Title: Revivals imply quantum many-body scars

Speakers: Alvaro Martin Alhambra

Series: Condensed Matter

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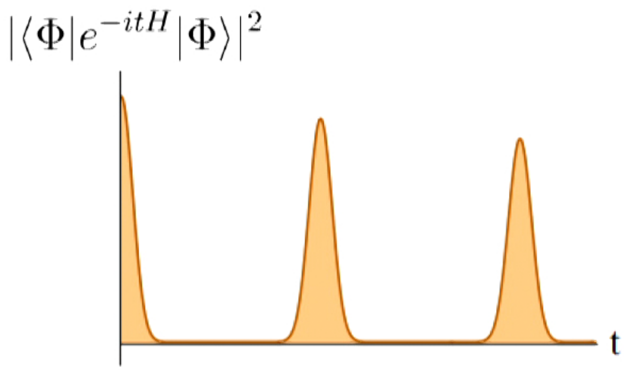
Abstract: We derive general results relating revivals in the dynamics of quantum many-body systems to the entanglement properties of energy eigenstates. For a D-dimensional lattice system of N sites initialized in a low-entangled and short-range correlated state, our results show that a perfect revival of the state after a time at most poly(N) implies the existence of "quantum many-body scars", whose number grows at least as the square root of N up to poly-logarithmic factors. These are energy eigenstates with energies placed in an equally-spaced ladder and with RÃ©nyi entanglement entropy scaling as log(N) plus an area law term for any region of the lattice. This shows that quantum many-body scars are a necessary condition for revivals, independent of particularities of the Hamiltonian leading to them. We also present results for approximate revivals, for revivals of expectation values of observables and prove that the duration of revivals of states has to become vanishingly short with increasing system size.

Work in collaboration with Henrik Wilming (ETH)

Revivals imply quantum many-body scars

Alvaro M. Alhambra
Perimeter Institute

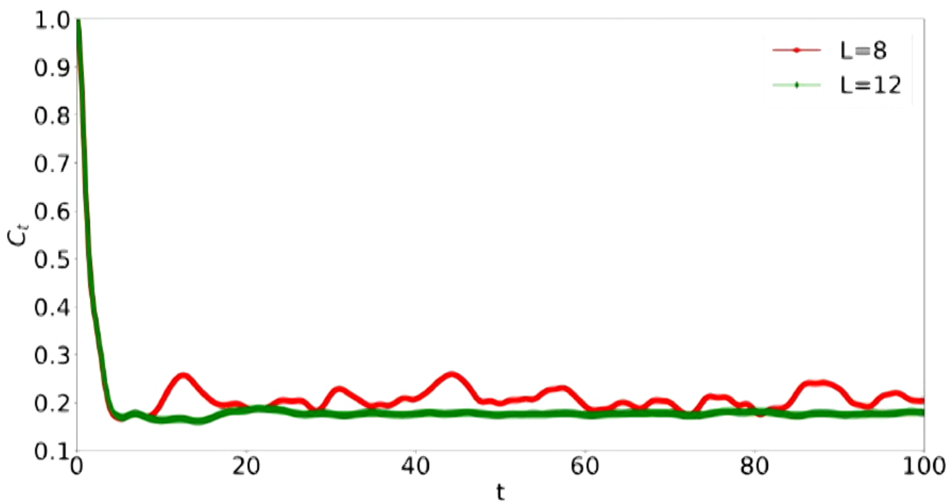
Henrik Wilming
ETH Zurich



arXiv:1911.05637

Setting: non-eq. dynamics of pure states

$$H = \sum_i^N h_i \qquad A(t) = e^{-itH} A e^{itH} \qquad C_t = \langle \Psi | A(t) | \Psi \rangle$$ (for instance)



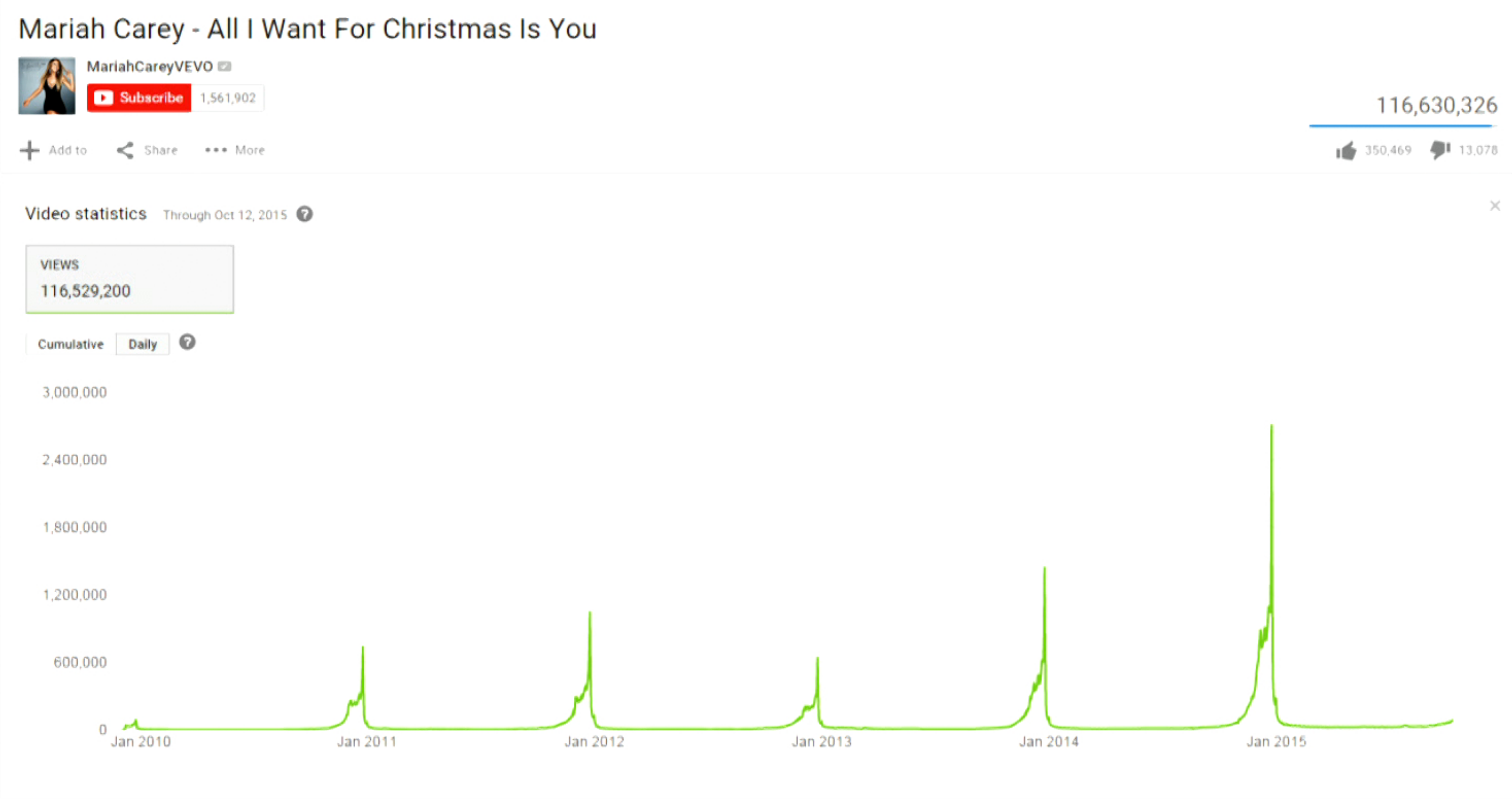
In "generic" situations that thermalize:

- Hamiltonian obeys ETH.
- Observable is local
- Initial state is "simple" (e.g. product or MPS)
- Recurrence time extremely large $\sim e^{cN}$

There are systems that behave in different ways:

- Integrable
- Many-body/Anderson localized
- Systems with scars
- ....?

Expected revivals:



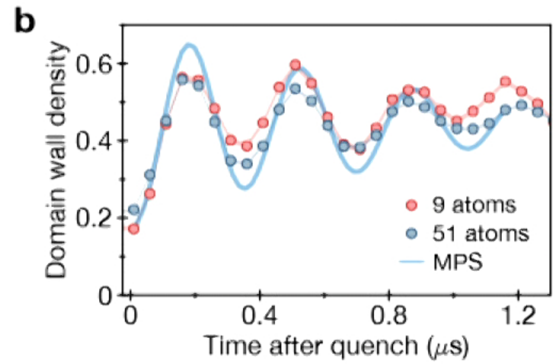
Unexpected revivals:

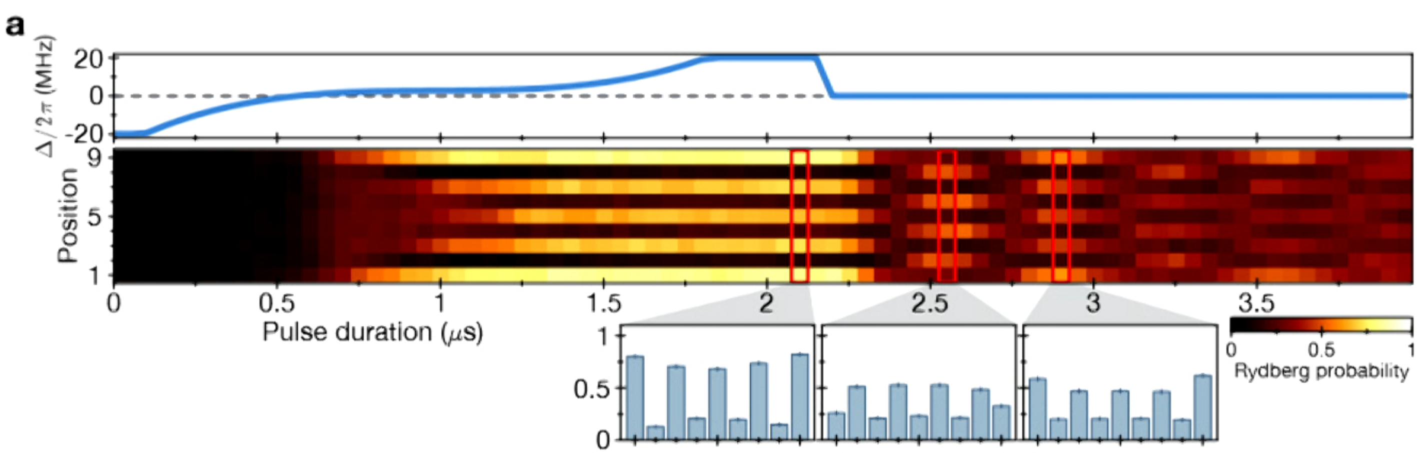
Article | Published: 30 November 2017

Probing many-body dynamics on a 51-atom quantum simulator

Hannes Bernien, Sylvain Schwartz, Alexander Keesling, Harry Levine, Ahmed Omran, Hannes Pichler, Soonwon Choi, Alexander S. Zibrov, Manuel Endres, Markus Greiner, Vladan Vuletić & Mikhail D. Lukin

Nature **551**, 579–584(2017) | Cite this article





Why? Many-body "scars"

Hamiltonian of Rydberg chain is effectively

$$H = \sum_i P_{i-1} X_i P_{i+1} \qquad P_i = |0\rangle\langle 0|_i$$

- There are a number of anomalous eigenstates in the spectrum (scars)
- The initial (product) state has high overlap with them
- First argued in Turner et al Nat. Phys. (2018)
- These eigenstates DO NOT obey the ETH ("weak" ergodicity breaking)

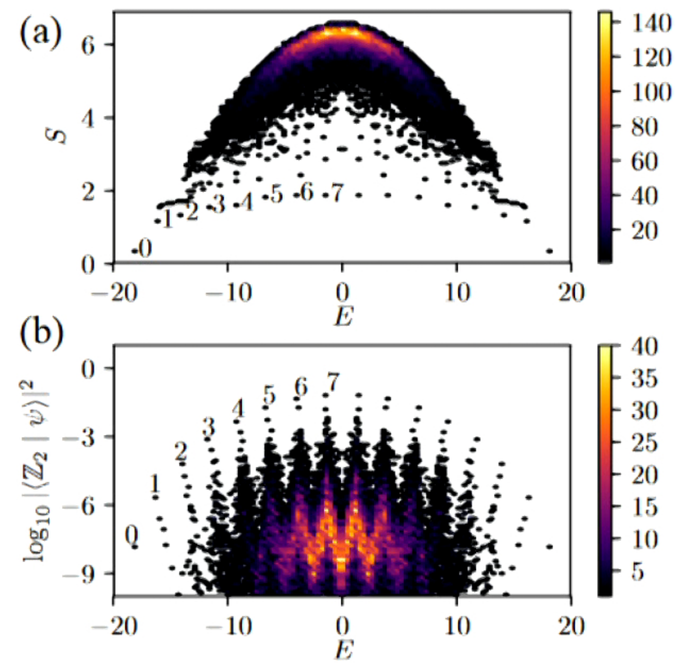


Figure from Turner et al Phys. Rev. B (2018)

Many-body scars: different models

PRL 119, 030601 (2017) PHYSICAL REVIEW LETTERS week ending 21 JULY 2017

Systematic Construction of Counterexamples to the Eigenstate Thermalization Hypothesis

Naoto Shiraishi
Department of Physics, Keio University, 3-14-1 Hiyoshi, Yokohama 223-8522, Japan

Takashi Mori
Department of Physics, The University of Tokyo, 7-3-1 Hongo, Bunkyo-ku, Tokyo 113-0033, Japan
(Received 4 February 2017; revised manuscript received 5 April 2017; published 21 July 2017)

Scars in strongly driven Floquet matter: resonance vs emergent conservation laws

Asmi Haldar[1], Diptiman Sen[2], Roderich Moessner[3], and Arnab Das[1]
[1]*Indian Association for the Cultivation of Science, 2A & 2B Raja S. C. Mullick Road, Kolkata 700032, India*
[2]*Centre for High Energy Physics and Department of Physics, Indian Institute of Science, Bengaluru 560012, India*
[3]*Max Planck Institute for the Physics of Complex Systems, Dresden, Germany*
(Dated: September 11, 2019)

Quantum scarred eigenstates in a Rydberg atom chain: Entanglement, breakdown of thermalization, and stability to perturbations

C. J. Turner,[1] A. A. Michailidis,[2] D. A. Abanin,[3] M. Serbyn,[2] and Z. Papić[1]
[1]*School of Physics and Astronomy, University of Leeds, Leeds LS2 9JT, United Kingdom*
[2]*IST Austria, Am Campus 1, 3400 Klosterneuburg, Austria*
[3]*Department of Theoretical Physics, University of Geneva, 24 quai Ernest-Ansermet, 1211 Geneva, Switzerland*

Collapse and revival of quantum scars via Floquet engineering

Bhaskar Mukherjee[1], Sourav Nandy[1], Arnab Sen[1], Diptiman Sen[2], and K. Sengupta[1]
[1]*School of Physical Sciences, Indian Association for the Cultivation of Science, Kolkata 700032, India*
[2]*Centre for High Energy Physics and Department of Physics, Indian Institute of Science, Bengaluru 560012, India*
(Dated: August 27, 2019)

Exact Localized and Ballistic Eigenstates in Disordered Chaotic Spin Ladders and the Fermi-Hubbard Model

Thomas Iadecola[1] and Marko Žnidarič[2,3]

PHYSICAL REVIEW LETTERS **123,** 147201 (2019)

Weak Ergodicity Breaking and Quantum Many-Body Scars in Spin-1 *XY* Magnets

Michael Schecter[1] and Thomas Iadecola[1,2]
[1]*Condensed Matter Theory Center and Joint Quantum Institute, Department of Physics, University of Maryland, College Park, Maryland 20742, USA*
[2]*Department of Physics and Astronomy, Iowa State University, Ames, Iowa 50011, USA*

Systematic Construction of Scarred Many-Body Dynamics in 1D Lattice Models

Kieran Bull,[1] Ivar Martin,[2] and Z. Papić[1]
[1]*School of Physics and Astronomy, University of Leeds, Leeds LS2 9JT, United Kingdom*
[2]*Materials Science Division, Argonne National Laboratory, Argonne, Illinois 60439, USA*

(Received 28 March 2019; published 15 July 2019)

Nonthermal States Arising from Confinement in One and Two Dimensions

Andrew J. A. James,[1,2,*] Robert M. Konik,[3,†] and Neil J. Robinson[4,‡]
[1]*London Centre for Nanotechnology, University College London, Gordon Street, London WC1H 0AH, United Kingdom*
[2]*School of Physical Sciences, The Open University, Milton Keynes, MK7 6AA, United Kingdom*
[3]*Condensed Matter Physics and Materials Science Division, Brookhaven National Laboratory, Upton, New York 11973-5000, USA*
[4]*Institute for Theoretical Physics, University of Amsterdam, Science Park 904, 1098 XH Amsterdam, Netherlands*

SciPost SciPost Phys. 3, 043 (2017)

Entanglement of exact excited eigenstates of the Hubbard model in arbitrary dimension

Oskar Vafek[1,2], Nicolas Regnault[1,3] and B. Andrei Bernevig[1]

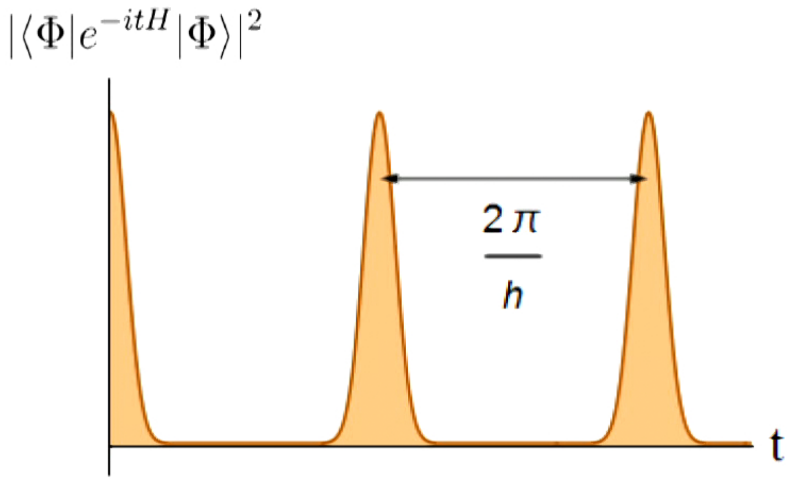
Exact Quantum Many-Body Scar States in the Rydberg-Blockaded Atom Chain

Cheng-Ju Lin and Olexei I. Motrunich
Department of Physics and Institute for Quantum Information and Matter, California Institute of Technology, Pasadena, California 91125, USA

Even perfect revivals in fidelity!

Some later models show initial states that perfectly come back after a short $\mathcal{O}(1)$ time.

-*S. Choi et al* 1812.05561
-*Schecter & Iadecola* 1906.10131*
-*S Chattopadhyay et al* 1910.08101*
-*Iadecola & Schecter* 1910.11350



These find expressions like:

$$|\langle\Phi|e^{-iH\tau}|\Phi\rangle|^2 = \cos(ht)^{2N}$$

As well as $\mathcal{O}(N)$ evenly-spaced eigenstates with entanglement entropy

$$S(\rho_A) \simeq c \log N_A$$


2π
h
t

Even perfect revivals in fidelity!

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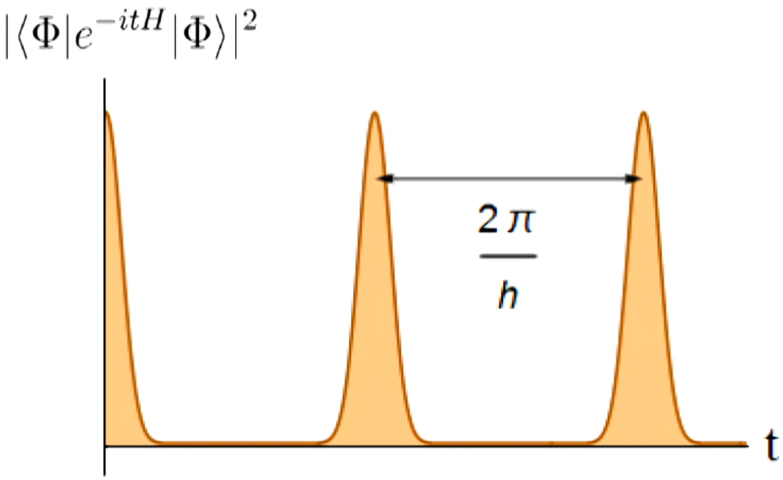


Starting assumptions:

- There is a perfect revival at a "short" time $\tau \sim \mathrm{poly}(N)$

$$|\langle\Phi|e^{-iH\tau}|\Phi\rangle| = 1$$

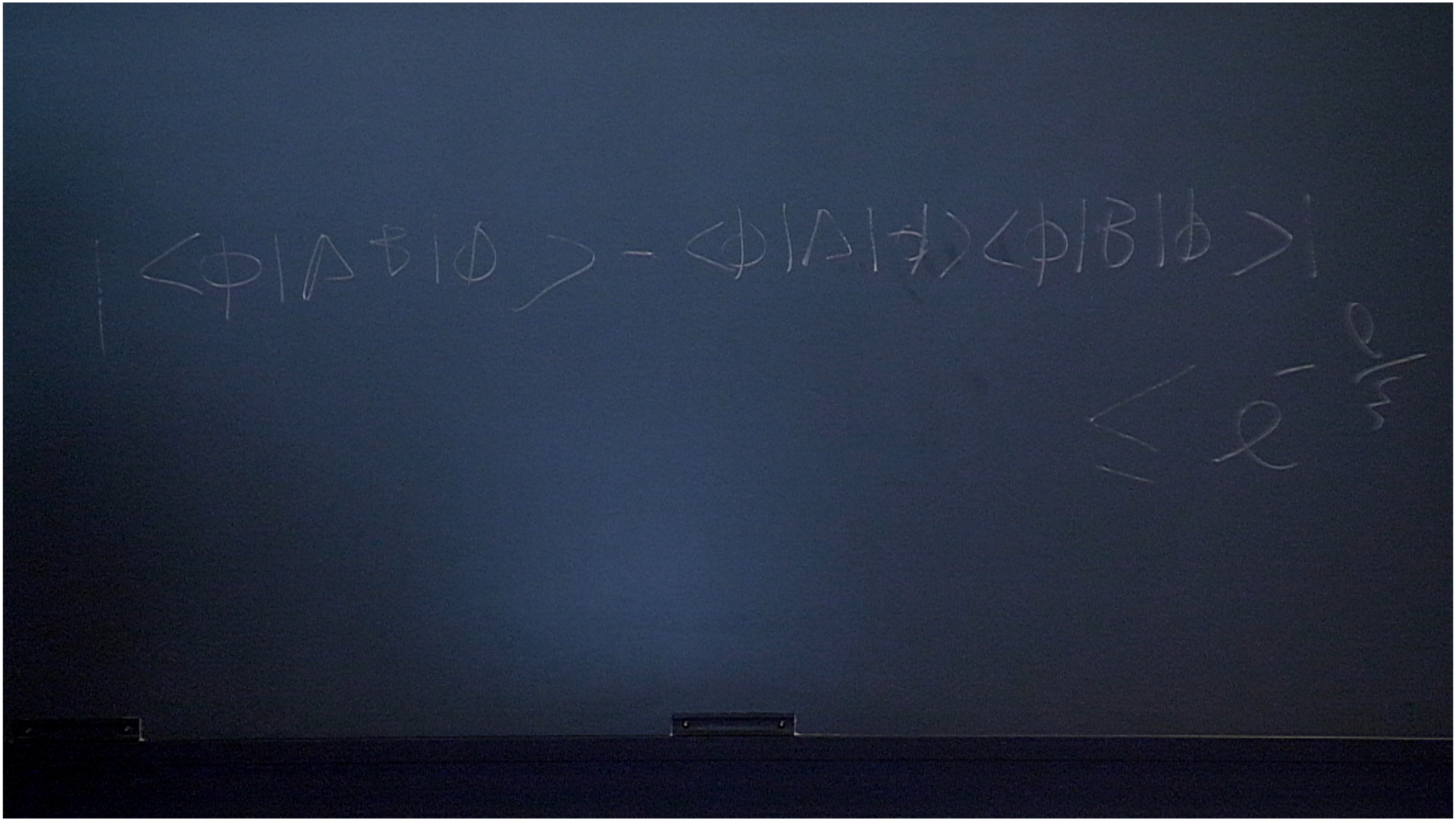
(we also have results for approximate revivals)

- The initial state $|\Phi\rangle$

  1-Is low-entangled: reduced state on region A has $\mathrm{rank}(\mathrm{tr}_{\setminus A}[|\Phi\rangle\langle\Phi|]) \leq \chi^{|\delta A|}$

  2-Is short-range correlated.

  3-Has standard deviation in energy $\sigma \equiv \sqrt{\langle H^2\rangle - \langle H\rangle^2} = s\sqrt{N}$

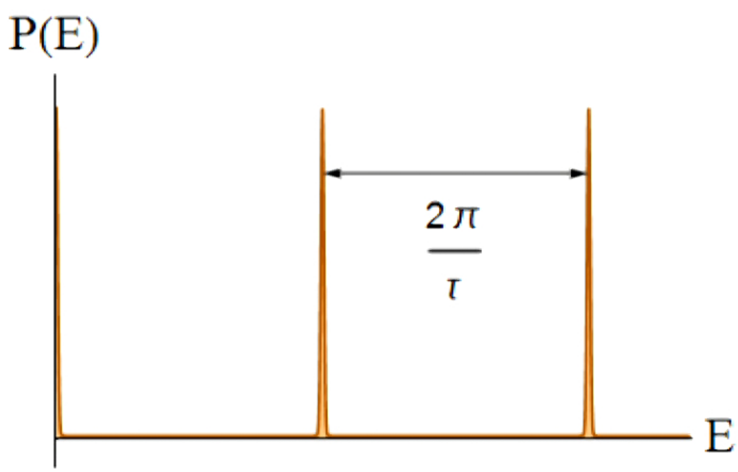

|⟨φ|A B|φ⟩ − ⟨φ|A|φ⟩⟨φ|B|φ⟩|
≤ e^(−ℓ/ξ)

Constraints on energy distribution

A perfect revival means that energy lies in equally-spaced levels:

$$\langle\Phi|e^{-iH\tau}|\Phi\rangle = \sum_j |c_j|^2 e^{-iE_j\tau} = 1 \;\Rightarrow\; E_j = \frac{2\pi l}{\tau}$$

Since $||H|| \leq hN$, there are **at most** $\frac{hN\tau}{2\pi}$ non-zero coefficients c_j *(S. Choi et al* 1812.05561)



Q: How is it distributed among the peaks?

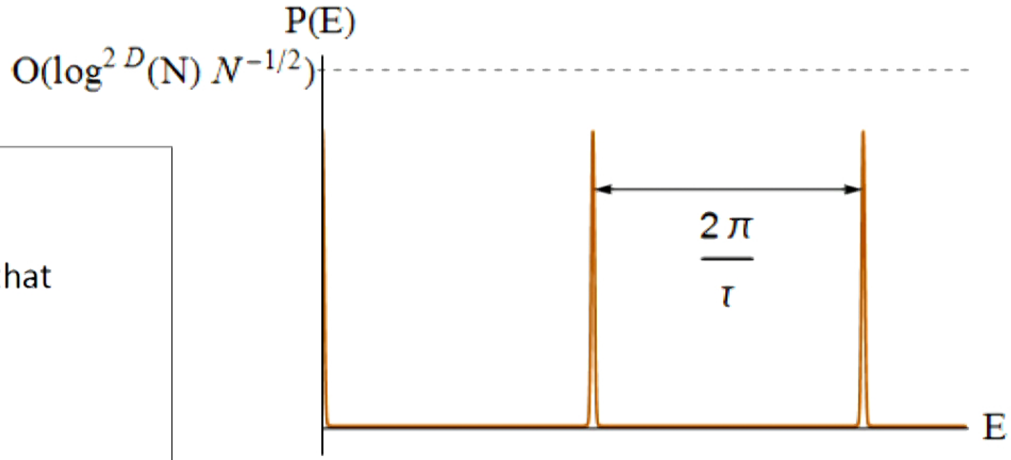
Constraints on energy distribution

We need the **Berry-Esseen theorem** (Brandao & Cramer 1502.03263).

If $|\Phi\rangle$ has short range correlations, and $\sigma \propto s\sqrt{N}$, then the energy distribution "is close to" a Gaussian, with an error vanishing as $\propto C\frac{\log^{2D} N}{\sqrt{N}}$

It follows that:

$$\max_j |c_j|^2 \leq C\frac{\log^{2D} N}{\sqrt{N}}$$



Result:

There are at least $\frac{\sqrt{N}}{C\log^{D} N}$ coefficients c_j such that

$$|c_j|^2 \geq \frac{1}{cN}$$

Where $c \geq \tau$

Berry-Esseen theorem

We need the **Berry-Esseen theorem** (Brandao & Cramer 1502.03263):

Lemma 8. *On $\Lambda = \{1, \ldots, n\}^{\times d}$ with $N = n^d > 1$ sites let H be a k-local Hamiltonian as in Eq. (1) and let ρ a state with (ξ, z)-exponentially decaying correlations. Let*

$$F(x) = \sum_{k:\, E_k \leq x} \langle k|\rho|k\rangle, \quad \mu = tr(\rho H), \quad \sigma^2 = tr\left(\rho(H-\mu)^2\right), \tag{82}$$

and

$$G(x) = \frac{1}{\sqrt{2\pi\sigma^2}} \int_{-\infty}^{x} \mathrm{d}y\, \mathrm{e}^{-\frac{(y-\mu)^2}{2\sigma^2}} \tag{83}$$

the Gaussian cumulative distribution with mean μ and variance σ^2. Then

$$\sup_x |F(x) - G(x)| \leq \Delta \frac{\ln^{2d}(N)}{\sqrt{N}}, \tag{84}$$

where

$$\Delta = C_d \frac{(\max\{k, \xi\}(z+1))^{2d}}{\sigma/\sqrt{N}} \max\left\{\frac{1}{\max\{k, \xi\}(z+1)\ln(N)}, \frac{1}{\sigma^2/N}\right\} \tag{85}$$

and $C_d \geq 1$ depends only on the dimension of the lattice.

Entanglement of “scars” from energy distribution

Result:

There are at least $\frac{\sqrt{N}}{C \log^D N}$ coefficients c_j such that $|c_j|^2 \geq \frac{1}{cN}$, where $c \geq \tau$

Notice that $|c_j|^2 = |\langle\Phi|E_j\rangle|^2$ and recall that $\mathrm{rank}(\mathrm{tr}_{\setminus A}[|\Phi\rangle\langle\Phi|]) \leq \chi^{|\delta A|}$

$$|\langle\Phi|E_j\rangle|^2 \leq \chi^{|\delta A|} e^{-\frac{1-\alpha}{\alpha} S_\alpha(\mathrm{tr}_{\setminus A}[|E_j\rangle\langle E_j|])}$$

(Wilming et al. 1802.02052)

Corollary:

There are at least $\frac{\sqrt{N}}{C \log^D N}$ evenly-spaced eigenstates such that:

$$S_\alpha(\mathrm{tr}_{\setminus A}[|E_j\rangle\langle E_j|]) \leq \frac{\alpha}{1-\alpha} (\log cN + |\partial A| \log \chi)$$

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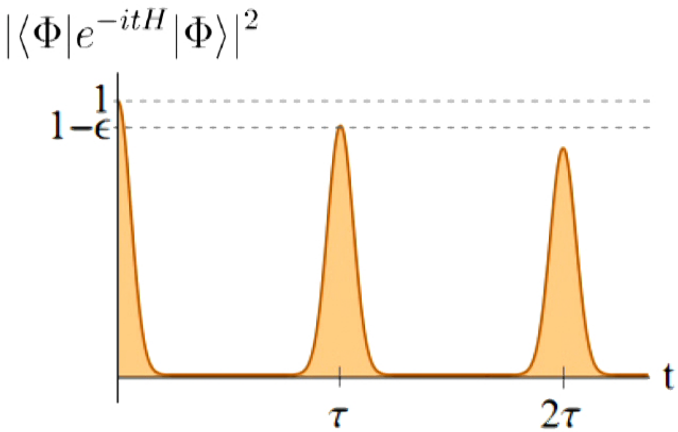
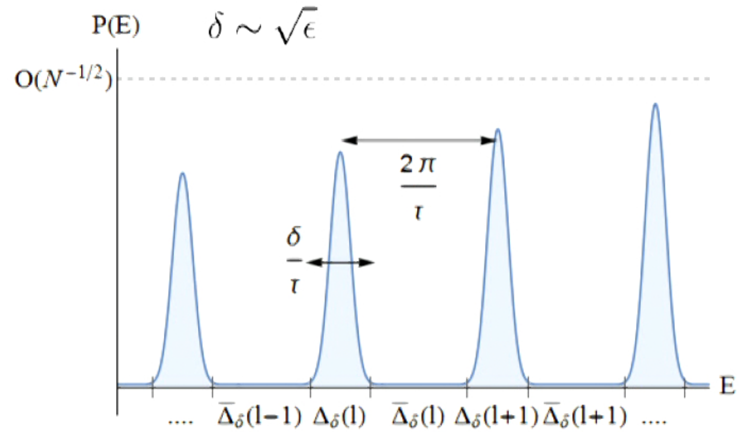
Approximate revivals





There are at least $\frac{\sqrt{N}}{C\log^D N}$ evenly-spaced "quasi-eigenstates": $|\hat{E}_j\rangle = \frac{1}{\sqrt{p\left(\Delta_\delta(j)\right)}} \sum_{E_l \in \Delta_\delta(j)} c_l |E_l\rangle$

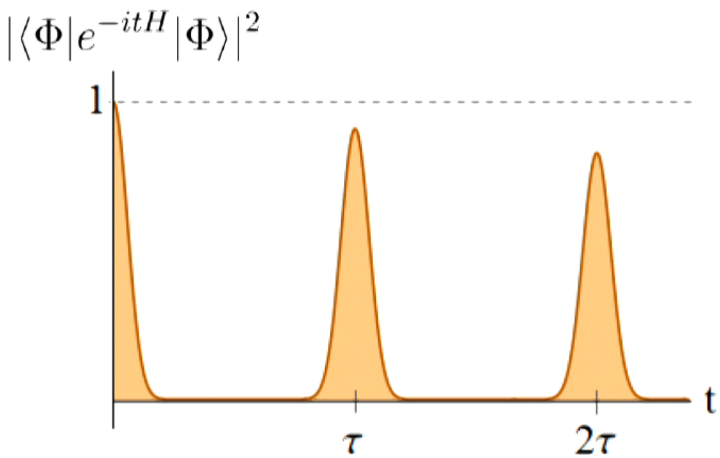
$$S_\alpha(\mathrm{tr}_{\backslash A}[|\hat{E}_j\rangle\langle\hat{E}_j|]) \leq \frac{\alpha}{1-\alpha}\left(\log cN + |\partial A| \log\chi\right)$$

Constraints on fidelity revivals

Result:

Let H be local, and $|\Phi\rangle$ have finite correlation length and $\sigma \propto s\sqrt{N}$

$$\int_0^T \frac{\mathrm{d}t}{T} |\langle\Phi|e^{-itH}|\Phi\rangle|^2 \leq \frac{5\pi}{2\sigma T} + K\frac{\log^{2D} N}{\sqrt{N}} \propto \frac{1}{\sqrt{N}}$$



- This uses the Berry-Esseen and some ideas from Malabarba et al 1402.1093
- Bound is saturated for

$$|\langle\Phi|e^{-iH\tau}|\Phi\rangle|^2 = \cos(ht)^{2N}$$

(since LHS goes as $\propto \frac{1}{\sqrt{N}}$)

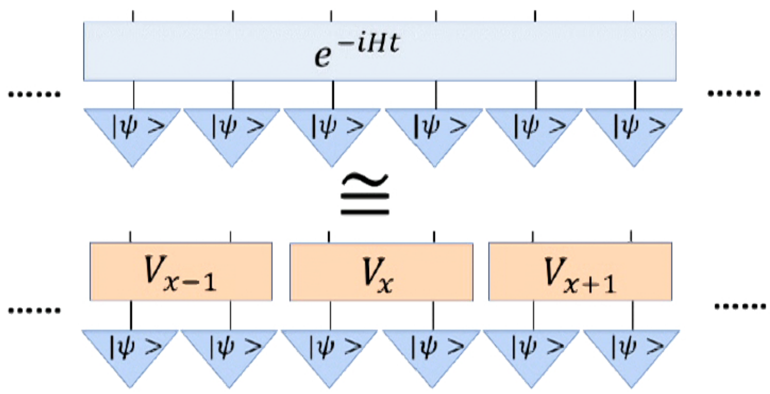
Constraints on fidelity revivals

Another feature of $|\langle\Phi|e^{-iHt}|\Phi\rangle = \cos(ht)^{2N}$ is that the value between revivals scales exponentially with N.

We can (almost) show this:

Result:
Let $|\Phi\rangle = |\phi\rangle^{\otimes N}$. For every $\delta > 0$ there exists a $\delta > t > 0$ such that $k(t) > 0$ and

$$|\langle\Phi|e^{-itH}|\Phi\rangle|^2 \leq \mathcal{O}\left(\exp\left(-\frac{1}{4}[Nk(t)]^{1/(1+D)}\right)\right)$$



With Lieb-Robinson bounds, one can show that

$$|\langle\Phi|e^{-itH}|\Phi\rangle|^2 \sim |\langle\phi|\phi'\rangle|^{2N}$$

Summary & Outlook

- We showed that perfect revivals imply that there are $\mathcal{O}(\sqrt{N})$ "scarred" eigenstates with bounded entanglement entropy

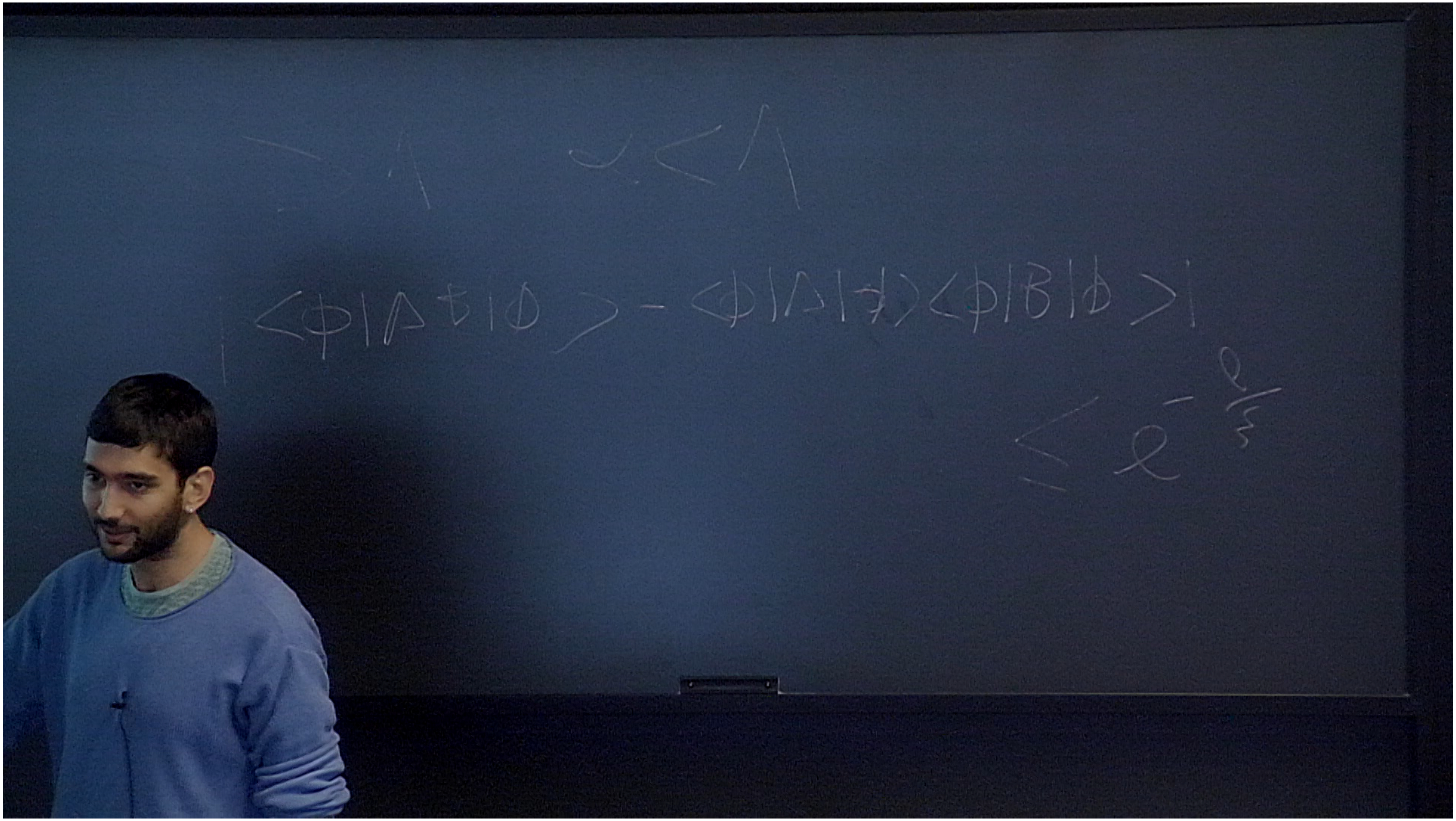
$$S_\alpha \leq \frac{\alpha}{1-\alpha}(\log cN + |\partial A| \log \chi) \qquad \alpha > 1$$

- This DOES NOT guarantee an efficient MPS description. (Schuch et al 0705.0292)

- The present argument is limited: given two regions, there exists a state with arbitrary high overlap with product state, but volume law in von Neumann entropy. (Wilming et al. 1802.02052)

- Open question: show there are $\mathcal{O}(N)$ evenly-spaced eigenstates with efficient MPS description (even from approx. revivals?).

- Many more open questions in scars: stability to perturbations (Lin et al 1910.07669), relation to integrability (Khemani et al 1807.02108),….



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THANKS!