdot H^2\varphi^2 + \cdot \varphi^4}$$

interactions

$$V(\vec{\Phi}) = -\frac{t^2}{2g} + (-t)H^2 + \cdot H^3 + \cdot H\varphi^2 + H^4 + \cdot H^2\varphi^2 + \cdot \varphi^4$$

interactions

+ interaction terms

$H^2 = (-2t) > 0$ curvature of V in radial direction

no curvature, potential is flat in transverse direction

$\vec{n}, \vec{n} \in S_2$ unit sphere in 3 dim.

$\vec{\Phi}_0$

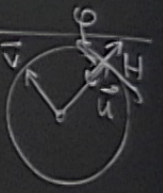
$$\left[\left(\Phi_{min} + H\right)^2 + \varphi^2\right] \cdot \cdot \left(\left(\Phi_{min} + H\right)^2 + \varphi^2\right)^2$$

$$\vec{\Phi}_0 = \vec{\Phi}_{min}(\cos\theta_0, \sin\theta_0)$$

$$\vec{\Phi} = \vec{\Phi}_0 + \vec{H} \quad \text{with } \vec{H} \parallel \vec{\Phi}_0 \qquad \vec{H} = \vec{u} \cdot H$$

$$+ \vec{\varphi} \quad \text{with } \vec{\varphi} \perp \vec{\Phi}_0 \qquad \vec{\varphi} = \vec{v} \cdot \varphi$$

$$\vec{\Phi}_0 = \Phi_{min}\vec{u} \qquad \vec{u} = (\cos\theta_0, \sin\theta_0) \qquad \vec{v} = (-\sin\theta_0, \cos\theta_0)$$

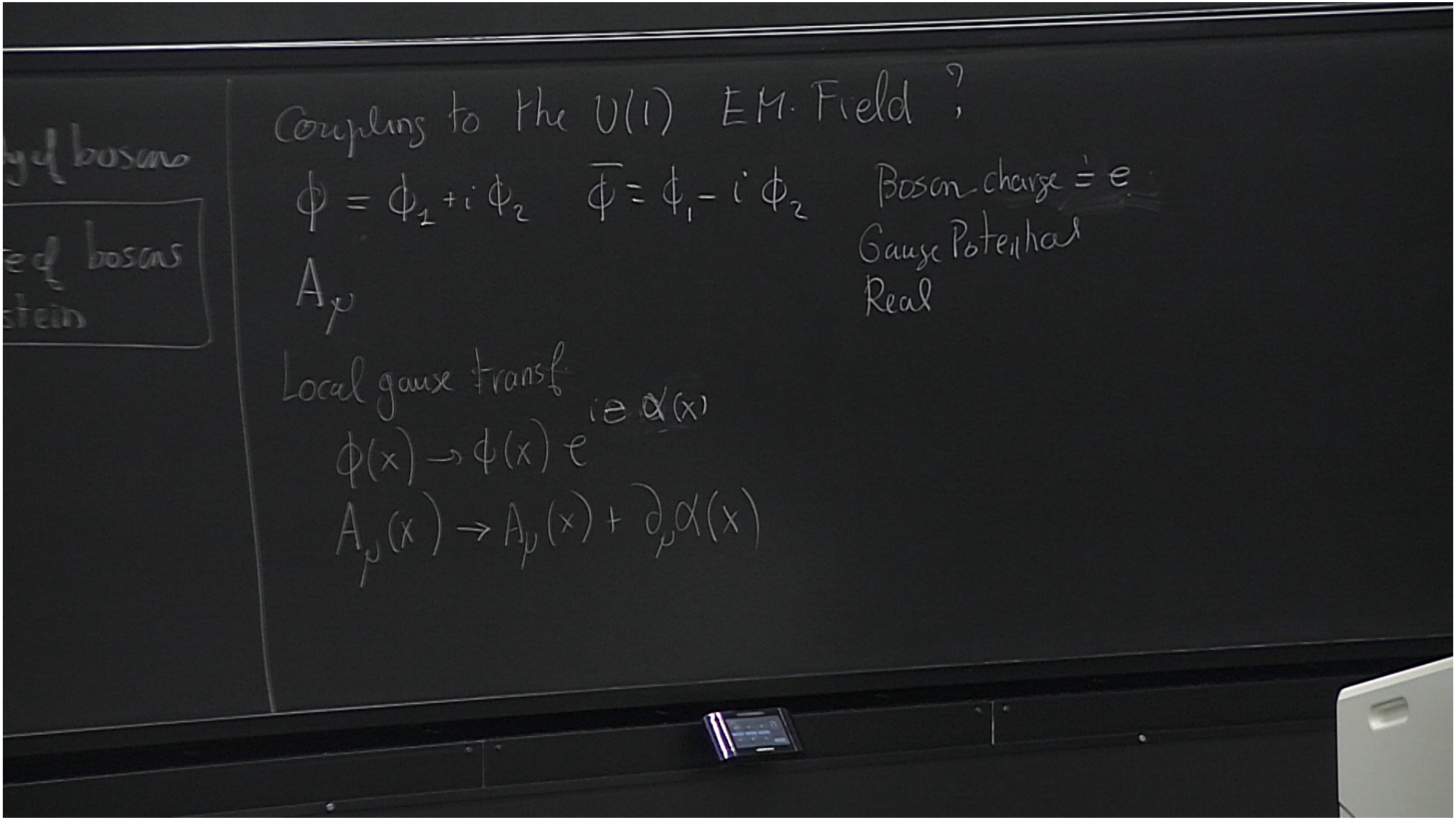


$\langle \Omega(\theta) | \bar{\Phi}\Phi(x) | \Omega(\theta) \rangle \simeq$ local density of bosons

$= \Phi_{min}^2 = \frac{(-2t)}{g} > 0$

condensate of bosons
Bose-Einstein

- Cooper pairs in BCS
- BE


Couplings to the U(1) EM. Field ?
$\phi = \phi_1 + i\phi_2$ $\bar{\phi} = \phi_1 - i\phi_2$
Boson charge = e
$A_\mu$
Gauge Potential
Real
Local gauge transf.
$\phi(x) \to \phi(x)\, e^{ie\alpha(x)}$
$A_\mu(x) \to A_\mu(x) + \partial_\mu \alpha(x)$

(1) EM-Field ?

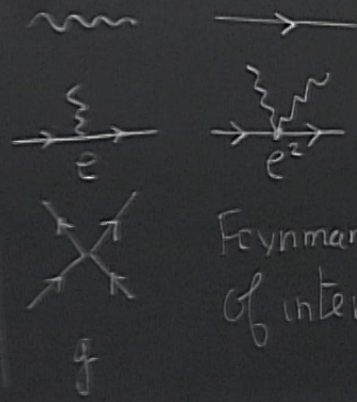
$= \phi_1 - i\phi_2$ Boson charge $= e$

Gauge Potential

Real

$\alpha(x)$

$\partial_\mu \alpha(x)$






Feynman Rules of interactions

g

$\partial_\mu \rightarrow D_\mu = \partial_\mu - ieA_\mu$ covariant derivatives

$\overline{D_\mu} = \partial_\mu + ieA_\mu$

Action for scalar QED

$$S = \int d^4x \; \frac{1}{2} D_\mu \phi \cdot \overline{D^\mu \phi} + \frac{1}{2}(\phi\bar{\phi}) + \frac{g}{8}(\phi\bar{\phi})^2 - \frac{1}{4} F_{\mu\nu} F^{\mu\nu}$$

$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$

$t<0$ Phase ?

$t>0$ Coulomb Phase

Coulomb phase

$1+1$

$\phi(x) = \phi_{min}\, e^{ie\theta(x)}$ $\Leftarrow$ Gauge Tr $\phi(x) = \phi_{min}$

$A_\mu(x) = \partial_\mu \theta(x)$ $A_\mu = 0$

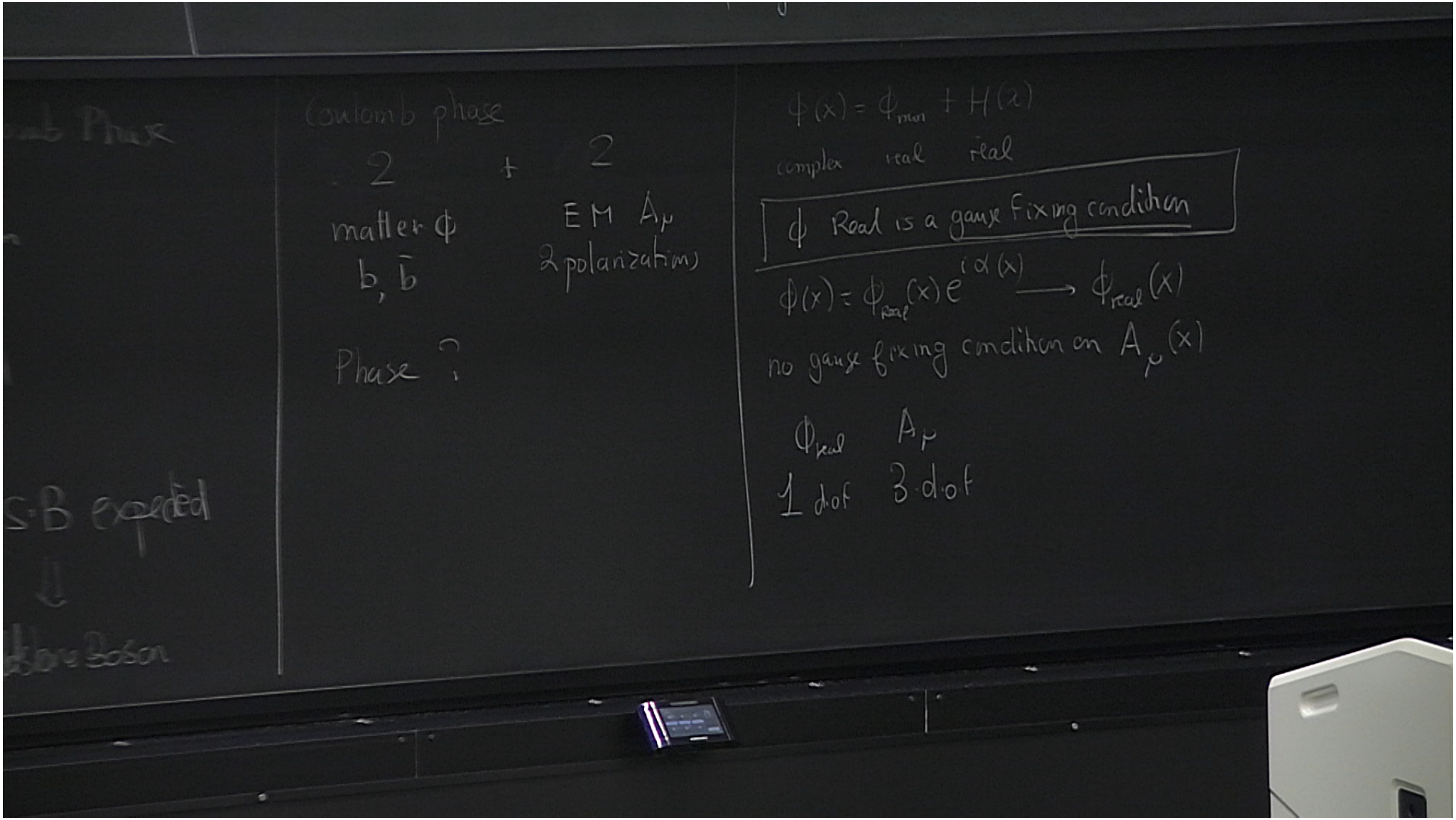
minimum of the action all gauge equivalent $\Leftrightarrow$ one physical vacuum

Symmetry is local (Gauge S.) $\Rightarrow$ no S.S.B expected

Formation of a condensate $\bar{\phi}\phi$ is gauge invariant

$\Downarrow$

no Goldstone Boson


Coulomb phase
2 + 2
matter $\phi$
$b, \bar{b}$
EM $A_\mu$
2 polarizations
Phase ?
$\phi(x) = \phi_{max} + H(x)$
complex real real
$\phi$ Real is a gauge fixing condition
$\phi(x) = \phi_{Real}(x) e^{i\alpha(x)} \longrightarrow \phi_{real}(x)$
no gauge fixing condition on $A_\mu(x)$
$\phi_{real}$ $A_\mu$
1 d.o.f 3 d.o.f

$\phi(x) = \phi_{min} + H(x)$

complex real real

$\phi$ Real is a gauge fixing condition

$\phi(x) = \phi_{Real}(x) e^{i\alpha(x)} \longrightarrow \phi_{real}(x)$

no gauge fixing condition on $A_\mu(x)$

$\phi_{real}$ $A_\mu$

1 d.o.f 3 d.o.f

$H$ $A_\mu$

$D_\mu \phi = \partial_\mu H - ie\phi_{min} A_\mu - ieA_\mu H$

new $\uparrow$ $\phi_{min} = \sqrt{\frac{2}{g}} \neq 0$

Action

$$\int dx \left[ \frac{1}{2}\left(\partial_\mu A_\nu \partial^\mu A^\nu - \partial_\mu A_\nu \partial^\nu A_\mu\right) + \frac{e^2\phi_{min}^2}{2} A_\mu A^\mu \right]$$

ordinary Maxwell — new

$$+\frac{1}{2}\partial_\mu H \partial^\mu H + g\frac{\phi_{min}^2}{2} H^2 + \cdot H^3 + \cdot H^4 + \cdots$$

pure H term of the theory without $A_\mu$

$$+ e^2 \phi_{min} A_\mu A^\mu H + \frac{e^2}{2} A_\mu A^\mu H^2$$

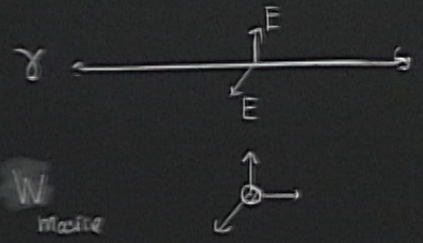
interactions between A and H

$\bar{\Phi}\Phi$ is gauge invariant

no Goldstone Boson

EM Field $A_\mu$ has a mass $M_A = e \cdot \Phi_{min} = e\sqrt{\frac{-2t}{g}} > 0$

massive gauge boson with 3 polarizations




because $v < c$

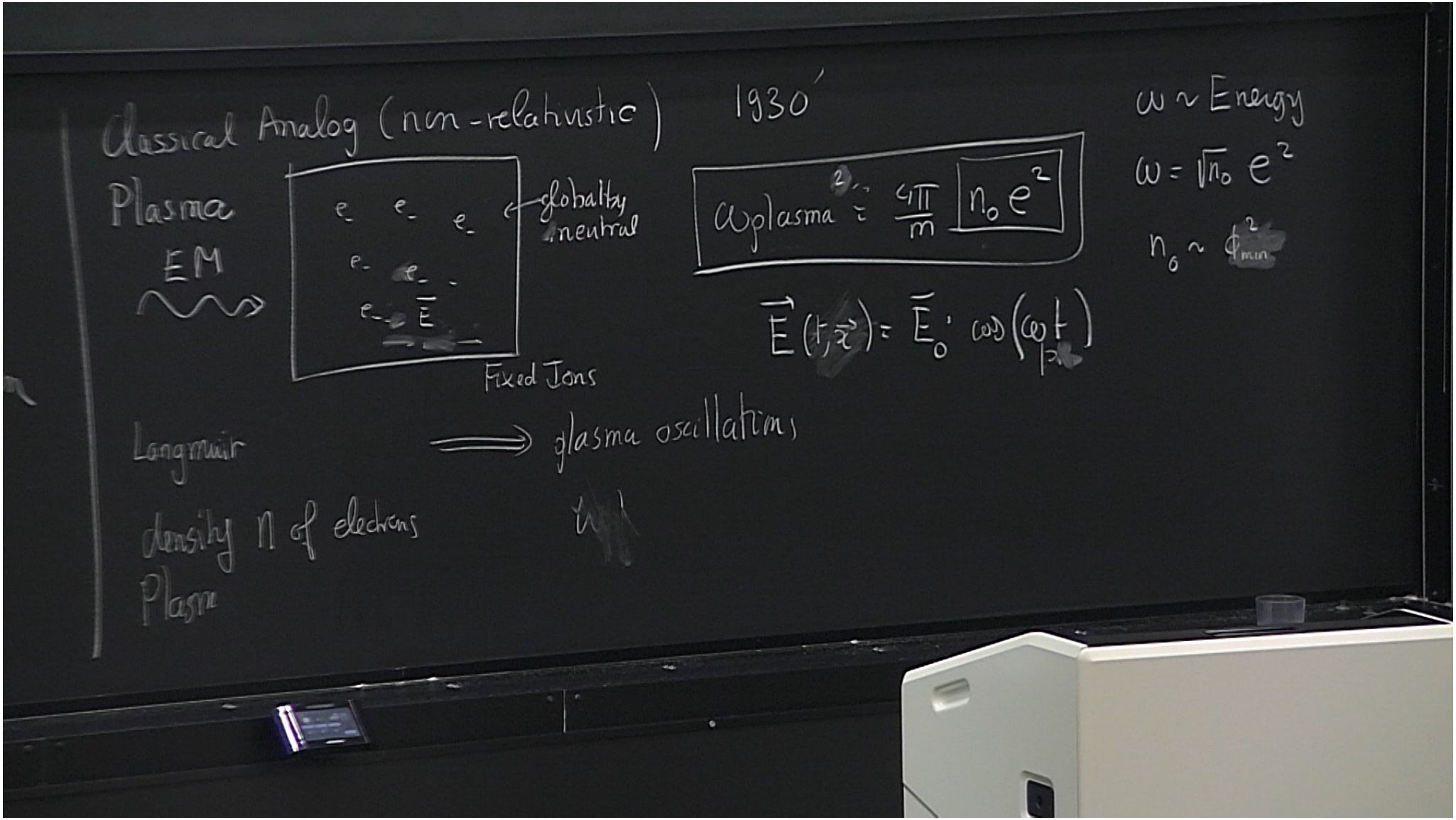
Condensate Formation

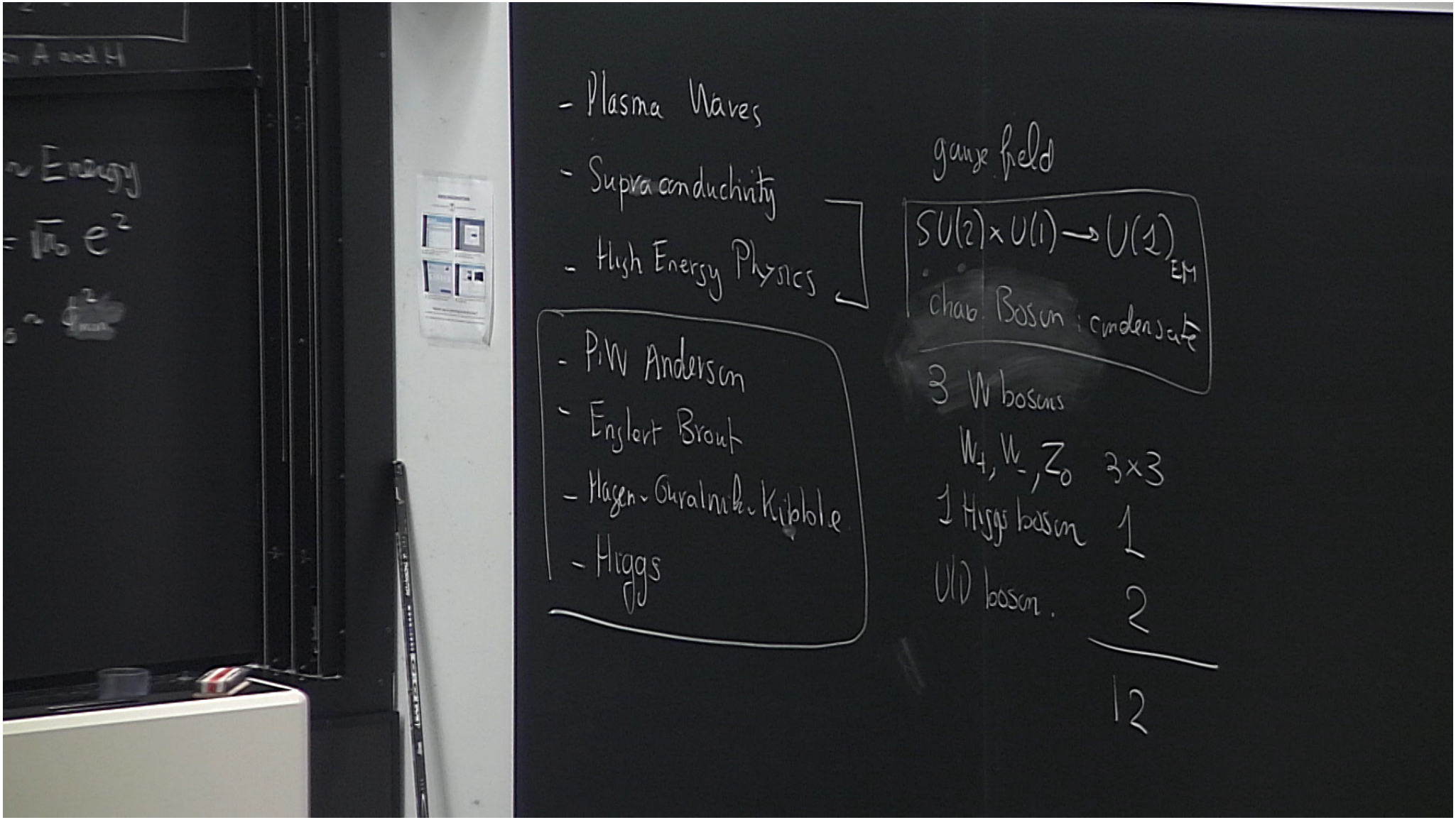
$A_\mu$ → massive gauge boson W 3

H → massive scalar Higgs Particle 1

$M_H = \sqrt{g}\,\Phi_{min}$ $M_W = e\,\Phi_{min}$

Classical A


Classical Analog (non-relativistic)
1930
ω ~ Energy
Plasma
globally neutral
$\omega_{plasma}^2 = \frac{4\pi}{m} n_0 e^2$
$\omega = \sqrt{n_0}\, e^2$
$n_0 \sim \phi_{min}^2$
EM
e⁻ e⁻ e⁻
e⁻ e⁻
e⁻ E
$\vec{E}(t,\vec{x}) = \vec{E}_0 \cos(\omega t - \vec{p}\cdot\vec{x})$
Fixed Ions
Langmuir
⟹ plasma oscillations
density n of electrons
Plasma


- Plasma Waves
- Superconductivity
- High Energy Physics
- P.W. Anderson
- Englert Brout
- Hagen - Guralnik - Kibble
- Higgs
gauge field
$SU(2) \times U(1) \rightarrow U(1)_{EM}$
chab. Boson : condensate
3 W bosons
$W_+, W_-, Z_0$ 3×3
1 Higgs boson 1
U(1) boson. 2
12