Title: Self-correction from symmetry

Speakers: Sam Roberts

Collection: Symmetry, Phases of Matter, and Resources in Quantum Computing

Date: November 28, 2019 - 9:30 AM

URL: http://pirsa.org/19110137

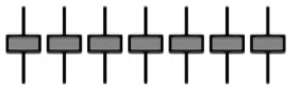
Abstract: A self-correcting quantum memory can store and protect quantum information for a time that increases without bound in the system size, without the need for active error correction. Unfortunately, the landscape of Hamiltonians based on stabilizer (subspace) codes is heavily constrained by numerous no-go results and it is not known if they can exist in three dimensions or less. In this talk, we will discuss the role of symmetry in self-correcting memories. Firstly, we will demonstrate that codes given by 2D symmetry-enriched topological (SET) phases that appear naturally on the boundary of 3D symmetry-protected topological (SPT) phases can be self-correcting -- provided that they are protected by an appropriate subsystem symmetry. Secondly, we discuss the feasibility of self-correction in Hamiltonians based on subsystem codes, guided by the concept of emergent symmetries. We present ongoing work on a new exactly solvable candidate model in this direction based on the 3D gauge color code. The model is a non-commuting, frustrated lattice model which we prove to have an energy barrier to all bulk errors. Finding boundary conditions that encode logical qubits and retain the bulk energy barrier remains an open question.

Self-correction from symmetry

Sam Roberts
PsiQuantum

joint work with
Stephen Bartlett, Tomas Jochym-O'Connor, John Preskill

Symmetries, phases of matter, quantum computation



Fault-tolerant logic gates

Yoshida 15,16
Heinrich *et al*. 16
Cheng *et al*. 17

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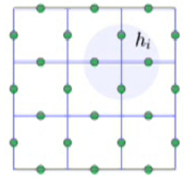

Topological phases with symmetry

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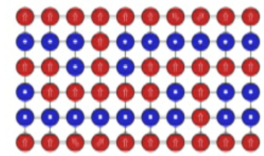

Resources for MBQC

Miyake 10
Else *et al*. 12
Prakash, Wei 15
Raussendorf *et al*. 19

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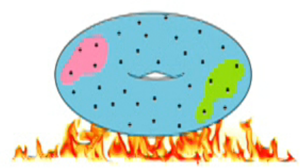
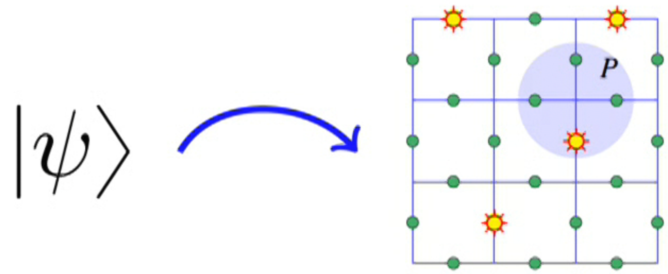


Quantum memories

Quantum memories: protecting quantum information

<u>Quantum information</u>
quantum error correcting code

<u>Condensed matter</u>
groundspace of topological phase



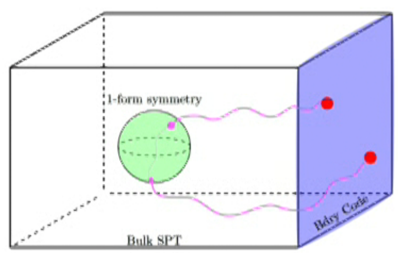
- local operators fix the code subspace
- errors can be diagnosed by measuring these operators
- <u>Self-correction</u>: protection without active error correction

Quantum memories through the lens of symmetry

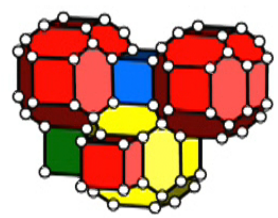
We show:

- Existence of self-correcting memories in 3D, protected by symmetry
- Candidate subsystem code where symmetry is emergent

Part I:
Symmetry-protected
self-correction



Part II:
Subsystem
quantum memories



Outline

Self-correcting quantum memories

- Background
- No-go results

Symmetry protected self-correcting quantum memories

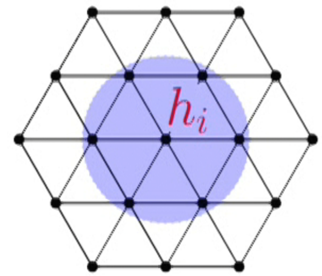
- The rules of the game
- Existence in 3D – example based on the cluster state model

Subsystem quantum memories

- A first step: confining model based on the 3D gauge color code

The (Caltech) rules for self correction

1. Finite density of spins in $\mathbb{R}^3$
2. Local Hamiltonian $H = \sum_i h_i$ with $\|h_i\| \leqslant 1$
3. Degenerate ground space, perturbatively stable
4. Coupled to a thermal bath, the lifetime τ of encoded information diverges (exponentially) with the system size
5. Efficient classical decoder



Open problem: existence in 3 dimensions or less?

The ~~(Caltech)~~ Poulin rules for self correction

1. Finite density of spins in $\mathbb{R}^3$
2. Local Hamiltonian $H = \sum_i h_i$ with $\|h_i\| \leqslant 1$
3. Degenerate ground space, perturbatively stable
4. Coupled to a thermal bath, the lifetime τ of encoded information diverges (exponentially) with the system size
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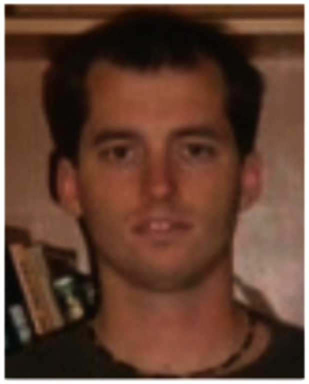
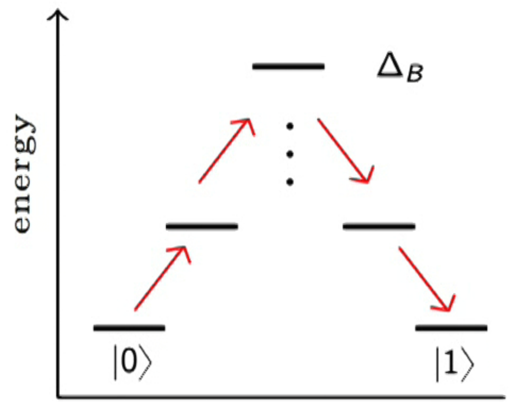


Open problem: existence in 3 dimensions or less?

Mechanism: the energy barrier

- Energy barrier: The minimal energy cost that needs to be overcome to implement a logical operator through local operations

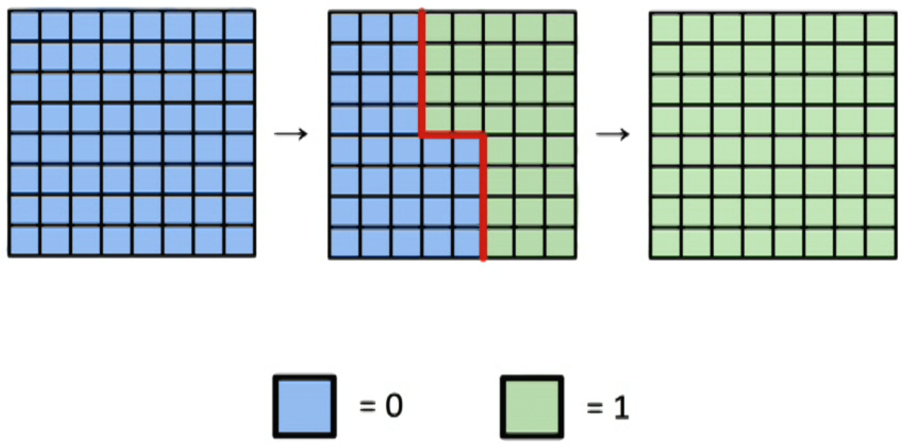


> Arrhenius Law
> for memory time
> $\tau \sim \exp(\frac{\Delta_B}{T})$
> (phenomenology)

- Necessary for stabilizer Hamiltonians (Temme 14, Temme & Kastoryano 15), 2D abelian quantum doubles (Komar *et al.* 16)
- General folklore: the no strings rule
- Can a macroscopic energy barrier exist in a 3D model?

The energy barrier: classical Ising example

$$H = -\sum_{f \sim f'} s_f s_{f'} \qquad s_f \in \mathbb{Z}_2$$



- Energy barrier: $O(L)$
- Classical lifetime: $\exp\left(\frac{L}{T}\right)$ for $T < T_c$

Dimensional constraints on the energy barrier

	Energy barrier	Memory time
2D toric code	O(1)	$O(1) = e^{c\beta}$
3D toric code	O(1)	$O(1) = e^{c\beta}$
4D toric code	$O(L)$	$\exp(\beta L)$

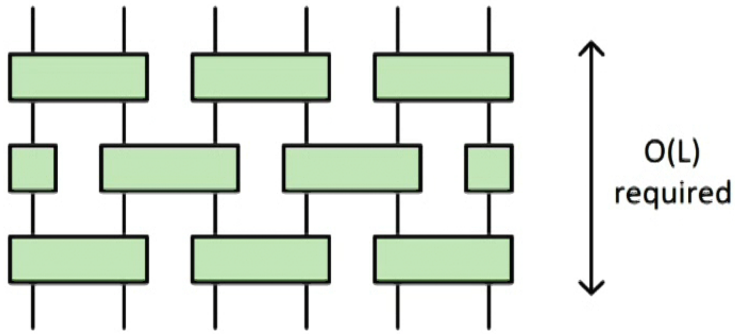
Models with constant energy barriers

- 2D stabilizer codes
  Bravyi & Terhal 09, Kay & Kolbeck 08, Haah & Preskill 12
- 2D commuting projectors
  - Landon-Cardinal & Poulin 12
- 3D Stabilizer models with translational and scale invariance
  Yoshida 11
- 3D Stabilizer models with non-Clifford gate
  Pastawski & Yoshida 15

A lesson from topological order

- Topologically protected ground states require long-range entanglement

Def $T = 0$ topological order: $|\psi\rangle$ is TO if we cannot prepare $|\psi\rangle$ with a low depth circuit



- Self correction: spends time not in ground space but in a mixture of low energy states

Def $T > 0$ topological order (Hastings): Gibbs state $\rho = e^{-\beta H}/\,\mathrm{Tr}(e^{-\beta H})$ is TO if we cannot prepare it from the Gibbs state of a classical Hamiltonian with a low depth circuit

A lesson from topological order

- No obvious candidates in dimensions $D \leqslant 3$.
- No known topologically ordered model at $T > 0$.
  - × Fractal models, Siva & Yoshida 17
  - × 3D stabilizer, translationally and scale invariant, Hastings 11

However....

> There exists symmetry protected topologically ordered phases at $T > 0$ in dimension 3.

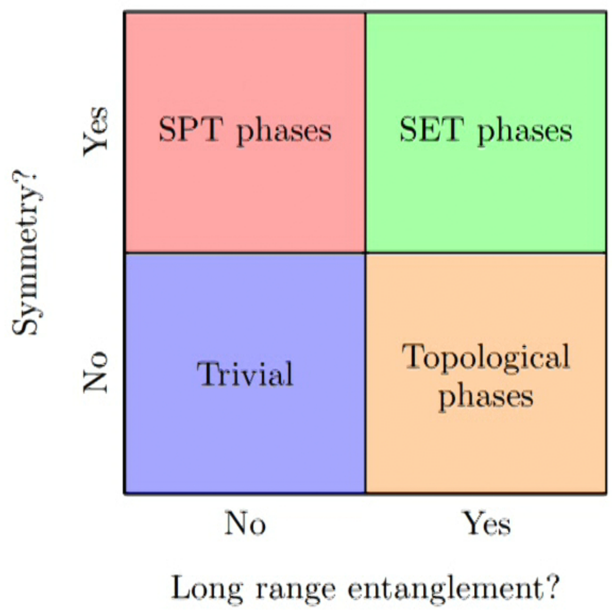
Goal:

- Understand stability
- Try to replicate in models without requiring symmetry

Phases with symmetry

Def (Topological order with symmetry $S(g)$): Cannot prepare the ground state (Gibbs state) from a product state (classical Gibbs state) using a low depth symmetric circuit.

- Symmetry-enriched (SET) or symmetry-protected (SPT) depending on entanglement in absence of symmetry.

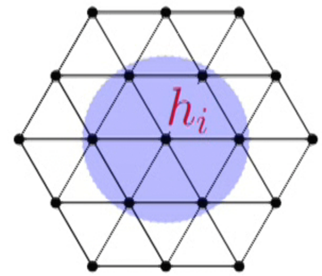


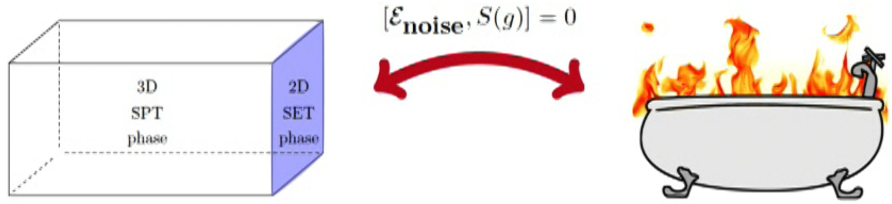
- SPT: No anyonic excitations, unique ground state
- SET Anyonic excitations, topology dependent ground degeneracy.
- SET found on boundary of SPT

Self correcting quantum memories with symmetries

Hamiltonian H invariant under representation of symmetry group G.

1. Finite density of spins in $\mathbb{R}^3$
2. Local Hamiltonian $H = \sum_i h_i$ with $\|h_i\| \leqslant 1$
3. Degenerate ground space, perturbatively stable
4. Growing symmetric energy barrier
5. Efficient classical decoder
6. Admit symmetric encoding circuit

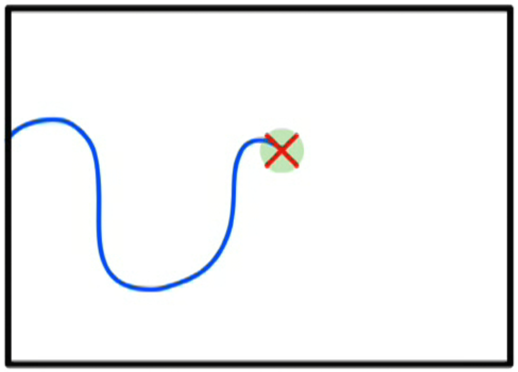




Symmetry protected memories: non examples

Admitting symmetric encoding circuits

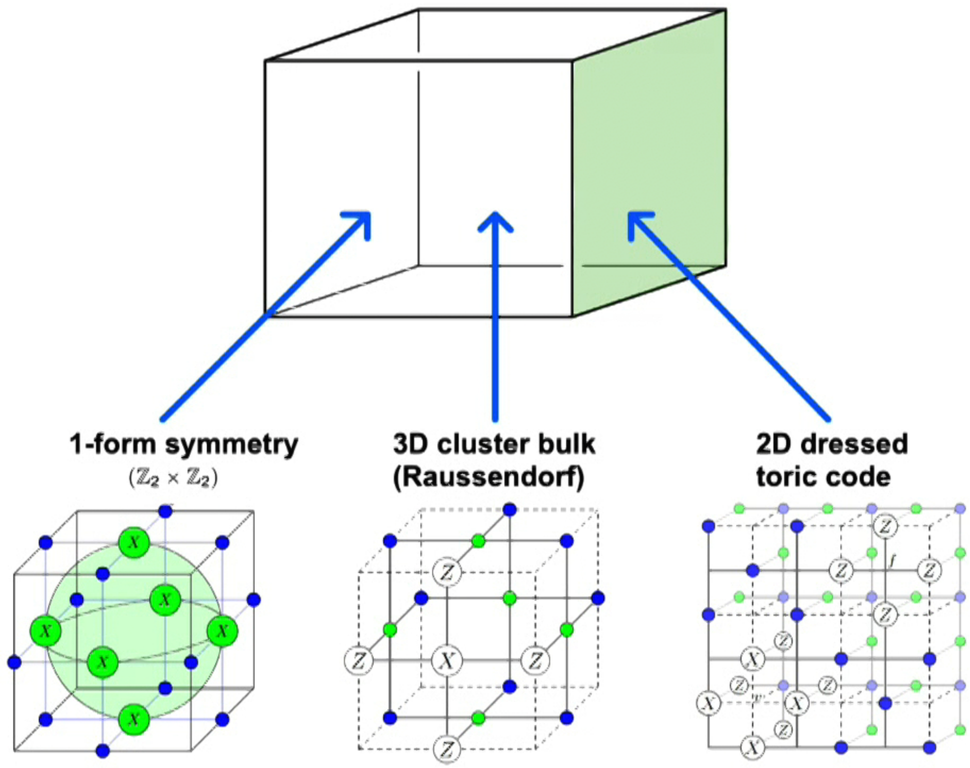
- All logical operators must admit symmetric local decompositions
- Prevents us from naively promoting stabilizers to symmetries



- Non examples:
  1. 2D stabilizer
  2. 3D stabilizer with translational and scale invariance

Existence of symmetry-protected self-correction

- The Raussendorf-Bravyi-Harrington model is self-correcting under 1-form symmetry



- Information encoded on the boundary, protected by the bulk

Understanding the model

Lemma
Bulk excitations are collections of loops (and therefore confined below a critical temperature)

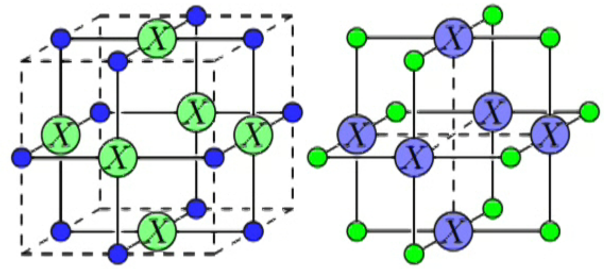
Lemma
Anyonic excitations can exist on the boundary if and only if by a bulk string excitation.

Lemma
Logical fault on the boundary requires traversing an anyon (syndrome) separation of $O(L)$

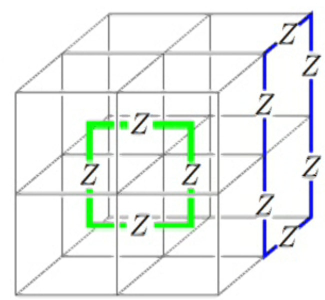
$\Longrightarrow$ Polynomial energy barrier + self correcting

1. Bulk confinement

Bulk excitations are collections of loops (and therefore confined below a critical temperature)



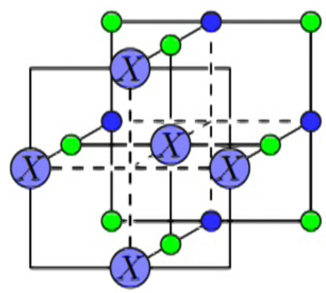
- Excitations are chains of Pauli-Z
- 1-form symmetry requires they pierce every closed 2D submanifold an even number of times.

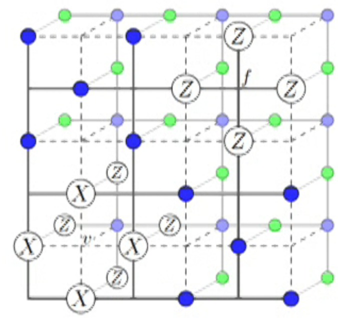


2. Boundary coupling

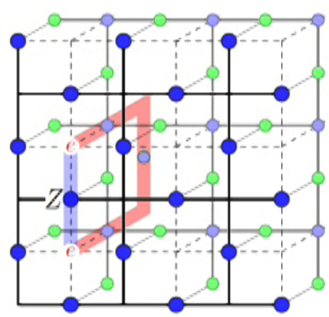
Anyonic excitations can exist on the boundary if and only if by a bulk string excitation.

- Symmetries on the boundary expressible as products of dressed toric code terms and bulk cluster terms



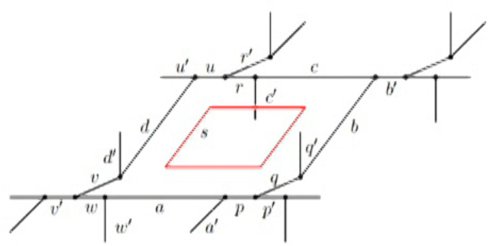


- Only valid configurations are those with paired excitations



Self-correction key properties

- Bulk confinement ✓
  - Property of 1-form symmetry
- Bulk boundary coupling ✓
  - Arises from SPT order of bulk
- Energy cost for separating anyons (syndromes) ✓
- Can be found in other models with symmetries, eg. (modular) Walker Wang



Summary of SPSCQM

- Self correction possible on boundary of SPT phases with subsystem symmetries (1-form)

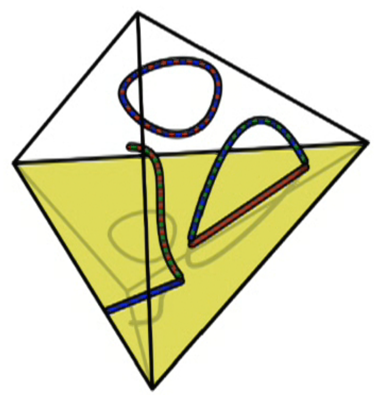
	Topological order $T>0$	Self-correction
SPT with onsite	×	×
Trivial with 1-form	×	×
SPT with 1-form	✓	✓
2D commuting proj.	× Hastings 11	× Landon-Cardinal & Poulin 12
3D fractal	× Siva & Yoshida 17	× Bravyi & Haah 13
4D toric	✓ Hastings 11	✓ Alicki *et al.* 10

- How to realise this mechanism in a model without explicitly enforcing symmetry.

Part II

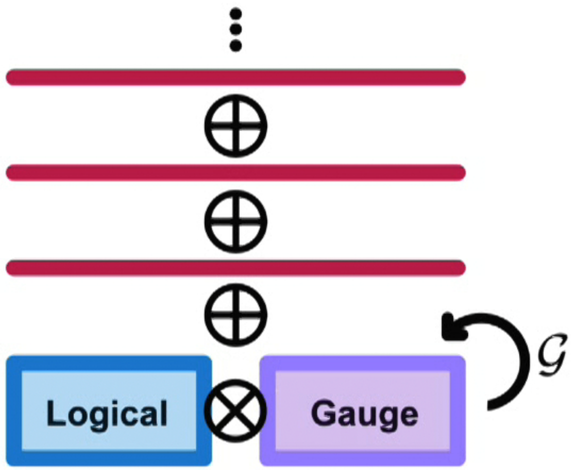
Subsystem code Hamiltonians

Goal: Study a phase of the gauge color code, try to replicate features of the 3D cluster model.



Bombin 15

Subsystem codes (Poulin 05)



- Gauge group

  $\mathcal{G} \subseteq$ Pauli

  (not necessarily abelian)

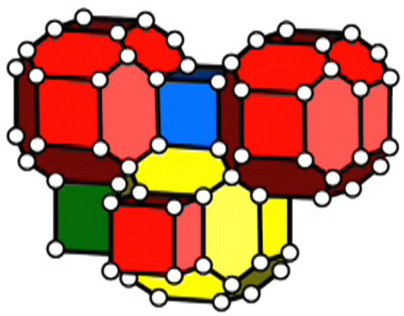
- Stabilizer group

  $\mathcal{S} \propto \mathcal{Z}(\mathcal{G})$

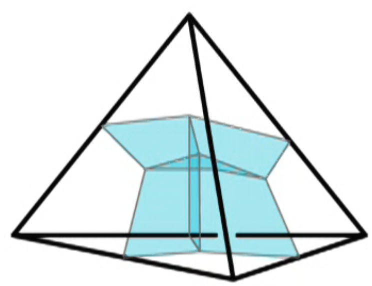
- Bare logicals $C(\mathcal{G})/\mathcal{S}$
- Dressed logicals $C(\mathcal{S})/\mathcal{G}$

The gauge color code (Bombin 15)

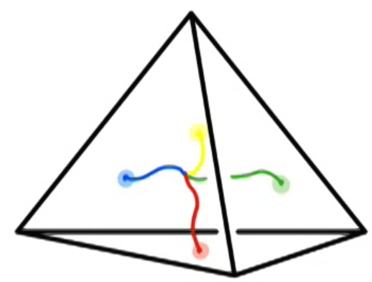
- Qubits on vertices of 4 colourable, 4 valent lattice in 3D



- Gauge generators X_f Z_f on plaquettes
  - hexagons and squares
- Stabilizer generators X_c Z_c on 3-cells
  - Cubes and soccer balls
- Encodes single logical qubit on 3-ball
- 2D bare logicals, 1D dressed logicals
- Logical operators supported on boundary



Dressed logical



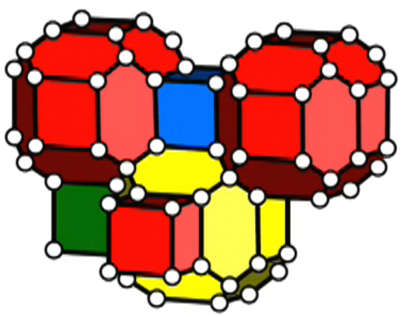
Bare logical

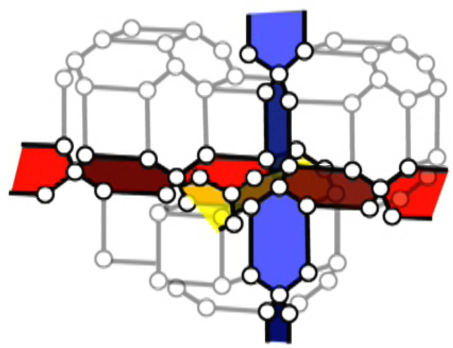
Lattice from Brown, Nickerson, Browne, 15

Hamiltonian: the cubic honeycomb model

- Want: Hamiltonian that has loop-like excitations

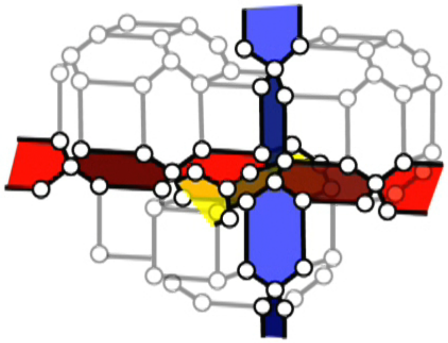
$$H(\lambda) = -\sum_{\hexagon \in F} \lambda X_{\hexagon} + (1-\lambda) Z_{\hexagon} - \sum_{c \in C} X_c + Z_c$$

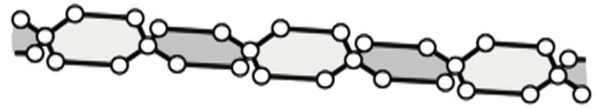




- Chains along different directions form a cubic honeycomb

Solving the model: frustration graphs



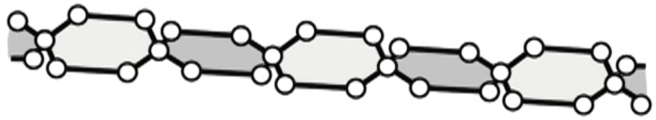


- Decompose gauge part of the model

$$H = H_{\mathcal{S}} + \sum_i H_i, \quad [H_{\mathcal{S}}, H_i] = [H_i, H_j] = 0, i \neq j$$

- Frustration graph is a number of linear graphs
- No product constraints amongst terms
- Map to independent XY-models solvable by free fermion (via JW)

Solving the model



Lemma

The cubic honeycomb model is dual to a number of independent 1D XY models

$$H_{XY}(\lambda) = -\sum_k (\lambda X_k X_{k+1} + (1-\lambda) Y_k Y_{k+1})$$

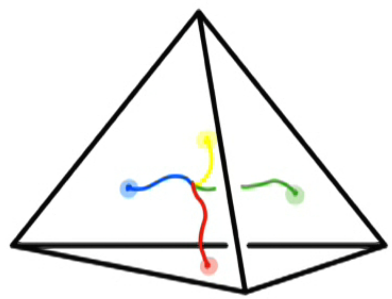
- Duality is a nonlocal unitary
- Each chain can be thought of as a $[n, 1, 1]$ code
  $\mathcal{G} = \langle X_i X_{i+1} . Z_i Z_{i+1} \rangle$
- Each solvable by JW transformation to free fermions

The ground space

Lemma

With tetrahedral boundary conditions, the groundspace of the cubic honeycomb model is equal to the codespace of the gauge color code (for a choice of fixed gauge)

$$H(\lambda) = -\sum_{\hexagon \in F} \lambda X_{\hexagon} + (1-\lambda) Z_{\hexagon} - \sum_{c \in C} X_c + Z_c$$

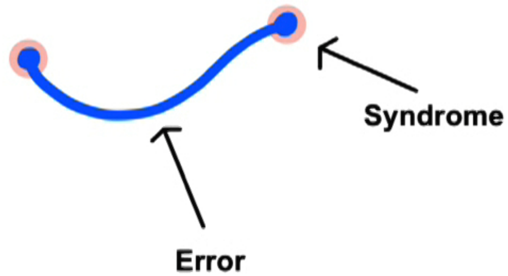


Lemma

The model is gapped for $\lambda \in [0, \frac{1}{2}) \cup (\frac{1}{2}, 1]$ and gapless at $\lambda = \frac{1}{2}$.

Errors

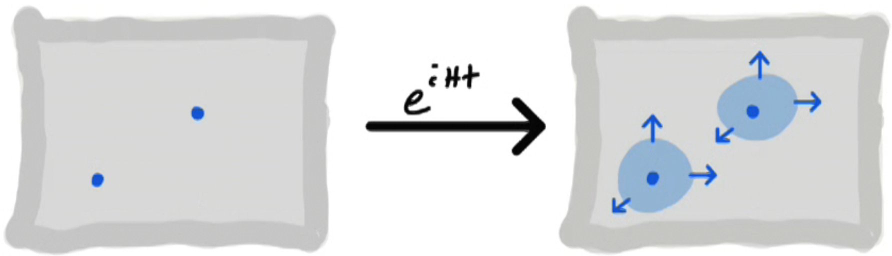
- To understand the energy barrier, we first need to understand what errors look like



- Pauli errors look like string segments, syndromes appear on their boundary
- Decoder fails on long strings
  - This process creates and separates a pair of syndromes
  - Goal: bound energy cost of such processes.

Subsystem energy barrier

- A subtlety: operator growth.



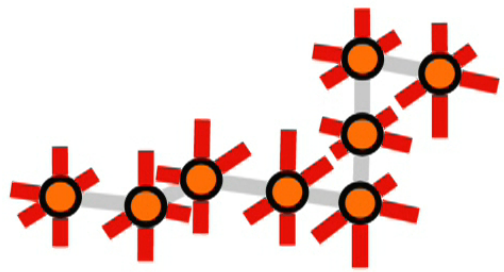
- But remain local up to gauge transformations

$\Longrightarrow$ We allow gauge transformations for free in the definition of energy barrier

- We consider energy barrier for Pauli errors
  - In order to determine harmful errors, we need a mapping: error $\mapsto$ syndrome

Confinement in the model: the bulk energy barrier

The gauge chains cover the model: any string l operator intersects $O(\mathrm{wt}(l))$ gauge chains.



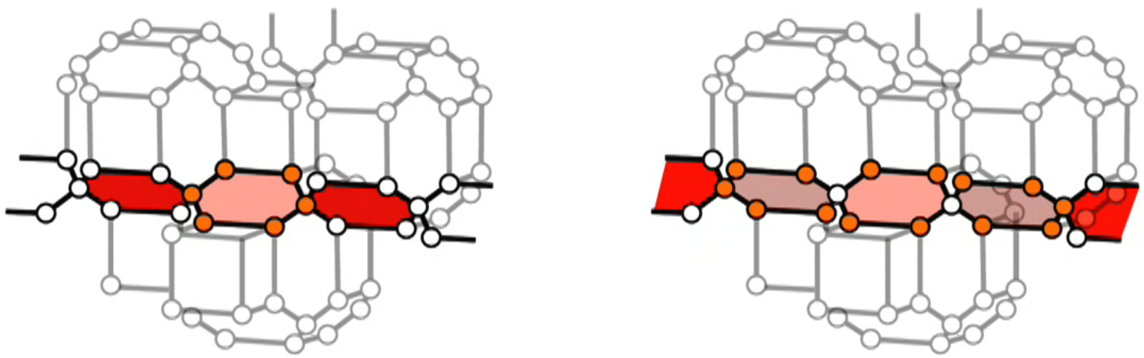
Lemma

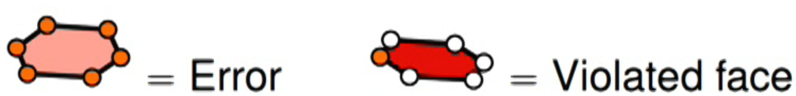
Pauli energy barrier to creating a pair of syndromes at v, w is proportional to $d(v, w)$

- Proportionality $c = 2\min\{\mathrm{Tr}(X_{\hexagon}\rho_0), \mathrm{Tr}(Z_{\hexagon}\rho_0)\} > 0$ for $\lambda \in (0, 1)$.

The boundary problem

- To encode logical qubits we need boundaries
  - All homologically nontrivial surface operators belong to the gauge group
- With boundaries, energy can be propagated down chain

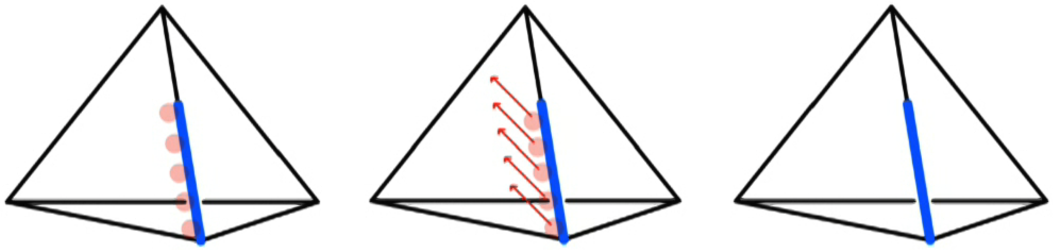




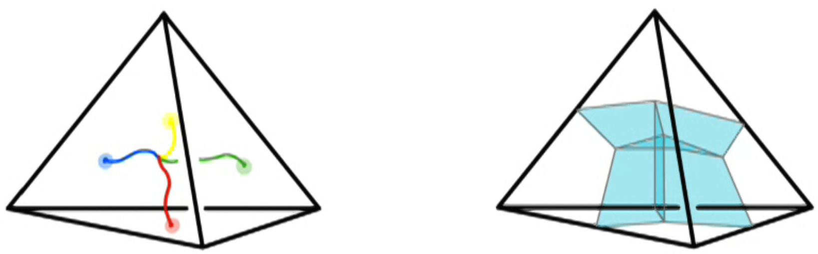
- And dissipated on the boundaries

The boundary problem

- Consider the operator that propagates a string error and dissipates energy along the way:



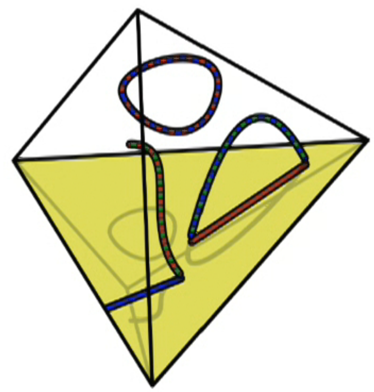
- Results in a bare logical



- Exist faults that couple all boundaries
- Can be implemented with constant energy cost

The boundary problem

- Bulk confinement ✓
- Energy cost for separating syndromes ✓
- Bulk boundary coupling ✗



- Possible avenues
  - Different topologies and geometries
  - Consider boundary Hamiltonians

Conclusion

Symmetry protected self-correction possible, can we replicate it in more reasonable models?

- Subsystem codes have rich physics
  - No strings rule not necessary?
- Can reproduce the bulk energy barrier as seen in the 3D cluster model, but not the boundary coupling
  - Look for other boundary conditions/Hamiltonians.
  - Look for other choices of Hamiltonians that realise different frustration graphs
- Understanding topological order in the GCC
  - SPT ordered?
  - Beyond TQFT?

	Open BCs	Periodic BCs
Energy barrier?	✗	✓
Codespace?	✓	✗

- Periodically switching between X and Z gauges.