Title: How Wilson lines in AdS redundantly compute CFT correlation functions

Speakers: Bartek Czech

Collection: Symmetry, Phases of Matter, and Resources in Quantum Computing

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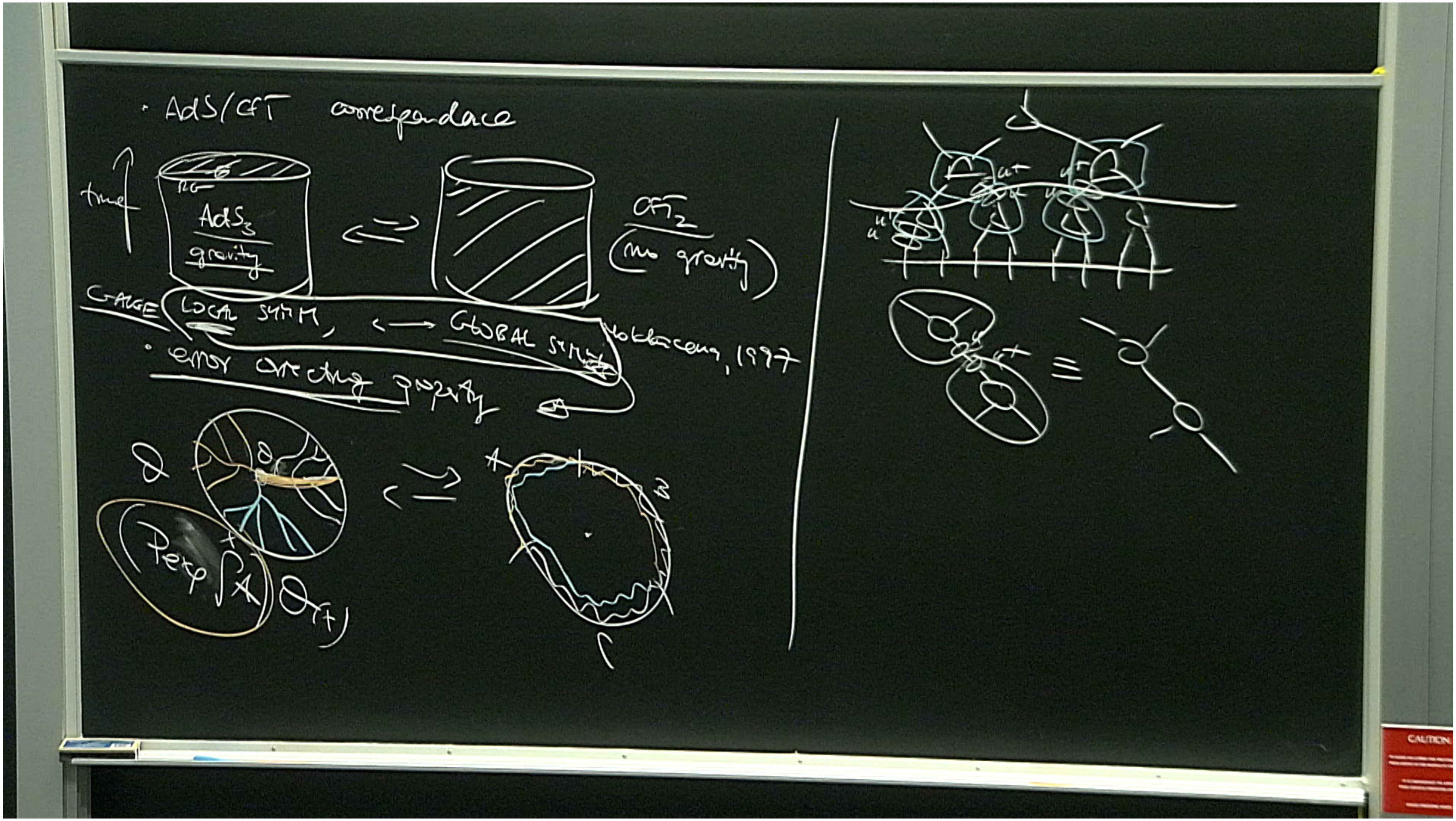
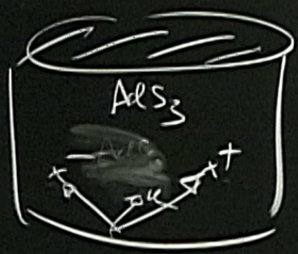
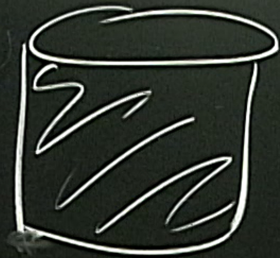
Abstract: In the AdS/CFT correspondence, global symmetries of the CFT are realized as local symmetries of AdS; this feature underlies the error-correcting property of AdS. I will explain how this allows AdS3 to realize multiple redundant computations of any CFT2 correlation function in the form of networks of Wilson lines. The main motivation is to rigorously define the CFT at a cutoff and study it as a model of computational complexity; in that regard we will find agreement with the holographic "Complexity = Volume" proposal. But the framework might be useful more generally.


AdS/CFT correspondance
time
AdS3
gravity
CFT2
(no gravity)
GAUGE
LOCAL SYMM. ⟷ GLOBAL SYMM
error correcting property
P exp ∫A
O(t)





CFT$_2$

$L_0, L_{\pm 1}$ are generators of $so(2,1)$

$\updownarrow$

$\sigma_x, \sigma_y, \sigma_z$

$su(2) \approx so(3)$

$so(2,2) \approx so(2,1) + so(2,1)$

$L_0 = \sigma_z$

$L_{-1} + L_{+1} = \sigma_x$

$L_{+1} - L_{-1} = -i\sigma_y$

$$\boxed{\begin{aligned} A &= L_{-1}\frac{dx^+}{u} + L_0\frac{du}{u} \\ \bar{A} &= L_{+1}\frac{dx^-}{u} - L_0\frac{du}{u} \end{aligned}} \Rightarrow \text{Metric:}$$

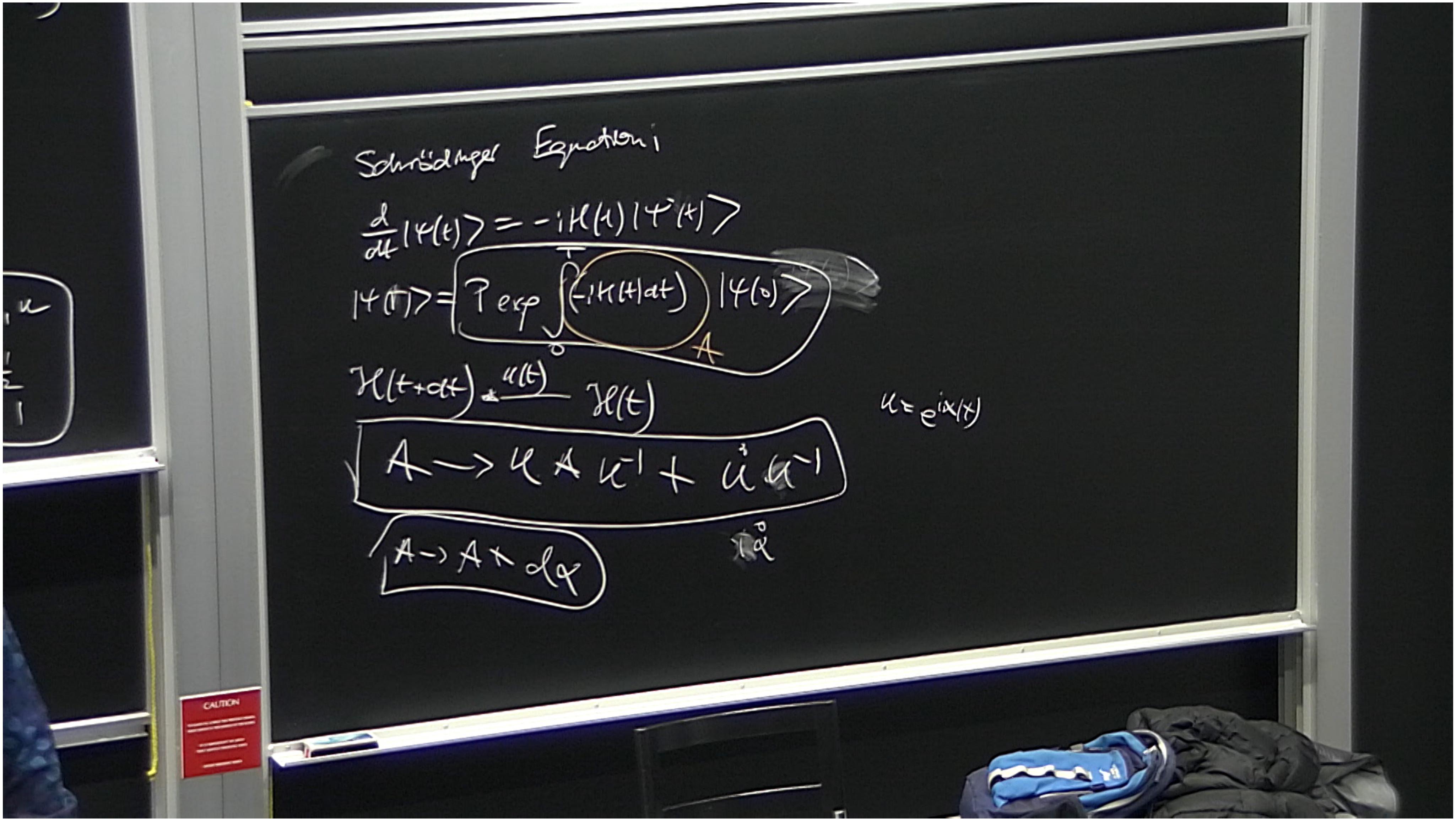
$$\Downarrow$$

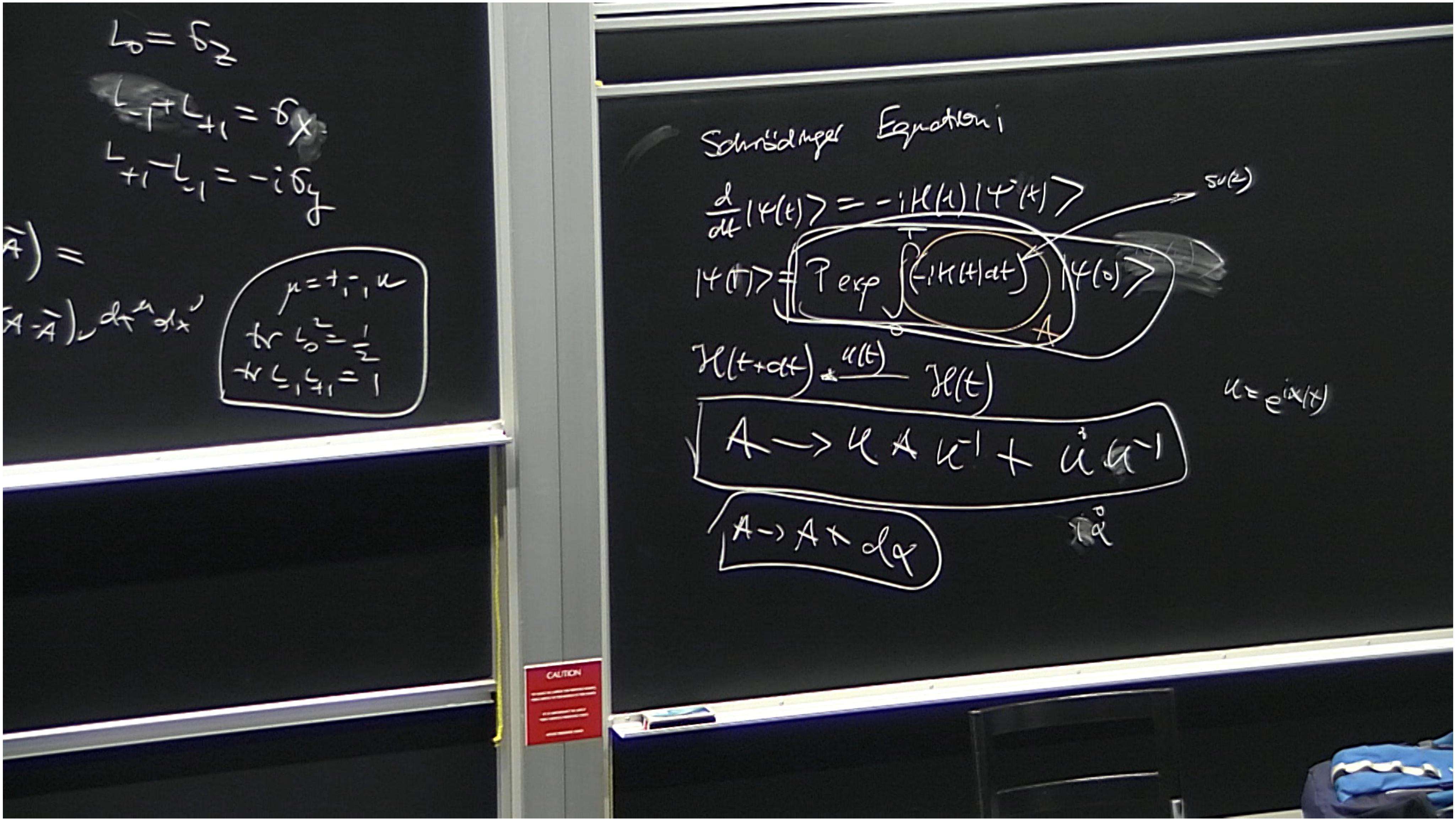
$$\boxed{ds^2 = \frac{dx^+ dx^- + du^2}{u^2}} \quad \text{AdS}_3$$

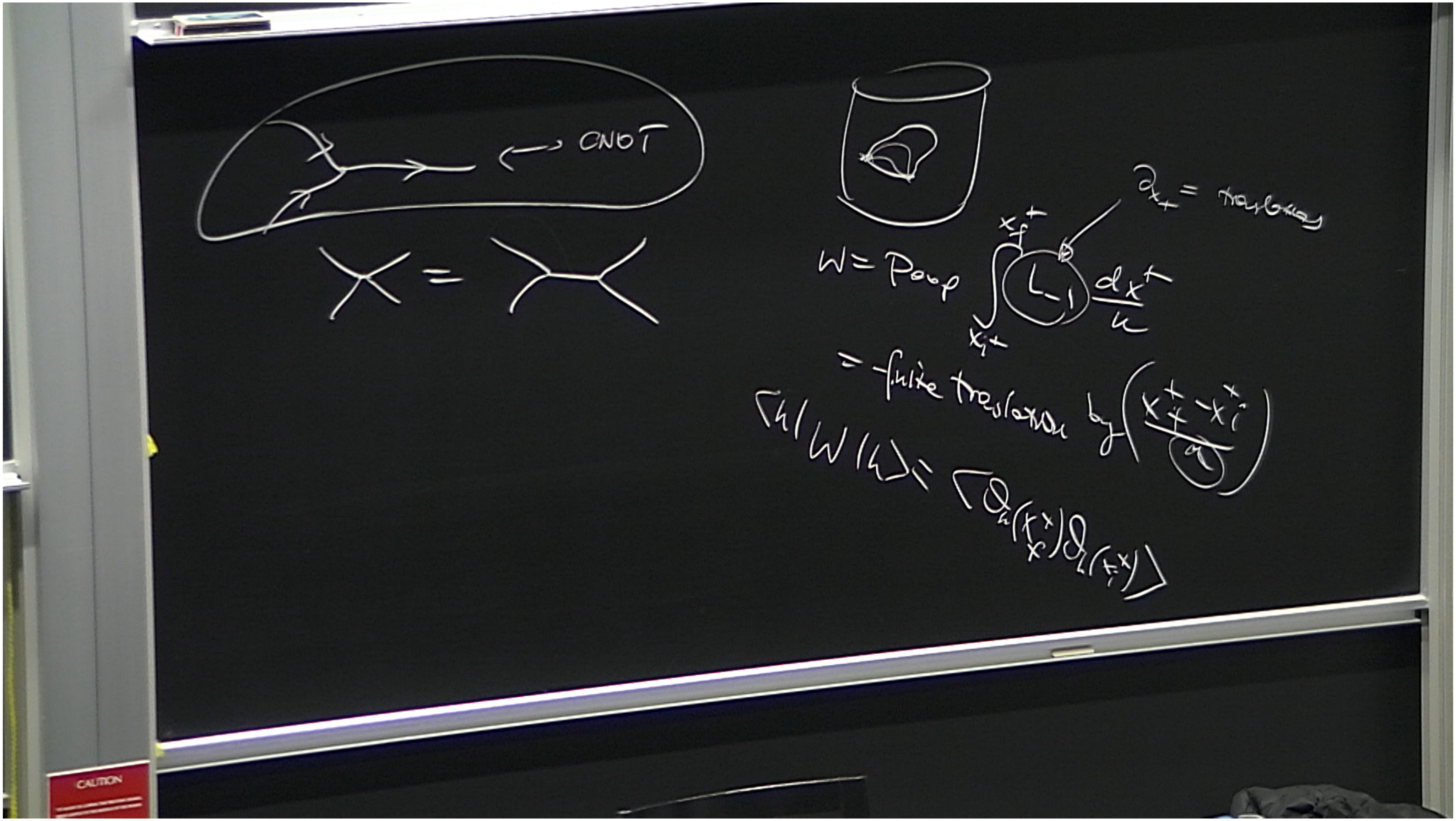
$$ds^2 = \frac{1}{2}\mathrm{tr}\,(A - \bar{A})(A - \bar{A}) =$$

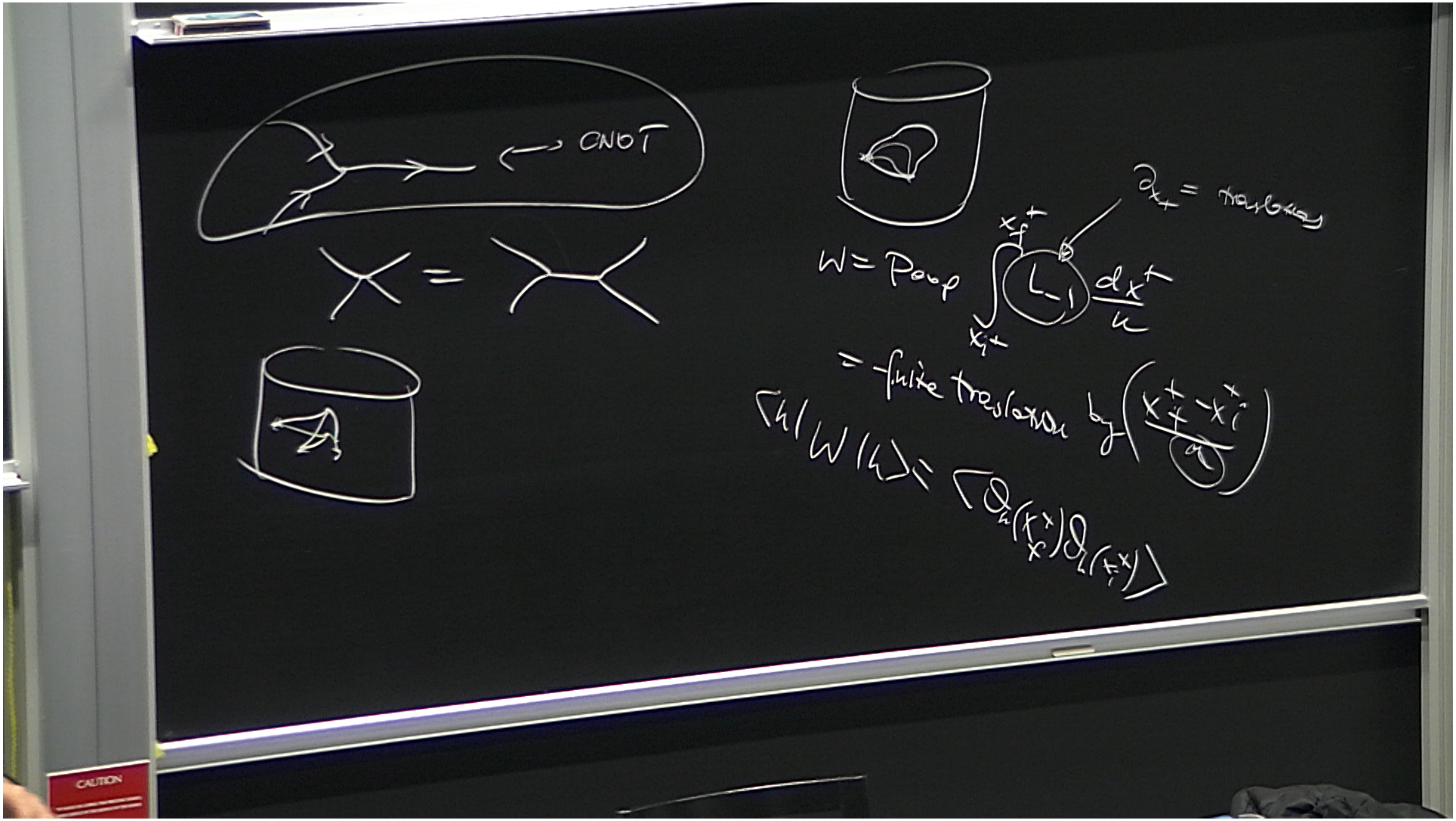
$$= \frac{1}{2}\mathrm{tr}\,(A - \bar{A})_\mu (A - \bar{A})_\nu\, dx^\mu dx^\nu$$

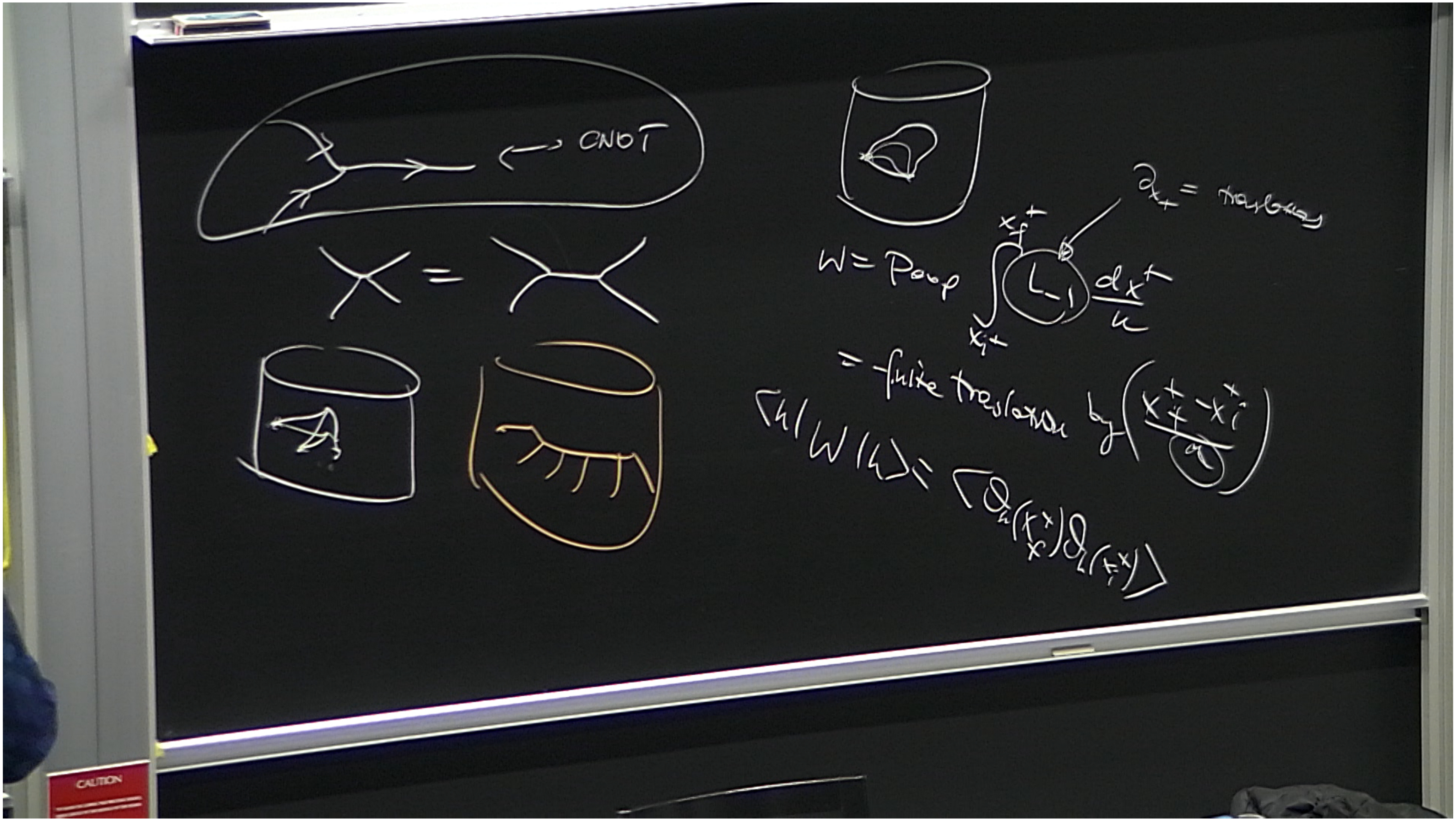
$$\boxed{\begin{aligned} &\mu = +, -, u \\ &\mathrm{tr}\, L_0^2 = \tfrac{1}{2} \\ &\mathrm{tr}\, L_{-1} L_{+1} = 1 \end{aligned}}$$

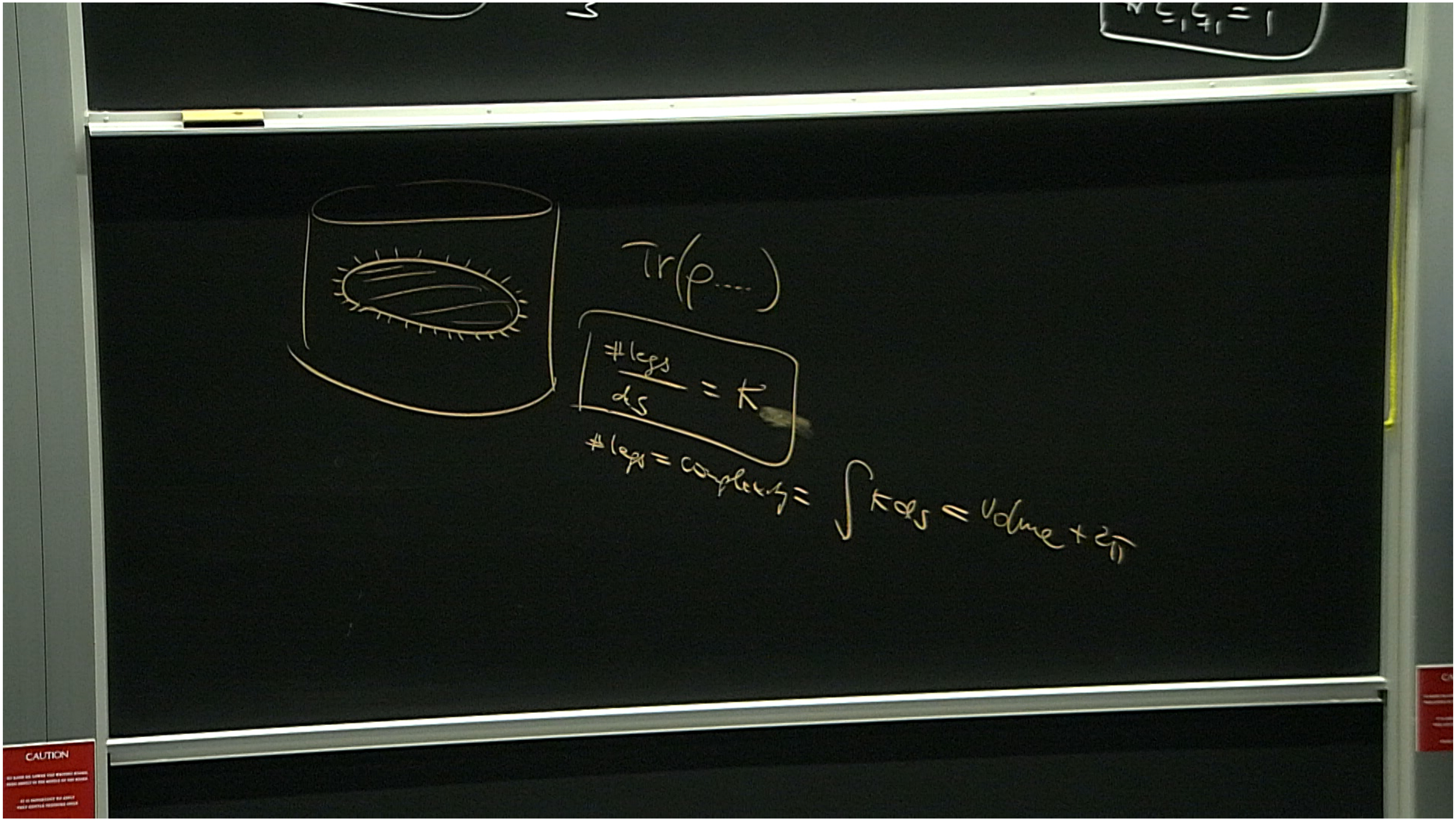

Schrödinger Equation:
$\frac{d}{dt}|\Psi(t)\rangle = -iH(t)|\Psi(t)\rangle$
$|\Psi(T)\rangle = P\exp\int_0^T(-iH(t)dt)\,|\Psi(0)\rangle$
A
$\mathcal{H}(t+dt) \xrightarrow{u(t)} \mathcal{H}(t)$
$u = e^{i\alpha(x)}$
$A \to uAu^{-1} + \dot{u}u^{-1}$
$A \to A + d\alpha$
$\dot{\alpha}$


$L_0 = \sigma_z$
$L_{-1} + L_{+1} = \sigma_x$
$L_{+1} - L_{-1} = -i\sigma_y$
$\mu = +, -, u$
$\mathrm{tr}\, L_0^2 = \frac{1}{2}$
$\mathrm{tr}\, L_{-1} L_{+1} = 1$
Schrödinger Equation:
$\frac{d}{dt}|\psi(t)\rangle = -iH(t)|\psi(t)\rangle$
$SU(2)$
$|\psi(t)\rangle = P\exp\int_0^T(-iH(t)dt)|\psi(0)\rangle$
$H(t+dt) \xrightarrow{u(t)} H(t)$
$u = e^{i\alpha(x)}$
$A \to uAu^{-1} + \dot{u}u^{-1}$
$A \to A + d\alpha$


CNOT
∂_{x^+} = translation
$W = \mathrm{Pexp} \int_{x_i^+}^{x_f^+} L_{-1} \frac{dx^+}{a}$
= finite translation by $\left(\frac{x_f^+ - x_i^+}{a}\right)$
$\langle h | W | h \rangle = \langle \mathcal{O}_h(t_f^x) \mathcal{O}_h(t_i^x) \rangle$


CNOT
∂_{x+} = translation
W = Pexp ∫_{x_i^+}^{x_f^+} L_{-1} dx^+/u
= finite translation by (x_f^+ − x_i^+)/u
⟨h| W |h⟩ = ⟨O_h(x_f^+) O_h(x_i^+)⟩


CNOT
W = Pexp
= finite translation by


Tr(ρ....)
#legs / ds = K
#legs = complexity = ∫ K ds = Volume + 2π
CAUTION