Title: Bootstrapping Inflationary Correlators

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Series: Particle Physics

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Abstract:
The central idea of the bootstrap philosophy is to constrain observables directly from consistency conditions alone, bypassing the intricacies of the Lagrangian formalism. In this talk, I will adopt this viewpoint and describe a boundary-centric approach to determine cosmological correlators, following a perspective familiar from the modern studies of scattering amplitudes. Specifically, I will describe the symmetries and singularities of three- and four-point functions in de Sitter space and inflation, and explain how these principles can be used to fully determine the final answer without reference to bulk time evolution. I will also highlight spectroscopic features encoded in these correlators, relevant for the search of primordial non-Gaussianity in future cosmological observations.

Bootstrapping Inflationary Correlators

Hayden Lee

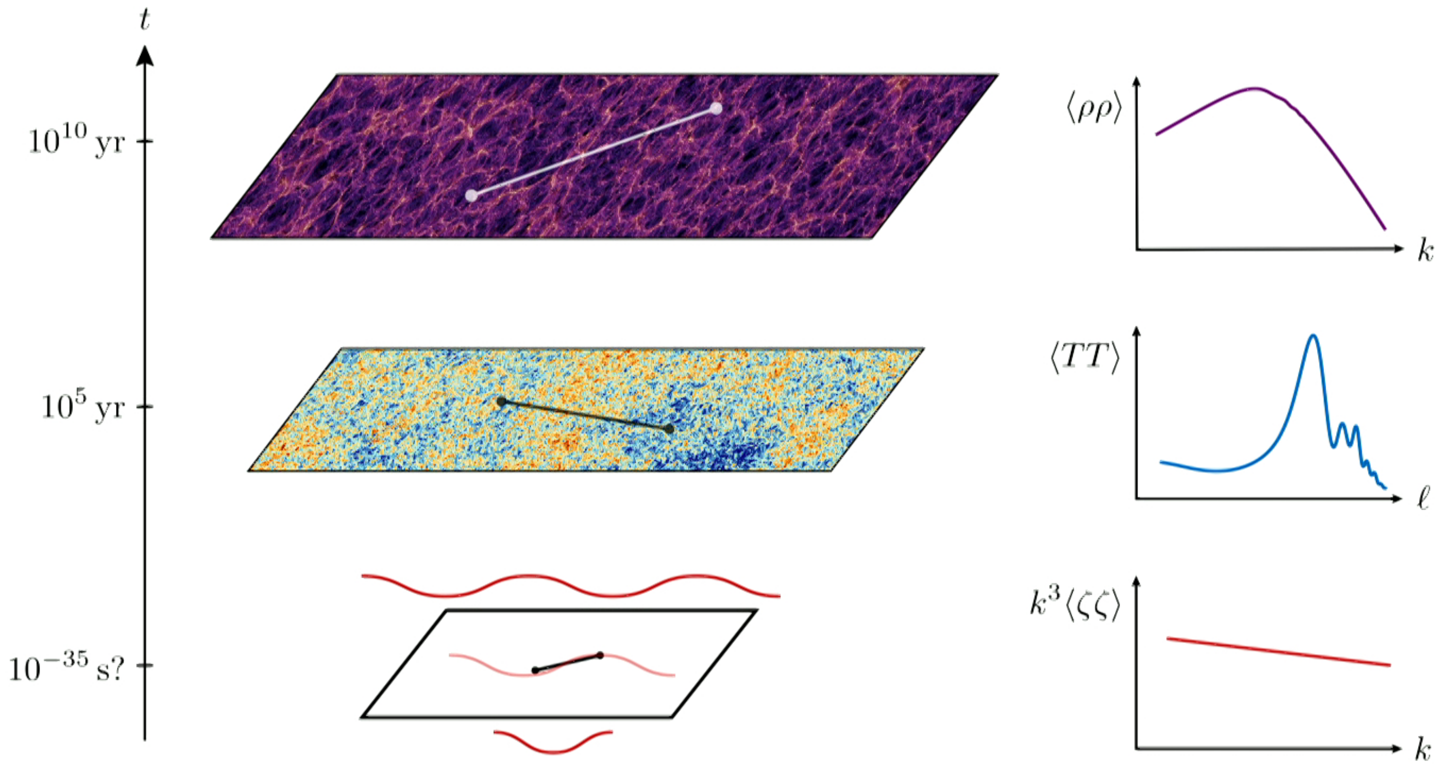
Harvard

w/ N. Arkani-Hamed, D. Baumann, G. Pimentel

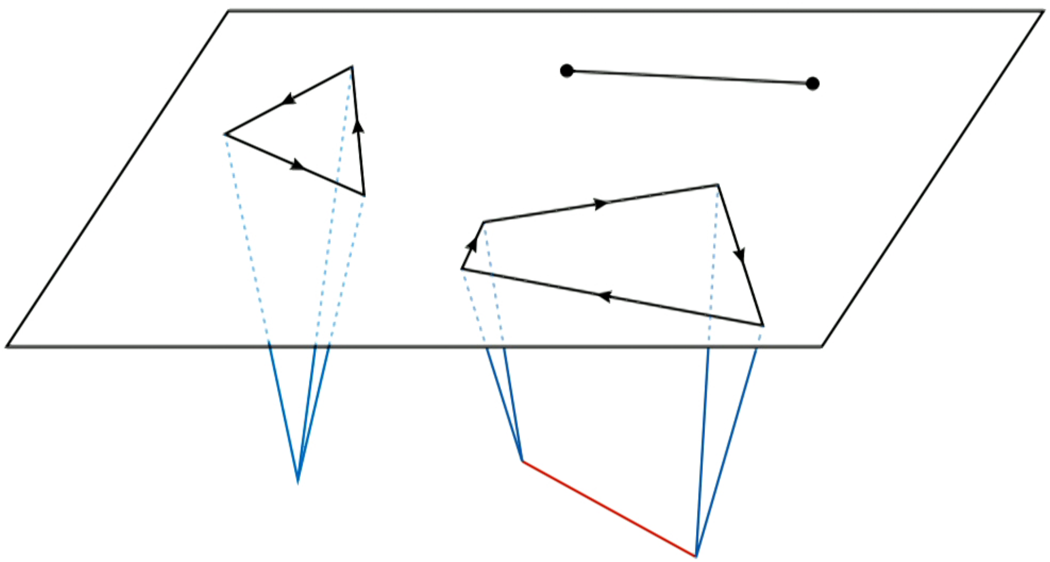
+ C. Duaso Pueyo, A. Joyce

[arXiv:1811.00024, 1910.14051]

Primordial Fluctuations



Cosmological Correlators

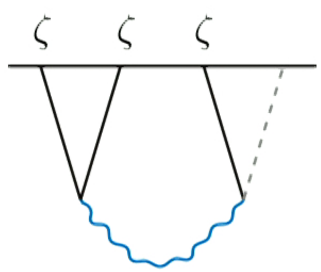


What particles exist up to 10^{14} GeV?

What are the rules for consistent correlators?

In-In Formalism

The in-in formalism is rather complicated.

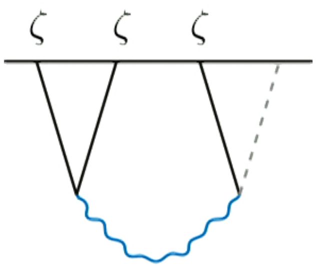


$\leftarrow$

$$
\begin{aligned}
\mathcal{L}_\zeta^{(3)} = {} & 3H^2\alpha^3 - \epsilon H^2\alpha^3 - 9H^2\alpha^2\zeta + 3\epsilon H^2\alpha^2\zeta + \frac{27}{2}H^2\alpha\zeta^2 \\
& - \frac{9}{2}\epsilon H^2\alpha\zeta^2 - \frac{27}{2}H^2\zeta^3 + \frac{9}{2}\epsilon H^2\zeta^3 - 6H\alpha^2\dot{\zeta} + 18H\alpha\zeta\dot{\zeta} \\
& - 27H\zeta^2\dot{\zeta} + 3\alpha(\dot{\zeta})^2 - 9\zeta(\dot{\zeta})^2 - \frac{2}{a^2}H\alpha b_i\partial_i\zeta + \frac{2}{a^2}H\zeta b_i\partial_i\zeta \\
& - \frac{2}{a^2}H\alpha\partial_i\beta\partial_i\zeta + \frac{2}{a^2}H\zeta\partial_i\beta\partial_i\zeta + \frac{2}{a^2}b_i\dot{\zeta}\partial_i\zeta + \frac{2}{a^2}\partial_i\beta\dot{\zeta}\partial_i\zeta \\
& - \frac{1}{a^2}\alpha(\partial_i\zeta)^2 - \frac{1}{a^2}\zeta(\partial_i\zeta)^2 - \frac{1}{a^4}b_j\partial_i\zeta\partial_j b_i - \frac{1}{a^4}\partial_j\beta\partial_i\zeta\partial_j b_i \\
& - \frac{1}{a^4}b_i\partial_j\zeta\partial_j b_i - \frac{1}{a^4}\partial_i\beta\partial_j\zeta\partial_j b_i - \frac{1}{4a^4}\alpha(\partial_j b_i)^2 - \frac{1}{4a^4}\zeta(\partial_j b_i)^2 \\
& - \frac{1}{4a^4}\alpha\partial_j b_i\partial_i b_j - \frac{1}{4a^4}\zeta\partial_j b_i\partial_i b_j + \frac{2}{a^2}H\alpha^2\partial^2\beta - \frac{2}{a^2}H\alpha\zeta\partial^2\beta \\
& + \frac{1}{a^2}H\zeta^2\partial^2\beta - \frac{2}{a^2}\alpha\dot{\zeta}\partial^2\beta + \frac{2}{a^2}\zeta\dot{\zeta}\partial^2\beta - \frac{2}{a^4}b_j\partial_i\zeta\partial_i\partial_j\beta \\
& - \frac{2}{a^4}\partial_j\beta\partial_i\zeta\partial_i\partial_j\beta - \frac{1}{a^4}\alpha\partial_j b_i\partial_i\partial_j\beta - \frac{1}{a^4}\zeta\partial_j b_i\partial_i\partial_j\beta \\
& - \frac{1}{2a^4}\alpha(\partial_i\partial_j\beta)^2 - \frac{1}{2a^4}\zeta(\partial_i\partial_j\beta)^2 + \frac{1}{2a^4}\alpha(\partial^2\beta)^2 + \frac{1}{2a^4}\zeta(\partial^2\beta)^2 \\
& - \frac{2}{a^2}\alpha\zeta\partial^2\zeta - \frac{1}{a^2}\zeta^2\partial^2\zeta - \frac{9}{2}V\alpha\zeta^2 - \frac{9}{2}V\zeta^3 \\
& \ldots
\end{aligned}
$$

In-In Formalism

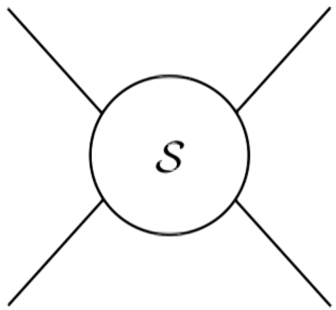
Yet, the final answer is remarkably simple.



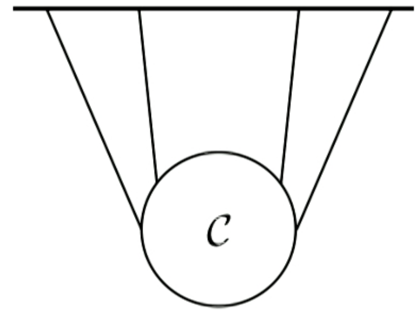
$$= \underbrace{\varepsilon \left[\sum_{n \neq m} k_n k_m^2 + \frac{8}{E} \sum_{n>m} k_n^2 k_m^2 \right]}_{\frac{\mathrm{Poly}[k_1, k_2, k_3]}{E}} + (n_s - 1) \underbrace{\sum_n k_n^3}_{\mathrm{Local}}$$

Maldacena [2002]

Bootstrap



"S-Matrix Bootstrap"



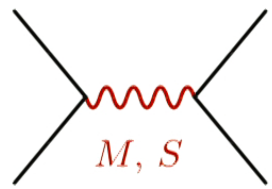
"Cosmological Bootstrap"

Idea: Use symmetries & singularities to determine observables without Feynman diagrams.

S-Matrix Bootstrap

In flat space, scattering amplitudes are constrained by Lorentz invariance, unitarity, locality.

Simple:



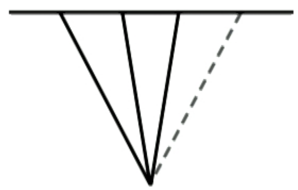
$$= \frac{g^2}{s - M^2} P_S \left(1 + \frac{2t}{M^2}\right)$$

Frontier: Multi-leg, multi-loop; needed for the LHC

Cosmological Bootstrap

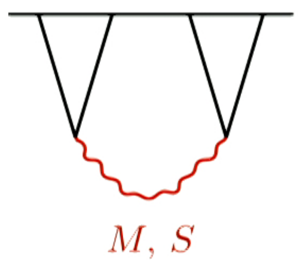
In near-de Sitter space, boundary correlators are constrained by (approximate) conformal symmetry, singularities.

Simple:



$$= \quad \varepsilon \frac{\mathrm{Poly}[k_1, k_2, k_3]}{E^{\#}}$$

Frontier:

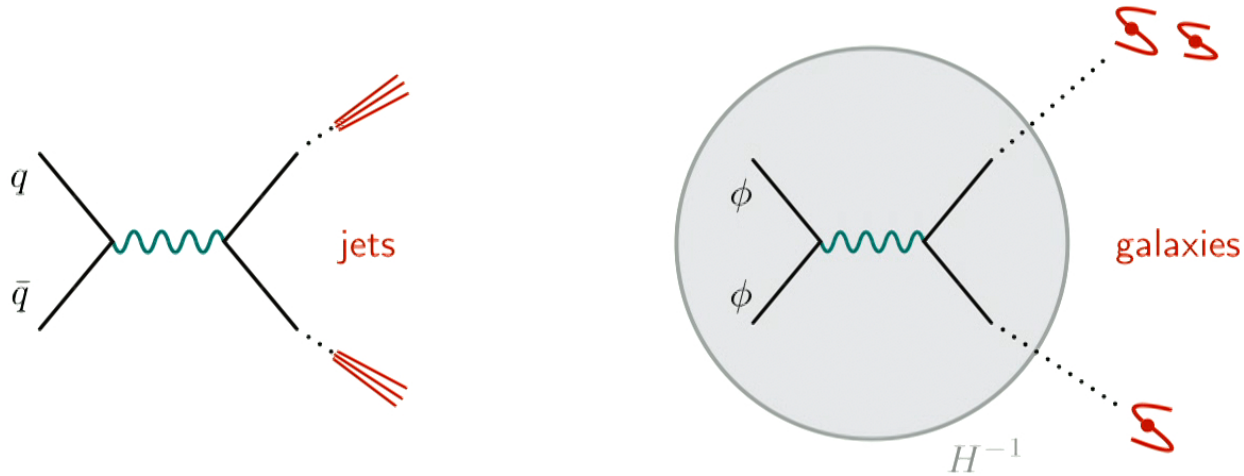


needed for next gen. CMB, LSS exp.

Outline

1. Cosmological Collider Physics
2. Cosmological Bootstrap
3. Weight-Shifting Operators
4. Future Directions

Cosmological Collider Physics

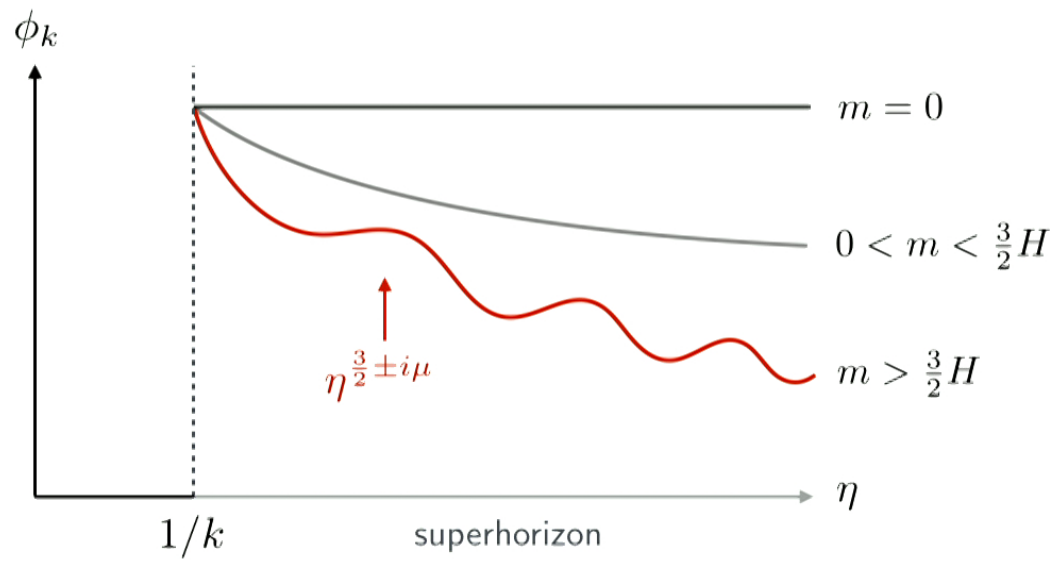


New particles can appear as special patterns in cosmological correlations.

Arkani-Hamed, Maldacena [2015]

Time Evolution

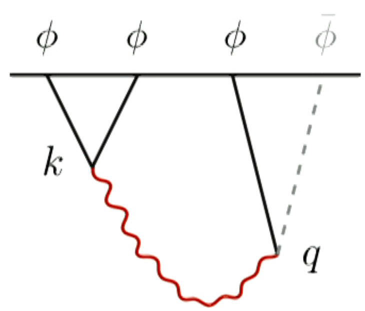
The late-time (boundary) behavior of a particle is



$$\phi(\eta, \mathbf{x}) \xrightarrow{\eta \to 0} \eta^{\Delta} \mathcal{O}_{\Delta}(\mathbf{x}) \qquad \Delta = \frac{3}{2} \pm \underbrace{\sqrt{\frac{9}{4} - \frac{m^2}{H^2}}}_{\equiv i\mu}$$

Squeezed Limit

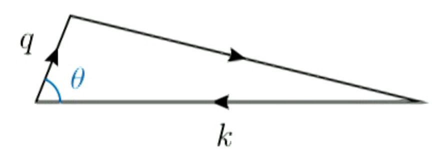
This scaling is reflected in the squeezed limit.



$$\xrightarrow{q \ll k} \quad e^{-\pi\mu}\left[e^{i\delta}\left(\frac{q}{k}\right)^{\frac{3}{2}+i\mu} + c.c.\right] P_S(\cos\theta)$$

dilution — oscillations

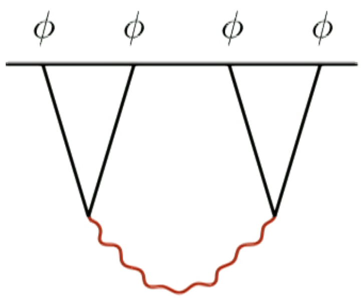
particle production — phase — angular dependence



Arkani-Hamed, Maldacena [2015]

Beyond the Squeezed Limit

The full calculation of the 4-point function leads to



bulk-to-boundary propagator

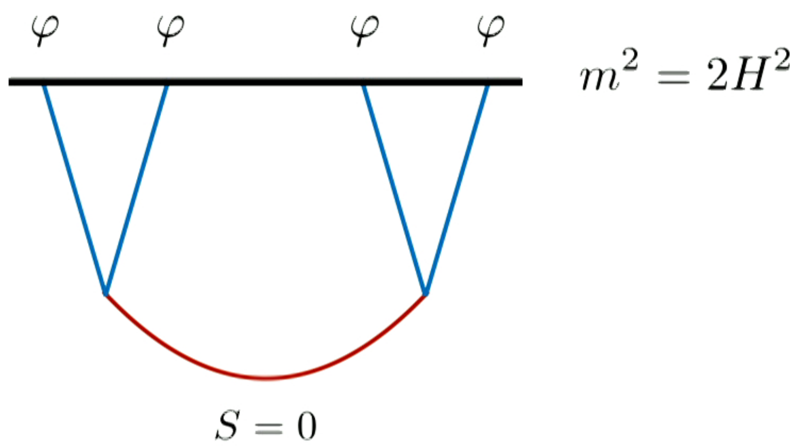
$$\sim \int \frac{\mathrm{d}\eta}{\eta^2}\frac{\mathrm{d}\eta'}{\eta'^2}\, e^{i(k_1+k_2)\eta} e^{i(k_3+k_4)\eta'}\, G(k_I,\eta,\eta')$$

bulk-to-bulk propagator (Hankel functions)

Q: Can we obtain an analytic solution for this correlator?

Basic Object

The basic object is the four-point function of conformally coupled scalars:

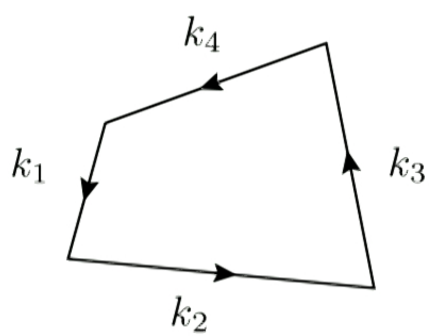


All other correlators can be reduced to this building block.

Kinematics

$$\langle \phi_{\vec{k}_1} \phi_{\vec{k}_2} \phi_{\vec{k}_3} \phi_{\vec{k}_4} \rangle = F(\vec{k}_1, \cdots, \vec{k}_4)\delta_{\rm D}(\vec{k}_1 + \cdots + \vec{k}_4)$$

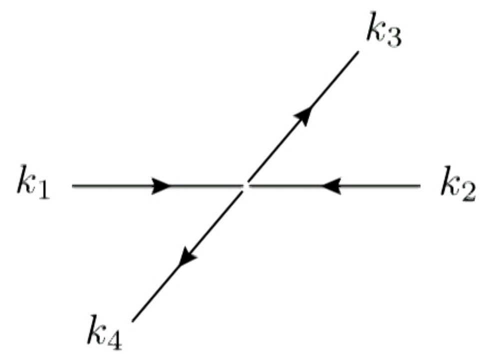
Cosmological correlator



$$\frac{A}{E^{\#}} + \cdots$$

Amplitude

Amplitude



$$\delta_{\rm D}(E = |\vec{k}_1| + \cdots + |\vec{k}_4|)$$

Symmetries

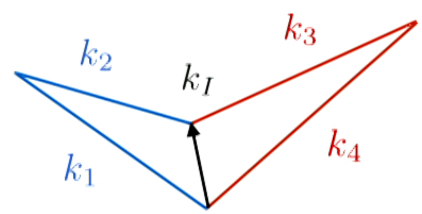
Invariance under **dilatations** and **SCTs** imply the following constraints:

$$0 = \sum_n \left[1 + \vec{k}_n \cdot \partial_{\vec{k}_n}\right] F$$

$$0 = \sum_n \left[\vec{k}_n (\partial_{\vec{k}_n} \cdot \partial_{\vec{k}_n}) - 2(\vec{k}_n \cdot \partial_{\vec{k}_n})\partial_{\vec{k}_n} - 2\partial_{\vec{k}_n}\right] F$$

The **dilatation** constraint is solved by

$$F = \frac{1}{k_I} \hat{F}(u, v)$$



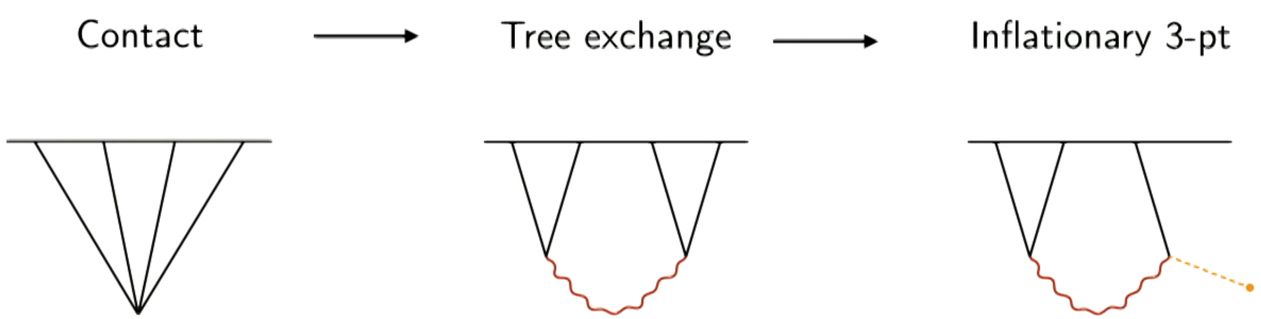
$$u \equiv \frac{k_I}{k_1 + k_2} \qquad v \equiv \frac{k_I}{k_3 + k_4}$$

Symmetries

For conformal scalars, the **SCT constraints** become

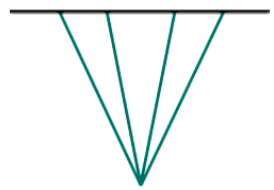
$$(\Delta_u - \Delta_v)\hat{F} = 0 \qquad \Delta_u = u^2(1-u^2)\partial_u^2 - 2u^3\partial_u$$

We'd like to classify the solutions according to their analytic structure:



Dynamics: Contact

Contact diagrams carry the "simplest possible" analytic structure.



$$= \sum_n c_n \frac{A_n(u,v)}{E^{2n+1}}$$

dS space

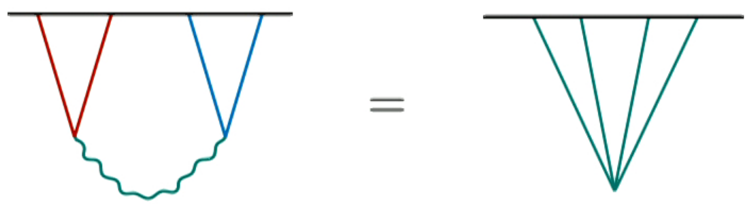
flat space

$$= \sum_n c_n s^n$$

Dynamics: Exchange

For **tree exchange**, the constraint separates into two ODEs:

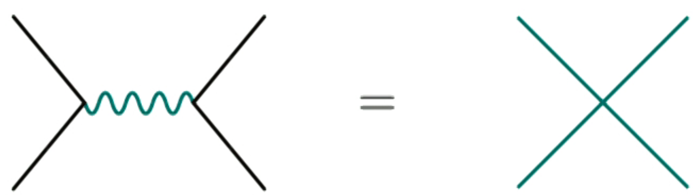
$(\Delta_u + M^2)$
$(\Delta_v + M^2)$



dS space

flat space

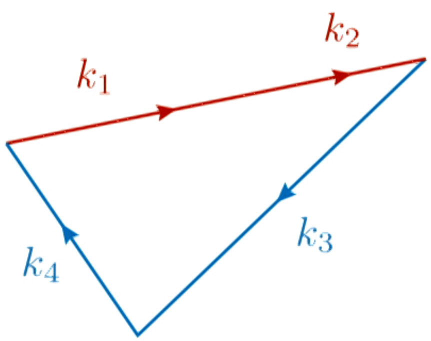
$(s - M^2)$



Singularities

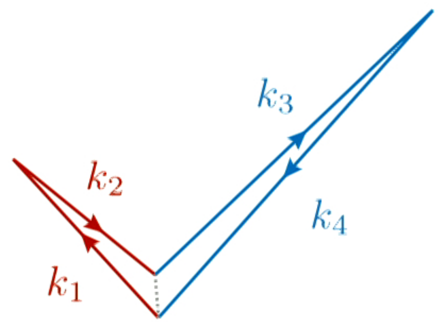
To fix the solution, we impose two boundary conditions:

folded limit



$$\hat{F} \xrightarrow{u,v\to 1} \text{regular}$$

collapsed limit



$$\hat{F} \xrightarrow{u,v\to 0} \text{3pt} \times \text{3pt}$$

Solution

For small u, the solution is

analytic

non-analytic

$$\hat{F} = \sum_n \frac{(-1)^n}{(n+\frac{1}{2})^2 + \mu^2} \left(\frac{u}{v}\right)^{n+1} + \frac{\pi}{\cosh \pi\mu} \frac{\sin(\mu \log(u/v))}{\mu}$$

$\sim H^2/M^2$

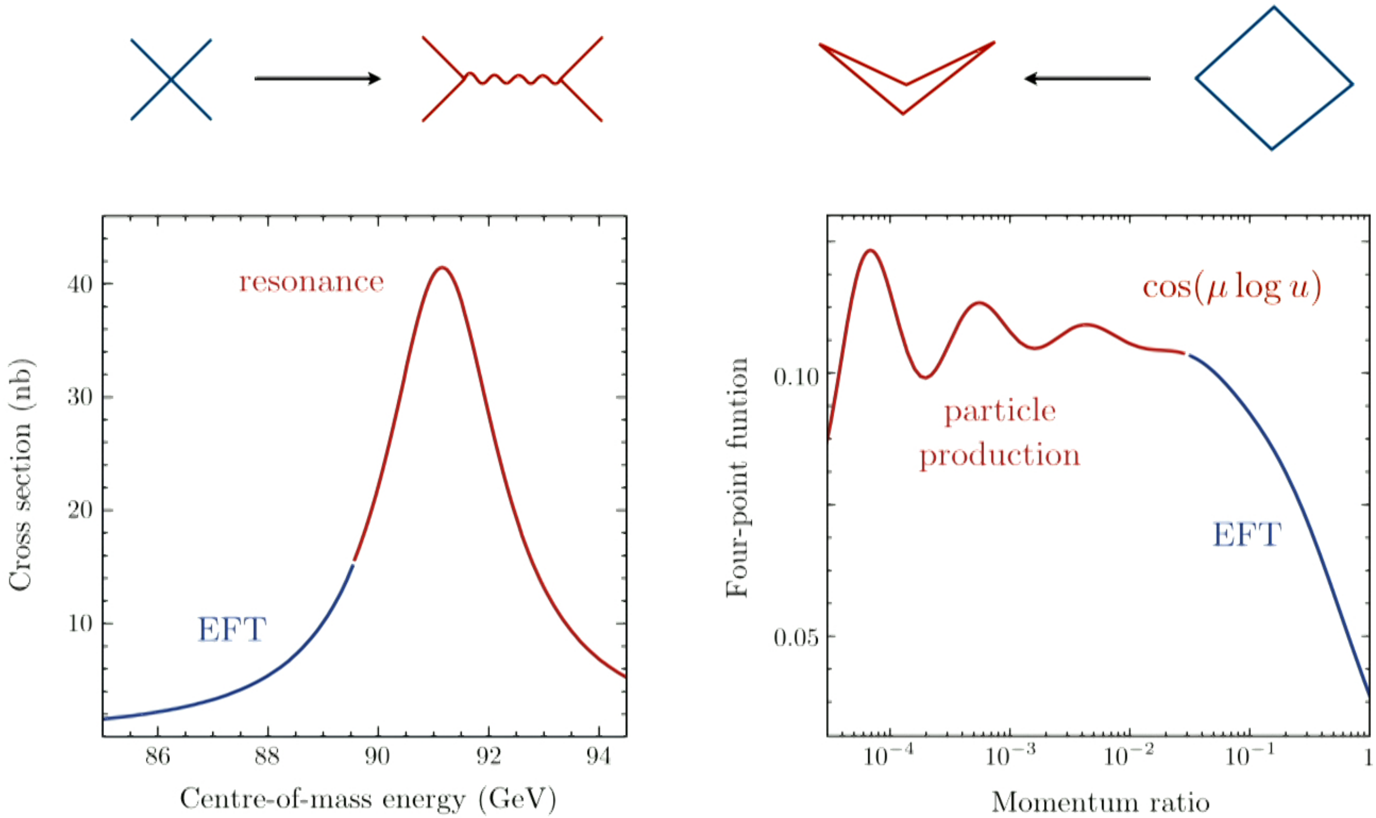
$\sim e^{-\pi M/H}$

EFT expansion

particle production

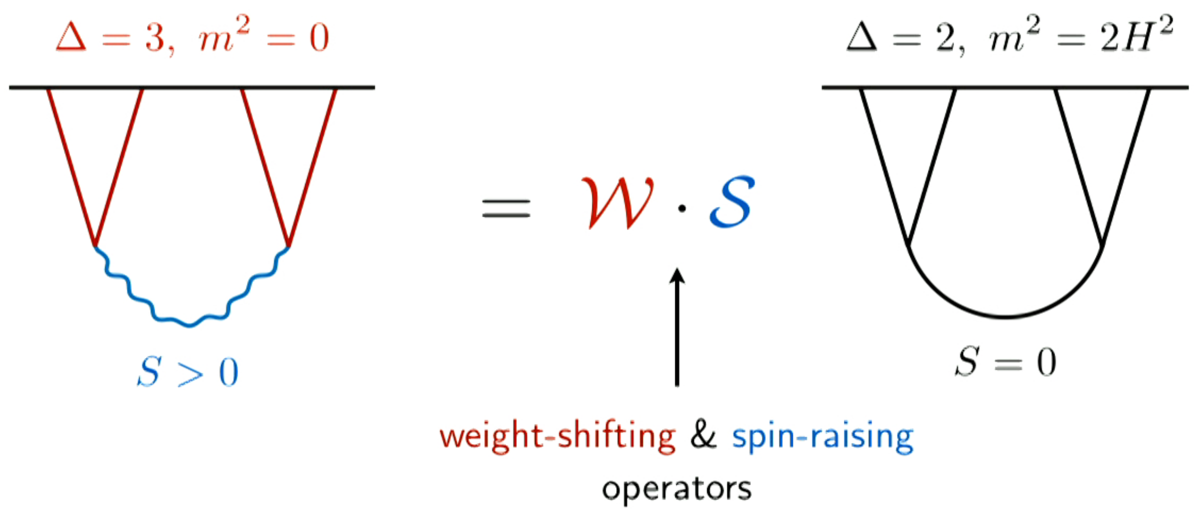
The full solution has a similar behavior.

Particle Spectroscopy



Strategy

We wish to find differential operators that shift the weights/spins of the external/internal particles.



These operators can be systematically constructed in embedding space.

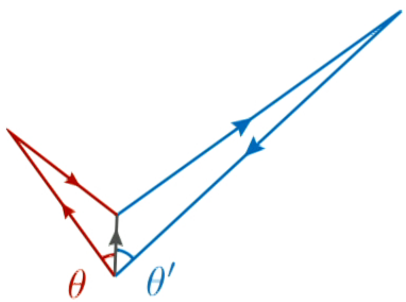
Spin Exchange

We can bootstrap spin-exchange solutions from the scalar-exchange one.

$$\hat{F}_S = \sum_{m=0}^{S} \Pi_m(\text{angles})\, D_{S,m}\, \hat{F}_0(u,v)$$

spin-exchange solution — polarization basis — weight-shifting operator — scalar-exchange solution

These solutions have a characteristic angular dependence:

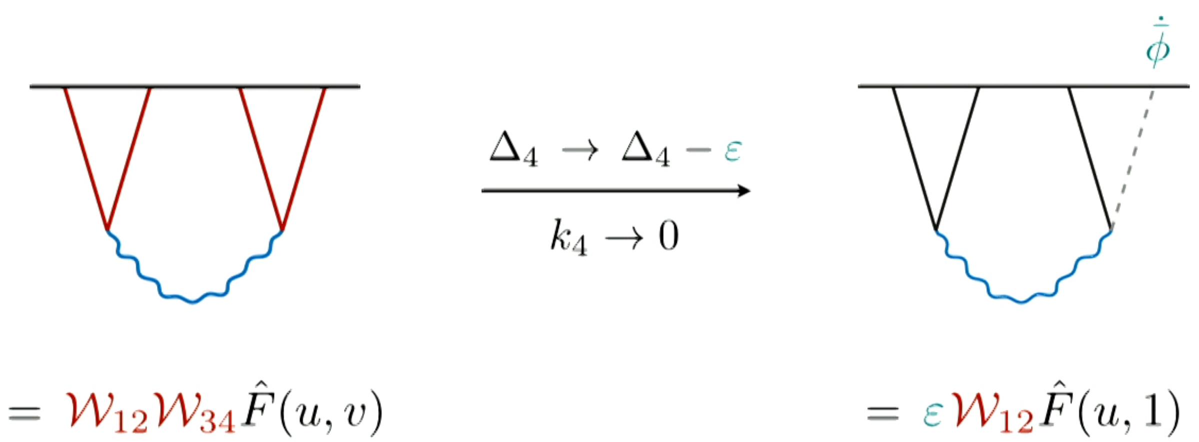


$$\xrightarrow{u,v\to 0} \quad (uv)^{\frac{3}{2}+i\mu} P_S(\cos\theta) P_S(\cos\theta')$$

Inflationary 3-pt

The inflaton is not exactly massless, but has a small mass.

This can be incorporated as a small deformation in the scaling dimension.



The three-point function can be obtained by taking the soft limit.

Inflationary 3-pt

The general inflationary three-point function depends on two seed functions:

coupling constants

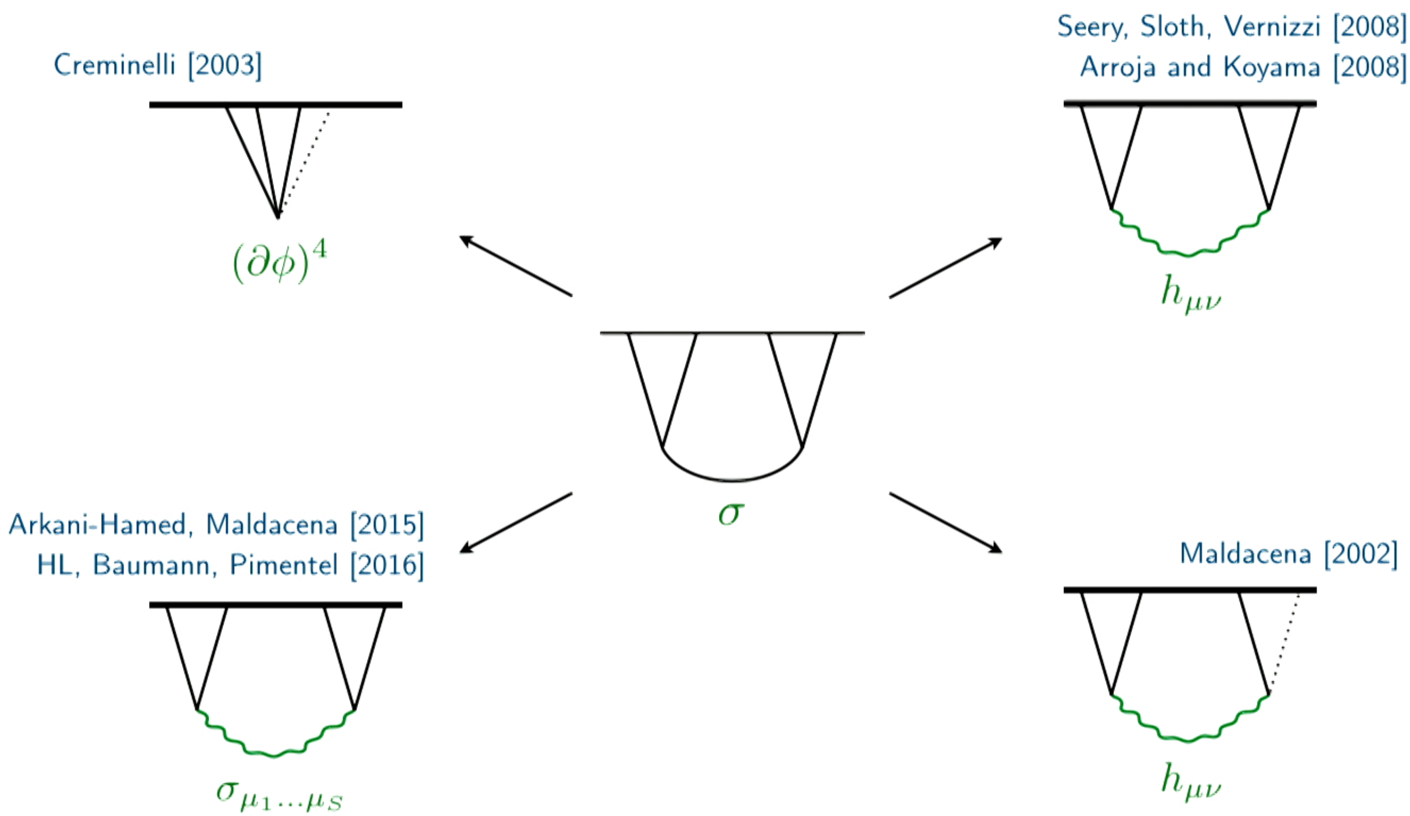
$$\langle \phi^3 \rangle_{\Delta=3} = \varepsilon\, \mathcal{W}_{12} \left[\sum_S a_S F^{(S)}_{\Delta=2}(u,1) + \sum_n b_n \Delta_u^n C(u,1) \right]$$

spin exchange

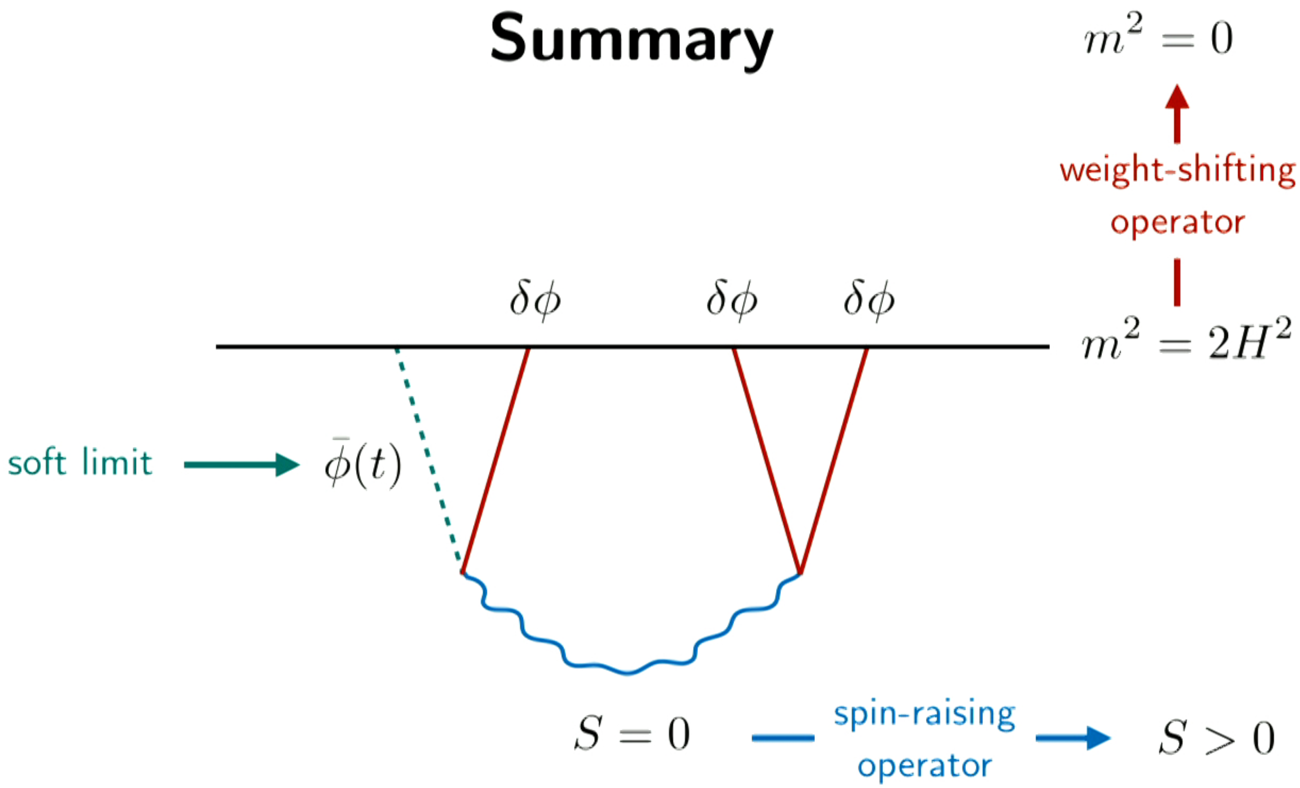
contact diagrams

Model details enter only through the spectrum and couplings.

Examples


Creminelli [2003]
$(\partial\phi)^4$
Seery, Sloth, Vernizzi [2008]
Arroja and Koyama [2008]
$h_{\mu\nu}$
σ
Arkani-Hamed, Maldacena [2015]
HL, Baumann, Pimentel [2016]
$\sigma_{\mu_1 \ldots \mu_S}$
Maldacena [2002]
$h_{\mu\nu}$

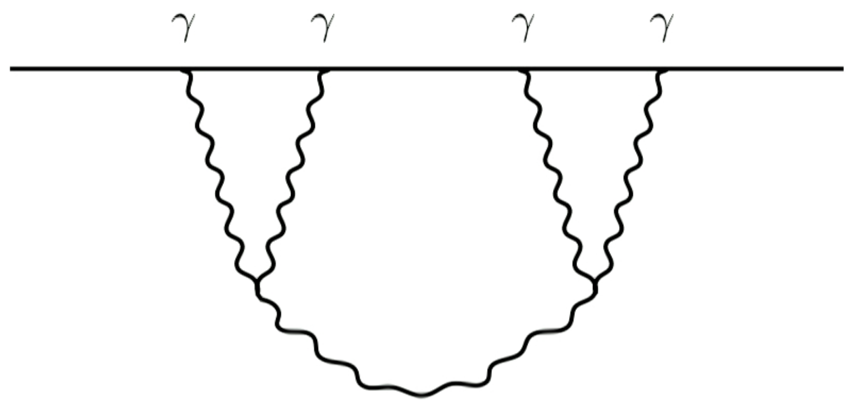
Summary



Symmetries and singularities fully determine correlation functions in inflation with (weakly broken) conformal symmetry.

Spinning Correlators

Very little is known about spinning correlators beyond three points.

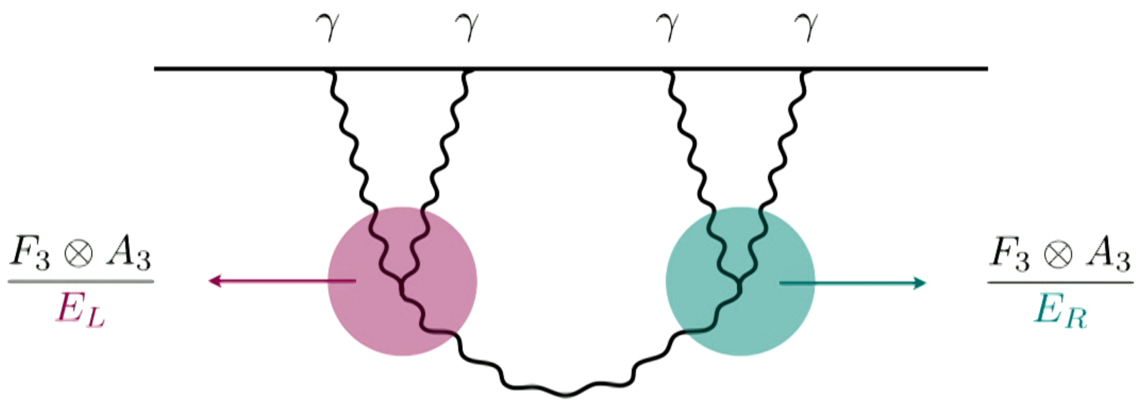


Can we bootstrap all spinning correlators from scalar ones?

[work in progress]

Factorization

Correlation functions must factorize consistently on all singularities.



Is there a cosmological analog of the four-particle test in flat space?

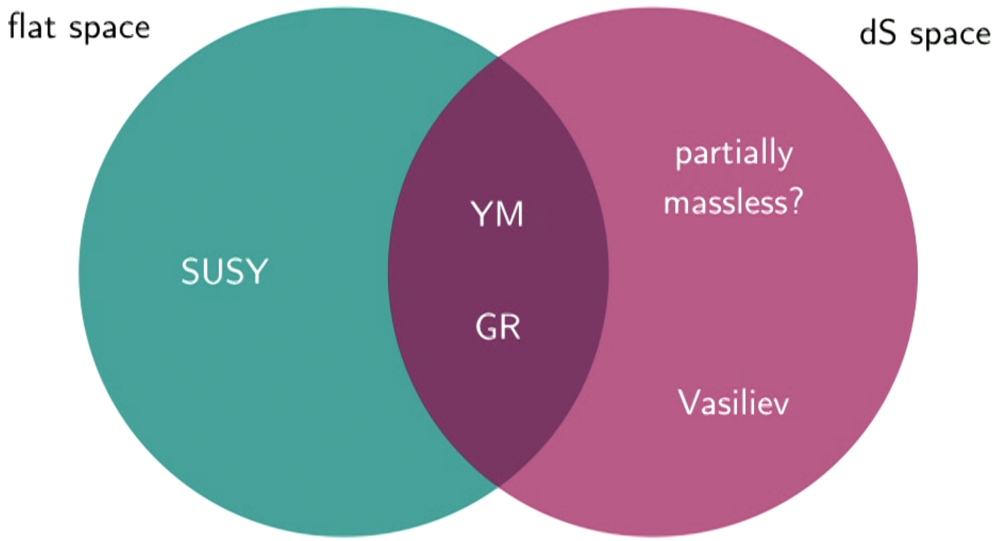
Cachazo, Benincasa [2007]

McGady, Rodina [2013]

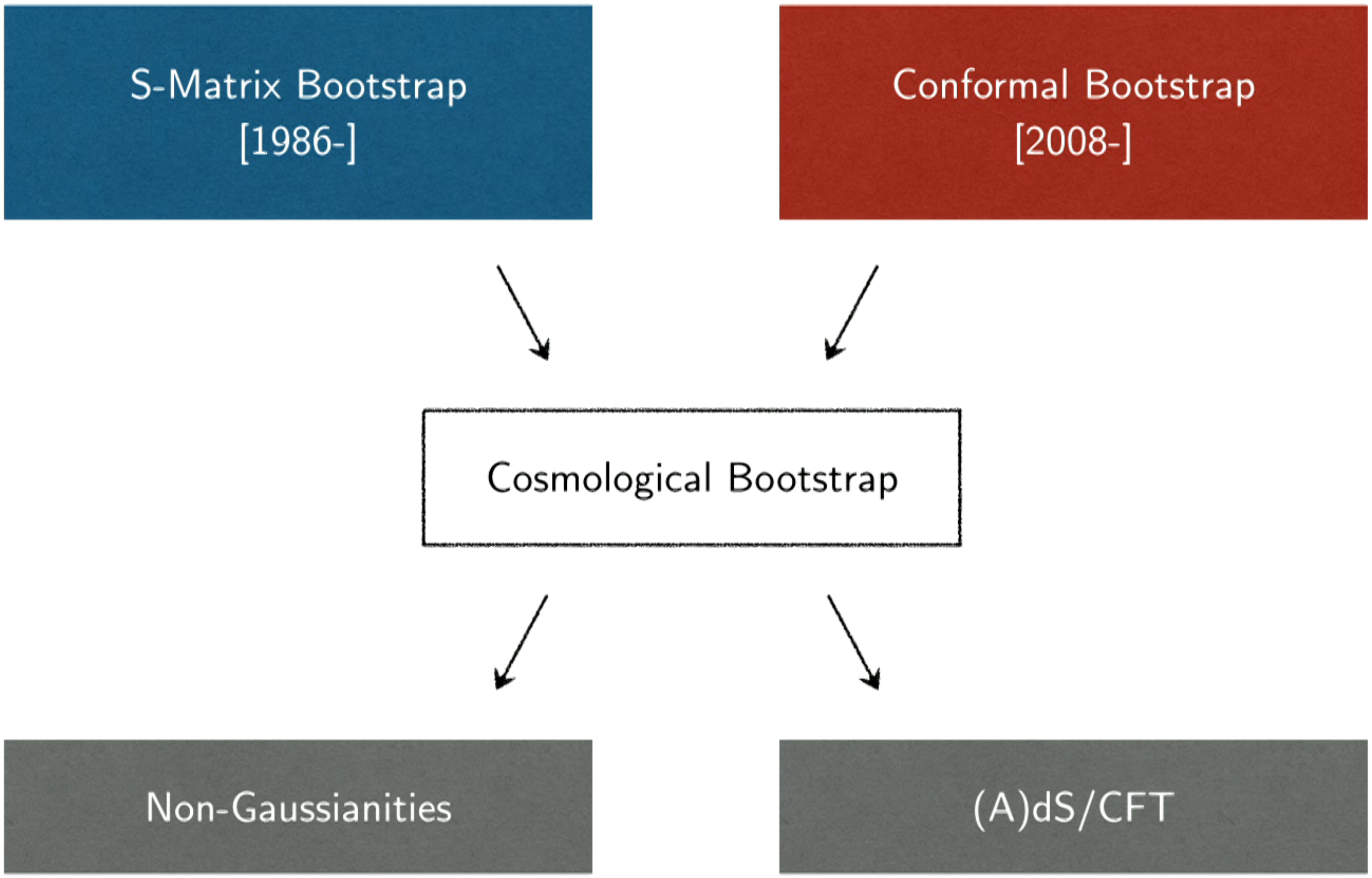
Arkani-Hamed, Huang, Huang [2017]

Carving Out Theory Space

We hope to use the bootstrap approach to rule out inconsistent theories in dS.



[work in progress]


S-Matrix Bootstrap
[1986-]
Conformal Bootstrap
[2008-]
Cosmological Bootstrap
Non-Gaussianities
(A)dS/CFT

Thank you!