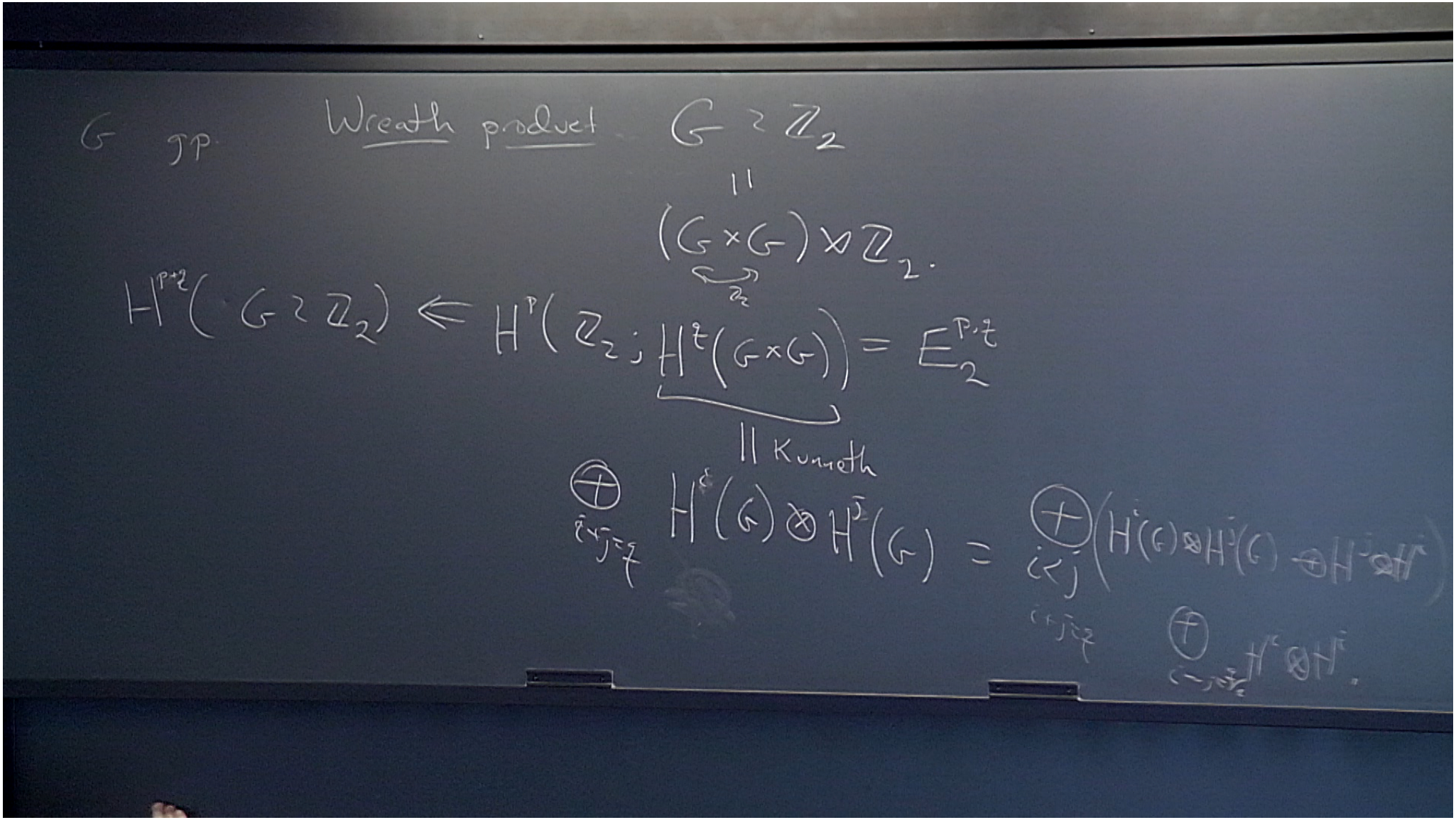
Title: The Cohomology of Groups (Johnson-Freyd/Guo) - Lecture 10

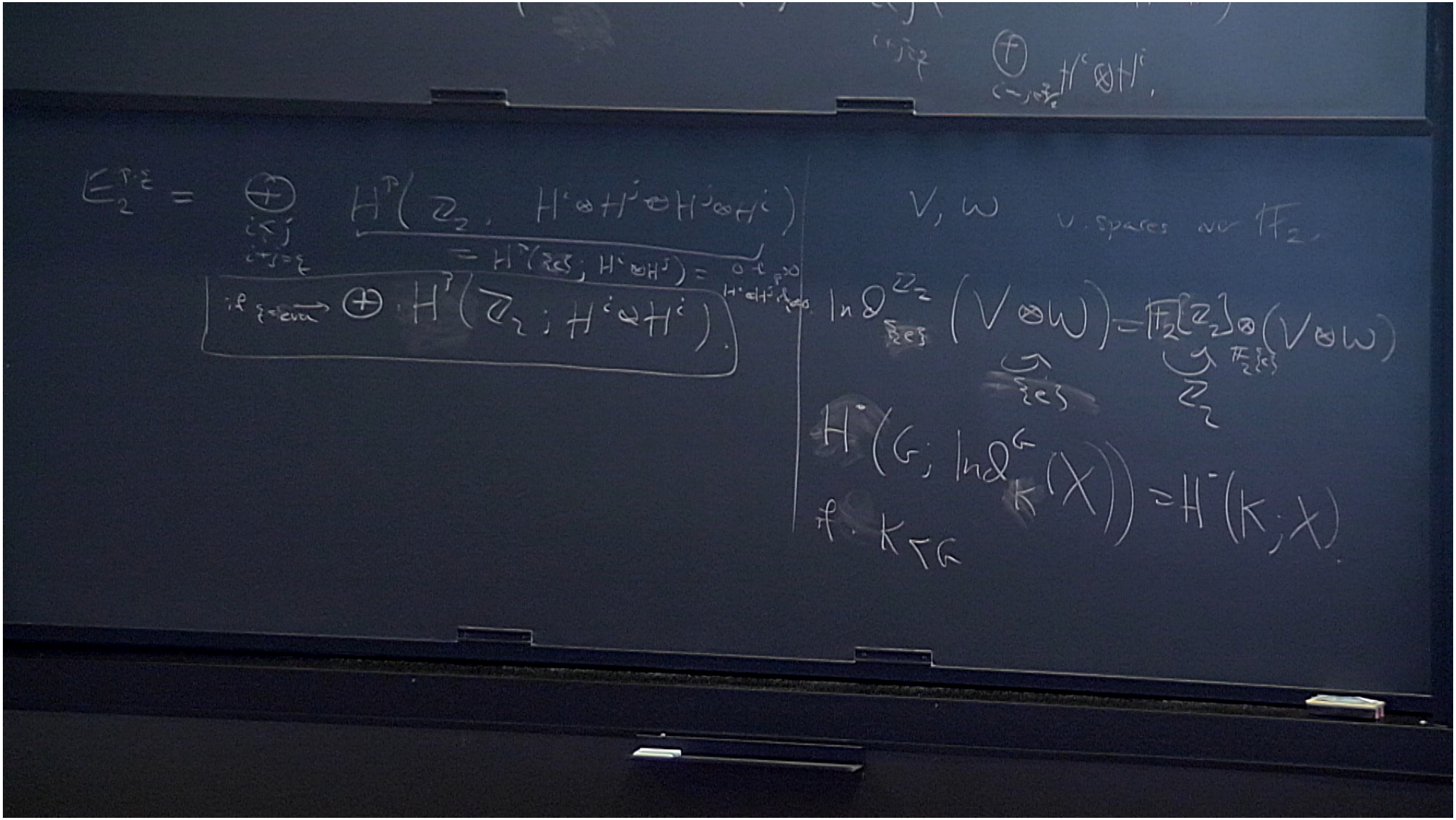
Speakers: Meng Guo, Theo Johnson-Freyd

Collection: The Cohomology of Groups (Johnson-Freyd/Guo)

Date: December 04, 2019 - 10:00 AM

URL: http://pirsa.org/19120001


G JP
Wreath product
$G \wr \mathbb{Z}_2$
$= (G \times G) \rtimes \mathbb{Z}_2$.
$\mathbb{Z}_2$
$H^{p+q}(G \wr \mathbb{Z}_2) \Leftarrow H^p(\mathbb{Z}_2; H^q(G \times G)) = E_2^{p,q}$
Kunneth
$\bigoplus_{i+j=q} H^i(G) \otimes H^j(G) = \bigoplus_{i<j} (H^i(G) \otimes H^j(G) \oplus H^j \otimes H^i) \oplus \bigoplus_{i=j} H^i \otimes H^i$.


$E_2^{p,q} = \bigoplus_{i<j,\ i+j=q} H^p(\mathbb{Z}_2, H^i \otimes H^j \otimes H^j \otimes H^i)$
$= H^p(\mathbb{Z}_2; H^i \otimes H^j) = \begin{cases} 0 & p \text{ odd} \\ H^i \otimes H^j & p = 0 \\ H^i \otimes H^j & p > 0 \end{cases}$
if q even $\longrightarrow \bigoplus_{i<j,\ i+j=q} H^p(\mathbb{Z}_2; H^i \otimes H^j)$
$\bigoplus_{i<j} H^i \otimes H^j$
V, W $\mathbb{F}_2$-spaces over $\mathbb{F}_2$.
$\ln \delta^{\mathbb{Z}_2}_{\mathbb{Z}\{\xi\}}(V \otimes W) = \mathbb{F}_2[\mathbb{Z}_2] \otimes_{\mathbb{F}_2\{\xi\}} (V \otimes W)$
$\mathbb{Z}\{\xi\}$
$\mathbb{Z}_2$
$H^\cdot(G; \ln \delta^G_K(X)) = H^\cdot(K; X)$
$K \subset G$

$H^* = H^*(-; \mathbb{F}_2)$.

Want to understand

N.B.: over $\mathbb{F}_2$, $\epsilon - 1 = \epsilon + 1$.

$H^*(\mathbb{Z}_2;\ V \otimes V)$ where V v.s. over $\mathbb{F}_2$.

favourite resolution of $\mathbb{Z}$ over $\mathbb{Z}[\mathbb{Z}_2]$

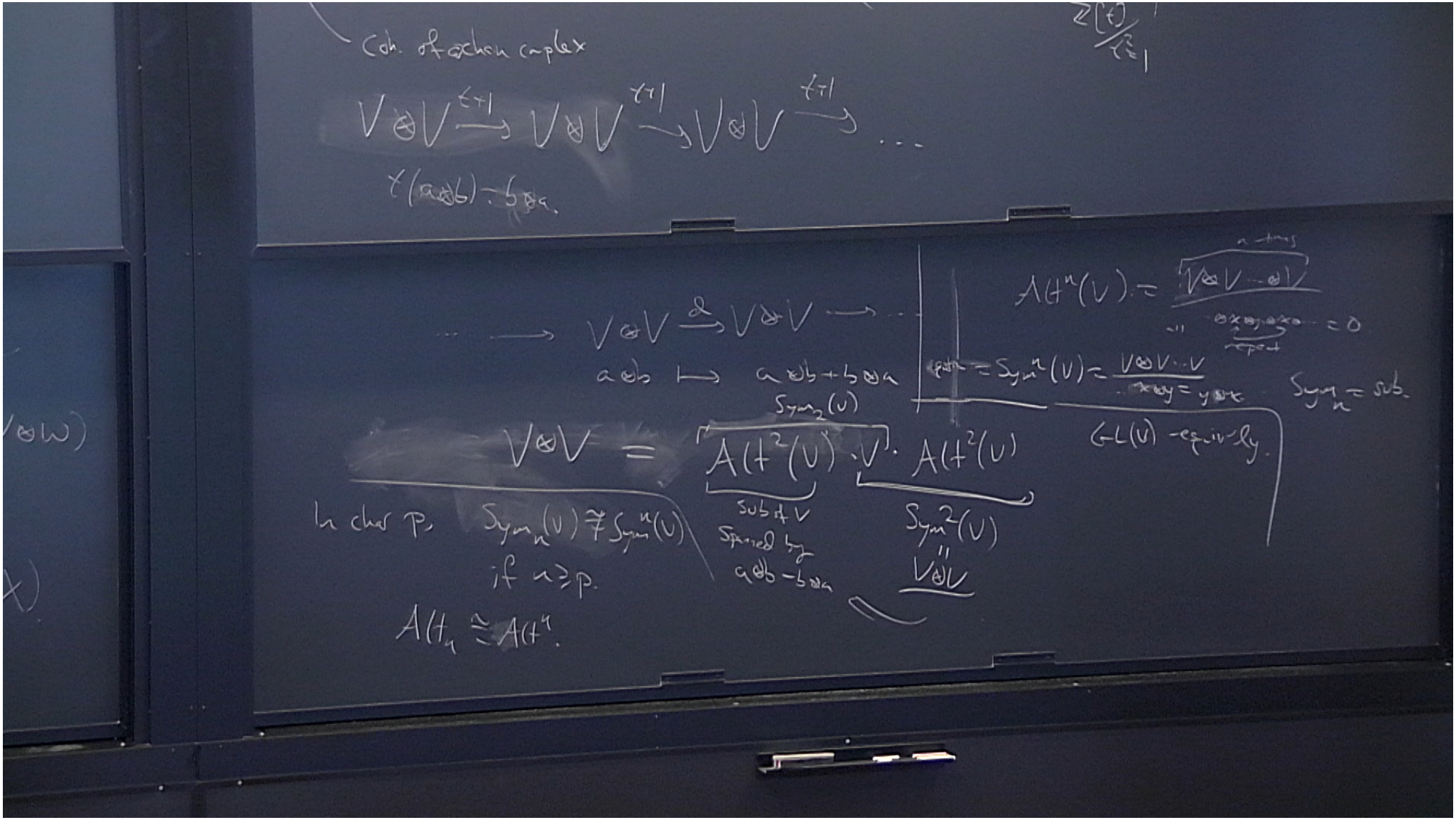
$\mathbb{Z} \xleftarrow{\epsilon} \mathbb{Z}[\mathbb{Z}_2] \xleftarrow{\chi(\epsilon-1)} \mathbb{Z}[\mathbb{Z}_2] \xleftarrow{\chi(\epsilon+1)} \mathbb{Z}[\mathbb{Z}_2] \xleftarrow{\chi(\epsilon-1)} \mathbb{Z}[\mathbb{Z}_2] \xleftarrow{t \mapsto 1} \mathbb{Z}$

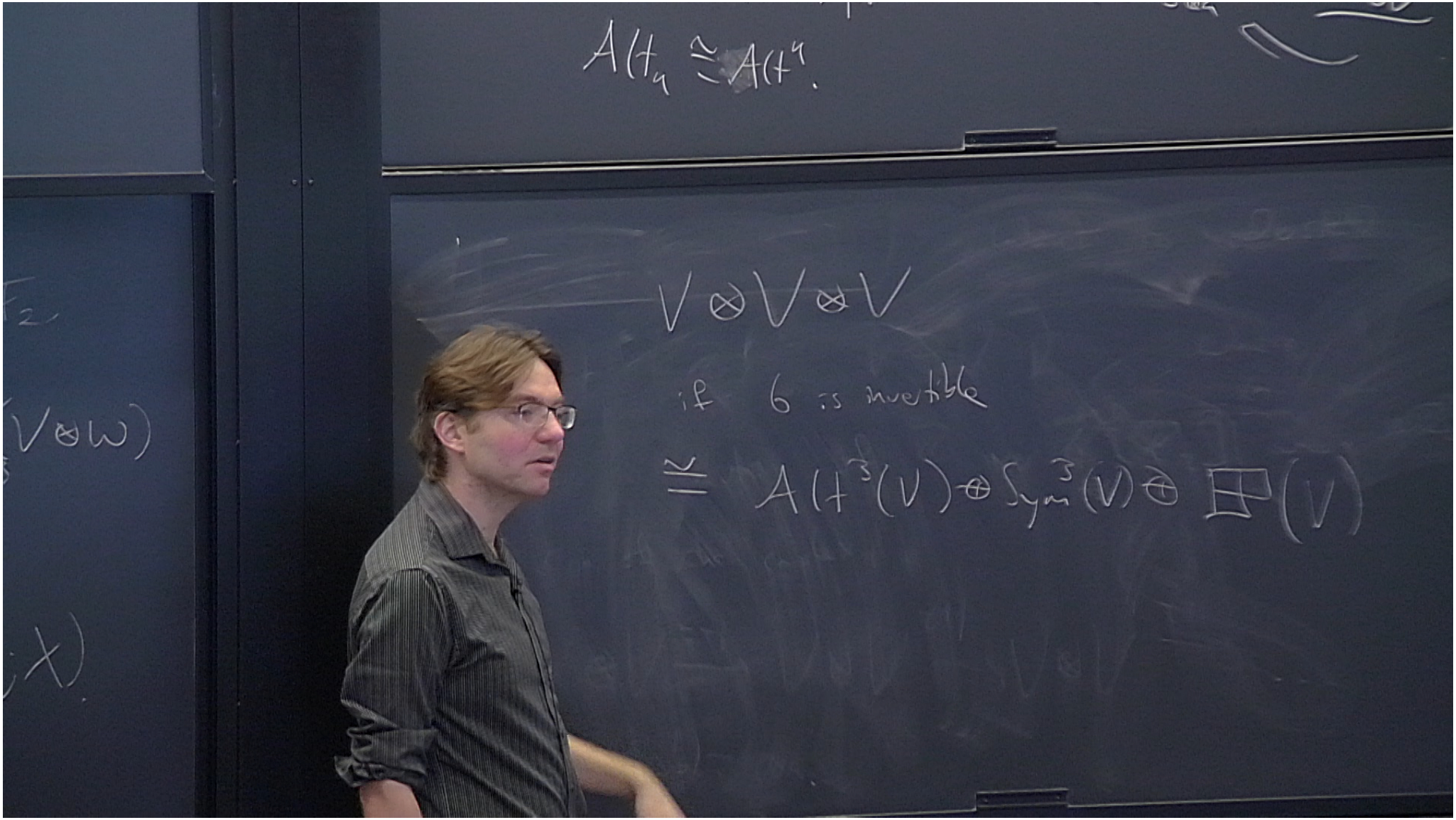
$\mathbb{Z}[t]/(t^2 = 1)$

Coh. of cochain complex

$V \otimes V \xrightarrow{\epsilon+1} V \otimes V \xrightarrow{\epsilon-1} V \otimes V \xrightarrow{\epsilon+1} \cdots$

$\epsilon(a \otimes b) = b \otimes a$.


Coh. of Cech complex
$V \otimes V \xrightarrow{\partial+1} V \otimes V \xrightarrow{\partial+1} V \otimes V \xrightarrow{\partial+1} \cdots$
$\partial(a \otimes b) = b \otimes a$.
$\cdots \longrightarrow V \otimes V \xrightarrow{\partial} V \otimes V \longrightarrow \cdots$
$a \otimes b \longmapsto a \otimes b + b \otimes a$
$\mathrm{Sym}_2(V)$
$V \otimes V = \mathrm{Alt}^2(V) \cdot V \cdot \mathrm{Alt}^2(V)$
$\mathrm{Sym}^2(V)$
$V \otimes V$
sub of V
spanned by
$a \otimes b - b \otimes a$
In char p. $\mathrm{Sym}_n(V) \neq \mathrm{Sym}^n(V)$ if $n \geq p$.
$\mathrm{Alt}_n \cong \mathrm{Alt}^n$.
$\mathrm{Alt}^n(V) = V \otimes V \cdots \otimes V$
n times
$x \otimes y \otimes x \otimes \cdots = 0$
repeat
$\mathrm{Sym}^n(V) = \frac{V \otimes V \cdots V}{x \otimes y = y \otimes x}$
$\mathrm{Sym}_n = \mathrm{sub}$.
$GL(V)$-equiv-ly.


$A(t_4 \cong A(t^4$.
$V \otimes V \otimes V$
if 6 is invertible
$\cong A(t^3(V) \oplus Sym^3(V) \oplus \square(V)$

if $n \geq p$.

$A(t_q) \cong A(t^q)$.

$V \oplus V$

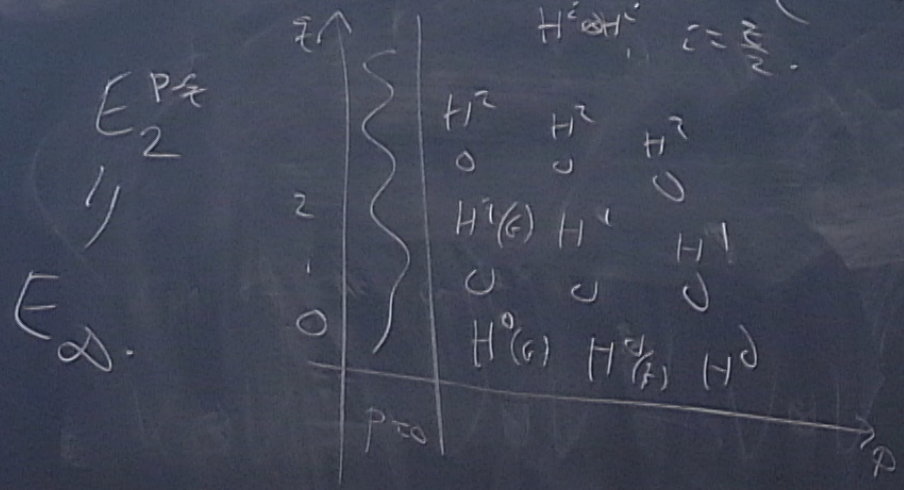
$\rightsquigarrow H^p(\mathbb{Z}_2; V \oplus V) = \begin{cases} Sym_2(V) & \text{if } p=0. \\ V & \text{if } p>0. \end{cases}$

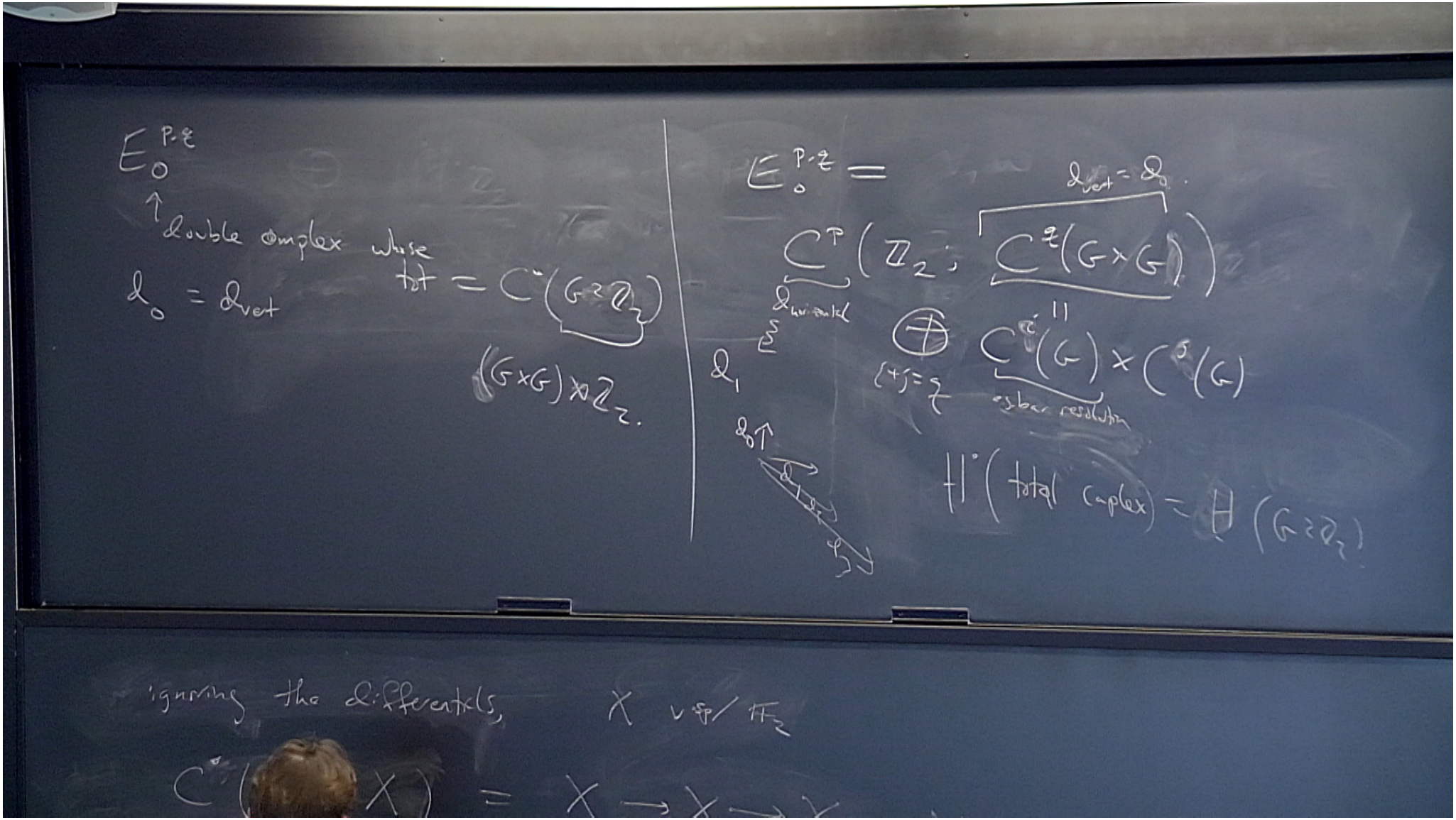
$H^i \oplus H^i$, $i = \frac{n}{2}$.

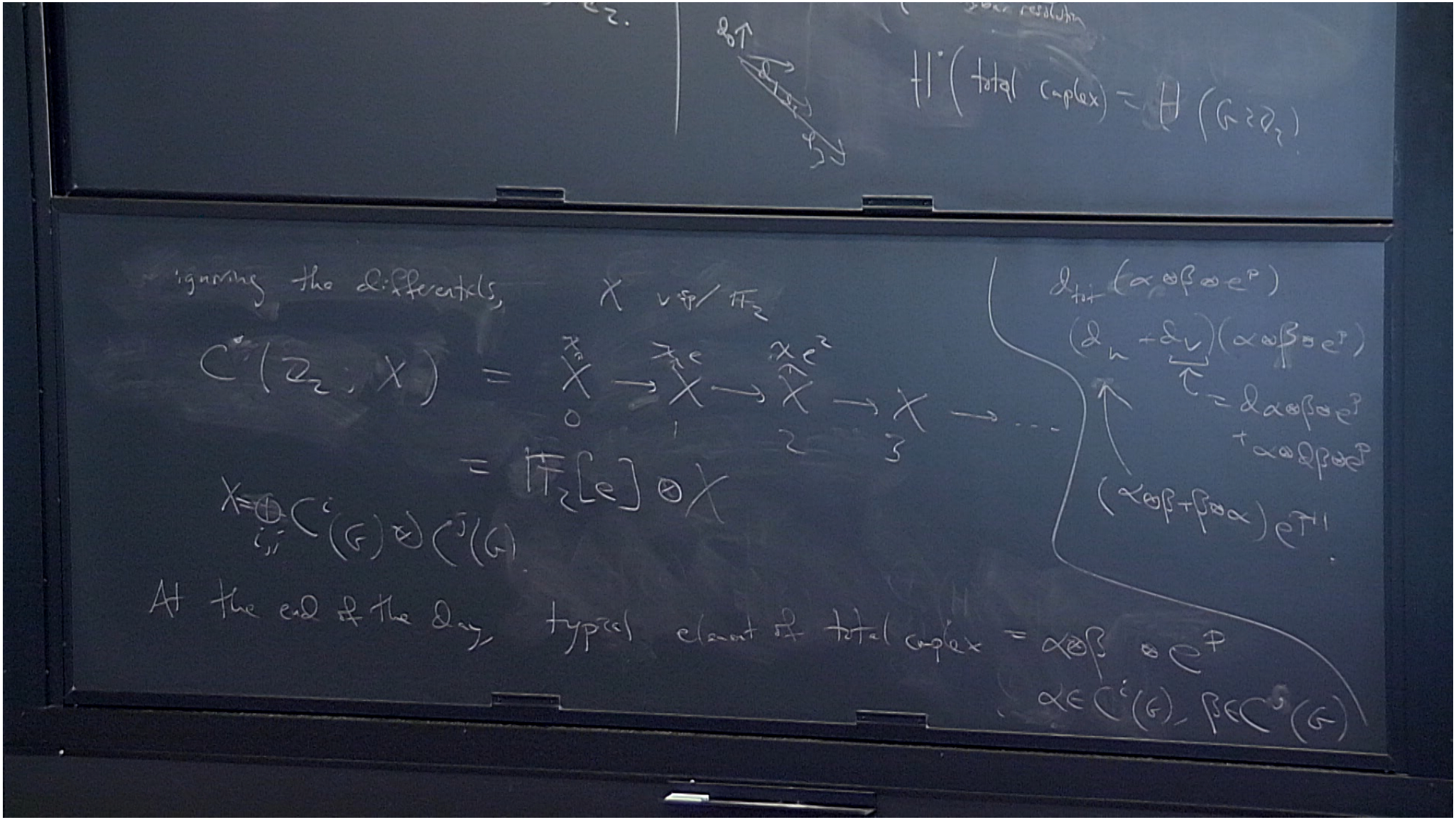
$E_2^{p,q}$

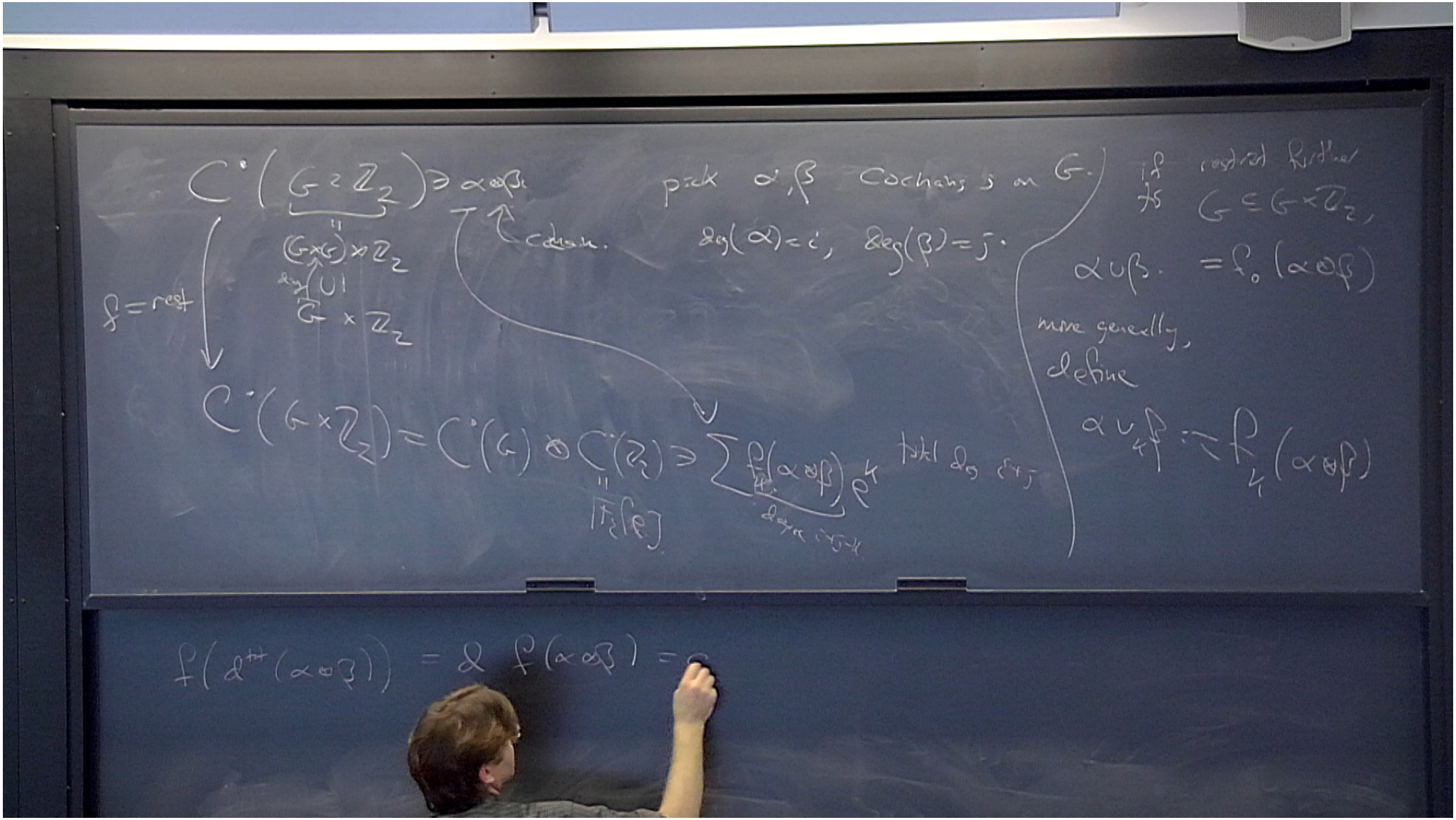
$\Downarrow$

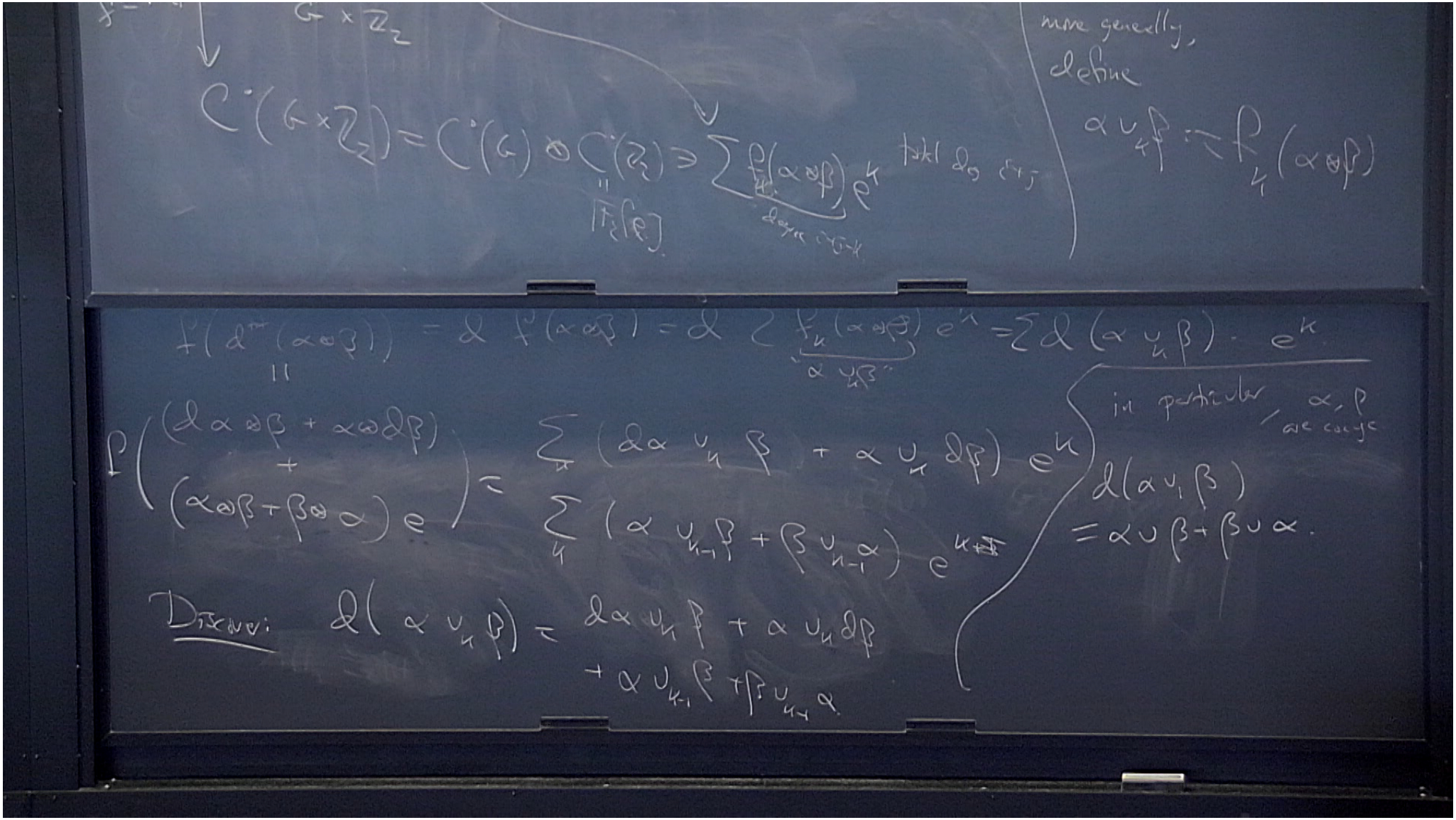
E_∞.




$E_0^{p,q}$
double complex whose tot $= C^\cdot(G \rtimes \mathbb{Z}_2)$
$d_0 = d_{vert}$
$(G \times G) \rtimes \mathbb{Z}_2$.
$E_0^{p,q} =$
$d_{vert} = d_0$
$C^p(\mathbb{Z}_2; C^q(G \times G))$
$d_{horizontal}$
$\bigoplus_{i+j=q} C^i(G) \times C^j(G)$
d_1
d_0
Bar resolution
$H^\cdot$(total complex) $= H(G \rtimes \mathbb{Z}_2)$
ignoring the differentials, X up to π_2
$C^\cdot(X) = X \to X \to$


$H^*(\text{total complex}) = H(G; \mathbb{Z}_2)$
ignoring the differentials, X via π_2
$C^*(\mathbb{Z}_2; X) = X \to X \to X \to X \to \cdots$
$= \mathbb{F}_2[e] \otimes X$
$X = \bigoplus_{i,j} C^i(G) \otimes C^j(G)$
$d_{tot}(\alpha \otimes \beta \otimes e^p)$
$(d_h + d_v)(\alpha \otimes \beta \otimes e^p)$
$= d\alpha \otimes \beta \otimes e^p + \alpha \otimes d\beta \otimes e^p$
$(\alpha \otimes \beta + \beta \otimes \alpha) e^{p+1}$
At the end of the day, typical element of total complex $= \alpha \otimes \beta \otimes e^p$
$\alpha \in C^i(G)$, $\beta \in C^j(G)$


$C^\bullet(G \times \mathbb{Z}_2) \ni \alpha \cup_i \beta$
pick α, β cochains on G.
$\deg(\alpha) = i$, $\deg(\beta) = j$.
if restrict further to $G \in G \times \mathbb{Z}_2$,
$\alpha \cup \beta = f_0(\alpha \otimes \beta)$
more generally, define
$\alpha \cup_k \beta := f_k(\alpha \otimes \beta)$
$f = \text{rest}$
$C^\bullet(G \times \mathbb{Z}_2) = C^\bullet(G) \otimes C^\bullet(\mathbb{Z}_2) \ni \sum f_k(\alpha \otimes \beta) e^k$
total deg $i+j$
$\mathbb{Z}_2[e]$
$f(d^{\text{tot}}(\alpha \otimes \beta)) = \delta f(\alpha \otimes \beta) =$


$G \times \mathbb{Z}_2$
$C^\cdot(G \times \mathbb{Z}_2) = C^\cdot(G) \otimes C^\cdot(\mathbb{Z}_2) \Rightarrow \sum f_k(\alpha \otimes \beta)\, e^k$
$= \mathbb{Z}[e]$
more generally, define
$\alpha \cup_k \beta := f_k(\alpha \otimes \beta)$
$\delta(\alpha \cup_k \beta) = \delta\alpha \cup_k \beta + (-1)^{|\alpha|} \alpha \cup_k \delta\beta + (-1)^{|\alpha|+|\beta|+k} \alpha \cup_{k-1} \beta + (-1)^{|\alpha|+|\beta|+k-1} \beta \cup_{k-1} \alpha$
$\delta(\alpha \cup_k \beta) = \delta\alpha \cup_k \beta + \alpha \cup_k \delta\beta + \alpha \cup_{k-1} \beta + \beta \cup_{k-1} \alpha$
$\delta(\alpha \cup_1 \beta) = \alpha \cup \beta + \beta \cup \alpha$
in particular, α, β are cocycles
$\delta(\alpha \cup_1 \beta) = \alpha \cup \beta + \beta \cup \alpha$.

Focus: $\alpha = \beta$ cocycle.

I'm working over $\mathbb{F}_2$.

$$d(\alpha \underbrace{\cup_k \alpha}) = \alpha \cup_{k-1} \alpha + \alpha \cup_{k-1} \alpha = 0.$$

$\leftarrow$ S_ξ^m takes cocycles to cocycles.

$\boxed{S_\xi \alpha}$ (raised deg by $k-1$) $\rightarrow$ deg $2k-1$

if $\deg \alpha = i$, define $S_\xi^m \alpha = S_{\xi^{i-m}} \alpha = \alpha \cup_{i-m} \alpha$.

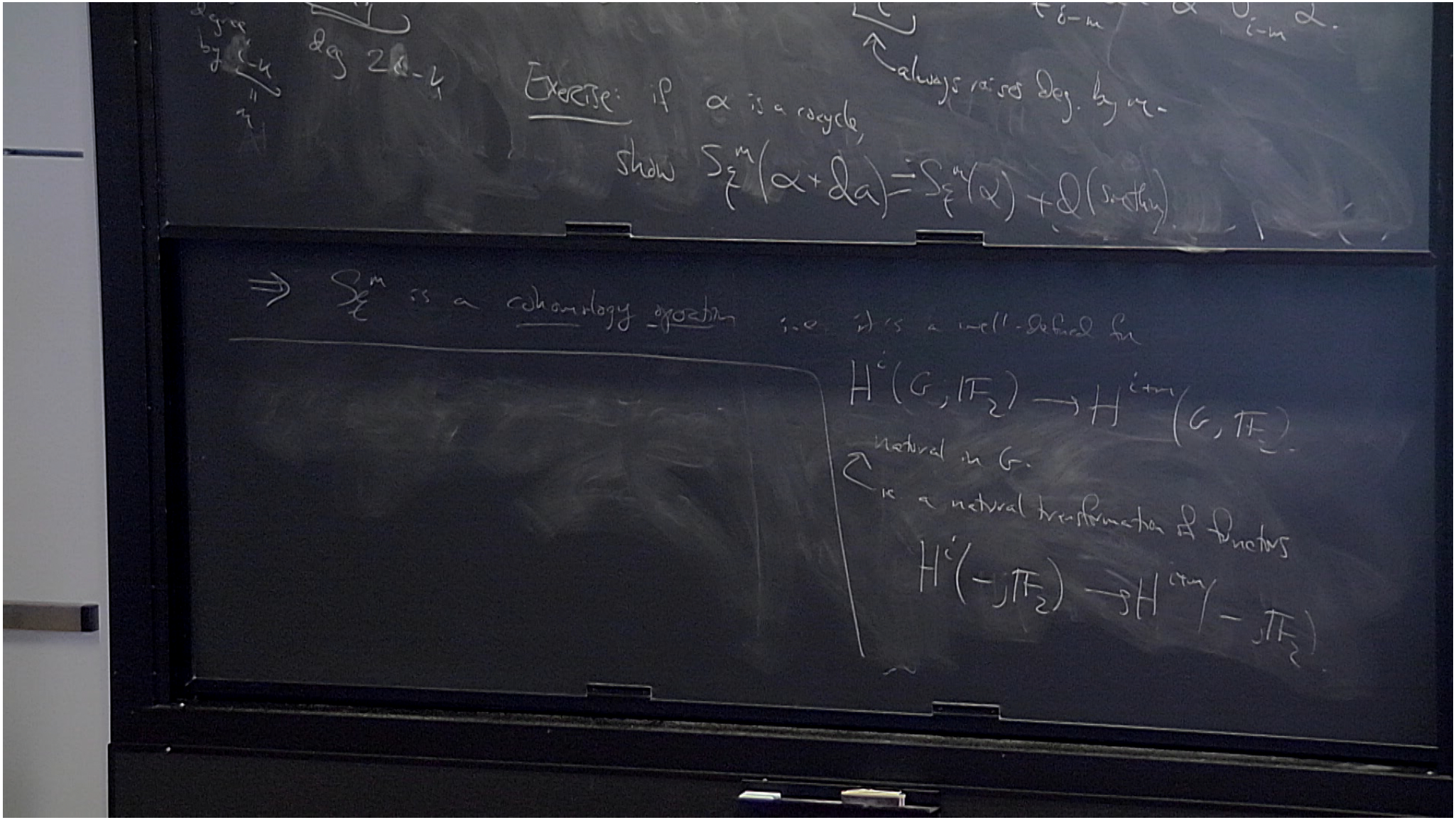
$\uparrow$ always raises deg. by m

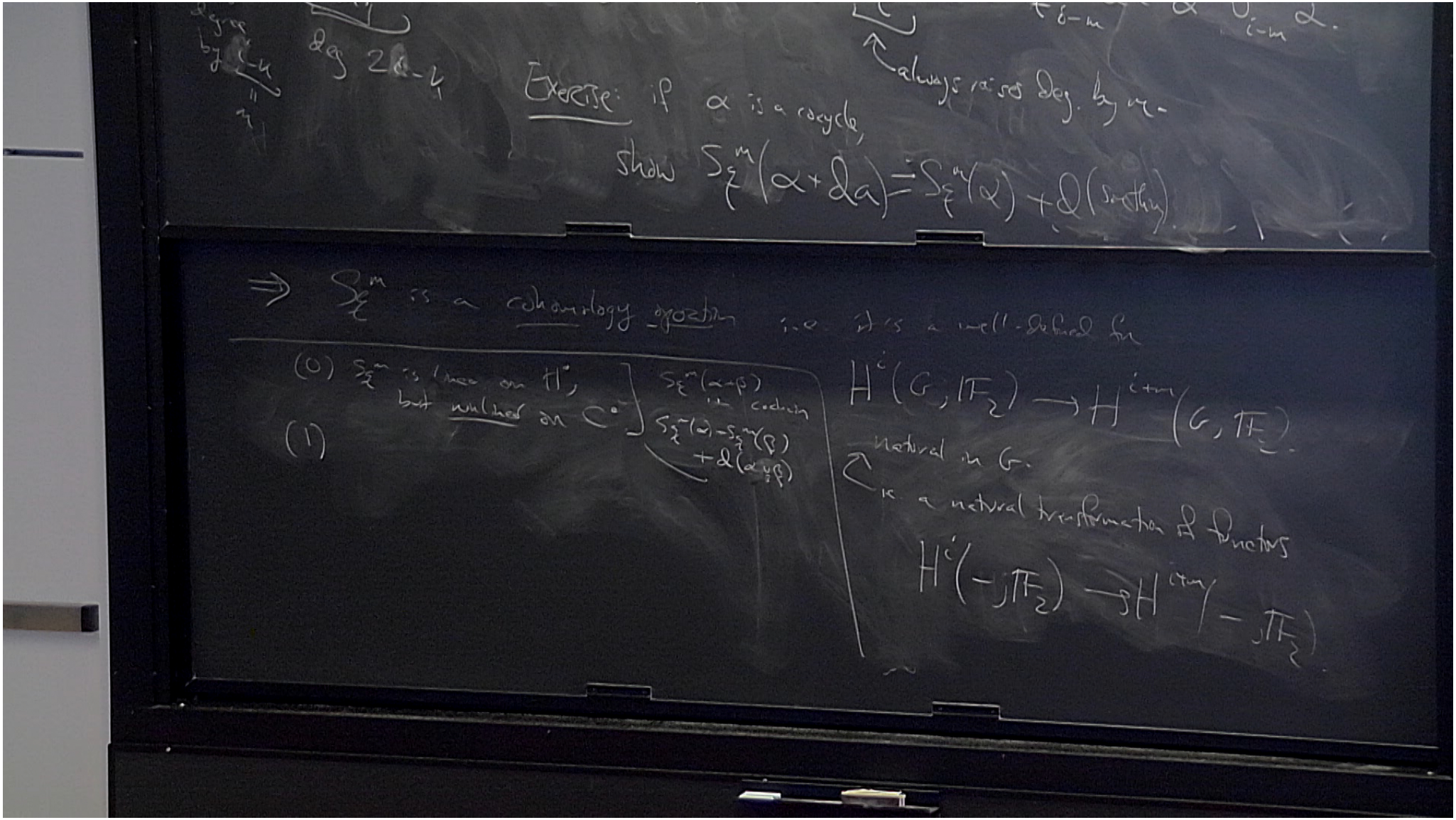
Exercise: if α is a cocycle,

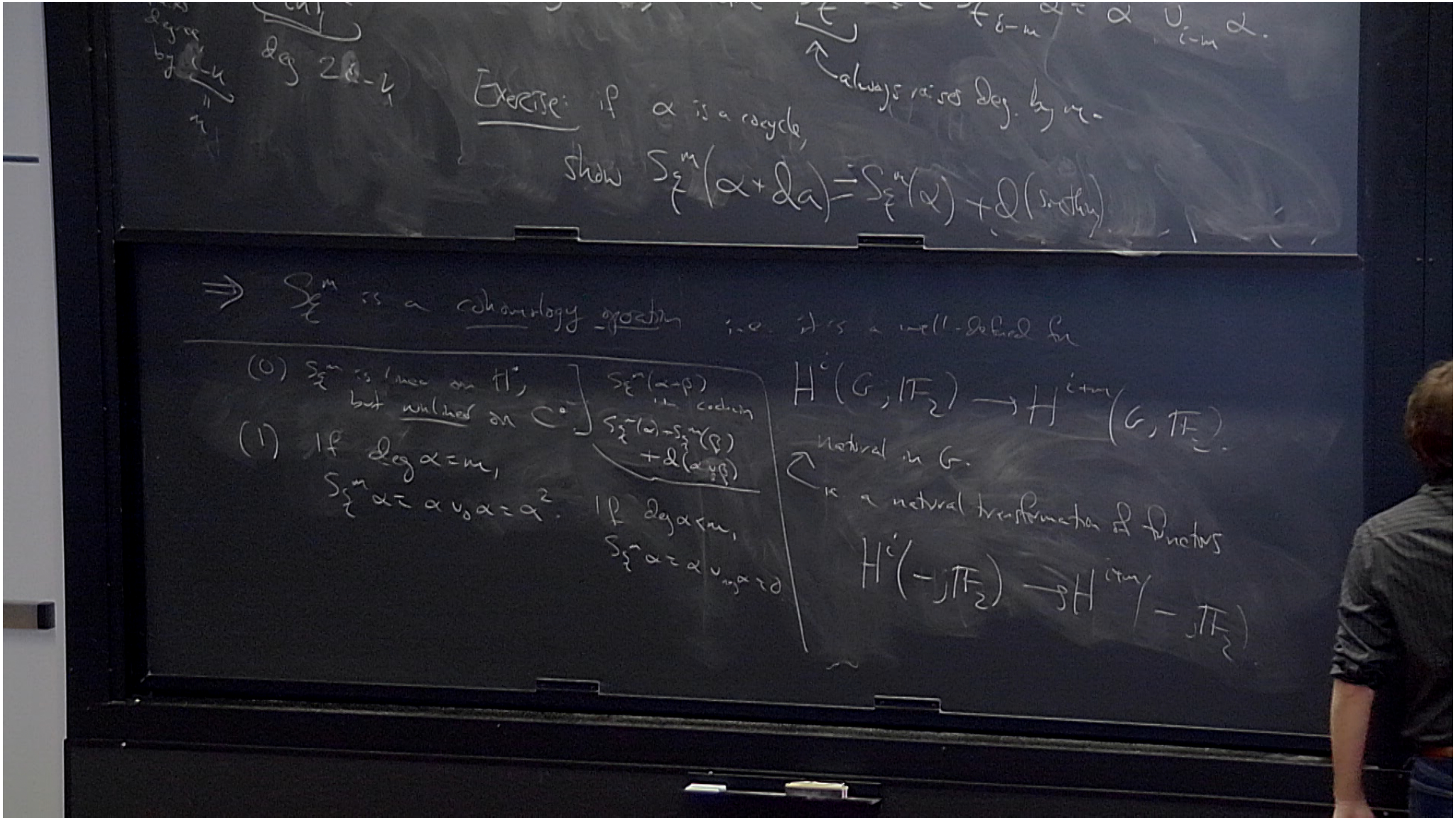
show $S_\xi^m(\alpha + da) = S_\xi^m(\alpha) + d(\text{something})$.

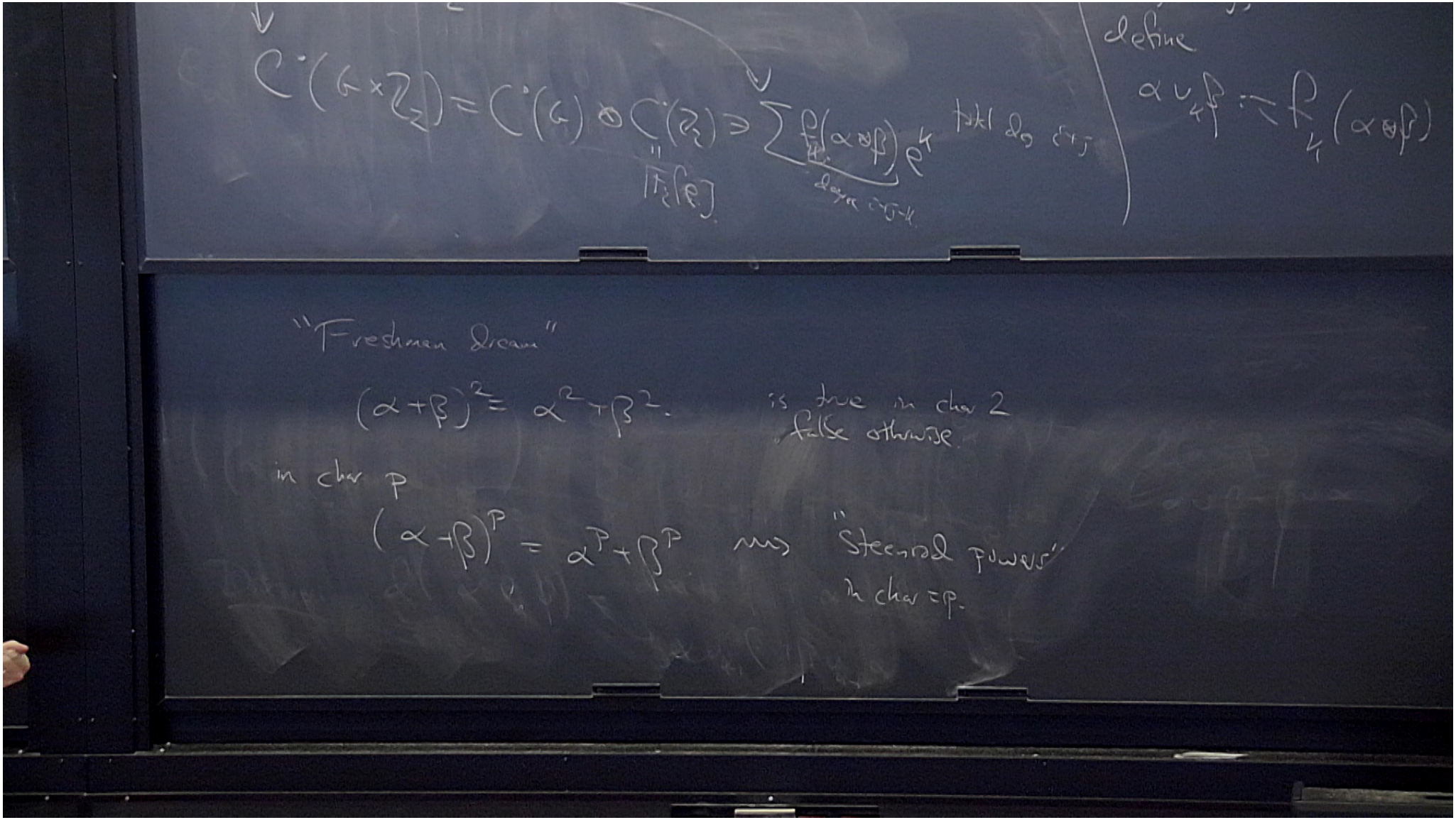
$E_0^{p,q}$ $\uparrow$

$E_0^{p,q} =$ $d_{out} = 0$


always raises deg. by m-
Exercise: if α is a cocycle,
show $S_\xi^m(\alpha + \delta a) = S_\xi^m(\alpha) + \delta(\text{something})$
$\Rightarrow S_\xi^m$ is a cohomology operation i.e. it is a well-defined, for
$H^i(G, \mathbb{F}_2) \to H^{i+m}(G, \mathbb{F}_2)$
natural in G.
is a natural transformation of functors
$H^i(-, \mathbb{F}_2) \to H^{i+m}(-, \mathbb{F}_2)$


Exercise: if α is a cocycle,
show $S_\xi^m(\alpha + d\alpha) = S_\xi^m(\alpha) + d(\text{something})$
$\Rightarrow S_\xi^m$ is a cohomology operation
(0) S_ξ^m is linear on Π^*,
but nonlinear on C^*
(1)
$S_\xi^m(\alpha-\beta)$ is cocycle
$S_\xi^m(\alpha) - S_\xi^m(\beta) + d(\alpha \cup_i \beta)$
$H^i(G, \mathbb{F}_2) \to H^{i+m}(G, \mathbb{F}_2)$
natural in G.
is a natural transformation of functors
$H^i(-, \mathbb{F}_2) \to H^{i+m}(-, \mathbb{F}_2)$


Exercise: if α is a cocycle,
always raises deg. by m-
show $S_\xi^m(\alpha + \delta a) = S_\xi^m(\alpha) + \delta(\text{something})$
$\Rightarrow S_\xi^m$ is a cohomology operation i.e. it is a well-defined fn
(0) S_ξ^m is linear on Π^*, but nonlinear on C^*
$S_\xi^m(\alpha+\beta)$ in coclass $S_\xi^m(\alpha)+S_\xi^m(\beta)+\delta(\alpha \cup_1 \beta)$
(1) If $\deg \alpha = m$, $S_\xi^m \alpha = \alpha \cup_0 \alpha = \alpha^2$. If $\deg \alpha < m$, $S_\xi^m \alpha = \alpha \cup_{m-1} \alpha = 0$
$H^i(G, \Pi_2) \to H^{i+m}(G, \Pi_2)$
natural in G.
is a natural transformation of functors
$H^i(-, \Pi_2) \to H^{i+m}(-, \Pi_2)$


$C^\cdot(G\times\mathbb{Z}_2) = C^\cdot(G)\otimes C^\cdot(\mathbb{Z}_2) \cong \sum_{i,j} f_i(\alpha)\otimes\beta\, e^j$
$\cong \Gamma_2[e]$
total deg $i+j$
deg $= j-i$
define
$\alpha \cup_4 \beta := f_4(\alpha\otimes\beta)$
"Freshman dream"
$(\alpha+\beta)^2 = \alpha^2+\beta^2$.
is true in char 2
false otherwise
in char p
$(\alpha+\beta)^p = \alpha^p+\beta^p$ ⇝ "Steenrod powers"
in char p.

$$C^\cdot(G\times\mathbb{Z}_2) = C^\cdot(G)\otimes C^\cdot(\mathbb{Z}_2) = \sum_{i+j=k} \underbrace{P_i(\alpha\otimes\beta)}_{\deg = 2i} e^j$$

$C^\cdot(\mathbb{Z}_2) = \mathbb{F}_2[e]$

$\deg = j$

define

$$\alpha \cup_k \beta := P_k(\alpha\otimes\beta)$$

"Freshman dream"

$$(\alpha+\beta)^2 = \alpha^2+\beta^2.$$

is true in char 2, false otherwise

in char p

$$(\alpha+\beta)^p = \alpha^p+\beta^p \rightsquigarrow \text{"Steenrod powers" in char } p.$$

$$H^\cdot(\mathbb{Z}_2, \mathbb{F}_2) = \mathbb{F}_2[e]$$

$\deg 1$

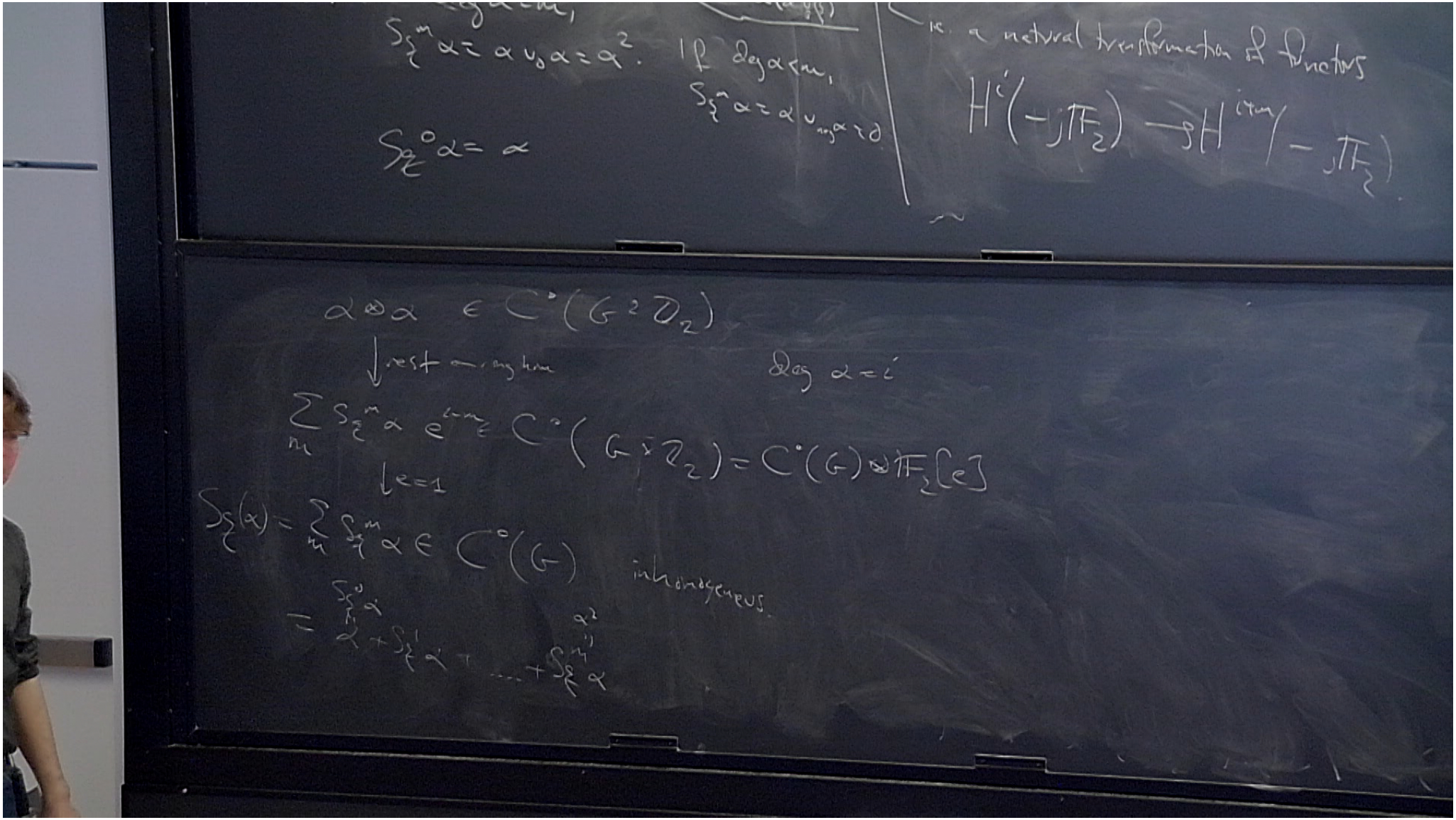
$p = 2$

$$H^\cdot(\mathbb{Z}_p, \mathbb{F}_p) = \mathbb{F}_2[x,y]/(x^2=0)$$

$\deg 1$ $\deg 2$

$x^2 = 0$? odd

$y = \beta x$.


$S_\xi^m \alpha = \alpha \cup_0 \alpha = \alpha^2$. If $\deg \alpha < m$,
$S_\xi^m \alpha = \alpha \cup_{m-\deg\alpha} \alpha = 0$
$S_\xi^0 \alpha = \alpha$
is a natural transformation of functors
$H^i(-; \mathbb{F}_2) \to H^{i+m}(-; \mathbb{F}_2)$
$\alpha \otimes \alpha \in C^*(G; \mathbb{Z}_2)$
restrict
$\deg \alpha = i$
$\sum_m S_\xi^m \alpha \, e^{+m} \in C^*(G; \mathbb{Z}_2) = C^*(G) \otimes \mathbb{F}_2[e]$
$e=1$
$S_\xi(\alpha) = \sum_m S_\xi^m \alpha \in C^*(G)$ inhomogeneous.
$= S_\xi^0 \alpha + S_\xi^1 \alpha + \dots + S_\xi^{\alpha^2} \alpha$

$S_\xi^m \alpha = \alpha \cup_0 \alpha = \alpha^2$. If $\deg \alpha < m$, $S_\xi^m \alpha = \alpha \cup_{m-\deg\alpha} \alpha = 0$

$S_\xi^0 \alpha = \alpha$

is a natural transformation of functors

$H^i(-; \mathbb{F}_2) \to H^{i+m}(-; \mathbb{F}_2)$.

$\alpha \otimes \alpha \in C^*(G; \mathbb{Z}_2)$

$\downarrow$ restriction homomorphism

$\deg \alpha = i$

$\sum_m S_\xi^m \alpha \, e^{\xi m} \in C^*(G; \mathbb{Z}_2) = C^*(G) \otimes \mathbb{F}_2[e]$

$\downarrow e=1$

$S_\xi(\alpha) = \sum_m S_\xi^m \alpha \in C^*(G)$ inhomogeneous.

$= \alpha + S_\xi^1 \alpha + \dots + S_\xi^{\deg\alpha} \alpha$

$S_\xi(\alpha \cup \beta)$

$=$

$\mathrm{rest}((\alpha \cup \beta) \otimes (\alpha \cup \beta))$

$=$

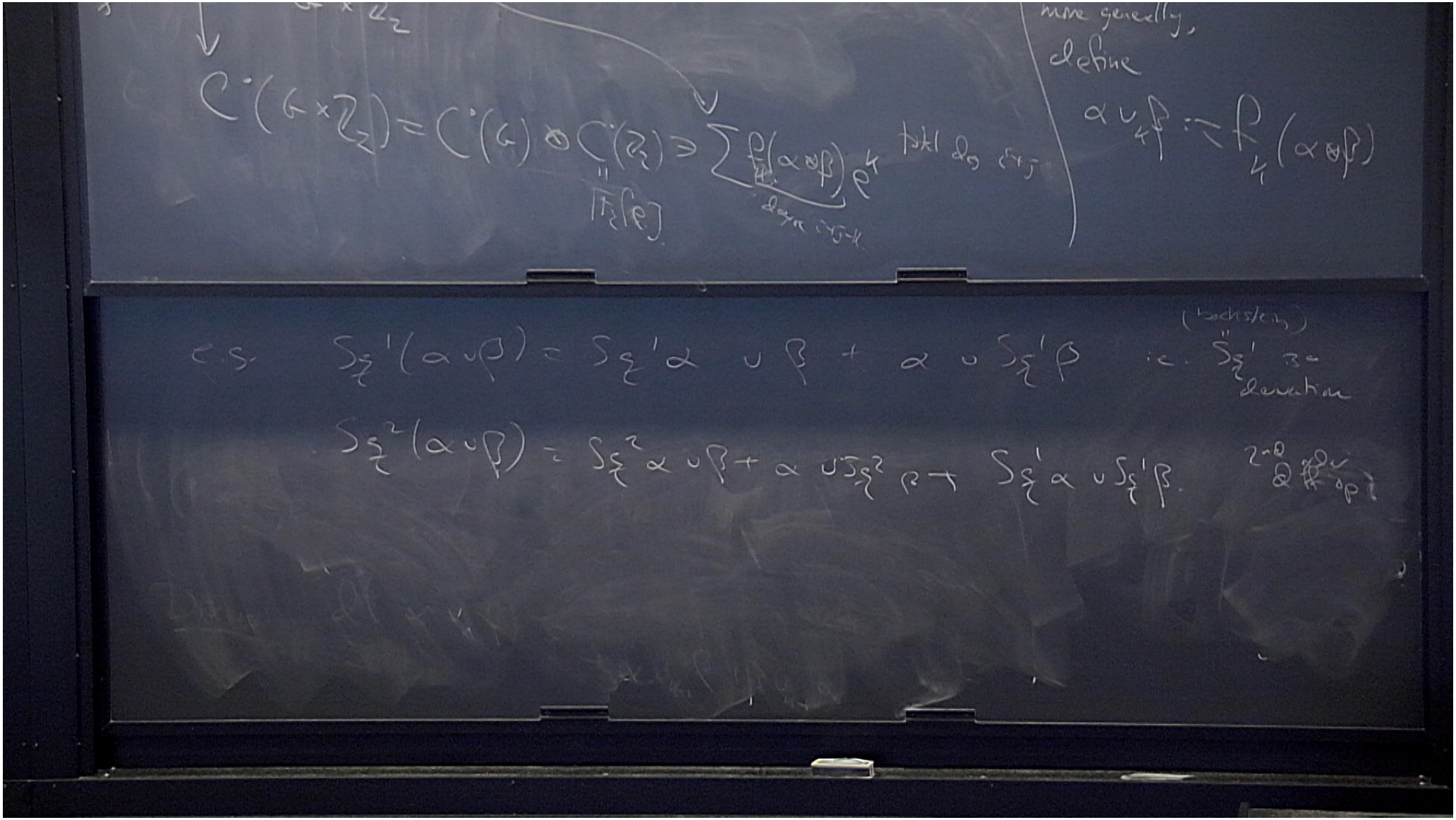
$\mathrm{rest}((\alpha \otimes \alpha) \cup (\beta \otimes \beta))$

$=$

$S_\xi(\alpha) \cup S_\xi(\beta)$

expand by degree. Cartan's formula

$S_\xi^m(\alpha \cup \beta) = \sum_{i+j=m} S_\xi^i \alpha \cup S_\xi^j \beta$


more generally,
define
(Bockstein)

e.g. $Sq^1(\alpha \cup \beta) = Sq^1\alpha \cup \beta + \alpha \cup Sq^1\beta$ i.e. $\tilde{Sq}^1$ = (bockstein) derivation

$Sq^2(\alpha \cup \beta) = Sq^2\alpha \cup \beta + \alpha \cup Sq^2\beta + Sq^1\alpha \cup Sq^1\beta$.

Adem relation: If $i < 2j$ then

$$Sq^i \circ Sq^j = \sum_{k=0}^{\lfloor i/2 \rfloor} \binom{j-k-1}{i-2k} Sq^{i+j-k} \circ Sq^k$$

Idea:

$$(\star)\quad Sq^i \circ Sq^j = \sum_{k=0}^{\lfloor i/2 \rfloor} \binom{j-k-1}{i-2k} Sq^{i+j-k} \circ Sq^k$$

Idea: Look at

$$(G \wr \mathbb{Z}_2) \wr \mathbb{Z}_2$$

$$\| $$

$$\left((G\times G)\rtimes \mathbb{Z}_2\right) \times \left((G\times G)\rtimes \mathbb{Z}_2\right) \rtimes \mathbb{Z}_2$$

$$\| $$

$$(G\times G\times G\times G) \rtimes Dih_8 \subseteq G \wr Sym_4$$

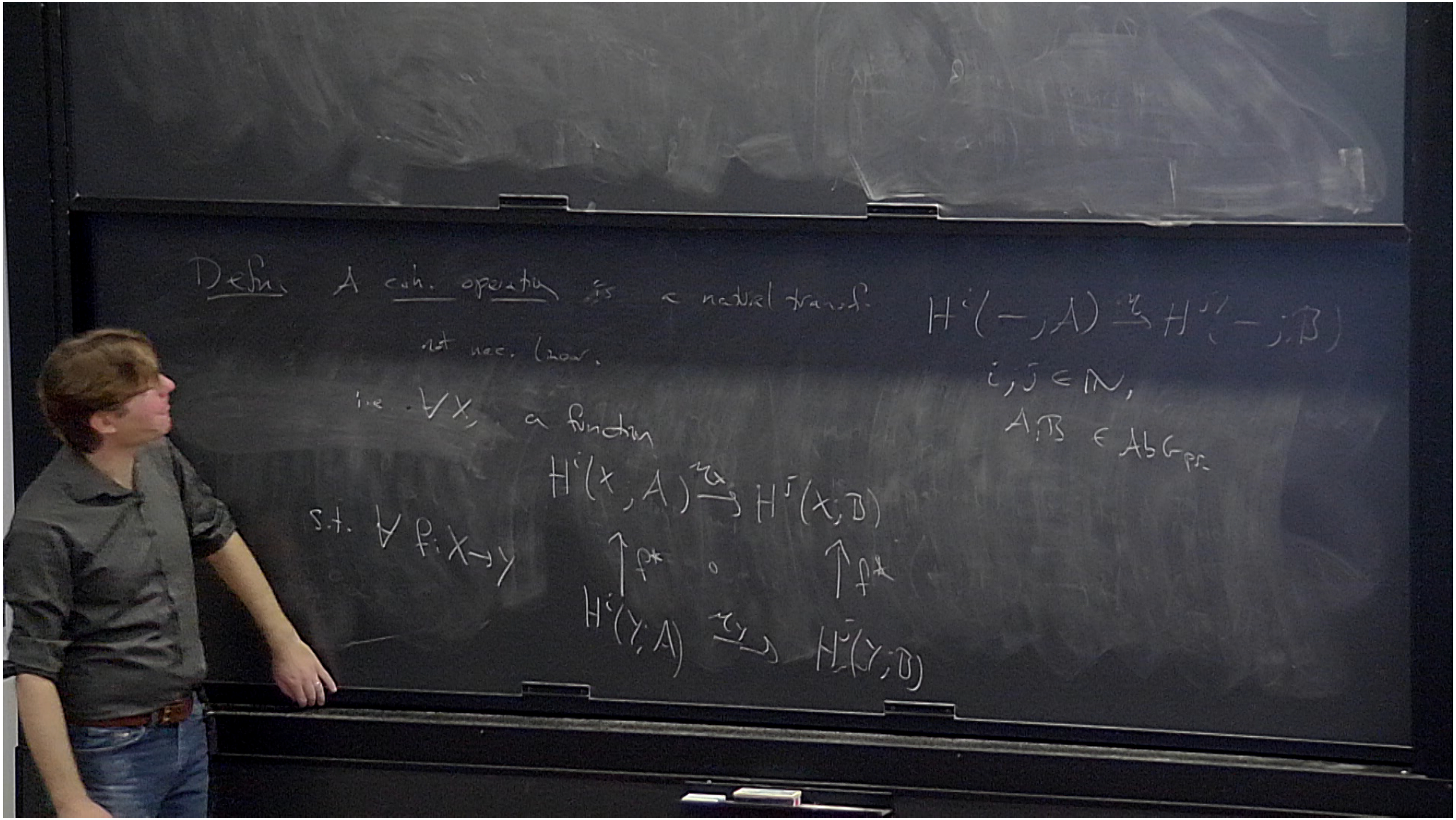
$\mathcal{A}_2 :=$ Steenrod algebra

$$= \mathbb{F}_2 \langle Sq^0, Sq^1, Sq^2, \dots \rangle$$

$$/ (Sq^0 = 1),\ (\star)$$

non com alg. which acts on $H^*(\text{anything})$

Cartan relation $\leftrightarrow$ $\mathcal{A}_2$: Hopf


Defn. A coh. operation is a natural transf.
not nec. linear.
$H^i(-,A) \Rightarrow H^j(-,B)$
$i,j \in \mathbb{N}$,
$A,B \in AbGps.$
i.e. $\forall X$, a function
$H^i(X,A) \to H^j(X;B)$
s.t. $\forall f: X \to Y$
f^* f^*
$H^i(Y;A) \to H^j(Y;B)$

s.t. $\forall f: X \to Y$ $\quad f^* \circ f^*$

$H^i(Y;A) \xrightarrow{\pi_Y} H^i(Y;B)$

e.g. $A = B = \pi_2$, $i = 2$, $j = 9$

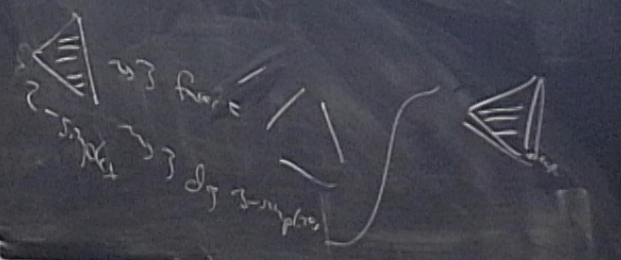
$X \longmapsto \Sigma^3 \vee S^1_\xi \Sigma X$.

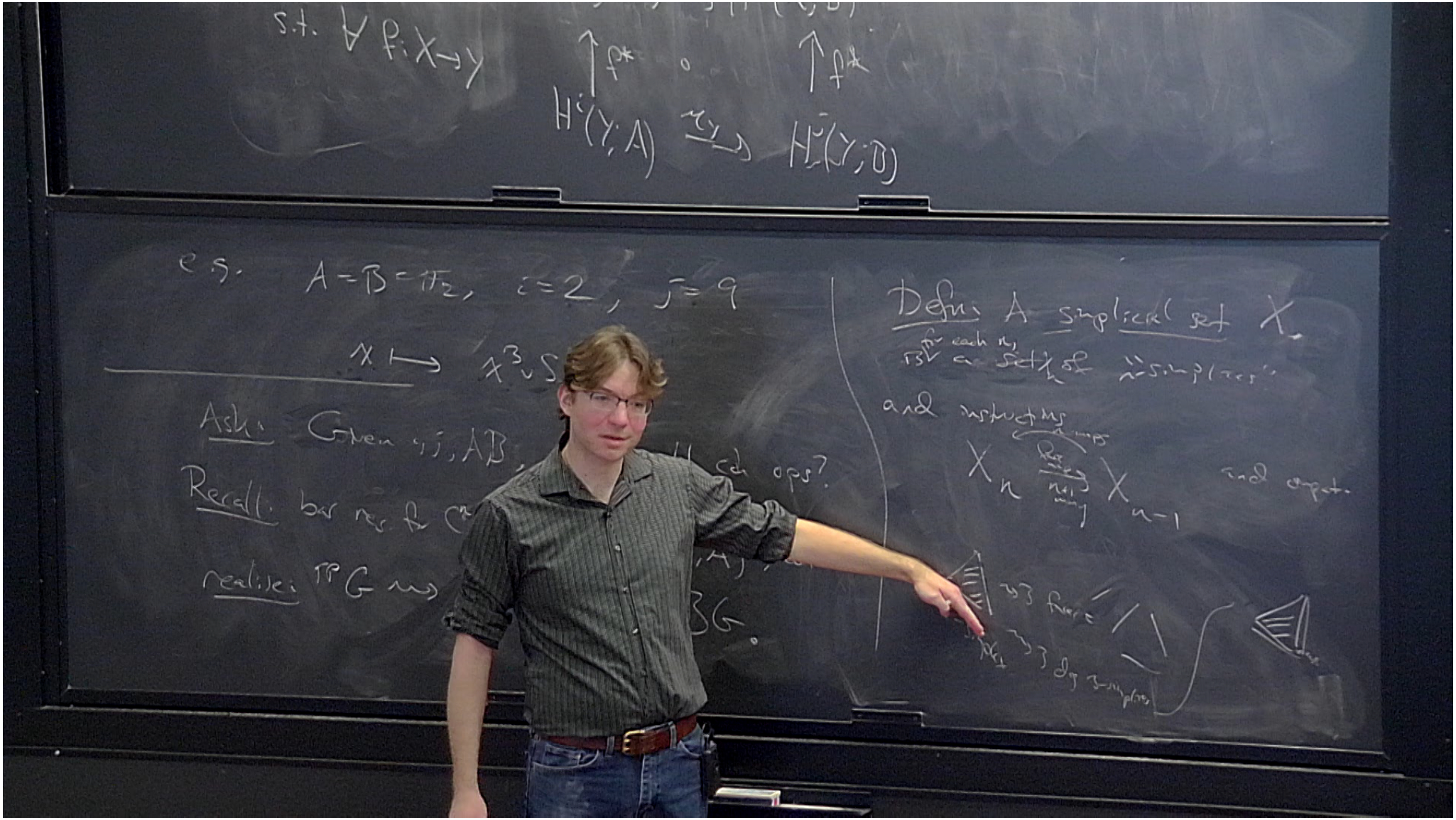
Ask: Given i, j, AB, what are all ch ops?

Recall: bar res. for $C^*(G, A) = \mathrm{Fun}(G^*, A)$, w/ diff.

realise: TFG $\rightsquigarrow$ simplicial set BG.

Defn: A simplicial set $X_\bullet$ is for each n, a set X_n of "n-simplices" and structure maps $X_n \to X_{n-1}$ and cmpt.




s.t. ∀ f: X → Y
f*
f*
H^i(Y; A) → H^i(Y; B)
e.g. A = B = π_2, i = 2, j = 9
Ask: Given i, j, AB
Recall: bar res. ft
realise: BG
Defn: A simplicial set X.
and maps
X_n
X_{n-1}
and cont.

i.e. if I tell you n compatible $(n-1)$-sides of a putative n-simplex then there is an actual n-simplex w/ those n-sides

Facts. - it makes sense to talk about homotopies of Kan simp. sets -

$X \rightrightarrows Y$ a homotopy is $X \star [0,1] \to Y$ interpolating

- homotopy cat of Kan s-sets $\simeq$ homotopy cat of nice top. spaces.

$[X,Y] = \pi_0(X,Y) =$ homotopy classes of maps $X \to Y$

- homotopy cat of Kan s.sets $\simeq$ homotopy cat of top spaces (n-types).

$[X,Y] = h_{\mathrm{ho}}(X,Y) =$ homotopy classes of maps $X \to Y$

- it underscores how little about $H^i(X;A)$ where X is $K(\pi,n)$.

. cocycles coboundaries

Dold-Kan corresp:

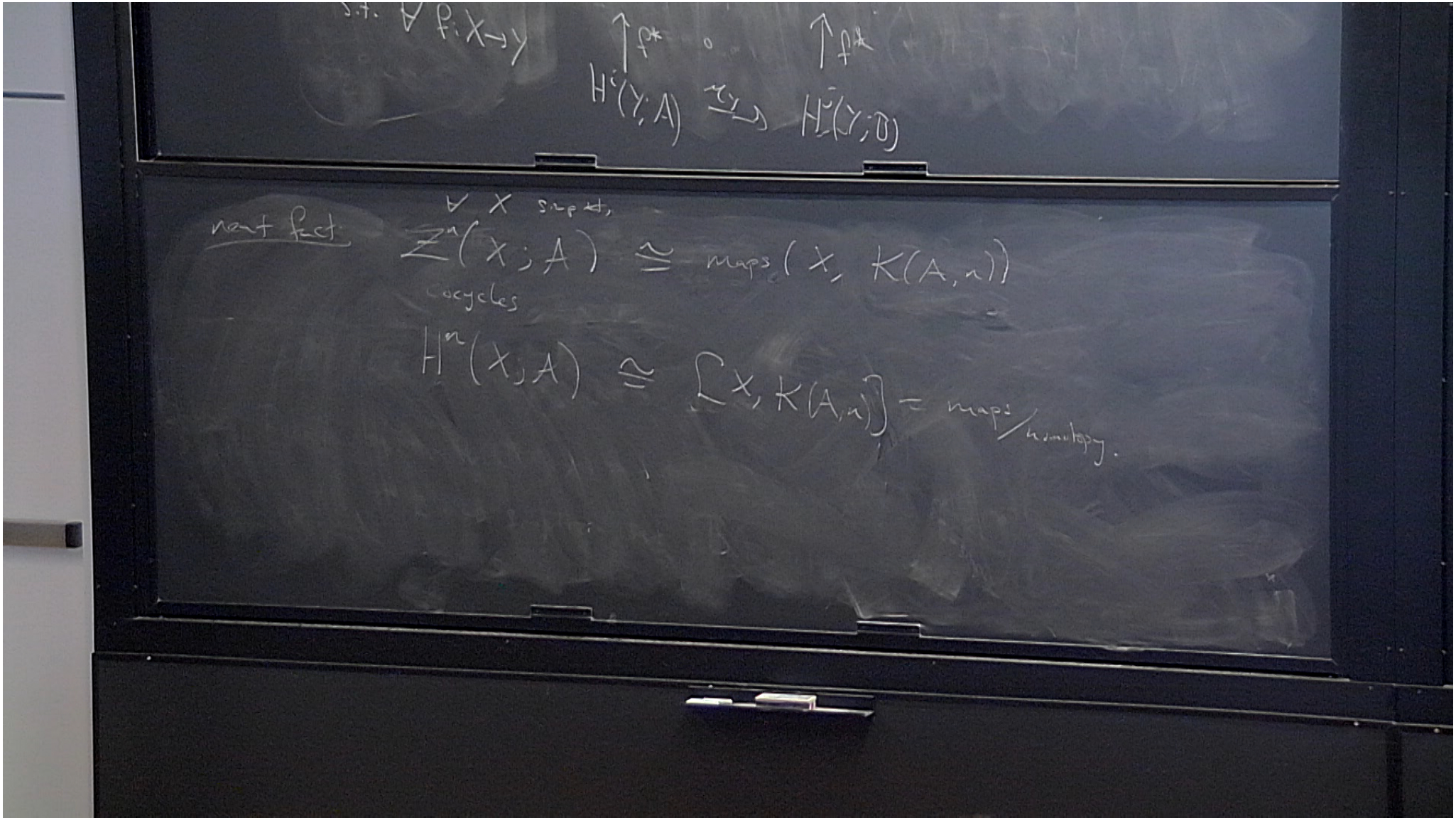
Chain complexes of ab. gps w/ hom deg ≥ 0.

$\overset{\text{cat}}{\simeq}$

simp(ssets) ab gps and every simp ab gp is $K(\pi,n)$.

So $0 \cdots, 0, \underset{n}{A}, 0 \cdots \longrightarrow K(A,n)$ simp ~~ab gp~~ set.

$\partial \circ \partial = 0$


s.t. $\forall f: X \to Y$
f^*
f^*
$H^i(Y;A) \to H^i(Y;B)$
$\forall$ X s.p.ct.
next fact
$Z^n(X;A) \cong \text{maps}(X, K(A,n))$
cocycles
$H^n(X;A) \cong [X, K(A,n)] = \text{maps}/\text{homotopy}$.

$H^i(Y;A) \xrightarrow{\cong} H^i(Y;B)$

$\forall$ X s.p. sets,

nat fact: $Z^n(X;A) \cong \text{maps}(X, K(A,n))$

cocycles

$H^n(X;A) \cong [X, K(A,n)] = \text{maps}/\text{homotopy}$

Yoneda Thm: $\{\text{ch. ops } H^i(-;A) \to H^j(-;B)\} = [K(A,i), K(B,j)] = H^j(K(A,i);B)$

$= H^j_{\pi_1}(\quad ; B)$

comb:

$K(A,1) = BA$

$K(A,2) \simeq B(A \to \quad \to \quad)$

Es. What are all coh. ops $H^i(-; \underline{\mathbb{F}_2}^{\mathbb{Z}_2}) \to H^j(-, \mathbb{F}_2)$

$= H^j(K(\mathbb{Z}_2, 1); \mathbb{F}_2)$

$= H^j_{gp}(\mathbb{Z}_2; \mathbb{F}_2) = \mathbb{F}_2[x]$ Today I

answer = $\{x \mapsto x^j, 0.\}$

$= H^*_{gp}(\mathbb{Z}_2; \mathbb{F}_2) = \mathbb{F}_2[x]$ $\quad \{x \mapsto x^3, 0.\}$

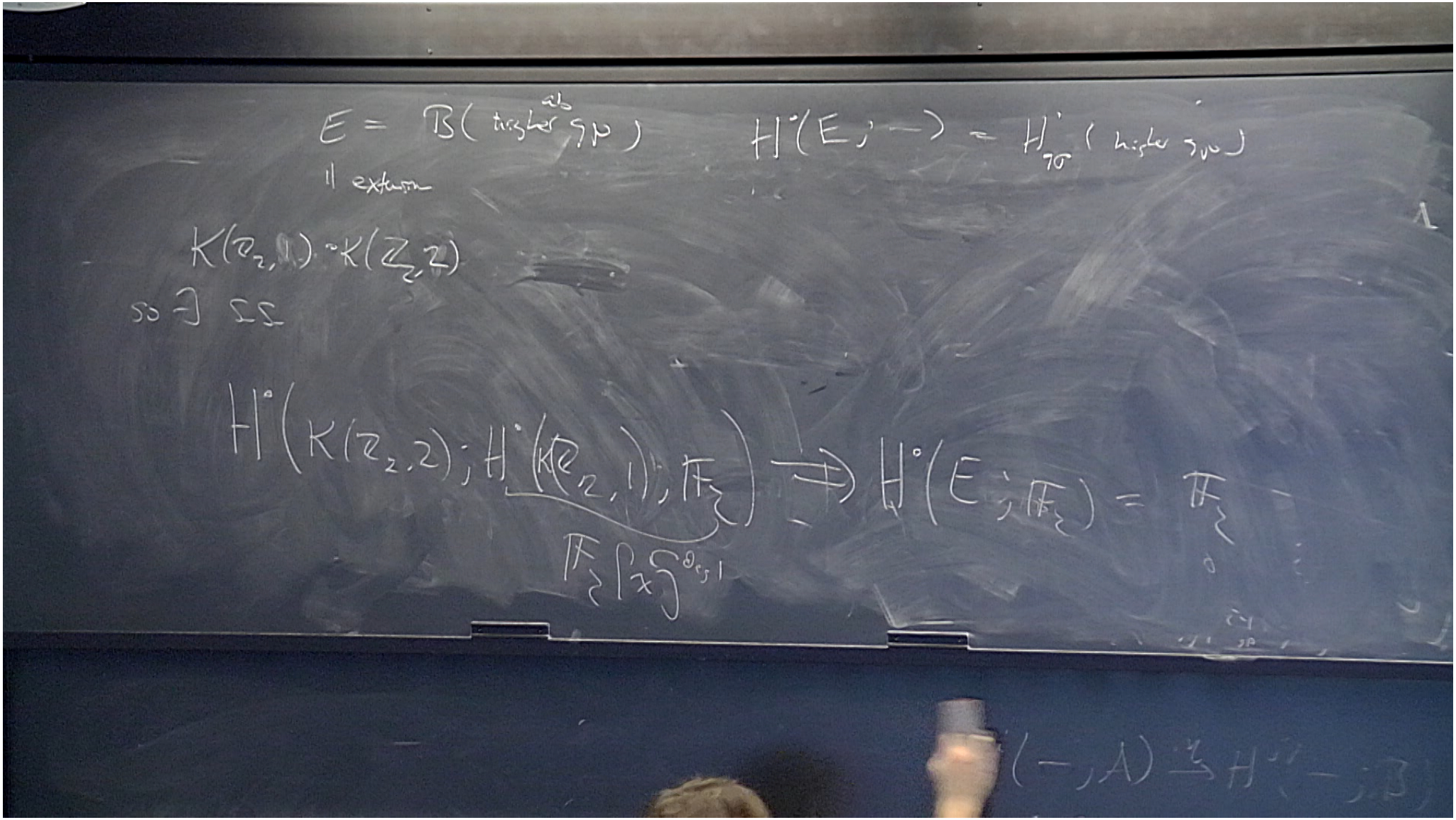
$\deg 1$

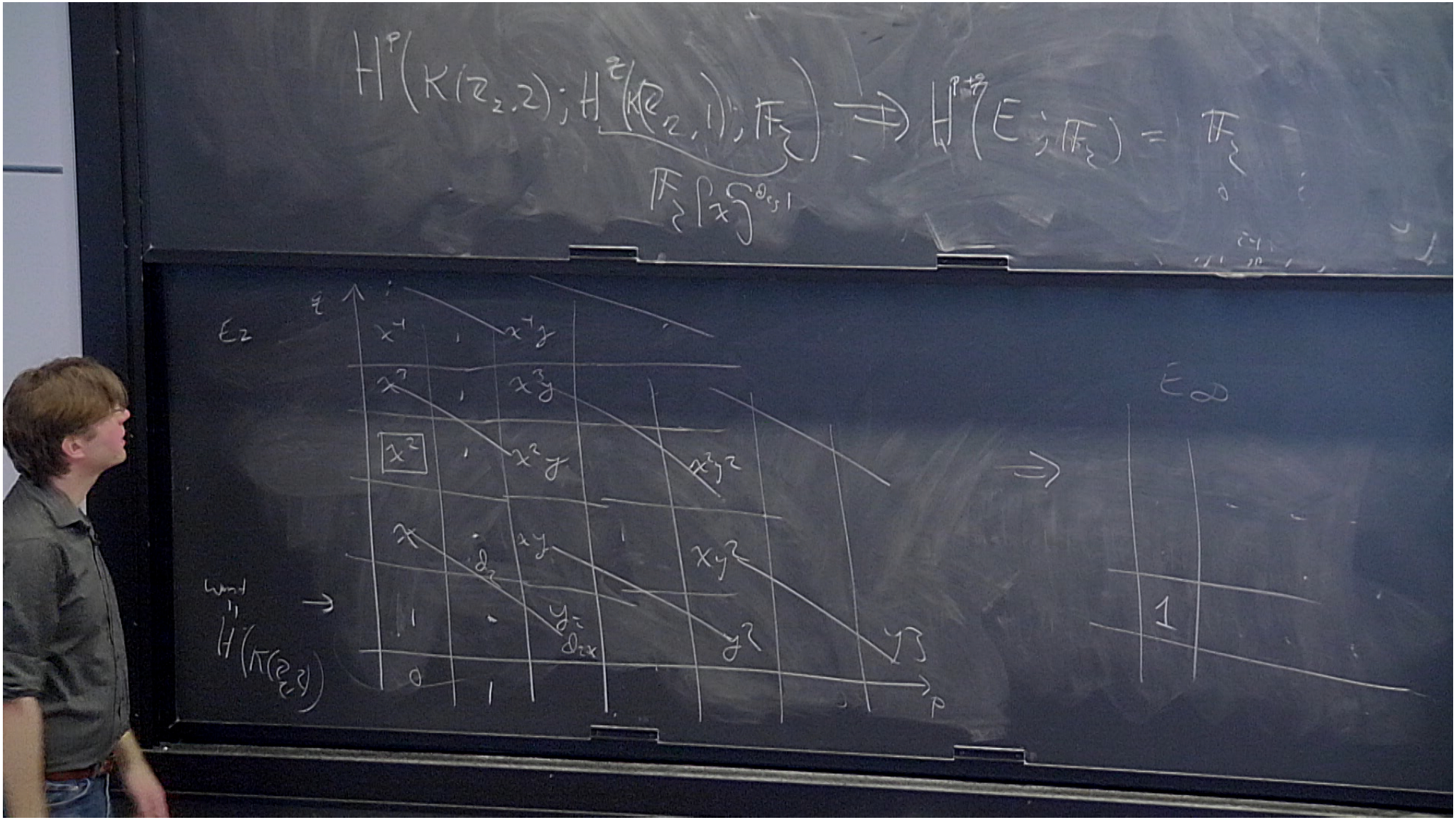
What are all coh ops $H^2(-, \mathbb{F}_2) \to H^*(-; \mathbb{F}_2)$?

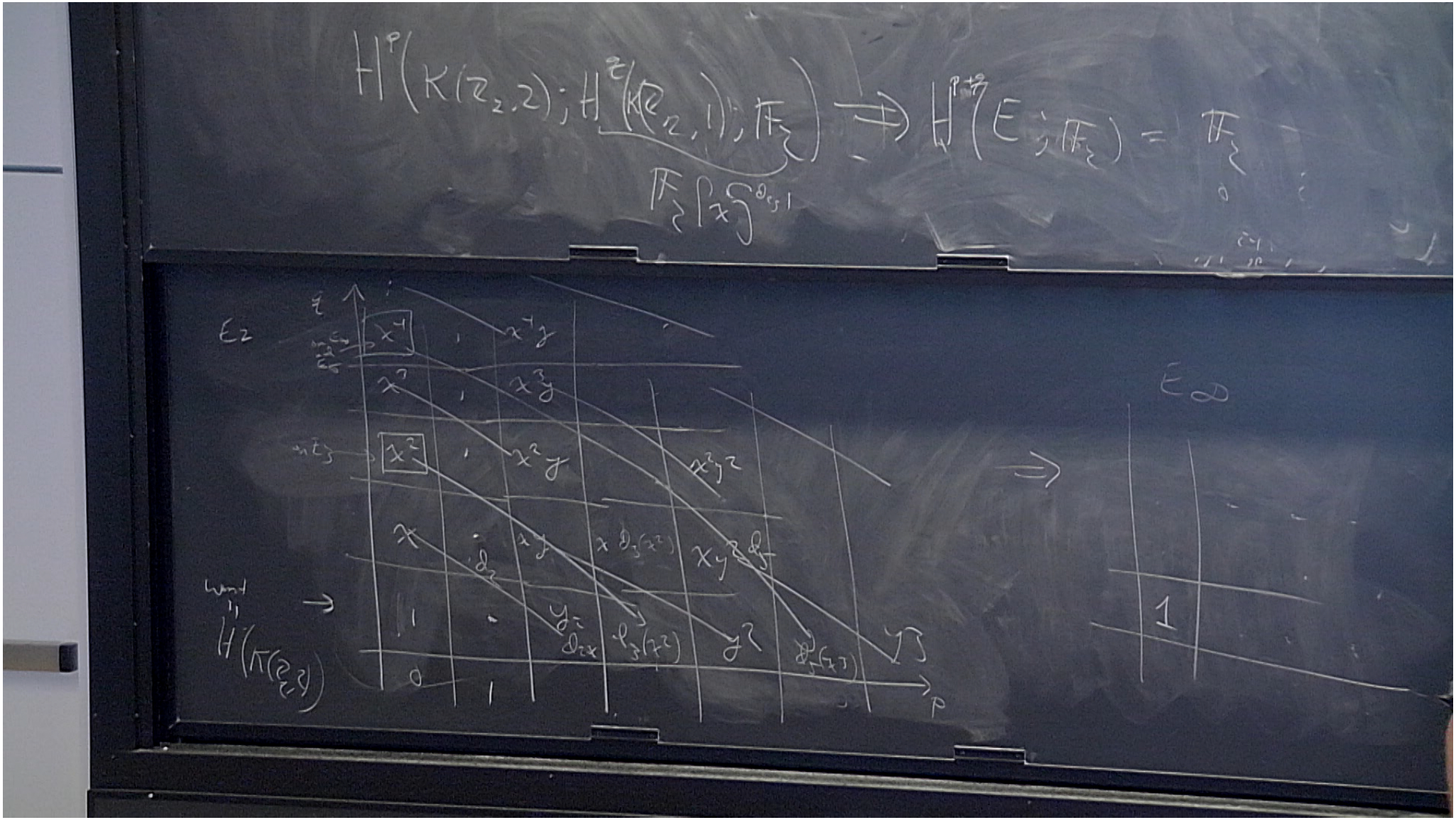
$= H^*(K(\mathbb{Z}_2, 2); \mathbb{F}_2)$?

Look at $\mathbb{Z}_2$ chain complex

$$\begin{array}{ccccccc} \cdots & 0 & 0 & \mathbb{Z}_2 & \xrightarrow{\mathrm{id}} & \mathbb{Z}_2 & 0 \\ & 3 & 2 & & 1 & & 0 \\ & & \downarrow \cong & & \downarrow \cong & & \downarrow \\ & & \mathbb{Z}_2 & & \mathbb{Z}_2 & & 0 \end{array} \xrightarrow{DK} E$$


$E = B(\text{higher } \overset{ab}{Sp})$
$\| \text{ extension}$
$H^\cdot(E, -) = H^\cdot_{Tot}(\text{higher } Sp)$
$K(\mathbb{Z}_2, 1) \cdot K(\mathbb{Z}_2, 2)$
so $\exists \Sigma$
$H^\cdot(K(\mathbb{Z}_2, 2); H^\cdot(K(\mathbb{Z}_2, 1); \pi_2) \Rightarrow H^\cdot(E; \pi_2) = \pi_2$
$\pi_2 \{\ast\}^{\deg 1}$
$(-, A) \to H^{\cdot+1}(-, B)$


$H^p(K(\mathbb{Z}_2,2); H^q(K(\mathbb{Z}_2,1); \mathbb{F}_2)) \Rightarrow H^{p+q}(E; \mathbb{F}_2) = \mathbb{F}_2$
$\mathbb{F}_2[x]^{\deg x = 1}$
E_2
E_∞
x^4
x^3y
x^2y^2
xy^2
y^3
$y = \delta_2 x$
1
$H(K(\mathbb{Z}_2,2))$


$H^p(K(\mathbb{Z}_2,2); H^q(K(\mathbb{Z}_2,1); \mathbb{F}_2)) \Rightarrow H^{p+q}(E; \mathbb{F}_2) = \mathbb{F}_2$
$\mathbb{F}_2[x]^{\deg x = 1}$
E_2
E_∞
$H(K(\mathbb{Z}_2,2))$
1

$= H^*_{gp}(\mathbb{Z}_2; \mathbb{F}_2) = \mathbb{F}_2[x]$ $\quad \deg 1$

$\{x \mapsto x^{\cdot}, 0.\}$

$H^*(K(\mathbb{Z}_2, 2); \mathbb{F}_2)$

"1-form gp"

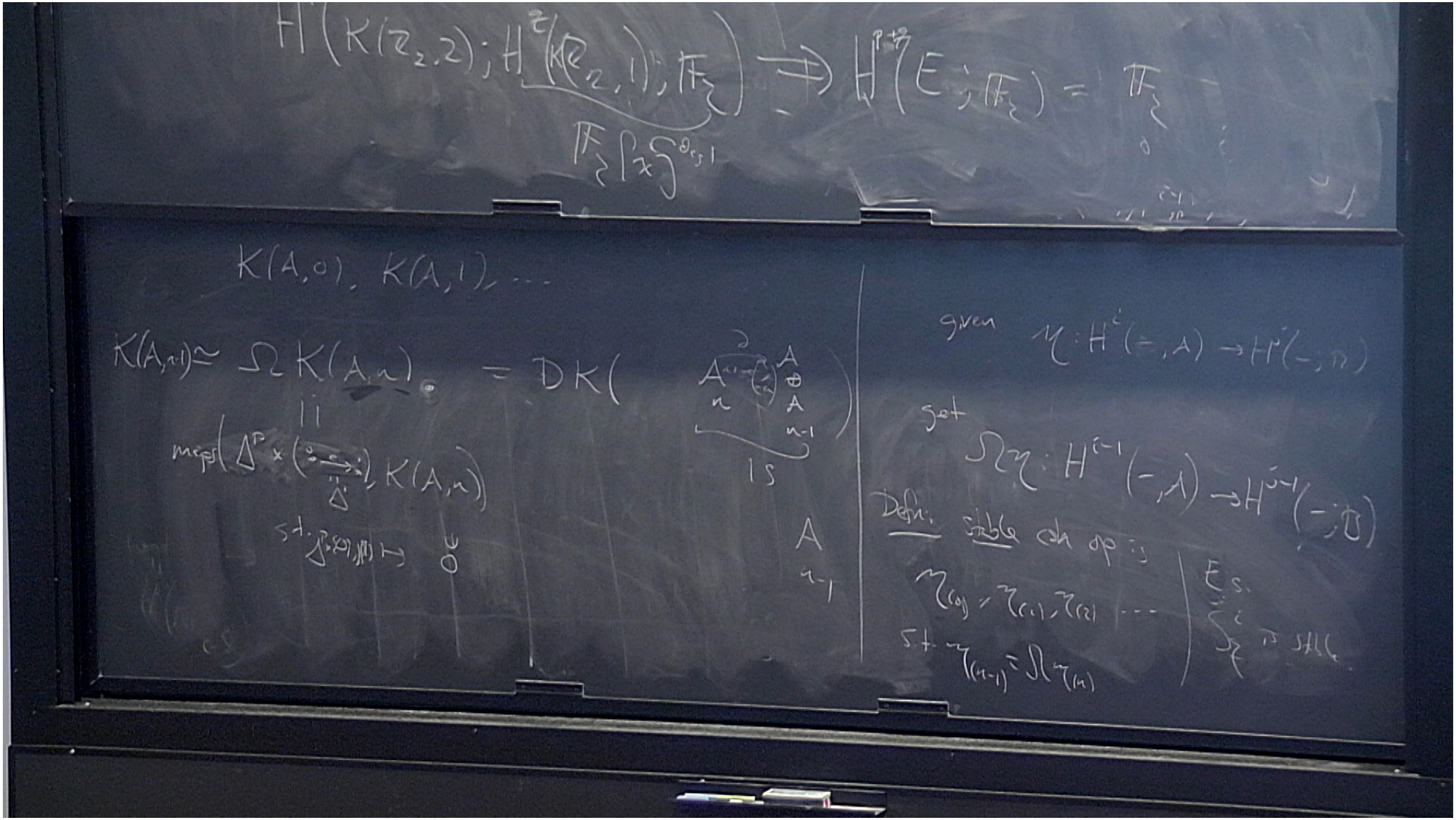
$= \mathbb{F}_2[y, Sq^1 y, Sq^2 Sq^1 y, Sq^4 Sq^2 Sq^1 y, Sq^8 \ldots]$

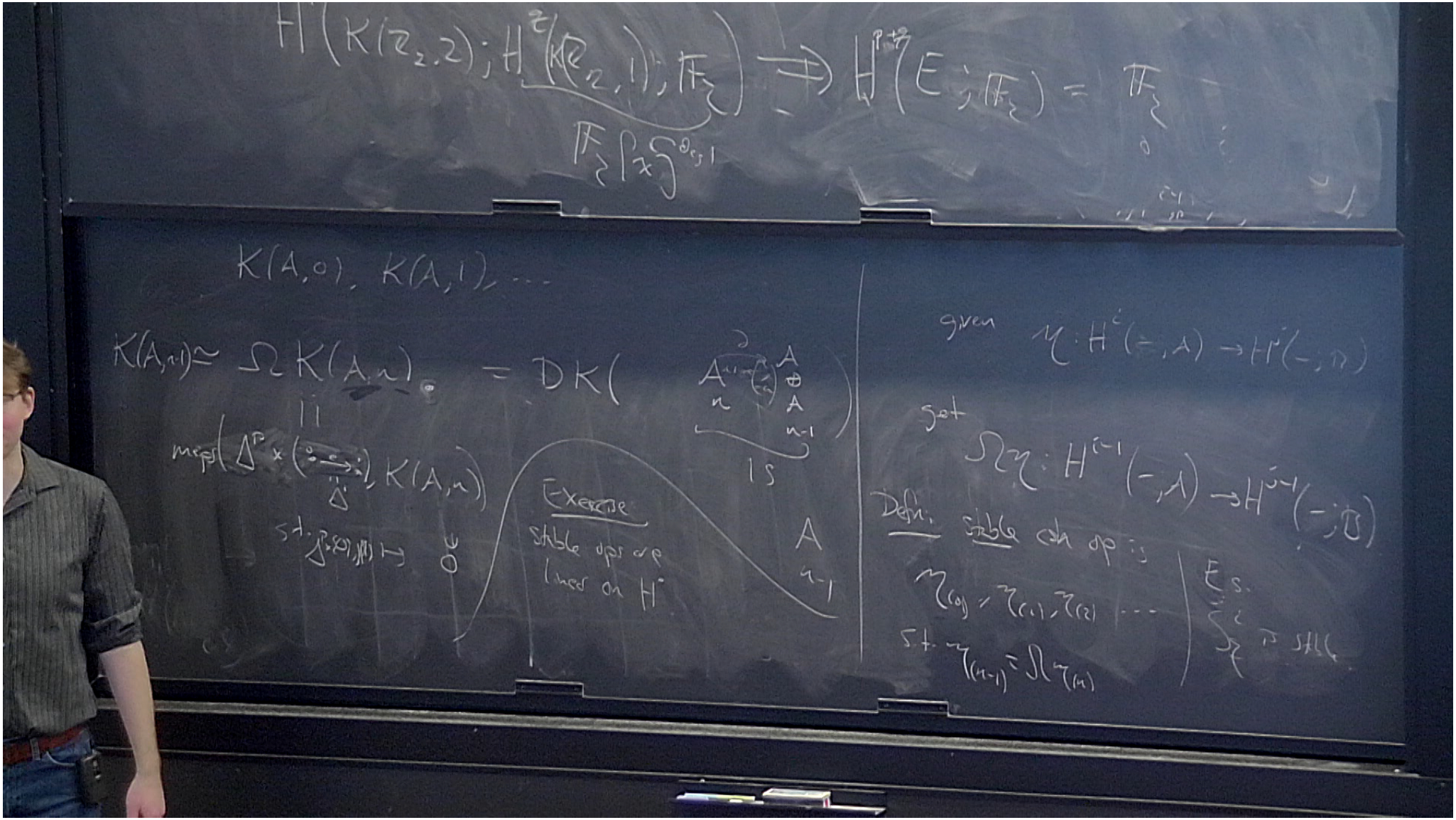
$\deg 2 \quad \deg 3 \quad \deg 5 \quad \deg 9$

only exh ops $\to H^*$

$H^2(-; \mathbb{F}_2)$

polys of Steenrod squares of input.


$H^*(K(\mathbb{Z}_2,2); H^*(K(\mathbb{Z}_2,1); \mathbb{F}_2)) \Rightarrow H^{p+q}(E; \mathbb{F}_2) = \mathbb{F}_2$
$K(A,0),\ K(A,1), \ldots$
$K(A,n+1) \simeq \Omega K(A,n)$
$= DK(\ A_n \to A_{n-1} \oplus A_{n-1} \to A_{n-1}\ )$
$\hom_{\mathrm{Sp}}(\Delta^n \times (\cdot \to \cdot), K(A,n))$
s.t. $\partial_i \mapsto 0$
given $\mathcal{H}: H^i(-,A) \to H^i(-,B)$
set $\Omega\mathcal{H}: H^{i-1}(-,A) \to H^{i-1}(-,B)$
Defn: stable coh op:
$\mathcal{H}_{(0)}, \mathcal{H}_{(1)}, \mathcal{H}_{(2)} \ldots$
s.t. $\mathcal{H}_{(n-1)} = \Omega \mathcal{H}_{(n)}$
Ex:


$H^*(K(\mathbb{Z}_2,2); H^*(K(\mathbb{Z}_2,1); \mathbb{F}_2)) \Rightarrow H^{p+q}(E; \mathbb{F}_2) = \mathbb{F}_2$
$K(A,0),\ K(A,1), \ldots$
$K(A,n+1) \simeq \Omega K(A,n)$
$= DK(\ A_n \oplus A_{n-1}\ )$
$\text{maps}(\Delta^n \times (\cdot \to \cdot), K(A,n))$
s.t. $\partial_i \sigma(x_0, \ldots, x_n) \mapsto 0$
Exercise
Stable ops are linear on H^*
A_{n-1}
given $\eta : H^i(-,A) \to H^i(-,B)$
get $\Omega\eta : H^{i-1}(-,A) \to H^{i-1}(-,B)$
Defn: stable coh op:
$\eta_{(0)} = \eta_{(1)}, \eta_{(2)} \ldots$
s.t. $\eta_{(n-1)} = \Omega \eta_{(n)}$
Ex:
$\mathbb{Z} \to$ stable