Title: Error correction with the color code

Speakers: Aleksander Kubica

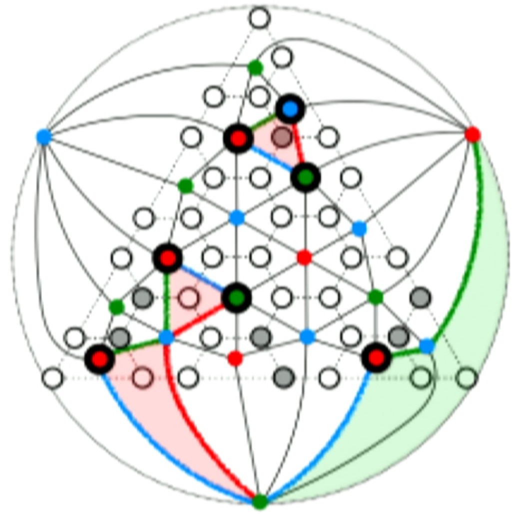
Collection: Symmetry, Phases of Matter, and Resources in Quantum Computing

Date: November 29, 2019 - 11:30 AM

URL: http://pirsa.org/19110136

Abstract: The color code is a topological quantum code with many valuable fault-tolerant logical gates. Its two-dimensional version may soon be realized with currently available superconducting hardware despite constrained qubit connectivity. In the talk, I will focus on how to perform error correction with the color code in d â‰¥ 2 dimensions. I will describe an efficient color code decoder, the Restriction Decoder, which uses as a subroutine any toric code decoder. I will also present numerical estimates of the storage threshold of the Restriction Decoder for the triangular color code against circuit-level depolarizing noise.
Based on arXiv:1905.07393 and arXiv:1911.00355.

Error correction with the color code



Aleksander Kubica

PERIMETER INSTITUTE

IQC Institute for Quantum Computing

work w/ N. Delfosse arXiv:**1905.07393**

work w/ C. Chamberland, T. Yoder, G. Zhu arXiv:**1911.00355**

Toward quantum computation

- We want to build a reliable, universal quantum computer.
- A path to fault-tolerant universal computation

physical system & operations → quantum error correction → quantum algorithms

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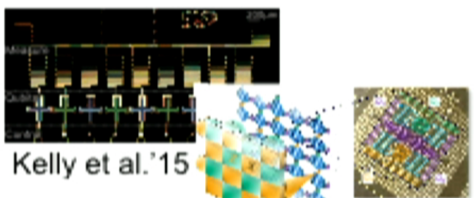
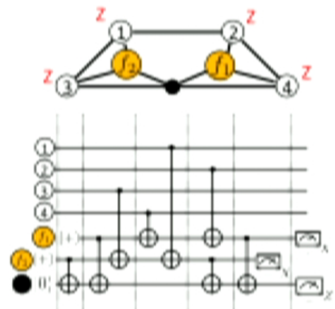


Kelly et al.'15

Córcoles et al.'15



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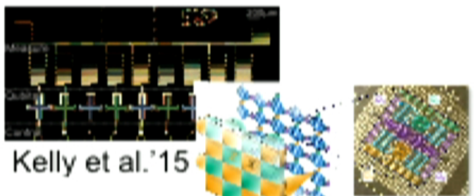

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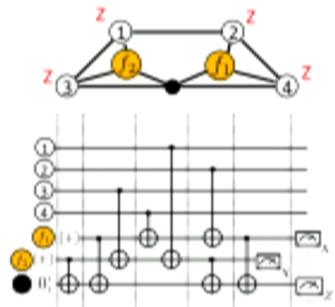

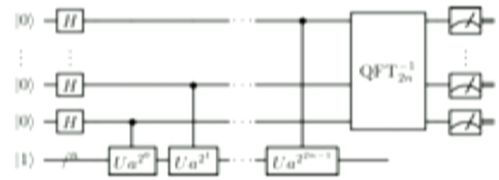

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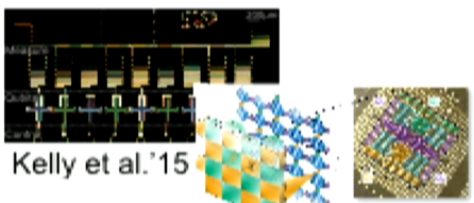

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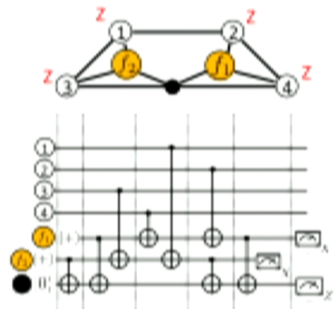

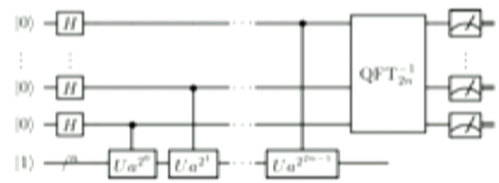

- Desired properties of quantum error-correcting codes:
  — can be implemented in the lab,
  — easy fault-tolerant logical gates,
  — efficient decoders w/ high thresholds.

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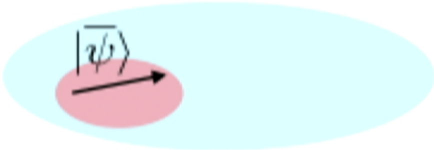
Decoding for stabilizer codes

- Stabilizer codes *[G96]*: commuting Pauli operators
  code space = (+1)-eigenspace of stabilizers.



Gottesman'96

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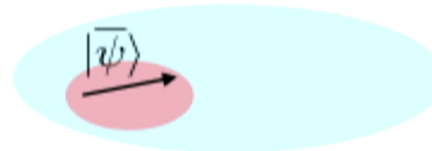

- Quantum error-correction game:

$$|\psi\rangle \xrightarrow{\text{encode}} |\overline{\psi}\rangle \xrightarrow{\text{noise}} \mathcal{E}(|\overline{\psi}\rangle) \xrightarrow{\text{recovery}} \mathcal{R} \circ \mathcal{E}(|\overline{\psi}\rangle) \xrightarrow{\text{read off}} |\psi'\rangle$$

move outside
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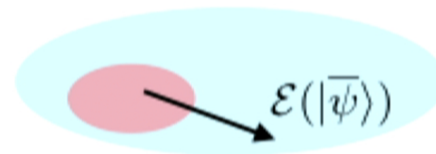

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move outside
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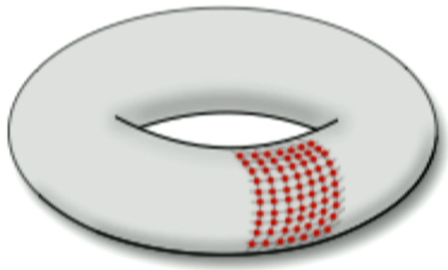
measure stabilizers to
discretize and diagnose errors

- **Decoding** = classical algorithm to find error correction from syndrome.

Gottesman'96

Topological quantum codes

- **Topological quantum codes**:
  — geometrically local generators,
  — logical info encoded non-locally.



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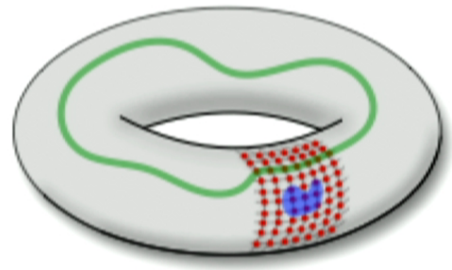
Topological quantum codes



- **Topological quantum codes**:
  — geometrically local generators,
  — logical info encoded non-locally.
- Examples: **toric code** & (gauge) **color code**.
- Locality comes at a price — limitations and no-go theorems!

Why color code?

- Leading approach to scalable q. computing — **2D toric code**.
- Difficulty: **fault-tolerant non-Clifford** gate (needed for universality).
- **Color code** as an alternative to toric code
  - 😀 more qubit efficient,
  - 😀😀 transversal gates — $Z(\pi/2^d)$ rotation in d dim *[B15,KB15]*,
  - 😀😀😀 avoiding magic state distillation *[B15,B18,BKS]*,

Bombin'15; Kubica&Beverland'15; Beverland et al. (in prep.); Bombin'18

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  - 😀😀😀 avoiding magic state distillation *[B15,B18,BKS]*,
- **Unfortunately**, color code
  - 🙁 seems challenging to decode,
  - 🙁🙁 seems to perform worse than toric code.

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Main results & outline

Results: efficient decoders for color code in $d \geq 2$ dim w/ high thresholds.

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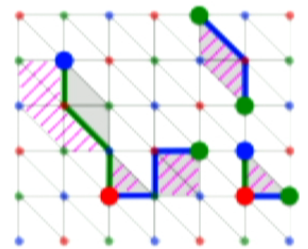
1. **Intro:** toric & color codes in 2D.

2. **Restriction Decoder:** color code decoding by using toric code decoding.



2D toric code & decoding

- For CSS codes, we can correct correct X and Z errors separately.

- **2D toric code** *[K97]*:
  — Z-errors = 1D strings,
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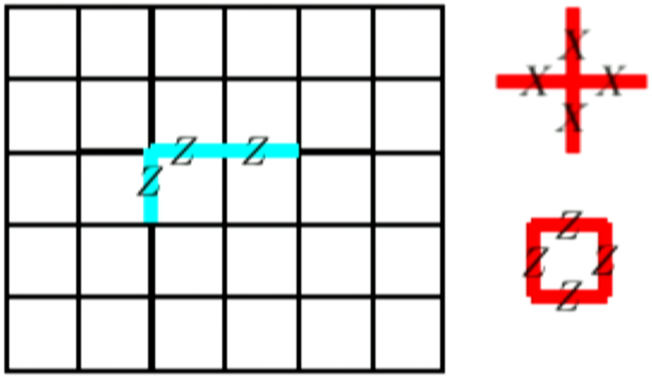

Kitaev'97; Dennis et al.'02; Harrington'04; Duclos-Cianci&Poulin'10; Bravyi et al.'14; Torlai&Melko'16

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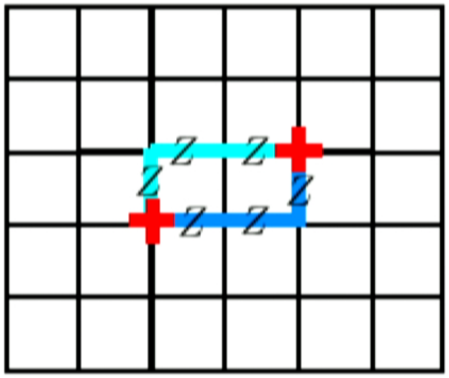
- Global Z_2 symmetry:
  excitations created in **pairs**.
- **Decoding =** finding position of errors
  from violated stabilizers = pairing up excitations!



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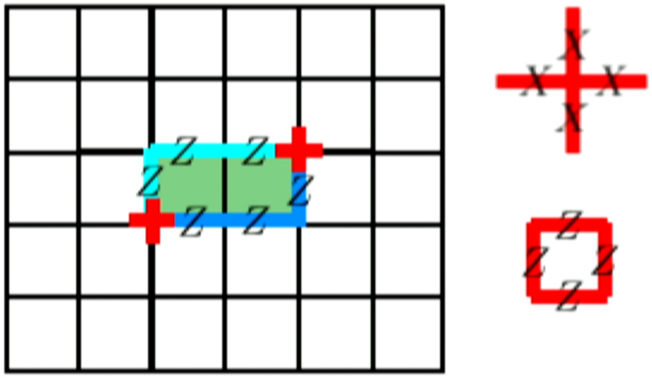


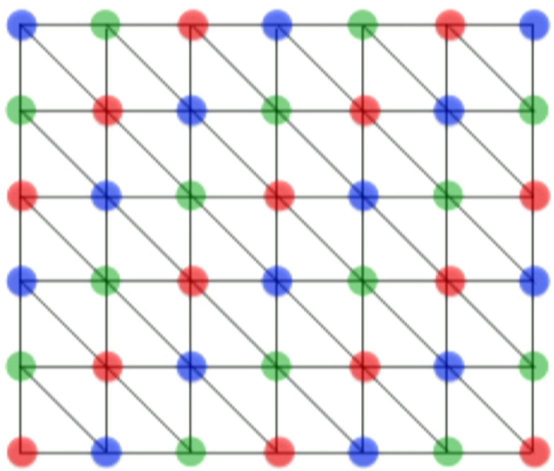
- Global Z_2 symmetry:
  excitations created in **pairs**.

- **Decoding =** finding position of errors from violated stabilizers = pairing up excitations!

- **Successful decoding** iff error and correction differ by stabilizer.

Kitaev'97; Dennis et al.'02; Harrington'04; Duclos-Cianci&Poulin'10; Bravyi et al.'14; Torlai&Melko'16

2D color code



- **Lattice**: triangles, 3-colorable vertices.
- **2D color code** *[BM06]*:
  — qubits = triangles,
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- Color and toric codes related *[KYP15]*…

Bombin&Martin-Delgado'06; Kubica et al.'15; Wang et al.'10; Landahl et al.'11

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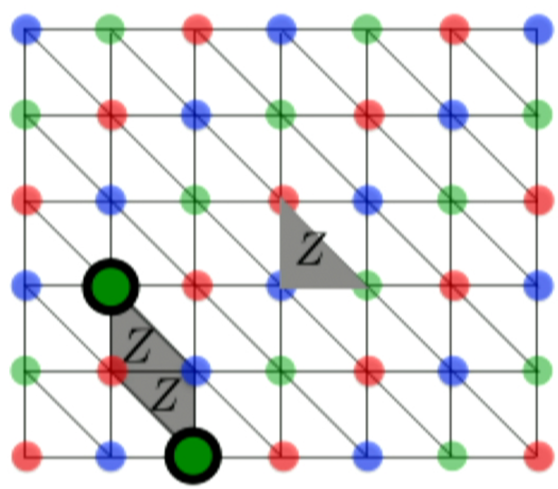

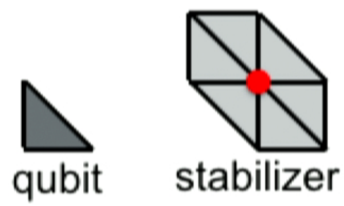

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- **2D color code** *[BM06]*:
  — qubits = triangles,
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- Color and toric codes related *[KYP15]*…
- …but decoding seems to be challenging *[WFHH10,LAR11]* as excitations created in **pairs** & in **triples**!

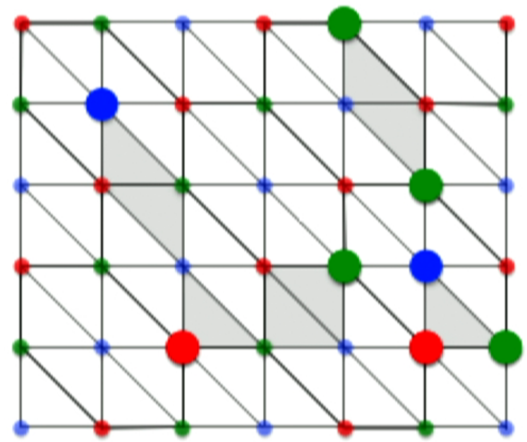
Bombin&Martin-Delgado'06; Kubica et al.'15; Wang et al.'10; Landahl et al.'11

From toric to color code decoder

- **Setup:** error 2D ← syndrome 0D → 1D TC decoder *[BDP12,D14]*

- Two notions: **restricted lattice** $\mathcal{L}_{RG}$ and **restricted syndrome** s_{RG}.

- **Restriction Decoder:**



Bombin et al.'12; Delfosse'14

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  local lift 2D ← 1D ← TC decoder [BDP12,D14] ← 0D

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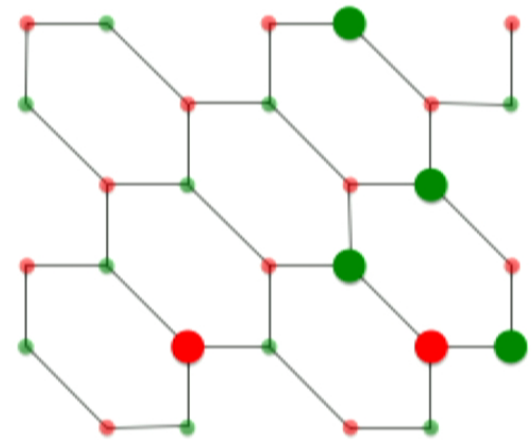
  1. Use toric code decoder for $\mathcal{L}_{RG}$ and s_{RG} to find blue pairings. Repeat for $\mathcal{L}_{RB}$ and s_{RB} to find green pairings.



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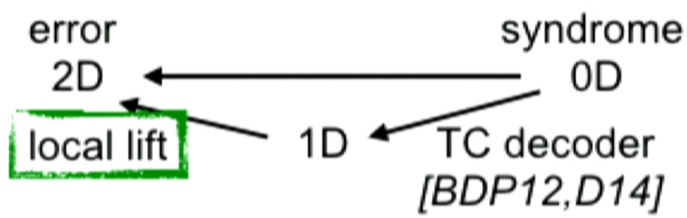

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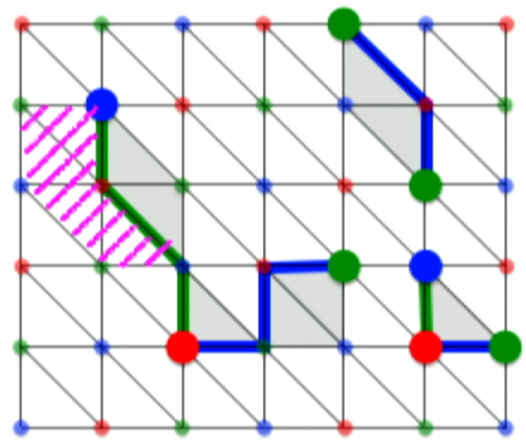
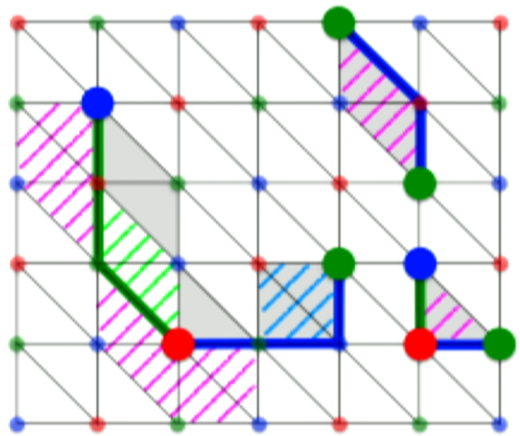
  2. For any R vertex v find neighboring faces $f(v)$, whose boundary locally matches blue/green pairings.



Bombin et al.'12; Delfosse'14

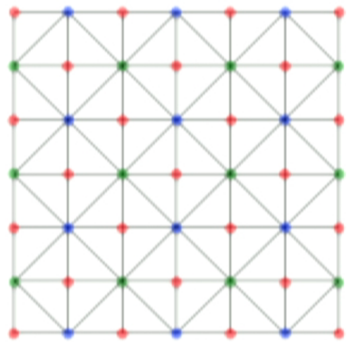
Comments on the Restriction Decoder

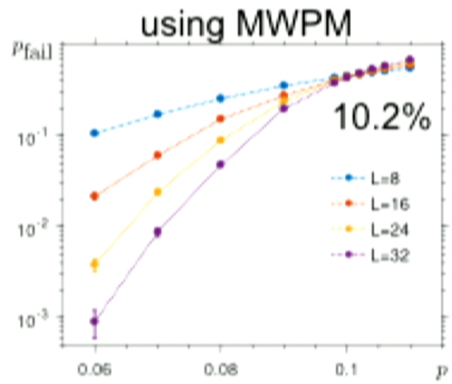
- **Any toric code decoder** can be used as a subroutine!
- Restriction Decoder threshold determined by **toric code threshold**!
- Improvements of Restriction Decoder over decoding by projection *[D14]*:
  — never aborts,
  — **two** (vs. three) restricted lattices,
  — **local** (vs. global) lift procedure,
  — can be generalized to d ≥ 2 dim.
- Various modifications, e.g., no need for restricted lattices. Adaptation to fracton models *[BW19]* or q. pin codes *[VB19]*?



Delfosse'14; Brown&Williamson'19; Vuillot&Breuckmann'19

Numerics





- Dual of 4.8.8. lattice on a torus, phase-flip noise, ideal measurements.
- Color code threshold ~ **10.2%** on a par w/ toric code MWPM ~ 10.3%.

Sarvepalli&Raussendorf'12; Bombin et al.'12; Delfosse'14; Delfosse&Nickerson'17

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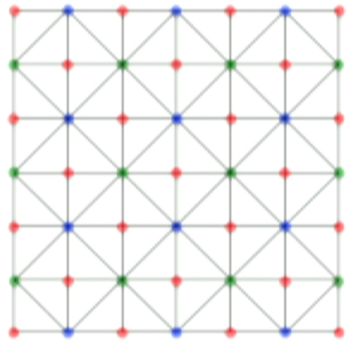

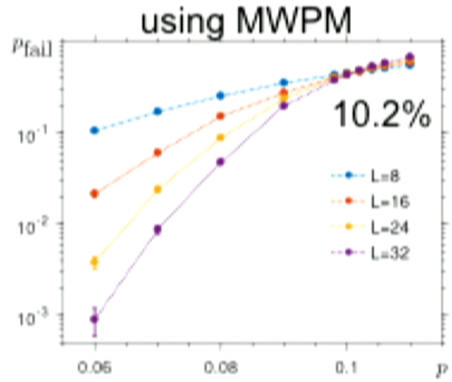

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- For **almost-linear time** decoder, instead of MWPM use UF *[DN17]*.

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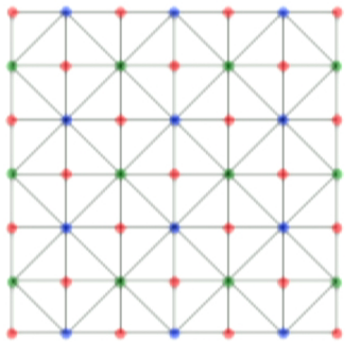

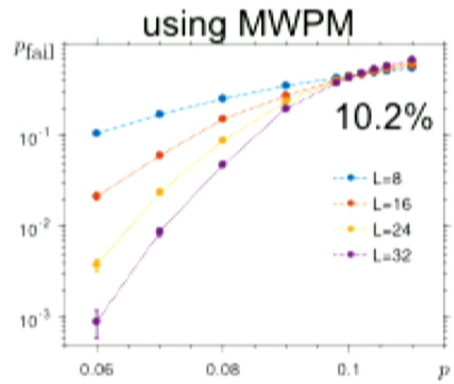

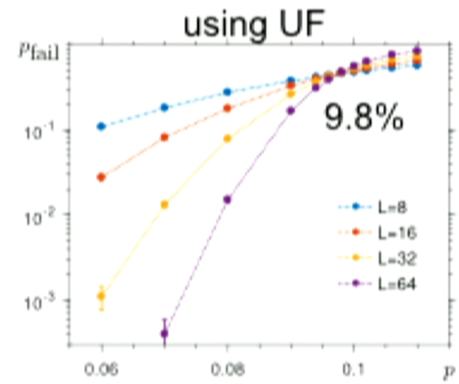

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Going beyond 2D

- **Restriction Decoder** for the d-dim color code on the lattice $\mathcal{L}$**:** toric code decoding on restricted lattices $\mathcal{L}_C$ + local lifting procedure.

- **Theorem 1:** the k^{th} homology groups of the color code lattice $\mathcal{L}$ and the restricted lattice $\mathcal{L}_C$ are isomorphic.

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- **Theorem 1:** the k^{th} homology groups of the color code lattice $\mathcal{L}$ and the restricted lattice $\mathcal{L}_C$ are isomorphic.

- **Lemma:** morphism between color and toric code chain complexes

$$\begin{array}{ccccc} C_{d-k-1}(\mathcal{L}) & \xrightarrow{\partial_{d-k-1,d}} & C_d(\mathcal{L}) & \xrightarrow{\partial_{d,k-1}} & C_{k-1}(\mathcal{L}) \\ \downarrow \pi_C^{(2)} & & \downarrow \pi_C^{(1)} & & \downarrow \pi_C^{(0)} \\ C_{k+1}(\mathcal{L}_C) & \xrightarrow{\partial_{k+1}^C} & C_k(\mathcal{L}_C) & \xrightarrow{\partial_k^C} & C_{k-1}(\mathcal{L}_C) \end{array}$$

Z-stabilizers qubits X-syndrome

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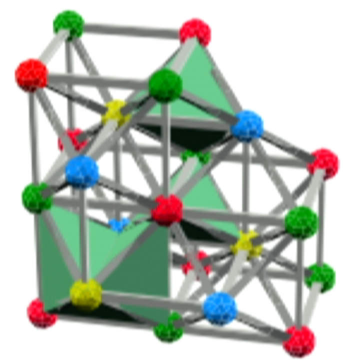
More comments & numerics

- Efficient solution to the color code decoding problem:

d-dim color code → *d*-dim toric code → 0D point-like excitations → MWPM

d-dim color code → *d*-dim toric code → higher-dim excitations → Sweep Decoder *[KP19]*

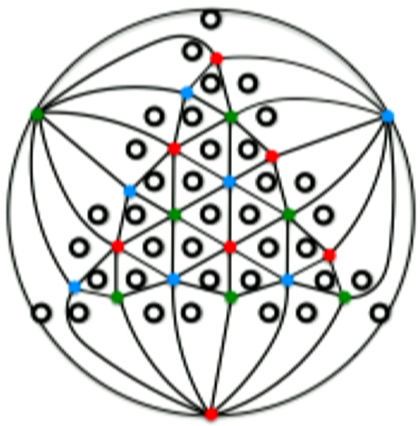
- Numerics for 3D bcc lattice, bit-/phase-flip noise, ideal measurements, two types of excitations: **0D point-like** and **1D loop-like**.



Kubica&Preskill'19; Kubica et al.'18

Realistic scenario

- A realistic setting:
  — a lattice w/ boundaries, e.g., triangular color code,
  — syndrome extracted via circuits w/ noisy components,
  — hardware-imposed limitations: 2D, connectivity,…



Tuckett et al.18; Maskara et al.'19

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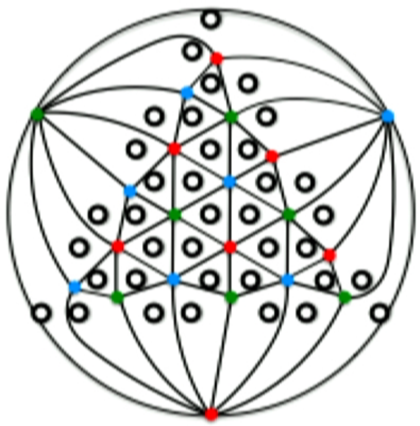

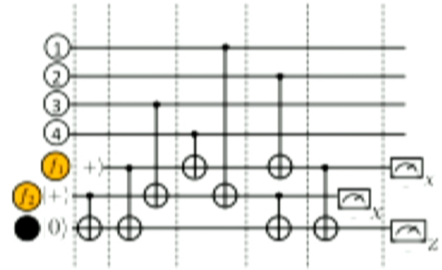

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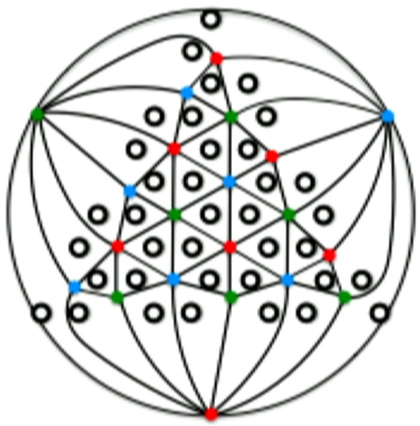
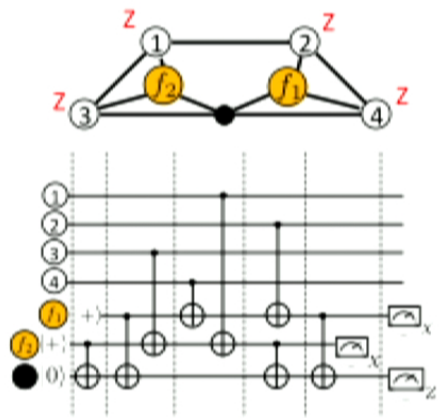




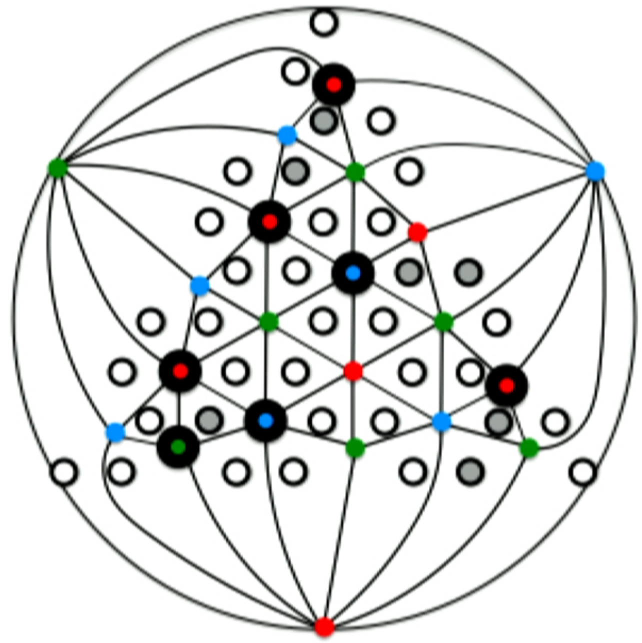
- Restriction Decoder can adapted to circuit-level noise!

- Not so easy for high-threshold decoders based on tensor networks *[TDCBBF18]* or neural networks *[MKJ19]*!

Tuckett et al.18; Maskara et al.'19

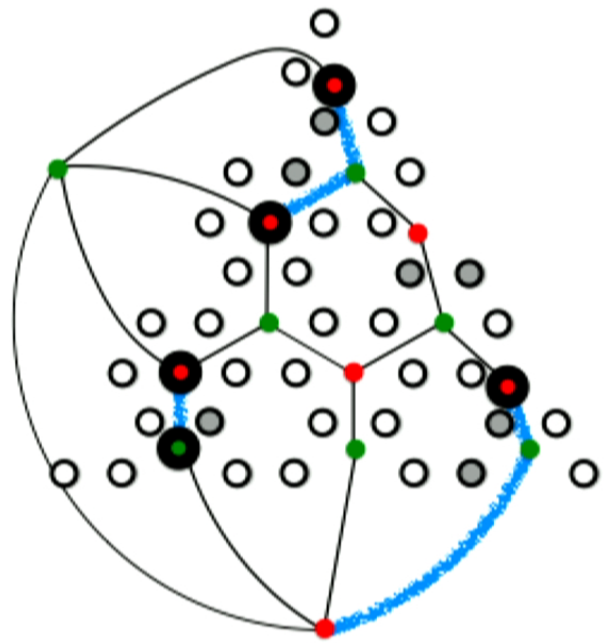
Incorporating boundaries

- A single excitation can be created near boundary!



Incorporating boundaries

- A single excitation can be created near boundary!
- Excitations either paired up or moved to the boundary.
- Naive approach: two restricted lattices & lifting the boundary vertex.



16

Incorporating boundaries

- A single excitation can be created near boundary!
- Excitations either paired up or moved to the boundary.
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- Better way:
  — use three restricted lattices,
  — find connected components,
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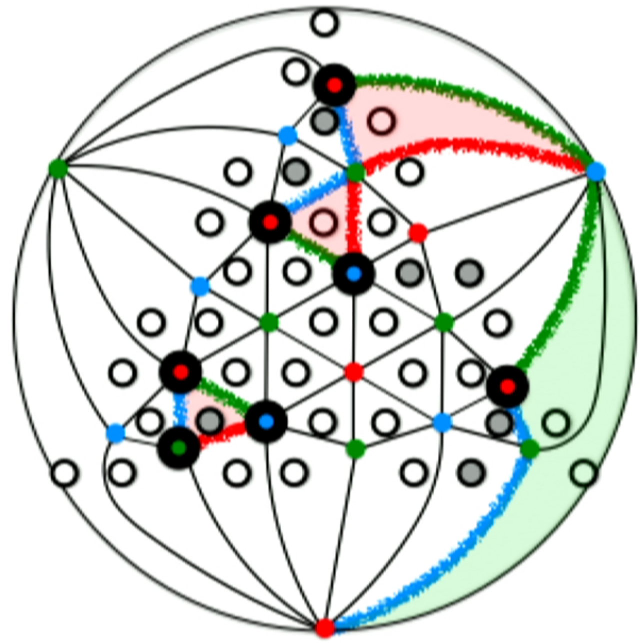

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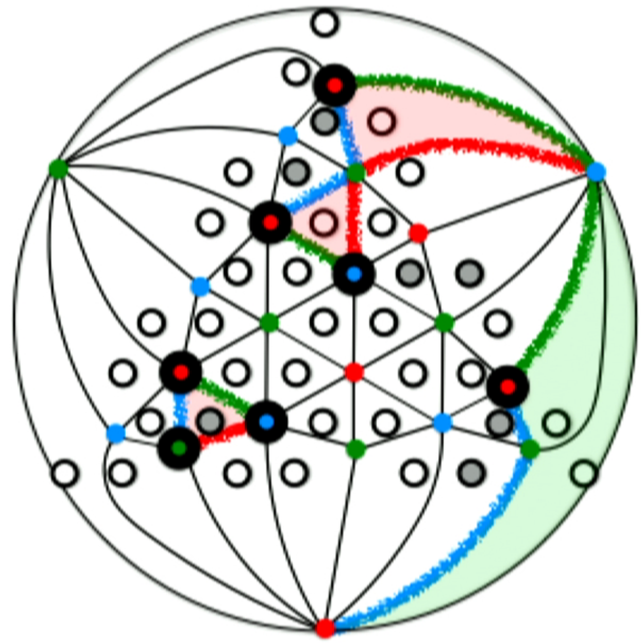


- For phenomenological noise: repeat stabilizer measurements and match excitations in (2+1)D.

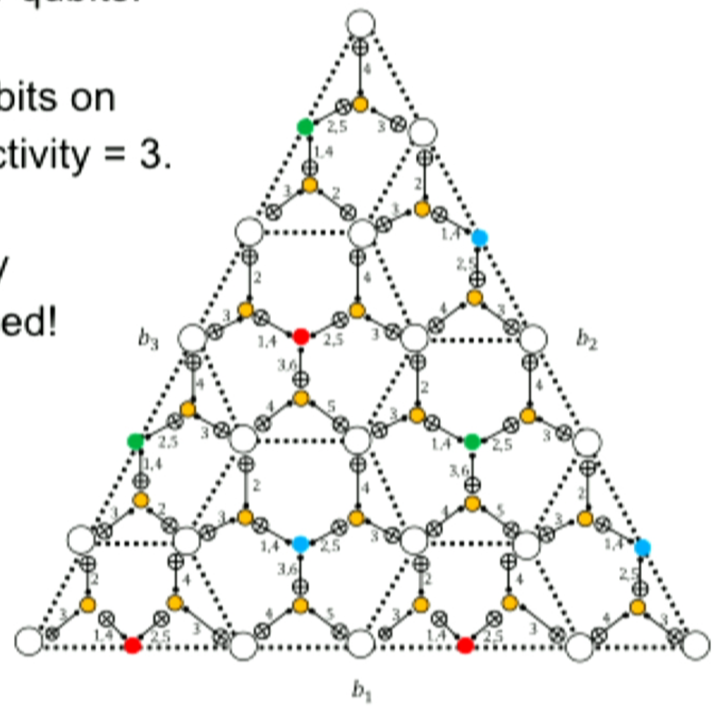
16

Hardware implementation

- Superconducting qubit architecture:
  — 2D layout,
  — CNOTs between nearest-neighbor qubits.

- Data (white) and ancilla (colored) qubits on the hexagonal lattice w/ qubit connectivity = 3.

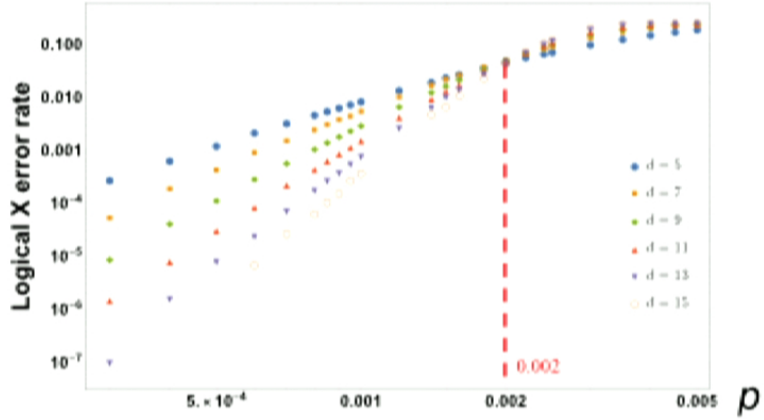
- For smaller connectivity — frequency collisions and cross-talk errors reduced!



Numerics

- Triangular color code & circuit-level depolarizing noise.
- Scaling of the logical error rate in the sub-threshold regime:

$$p_L \propto p^{(d-1)/2}$$

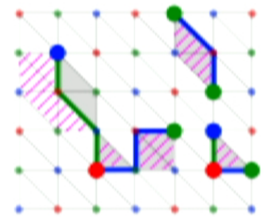


code	connectivity	#qubits	threshold
rotated surface	{4,4}	$2d^2$-1	0.7%
heavy hexagon	{12/5,3}	($5d^2$-2d-1)/2	0.45% *[CZYHC19]*
heavy square	{8/3,4}	$3d^2$-2d	0.3% *[CZYHC19]*
triangular color	{3,3}	($3d^2$-1)/2	0.2%

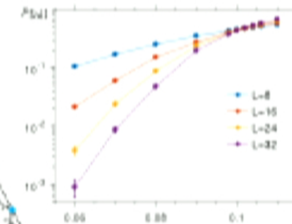
Chamberland et al.'19

Discussion

- Plug & play efficient **Restriction Decoder in $d \geq 2$ dim:** color code decoder = toric code decoding + local lift.



- Restriction Decoder threshold ~ **10.2%**
  — better than efficient decoders for 2D color code,
  — on a par w/ 2D toric code MWPM ~ 10.3%.



- Adaptable to boundaries and circuit-level noise!