Title: Entanglement and extended conformal field theory (or how to get a tensor network from a CFT path integral)Â

Speakers: Gabriel Wong

Collection: Symmetry, Phases of Matter, and Resources in Quantum Computing

Date: November 29, 2019 - 9:30 AM

URL: http://pirsa.org/19110134

Abstract: In a continuum field theory the Hilbert space does not factorize into local tensor products. How then can we define entanglement and the basic protocols of quantum information theory? In this talk we will show how the factorization problem can be solved in a class of 2D conformal field theories by directly appealing to the fusion rules. The solution suggests a tensor network description of a CFT path integral using the OPE data.

Entanglement and Extended Conformal field theory

(or how to get a tensor network from a CFT path integral)

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Based on ongoing work with Janet Hung and hep-th 1811.10785
with William Donnelly

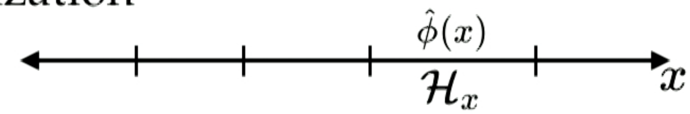
What is a QFT?

$$Z[J] = \int D[\phi] e^{-S(\phi,J)}$$

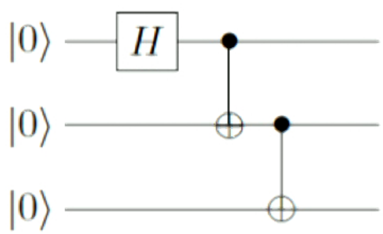
Correlators /Scattering Amplitudes
Topological invariants
Replica trick computation of EE

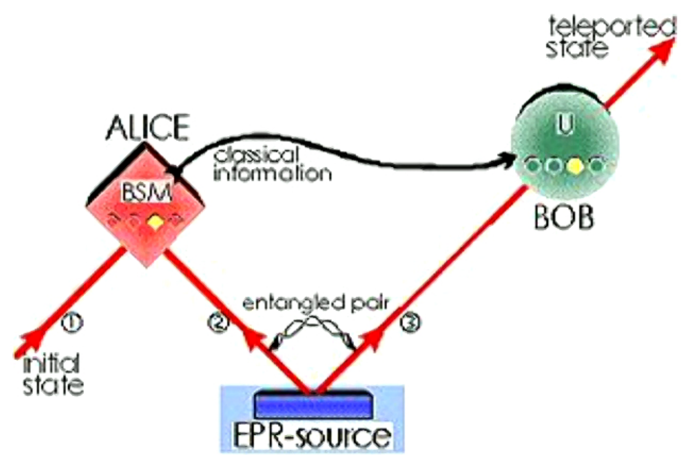
~~Local Hilbert Space factorization~~

$$\mathcal{H} = \otimes_x \mathcal{H}_x$$



How do we define quantum circuits in QFT ?

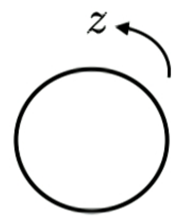




Locality is Not canonical

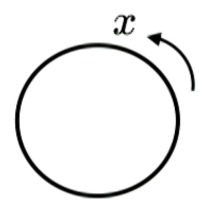
$$\mathcal{H} = \{\text{Anti-sym} \quad \Psi(z_1, \cdots z_N)\}$$

N non-relativisitic fermions on a spatial circle



U(N) Yang Mills on a spatial circle .

$$U = \mathrm{P}\exp(i\int A_x\, dx)$$

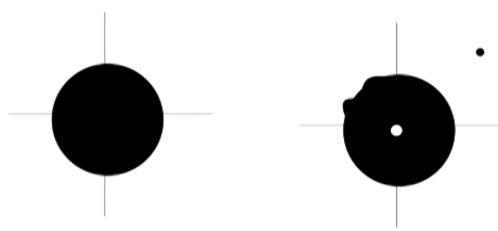


A free fermion Hilbert space in N=4 SYM

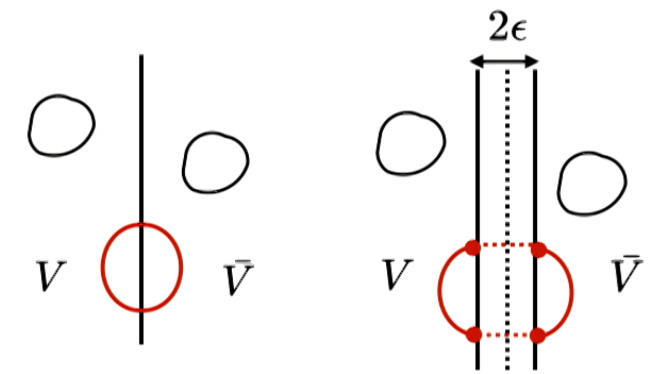
Ten- Dimensional Geometry for the IIB
(Lin, Lunin, Maldacena)

$$\begin{aligned}
ds^2 &= -h^{-2}(dt + V_i dx^i)^2 + h^2(dy^2 + dx^i dx^i) + ye^G d\Omega_3^2 + ye^{-G} d\tilde{\Omega}_3^2 \\
h^{-2} &= 2y\cosh G, \\
y\partial_y V_i &= \epsilon_{ij}\partial_j z, \qquad y(\partial_i V_j - \partial_j V_i) = \epsilon_{ij}\partial_y z \\
z &= \frac{1}{2}\tanh G \\
F &= dB_t \wedge (dt + V) + B_t dV + d\hat{B}\ , \\
\tilde{F} &= d\tilde{B}_t \wedge (dt + V) + \tilde{B}_t dV + d\hat{\tilde{B}} \\
B_t &= -\frac{1}{4}y^2 e^{2G}, \qquad\qquad \tilde{B}_t = -\frac{1}{4}y^2 e^{-2G} \\
d\hat{B} &= -\frac{1}{4}y^3 *_3 d\left(\frac{z + \frac{1}{2}}{y^2}\right)\ , \qquad d\hat{\tilde{B}} = -\frac{1}{4}y^3 *_3 d\left(\frac{z - \frac{1}{2}}{y^2}\right)
\end{aligned}$$

1/2 BPS sector of N=4
Free Fermions in a Magnetic field in the LLL



Extended Hilbert space for gauge theories



$$\mathcal{H}_{\text{physical}} \subset \mathcal{H}_V \otimes \mathcal{H}_{\bar{V}}$$

Contains edge modes transforming under boundary symmetry group $G \otimes \bar{G}$

Gauss law $\quad (J_n \otimes 1 + 1 \otimes \bar{J}_{-n})|\psi\rangle = 0 \qquad |\psi\rangle \in \mathcal{H}_V \otimes \mathcal{H}_{\bar{V}}$

(Can be viewed as an equivalence relation)

Entangling product
(Freidel, Donnelly)

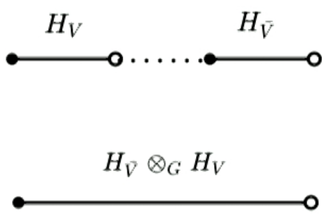
$$\mathcal{H}_{\text{physical}} = \mathcal{H}_V \otimes_G \mathcal{H}_{\bar{V}}$$

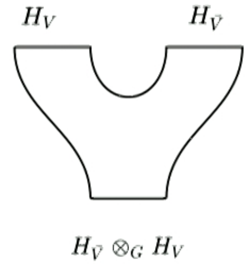
Tensor product of modules over G

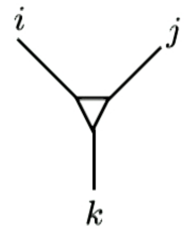
Extended Hilbert space and extended QFT

- In 1+1 D, Hilbert space extension is part of the data of the extended topological quantum field theory (Donnelly-Wong 2018)

- Key insight: View the entangling product as a cobordism

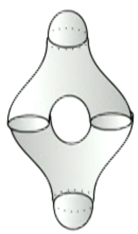


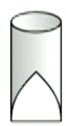


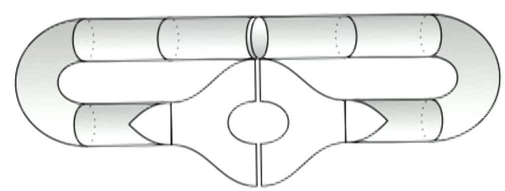


Extended TQFT (Atiyah, Segal, Freed, Baez,...)

Cut path integral along surfaces of increasing codimension







2D Closed TQFT(Frobenius Algebra)

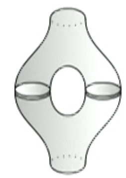
A Closed TQFT is a rule Z assigning

$Z($  $) = \mathcal{H}_{S_1}$

$Z($  $) \in H_{S^1}$

$Z($  $) =$ **linear map** $\mathcal{H}_{S^1} \otimes \mathcal{H}_{S^1} \longrightarrow \mathcal{H}_{S^1}$

Gluing Cobordisms =
Composing linear maps



$$\mathcal{H}_{S^1} \longrightarrow \mathcal{H}_{S^1} \otimes \mathcal{H}_{S^1} \longrightarrow \mathcal{H}_{S^1}$$

2D Closed TQFT(Frobenius Algebra)

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Gluing Cobordisms =
Composing linear maps

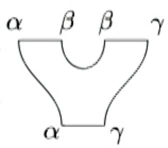
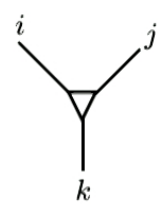
$$\mathcal{H}_{S^1} \longrightarrow \mathcal{H}_{S^1} \otimes \mathcal{H}_{S^1} \longrightarrow \mathcal{H}_{S^1}$$

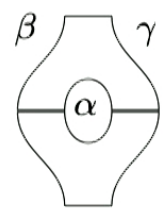
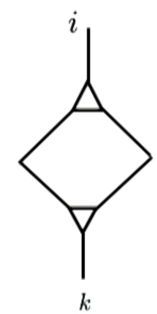
Extension to open-closed TQFT (non-commutative Frobenius Alg.)

$Z({}^{\alpha}\bullet\!\!-\!\!-\!\!\bullet^{\beta}) = \mathcal{H}_{\alpha\beta}$

$Z($  $) = \mathcal{H}_{S_1} \longrightarrow \mathcal{H}_{\alpha\alpha}$

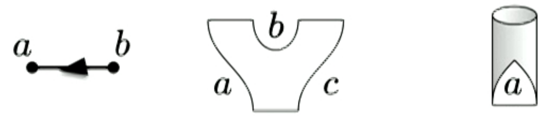
Boundary labels must match !

$Z($  $) = \mathcal{H}_{\alpha\beta} \otimes \mathcal{H}_{\beta\gamma} \longrightarrow \mathcal{H}_{\alpha\gamma} \sim$ 

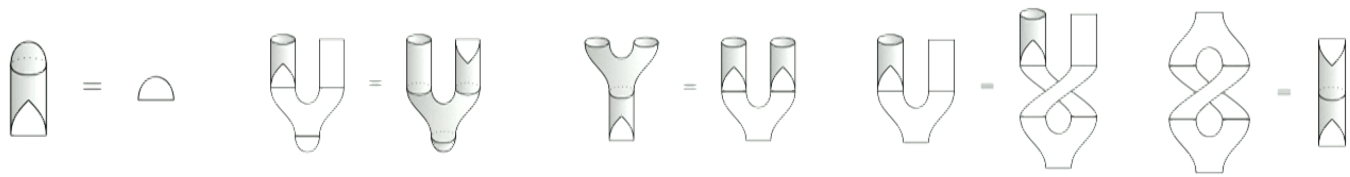
 $\sim$ 

Extensions are constrained by sewing relations

Extension to open strings:



Sewing relations:

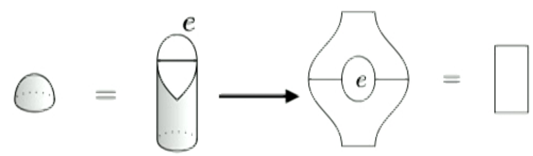


Entanglement brane extension

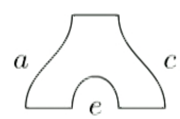
Factorization:

$H_{\bar{V}} \otimes_G H_V$

E brane axiom



e generically a sum over BC:

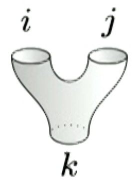


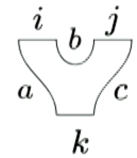
$: \mathcal{H}_{ac} \to \bigoplus_b \mathcal{H}_{ab} \otimes \mathcal{H}_{bc}$

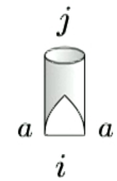
edge modes

Extended Conformal field theory

In a open-closed CFT: Elementary Cobordisms ~ OPE coefficient !

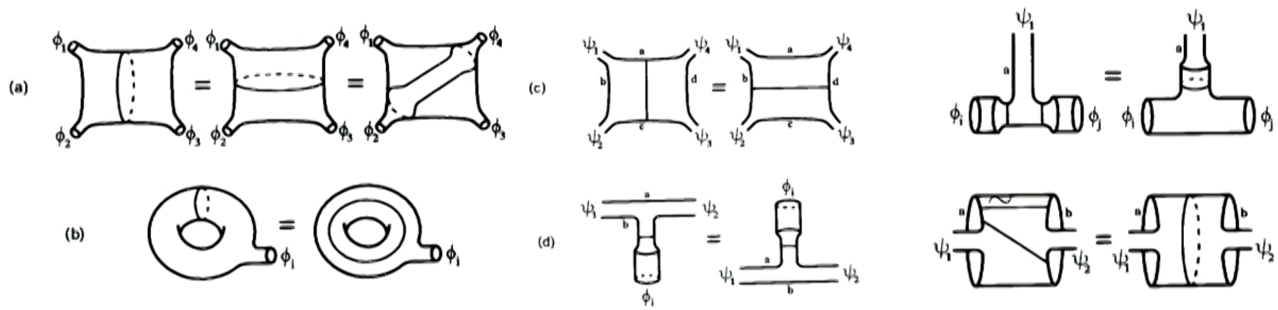
$C_{ijk} \sim$ 

$C^{abc}_{ijk} \sim$ 

$C^{a}_{ij} \sim$ 

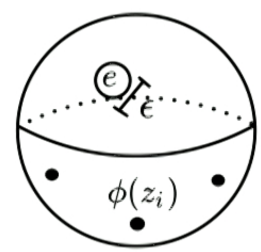
$a, b =$ conformally invariant B.C.

Sewing axioms (Lewellen 1992)



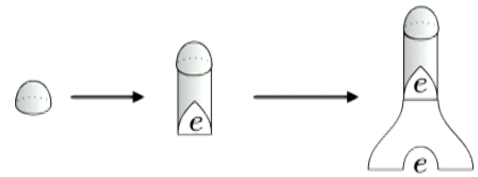
Entanglement brane axiom

$$\lim_{\epsilon\to 0}\langle \phi(z_1)\dots\phi(z_n)\rangle_e = \langle \phi(z_1)\dots\phi(z_n)\rangle_{\text{bulk}}$$

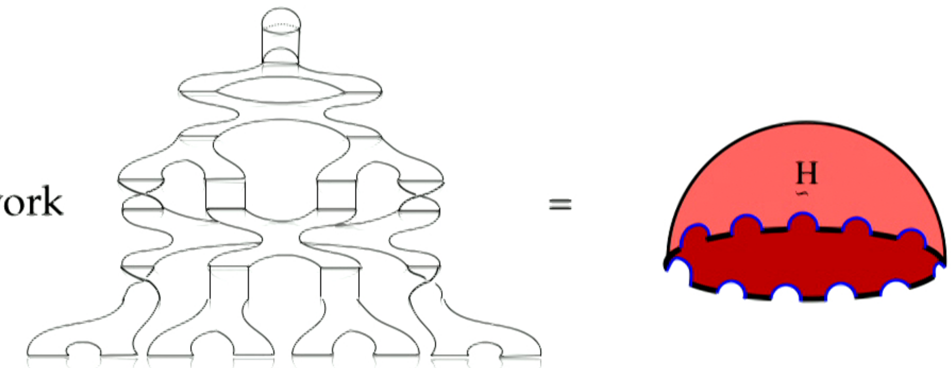


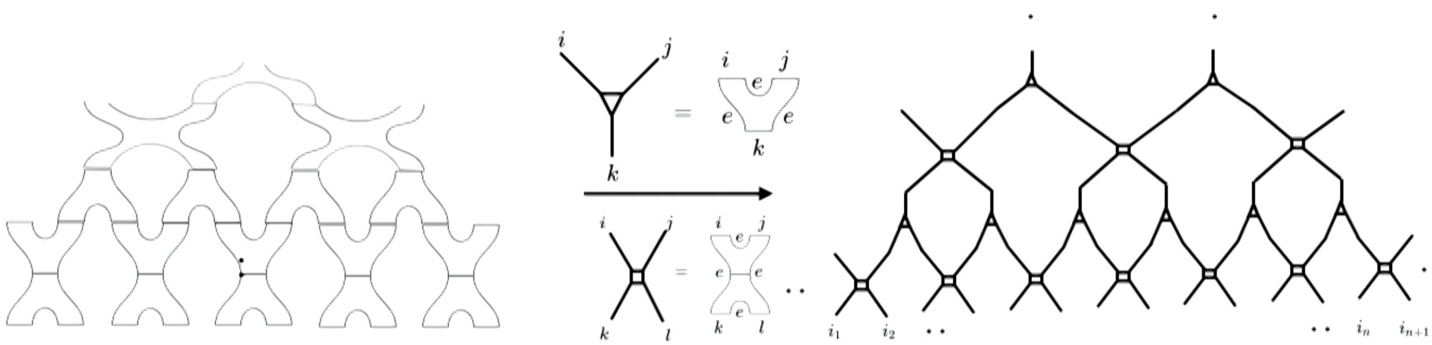
Tensor Networks from OPE's

Factorization the vacuum state



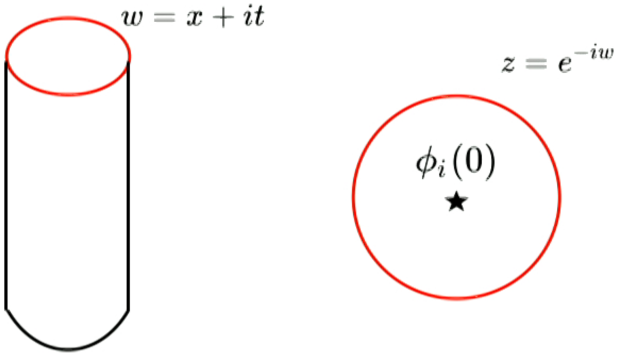
Cobordism MERA network





- Hilbert spaces and basic CFT cobordisms
- Entanglement Brane for a free boson
- CFT factorization from co-product of symmetry algebras
- Comparison with the unruh effect and Bogoliubov transformation
- Conclusion and applications

Local Hilbert space for a closed CFT



Primaries/Highest Weight

$|i\rangle = \phi_i(0)|0>_{\text{disk}}$

Representation of Kacs-Moody

$$J_n^a = \oint dz\, z^n\, J^a(z)$$

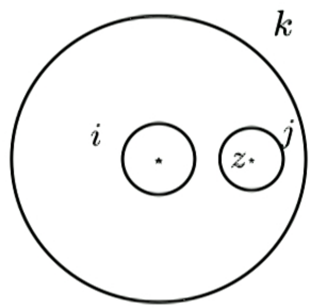
$$J^a_{-n_1} \cdots J^a_{-n_k}|i\rangle$$

$$[J_n^a, J_m^b] = i f_{abc} J^c_{m+n} + k\, m\, \delta^{ab} \delta_{m+n,0}$$

(Also an anti-chiral copy: $\bar{J}(\bar{z})$)

Pair of pants splits local Hilbert spaces

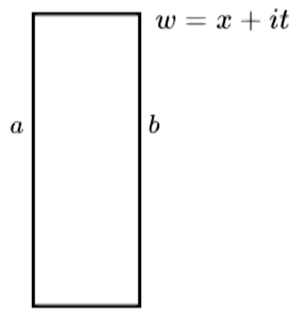
Defined by Bulk OPE's ~three point functions



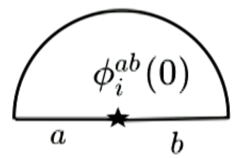
$$V_{ij}^k(z) : H_i \otimes H_j \to H_k$$

$$V_{ij}^k(z) = \frac{C_{ijk}}{z^{\Delta_i + \Delta_j - \Delta_k}}$$

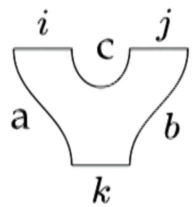
Extension to Open-closed CFT

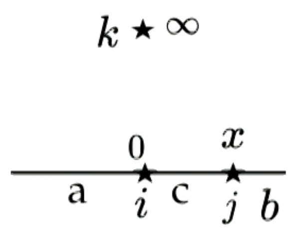


$$z = e^{-iw}$$

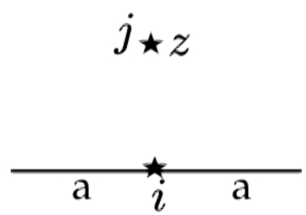


$$|i >_{ab} = \phi_i^{ab}(0)|0 >_{\text{half disk}}$$



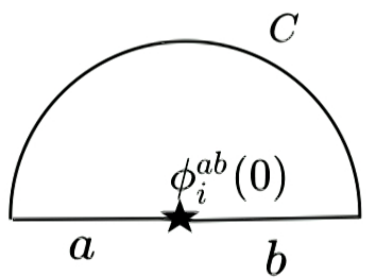


$$= \frac{C_{ijk}^{abc}}{x^{\Delta_i + \Delta_j - \Delta_k}}$$



$$= (2\mathrm{Im} z)^{\Delta_i - \Delta_j - \bar{\Delta}_j} C_{ji}^{a}$$

Conformal BC,unfolding and open string states

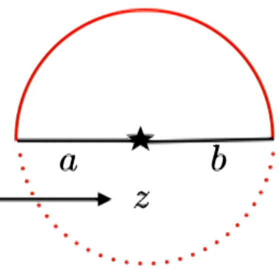


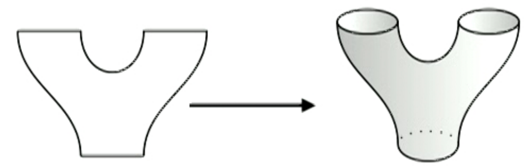
a,b = conformally invariant boundary conditions

$$T(z) = \bar{T}(\bar{z}) \text{ for real } z$$
$$J(z) = \pm \bar{J}(\bar{z}) \text{ for real } z$$

Analytic cont. into LHP

$$J(z) = \bar{J}(\bar{z}^*)$$





Preserves half the symmetry generators

$$J_n = \int_C dz\, z^n J(z) \mp \int_C d\bar{z}\, \bar{z}^n \bar{J}(\bar{z}) = \oint dz\, z^n J(z)$$

Standing waves

$$z = e^{-iw}$$

$$J_{-n_1} \cdots J_{-n_k} |i>_{ab}$$

States unfold into a representation of one chiral algebra

- Hilbert spaces and basic CFT cobordisms
- Entanglement Brane and vacuum factorization
- CFT factorization from co-product of symmetry algebras
- Comparison with the unruh effect and Bogoliubov transformation
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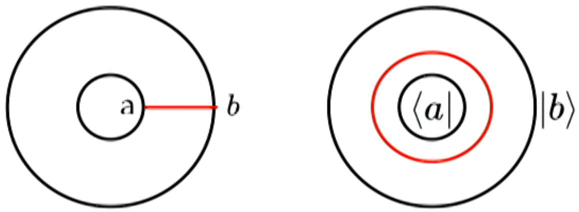
Entanglement Brane boundary condition for a CFT

"Closed string" description of conformally invariant boundary condition.



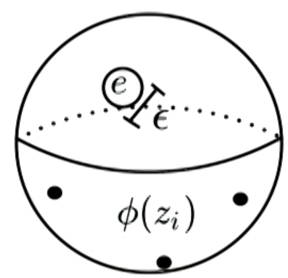
$$T(z) = \bar{T}(\bar{z})$$

$$(L_n^C - \bar{L}_{-n}^C)|a\rangle = 0$$

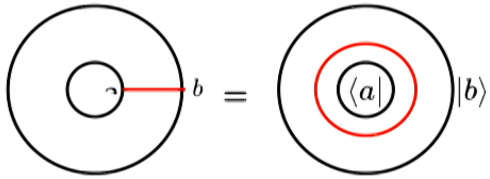
Solutions: Ishibashi states $\quad |h\rangle\rangle = \sum_N |h, N\rangle \otimes |\bar{h}, \bar{N}\rangle \qquad h = \text{Bulk Primary}$

Proposal for the Entanglement brane $\qquad |e\rangle = |0\rangle\rangle$

$$\lim_{\epsilon \to 0} \langle \phi(z_1) \ldots \phi(z_n) \rangle_e = \langle \phi(z_1) \ldots \phi(z_n) \rangle_{\text{bulk}}$$



E Brane in the open string channel



Cardy condition -> local B.C.

$$\langle a|e^{-TH_{\text{closed}}}|b\rangle = \text{tr}_{ab}\, e^{-\frac{1}{T}H_{open}}$$

e.g. for a RCFT

$$|0\rangle\rangle = \sum_j \frac{S_{0a}}{\sqrt{S_{00}}}|a\rangle \longrightarrow$$

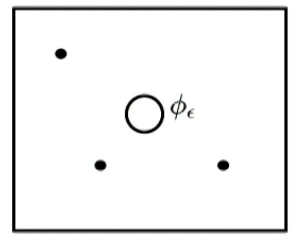
Sum over complete set of local boundary conditions !

Dirichlet E brane for the compact boson

$$\phi \sim \phi + 2\pi R$$

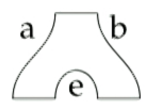
$$|e\rangle = |0\rangle\rangle \sim \int_0^{2\pi R} d\phi_\epsilon ||\phi_\epsilon\rangle\rangle$$

Restore shift invariance



Sum over superselection sectors

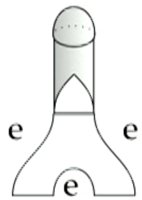
$$: \mathcal{H}_{S^1} \to \bigoplus_{\phi_\epsilon} \mathcal{H}_{\phi_\epsilon \phi_\epsilon}$$

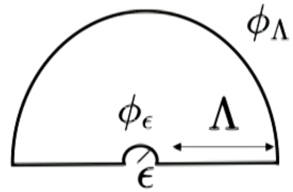


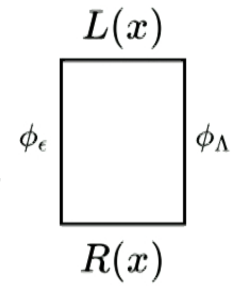
$$: \mathcal{H}_{ac} \to \bigoplus_{\phi_\epsilon} \mathcal{H}_{a\phi_\epsilon} \otimes \mathcal{H}_{\phi_\epsilon c}$$

Vacuum factorization and edge mode EE

Factorizing the close string vacuum:

$|\Psi\rangle_D =$ 

$\longrightarrow \int d\phi_\epsilon \int d\phi_\Lambda$ 

$\xrightarrow{w = \log z}$ 

Entanglement edge modes

$$\phi(x,t) = \phi_0 + \phi_1 x + \cdots \qquad \phi_0 \in [0, 2\pi R] \qquad \phi_1 \in \mathbb{R}$$

$$\phi_\epsilon = \phi_0 + \phi_1 \log \epsilon$$
$$\phi_\Lambda = \phi_0 + \phi_1 \log \Lambda$$

Factorized Wavfunctional

$$|\Psi\rangle_D = \int d\phi_\epsilon d\phi_\Lambda \int D[R(x)] D[L(x)] e^{-S_{\text{on-shell}}} |\phi_\Lambda, \phi_\epsilon, L(x)\rangle \otimes |\phi_\epsilon, \phi_\Lambda, R(x)\rangle$$

$$Z(\beta) = \mathrm{tr}_V \rho_V^{\frac{\beta}{2\pi}} = l \int_0^{2\pi R} d\phi_0 \int_{-\infty}^{\infty} d\phi_1 e^{-\beta\, l \phi_1^2} Z_{\text{osc}} \qquad l = \log \frac{\Lambda}{\epsilon}$$

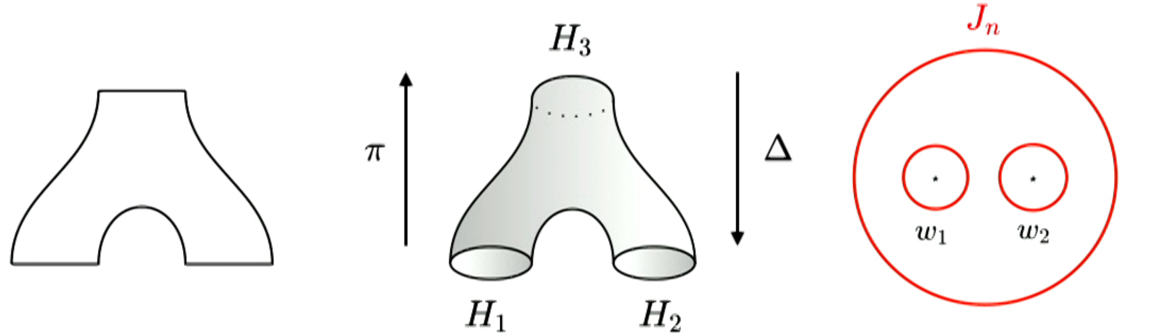
Entanglement Entropy

$$S = -\mathrm{tr} \rho_V \log \rho_V = \frac{1}{3} \log \frac{\Lambda}{\epsilon} + \boxed{\frac{1}{2} \log \pi R^2} + \cdots \longleftarrow$$

Michel-Srednicki
"Edge mode EE"

Tensor product factorization from CFT fusion

Gaberdiel, Moore-Seiberg



$$\left.\begin{array}{lll} \text{Intertwiner} & & \pi : H_1 \otimes H_2 \to H_3 \\ \text{Co product} & & \Delta : \mathcal{A}(H_3) \to \mathcal{A}(H_1) \otimes \mathcal{A}(H_2) \end{array}\right\} \quad J_n \pi = \pi \Delta(J_n)$$

This defines factorization of all states in H_3 from $|0\rangle_3 = \pi|\Omega_{12}\rangle$

$$J_n|0\rangle_3 = 0 \to \Delta(J_n)|\Omega_{12}\rangle = 0 \quad n > 0$$

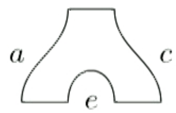
Highest weight
(e.g. vacuum)

"Thermofield
Double"

$$J_{-n}|0\rangle_3 = 0 \to \Delta(J_{-n})|\Omega_{12}\rangle$$

Summary

We propose an extension of 2D CFT incorporating the E brane BC ~ vacuum Ishibashi state.



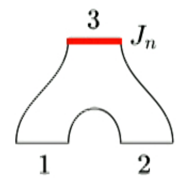
$$: \mathcal{H}_{ac} \to \bigoplus_{b} \mathcal{H}_{ab} \otimes \mathcal{H}_{bc}$$

edge modes

Factorization of CFT Hilbert space via a co- product formula (Gaberdiel/Moore Seiberg)

$$J_n|0\rangle_3 = \Delta(J_n)|\Omega\rangle_{12} = 0$$

$$J_{-n}|0\rangle_3 = \Delta(J_{-n})|\Omega\rangle_{12}$$



$|\Omega\rangle_{12} =$

Applications to Tensor Networks?