Title: Floquet quantum criticality

Speakers: Will Berdanier

Series: Condensed Matter

Date: November 21, 2019 - 3:30 PM

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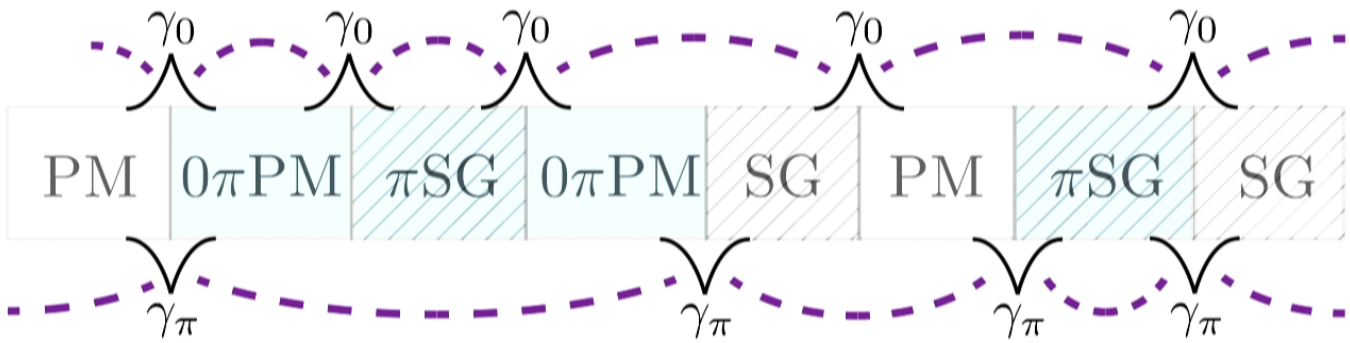
Abstract: It has recently been shown that quenched randomness, via the phenomenon of many-body localization, can stabilize dynamical phases of matter in periodically driven (Floquet) systems, with one example being discrete time crystals. This raises the question: what is the nature of the transitions between these Floquet many-body-localized phases, and how do they differ from equilibrium? We argue that such transitions are generically controlled by infinite randomness fixed points. By introducing a real-space renormalization group procedure for Floquet systems, asymptotically exact in the strong-disorder limit, we characterize the criticality of the periodically driven interacting quantum Ising model, finding forms of (multi-)criticality novel to the Floquet setting. We validate our analysis via numerical simulations of free-fermion models sufficient to capture the critical physics.

Floquet quantum criticality



William Berdanier, UC Berkeley

Perimeter Institute CMT Seminar, 21 November 2019

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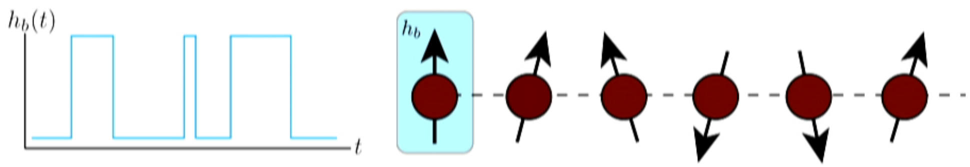
WB, MK, SAP and RV, *PNAS* 115 (38) 9491-9496 (2018)
WB, MK, SAP and RV, *PRB* 98,174203 (2018)

First…some other projects

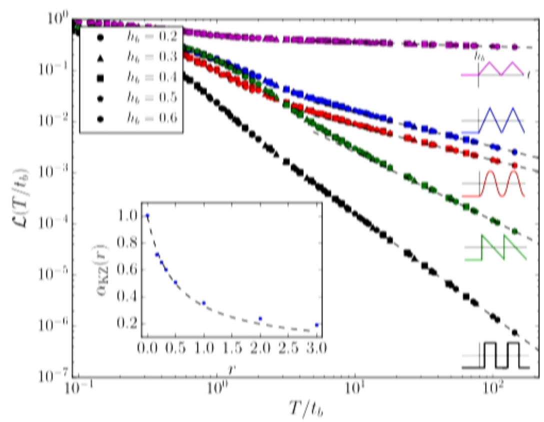
Theme: Universal dynamics in non-equilibrium quantum systems

Boundary driving & CFTs



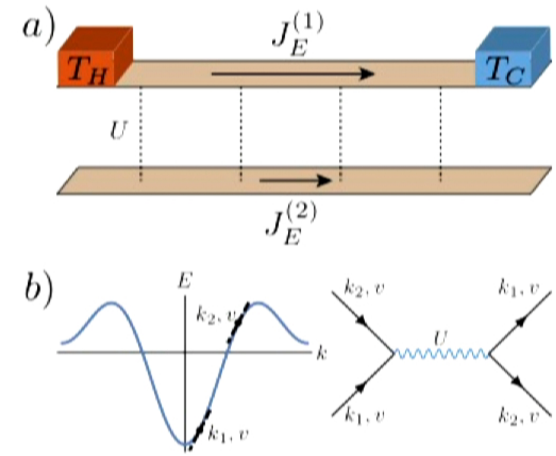
Periodic driving:
WB et al, *PRL* 118, 260602 (2017)

Stochastic driving:
WB et al, *PRL* 2019 (in press)



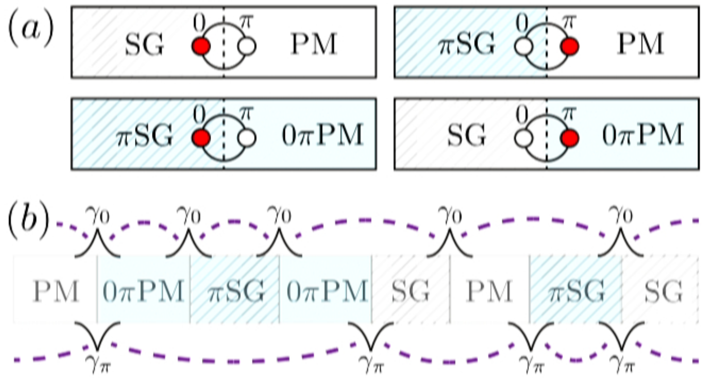
Hydrodynamics & Thermal Coulomb drag

WB et al, arXiv:1909.05251



Outline

- Motivation & Floquet theory
- Floquet RSRG
  - Argument from domain walls
  - Microscopic RG
  - Numerics
- Conclusions



Dynamical phases of matter

- Central goal of condensed matter physics: classification of phases
- Generally concerned with static H. Why not H(t)?
  - Non-equilibrium (limited tools)
  - "No eigenstates" (hence no ground state)
  - No conservation of energy, generically expect destroys order (entropy maximized)
- Floquet MBL systems sidestep these issues! Allow for intrinsically dynamical, non-equilibrium phases of matter.

Floquet phases

- Recently discovered that periodically driven ("Floquet") systems can host new types of order:

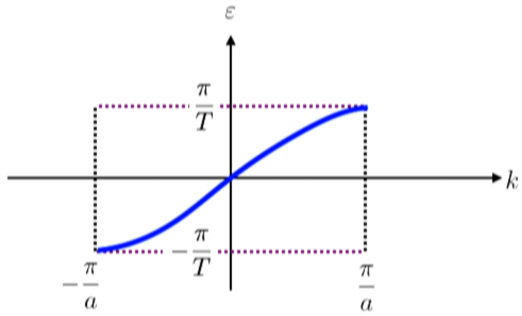
  - chiral lattice models [Kitagawa et al '10]

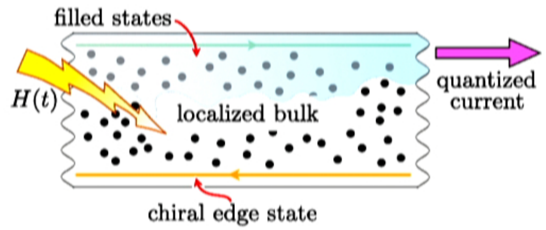
  - Anomalous Floquet Anderson Insulator [Titum et al '16]

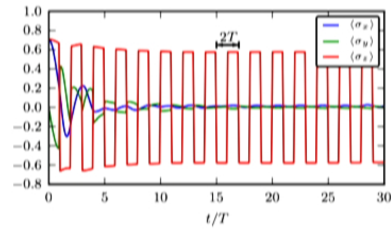
  - Floquet TIs & symmetry-protected topological phases
    [Oka & Aoki '09] [Potter et al '16; Roy et al '16; many others]

  - "time crystals" (time translation symmetry breaking)
    [Khemani et al '16; Else et al '16; Yao et al '17]

New to the non-equilibrium setting; no equilibrium counterparts!







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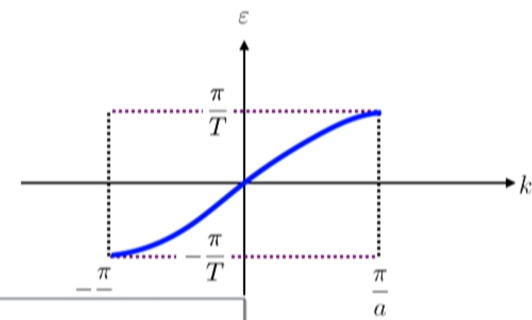
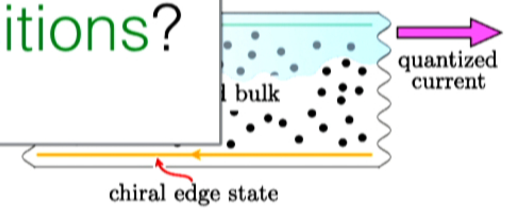


  - chiral
  - Anom

What is the nature of Floquet phase transitions?



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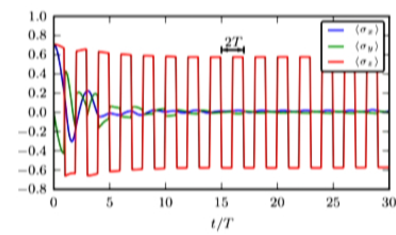

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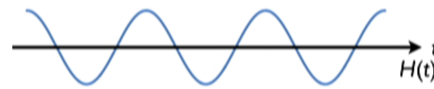
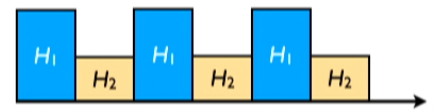
Floquet systems

- Time-periodic Hamiltonian $H(t) = H(t+T)$



  - Different classes of driving: sinusoid, two-step, …



- Central object of interest: Evolution operator $F = U(T) = \mathcal{T} e^{-i \int_0^T H(t) dt}$
- No energy conservation ⇒ No notion of ground state! (but *do* have eigs)
- Floquet's theorem: $|\psi_\alpha(t)\rangle = e^{-i\epsilon_\alpha t} |\phi_\alpha(t)\rangle$ where $|\phi_\alpha(t)\rangle = |\phi_\alpha(t+T)\rangle$ (~Bloch)

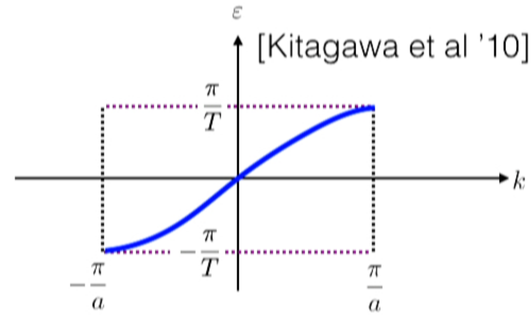
$\Rightarrow \quad U(T)|\psi_\alpha(t)\rangle = e^{-i\epsilon_\alpha T} |\psi_\alpha(t)\rangle$

"quasienergy"

$$F = e^{-iTH_F}$$

Input: DVI - 1920x1080p@60Hz
Output: SDI - 1920x1080i@60Hz

Heating and localization



- Periodic driving allows new possibilities,
  e.g. chiral phases on 1D lattice, Floquet SPTs, …
  (note: not interested in Floquet engineering, small T limit)

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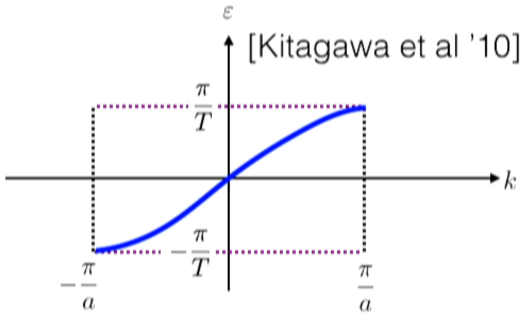

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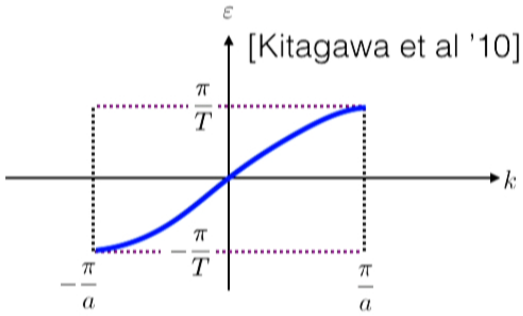

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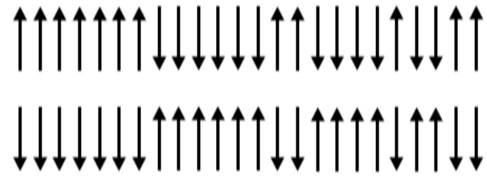
- 3 ways out (in 1D):

Integrability	Open system	Disorder
Extensive # of conserved quantities (forms GGE)	Coupling to bath allows for nontrivial steady state	MBL: excitations localize
Non-generic ("fine-tuned")	Study Lindblad dynamics	Robust to rapid driving & generic perturbations

Floquet MBL

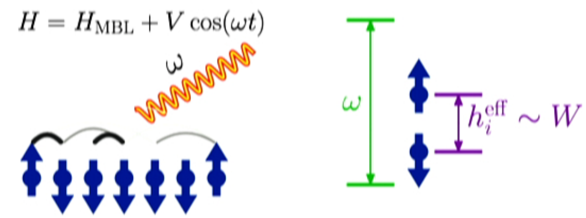
- Localization "protects" of order: e.g., domain walls localize in Ising, forming a spin-glass



- In MBL, all eigenstates "look like" ground states: area law entanglement
- Hence, even infinite temperature is nontrivial!
  [Huse et al '13; Bauer & Nayak '13]

[Ponte et al '14; Abanin et al '14]

- MBL robust to (rapid) periodic driving: Floquet MBL



- Study Floquet phases by studying spectrum
  [Khemani et al '16]

Driven Ising model

$$F = \exp(-i\sum_i J_i Z_i Z_{i+1})\exp(-i\sum_i h_i X_i)$$

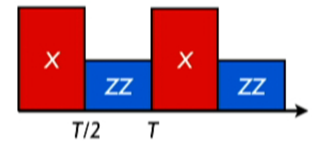
$(T=1)$

Jordan-Wigner transformation

$$F = \exp(\sum_i J_i b_i a_{i+1})\exp(\sum_i h_i a_i b_i)$$

Interactions:

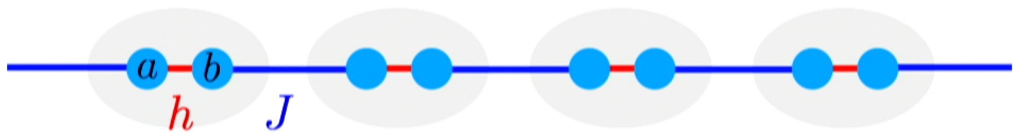
$\Gamma^Z Z_i Z_{i+2}, \Gamma^X X_i X_{i+1}$



$\Gamma = 0$

$\{a_i, b_j\} = 2\delta_{ab}\delta_{ij} \qquad a_i^2 = b_i^2 = 1$

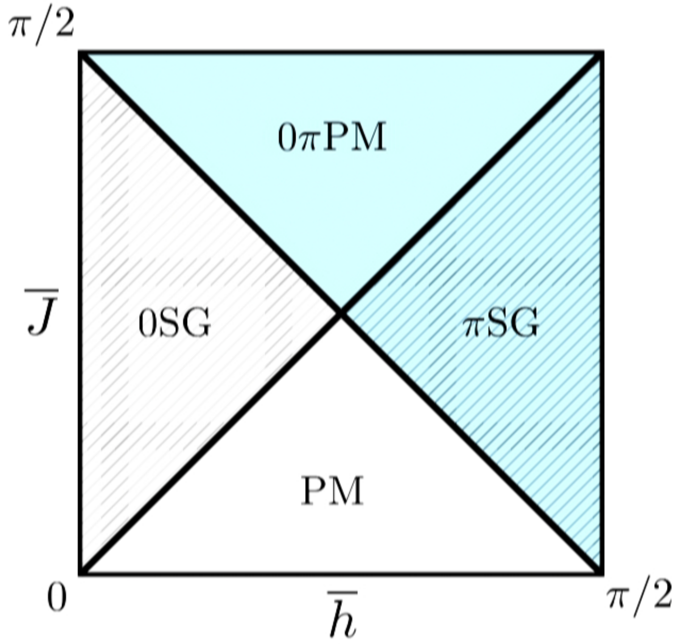
$\Gamma^Z b_i a_{i+1} b_{i+1} a_{i+2}, \Gamma^X a_i b_i a_{i+1} b_{i+1}$



Symmetries: Ising (parity) $G = \prod_i X_i$, time translation

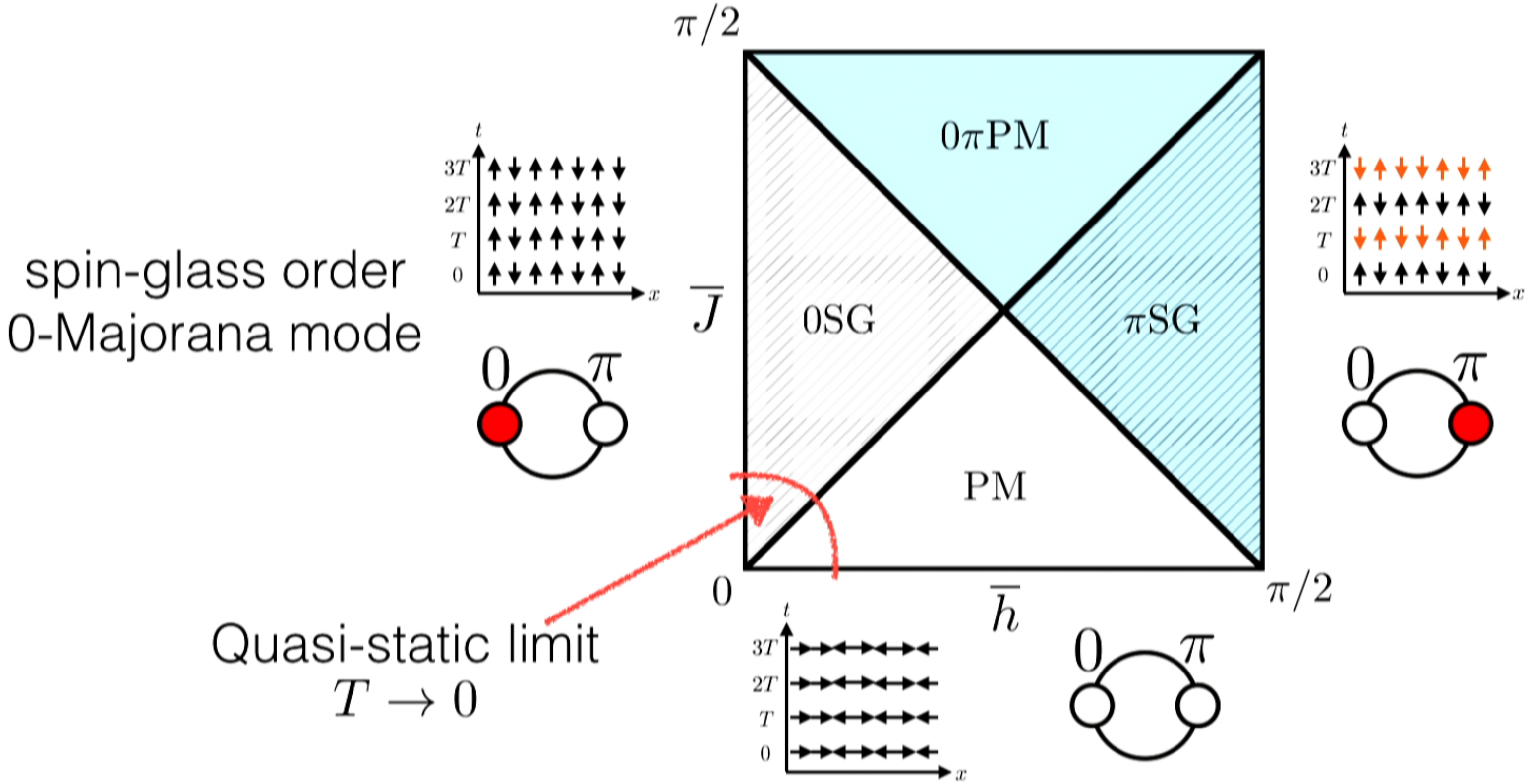
Dualities: Bond-field (even/odd), global pi/2 phase shift $J_i, h_i \to J_i, h_i + \pi/2$

Driven Ising model



Driven Ising model

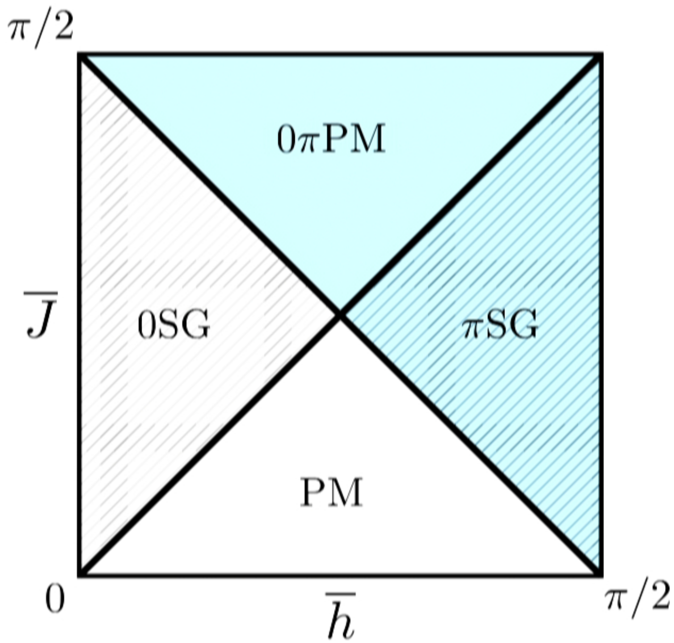
$$F\gamma_0 F^\dagger = \gamma_0$$
$$F\gamma_\pi F^\dagger = -\gamma_\pi$$
$$(E = -E \mod 2\pi/T)$$



[interacting case:
Khemani et al '16;
von Keyserlingk et al '16;
Else et al '16, others]

[free fermions:
Jiang et al '11, others]

Driven Ising model

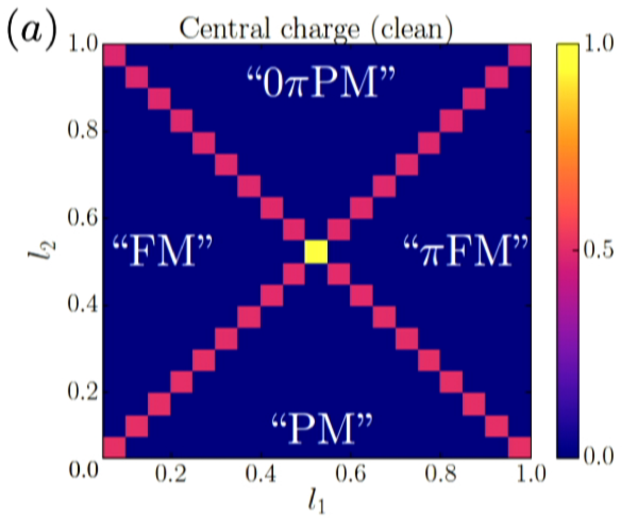


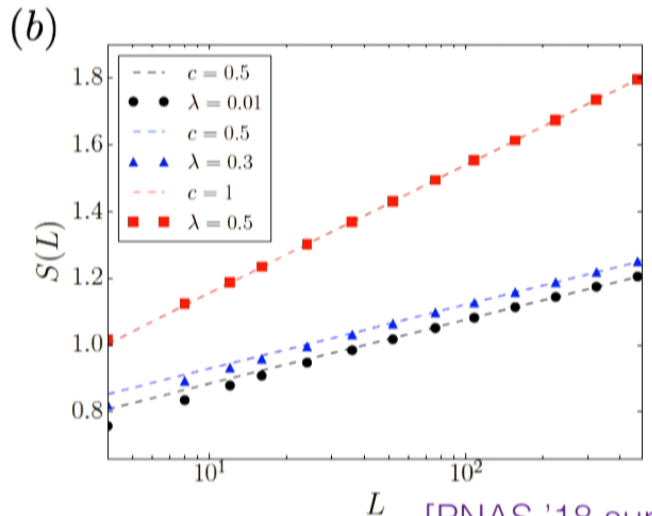
What is the nature of the transition lines?

What happens at the multicritical point?

Clean criticality

- Clean systems with interactions heat (so non-generic), but define "Floquet ground state" $|FGS\rangle = \prod \alpha_i^\dagger |0\rangle, \quad \epsilon_i < 0$ $\quad |FGS\rangle \xrightarrow{T\to 0} |GS\rangle$
- Ansatz: entropy scales as in a CFT ground state $S_{L/2} \sim \frac{c}{6}\log L$





c=1/2 along lines

[PNAS '18 supplement; see also Yates et al '18]

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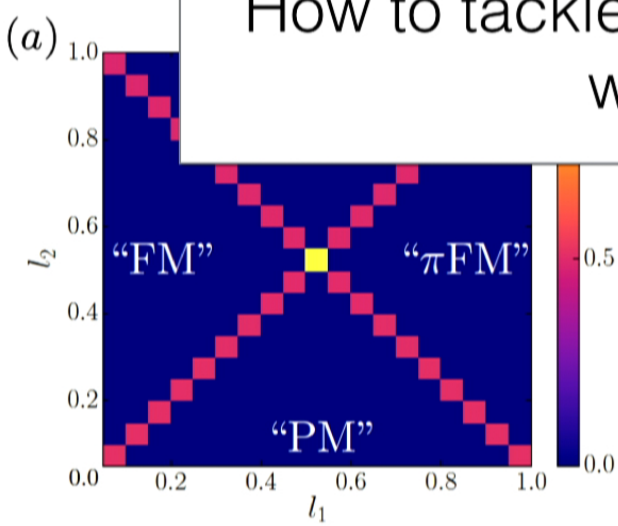
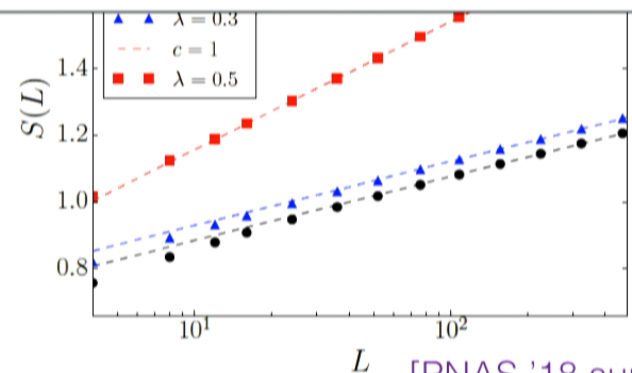
How to tackle the disordered (generic) case, with no CFT picture?





c=1/2 along lines

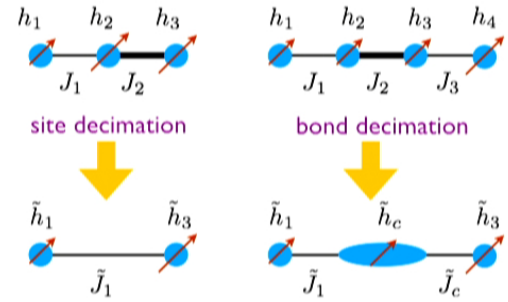
c=1 multicritical point

[PNAS '18 supplement; see also Yates et al '18]

Real space renormalization group

- Method originally invented for describing ground states of random spin chains
- Disorder strongly relevant (Harris criterion)
- Workhorse: Schrieffer-Wolff transformation $[e^{iS}He^{-iS}, H_0] = 0$
  - Idea: Pick strongest coupling in chain at step 0
  - "Lock" spin into GS of that coupling. Then use perturbation theory for neighbors. Repeat!
  - Flow distribution of couplings with the RG scale…

Infinite Randomness Fixed Point



[Ma, Dasgupta, Hu '79; Fisher '92, '95]

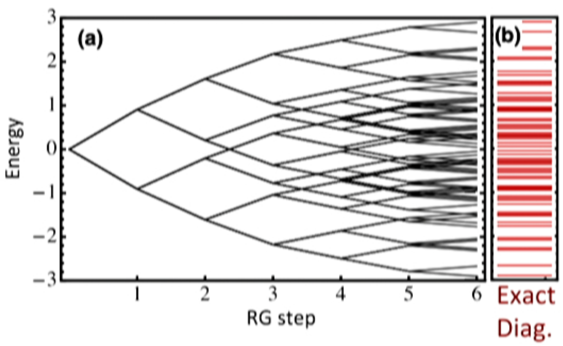
$$\tilde{J} = \frac{J_L J_R}{\Omega}$$

$$P_\Gamma(J) \sim \frac{1}{J^{1-1/w(\Gamma)}}, w \to \infty$$

asymptotically exact

RSRG-X and Floquet

- With MBL, can do RSRG to obtain entire spectrum (RSRG-X). RSRG much more useful than initially realized!
  - Idea: instead of projecting onto GS, project onto excited states — decimate smaller *energy differences*

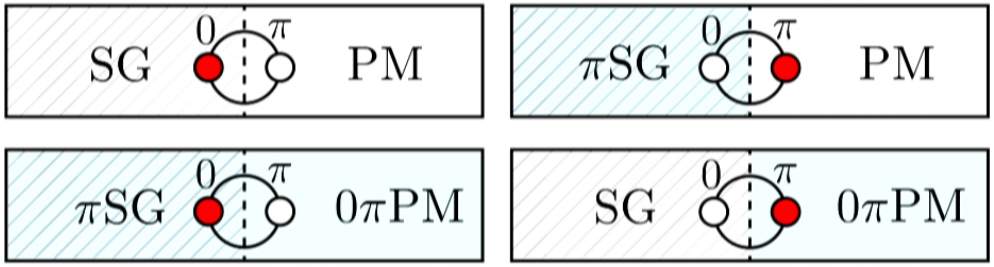


[Pekker et al '14]

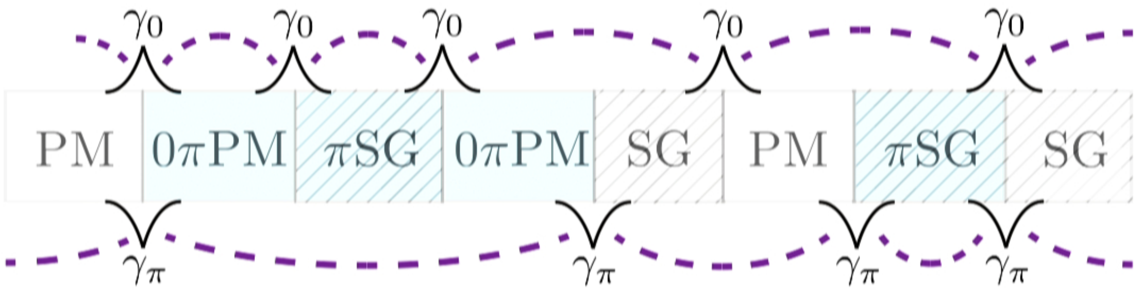
- Challenge for Floquet: what is "small" and "large"? $|e^{i\lambda\mathcal{O}}|\ldots?$

Intuition: Domain walls

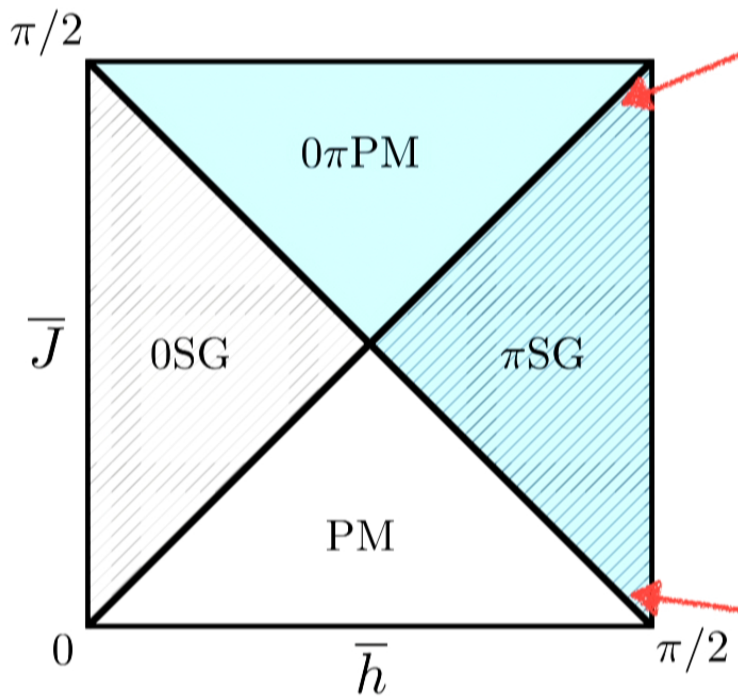
- Infinite randomness fixed points: composed locally of "puddles" deep in neighboring phases (in contrast to clean critical points!) [Damle & Huse '02]
- Idea: coarse grain at the critical point. If IRFP: 4 types of domain wall



- Hence a typical configuration will be composed of chains of 0 and pi majoranas!

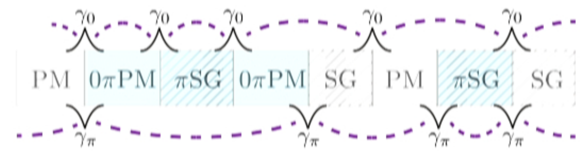


Criticality: DW argument



0-Majoranas critical
Ising $\mathbb{Z}_2$ transition

π-Majoranas critical
Ising $\mathbb{Z}_2$ transition



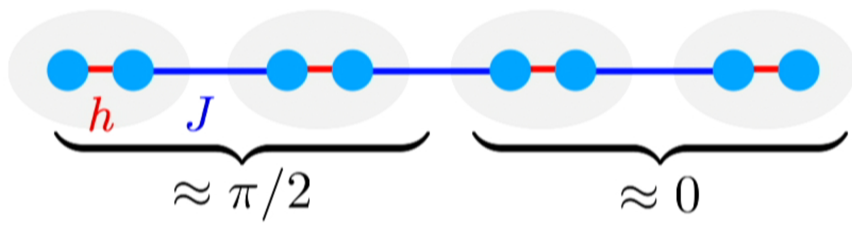
Emergent symmetry

Consider the 2-period evolution operator: $U(2T) = F^2$ [Yao et al '17]

Expect the form: $F = De^{-i\tilde{H}}$ $[D, \tilde{H}] = 0$ [Else et al '16]

$$\Rightarrow D = F\sqrt{F^2}^\dagger$$

Explicit calculation:



$$D = e^{-\frac{1}{2}(\delta_0 a_0 b_0 - \epsilon_{-1} b_{-1} a_0 + \epsilon_{-1}\delta_{-1} a_{-1} a_0 - \epsilon_0 \delta_0 a_0 a_1 + \dots)} a_0 e^{\frac{1}{2}(\delta_0 a_0 b_0 - \epsilon_{-1} b_{-1} a_0 + \epsilon_{-1}\delta_{-1} a_{-1} a_0 - \epsilon_0 \delta_0 a_0 a_1 + \dots)}$$
$$= \tilde{a}_0,$$

i.e., D is the parity of the π-chain! $D = \prod_{i \in \{DWs\}} \tilde{\gamma}_i$

Microscopic RG

Work in Majorana picture. Key insight: **both** 0 and π are "small"

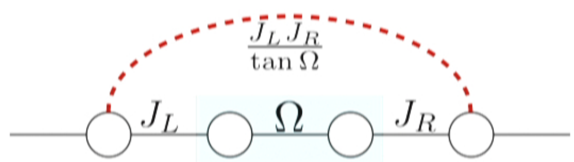
either 1) $e^{Jab} = e^{\epsilon ab}$ or 2) $e^{Jab} = e^{(\pi/2+\epsilon)ab} = abe^{\epsilon ab}$

By factoring out these exact π pulses, everything is a small exponential!

These small exponentials control the RG.

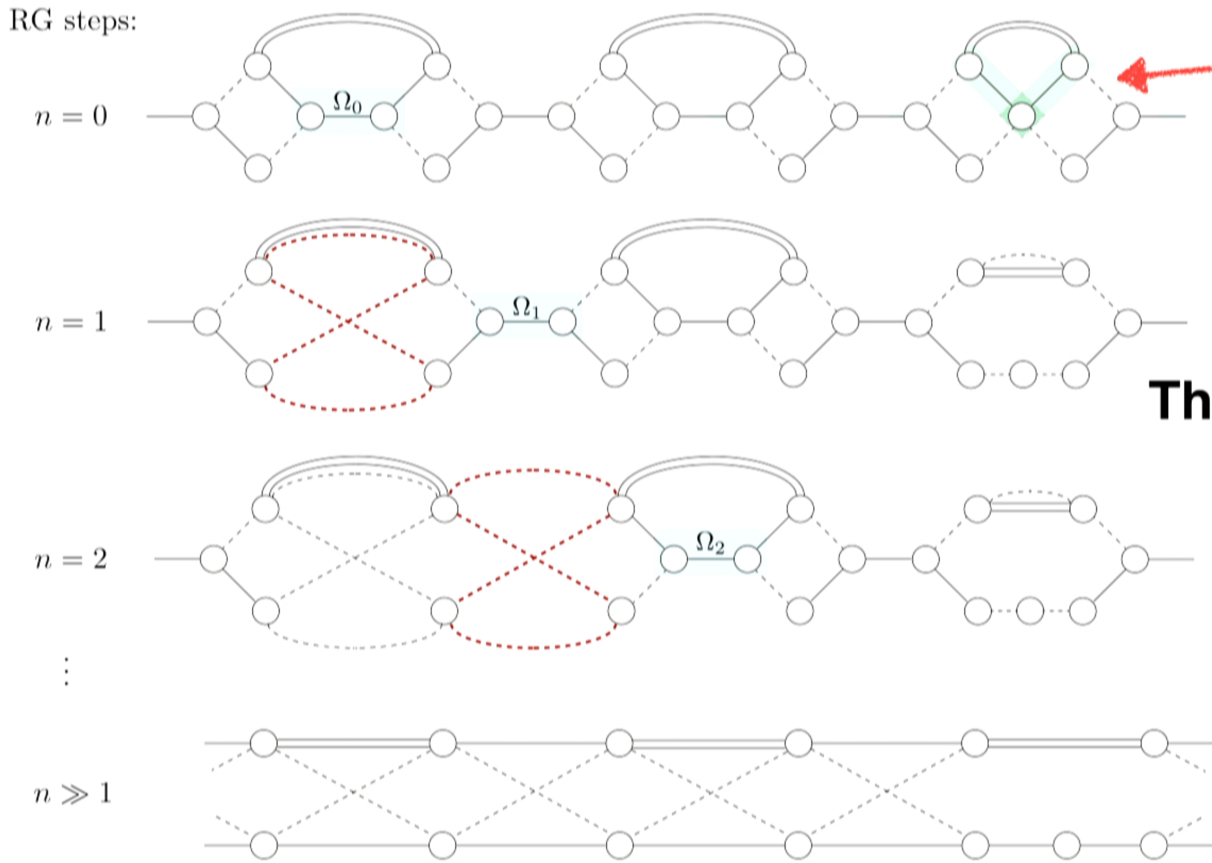
Floquet Schrieffer-Wolff: $[e^{iS}Fe^{-iS}, F_0] = 0$ (series in ε)

RG rule: $\tan \tilde{J} = \dfrac{J_L J_R}{\tan \Omega}$



$\log \tan \tilde{J} = \log J_L + \log J_R - \log \tan \Omega$ → Ising infinite randomness fixed point

Microscopic RG flow



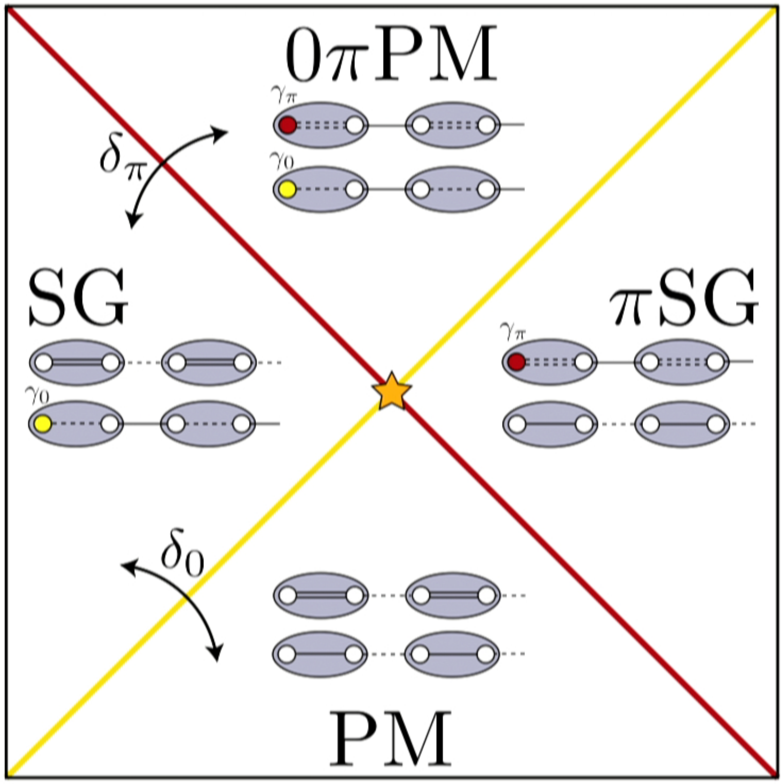
Domain walls dominate late in the RG flow

These are the 0 and π chains emerging from the RG!

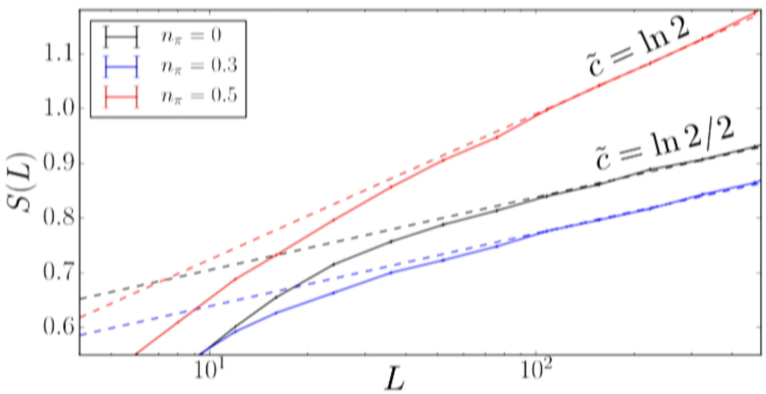
0/pi alternating

all bonds near 0

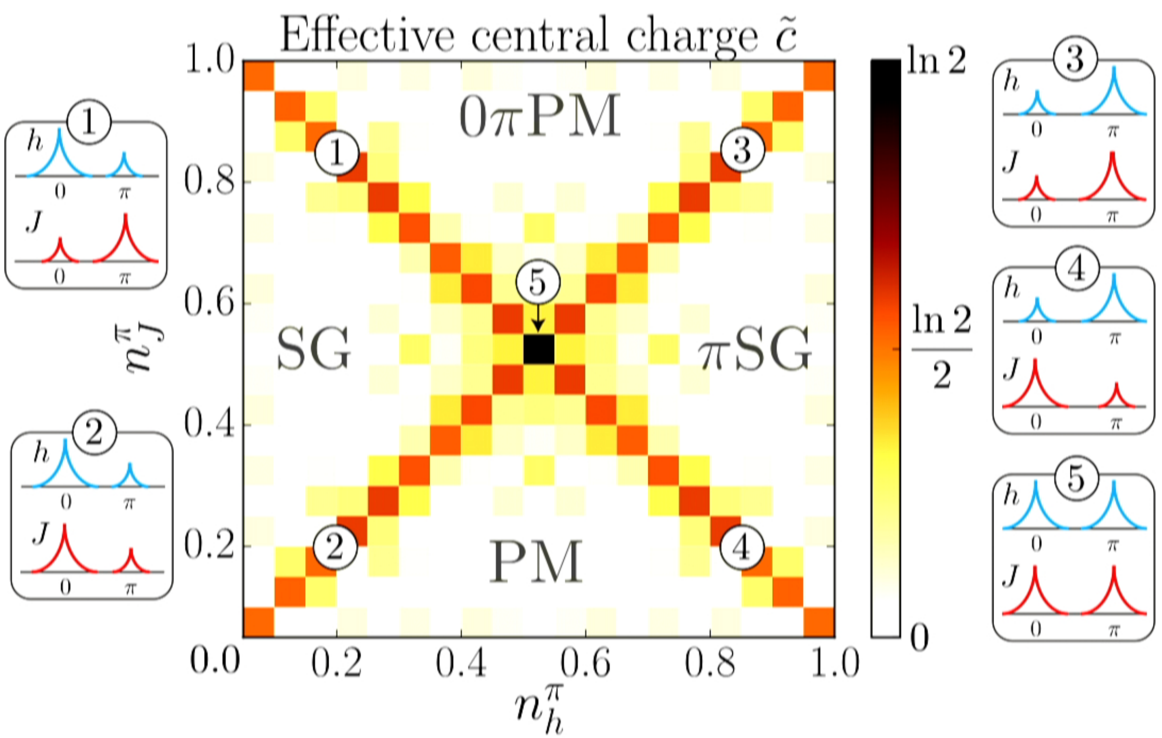
Microscopic RG - phase diagram



Numerics



$$\overline{S_{L/2}} \sim \frac{\tilde{c}}{6} \log L$$



Numerics

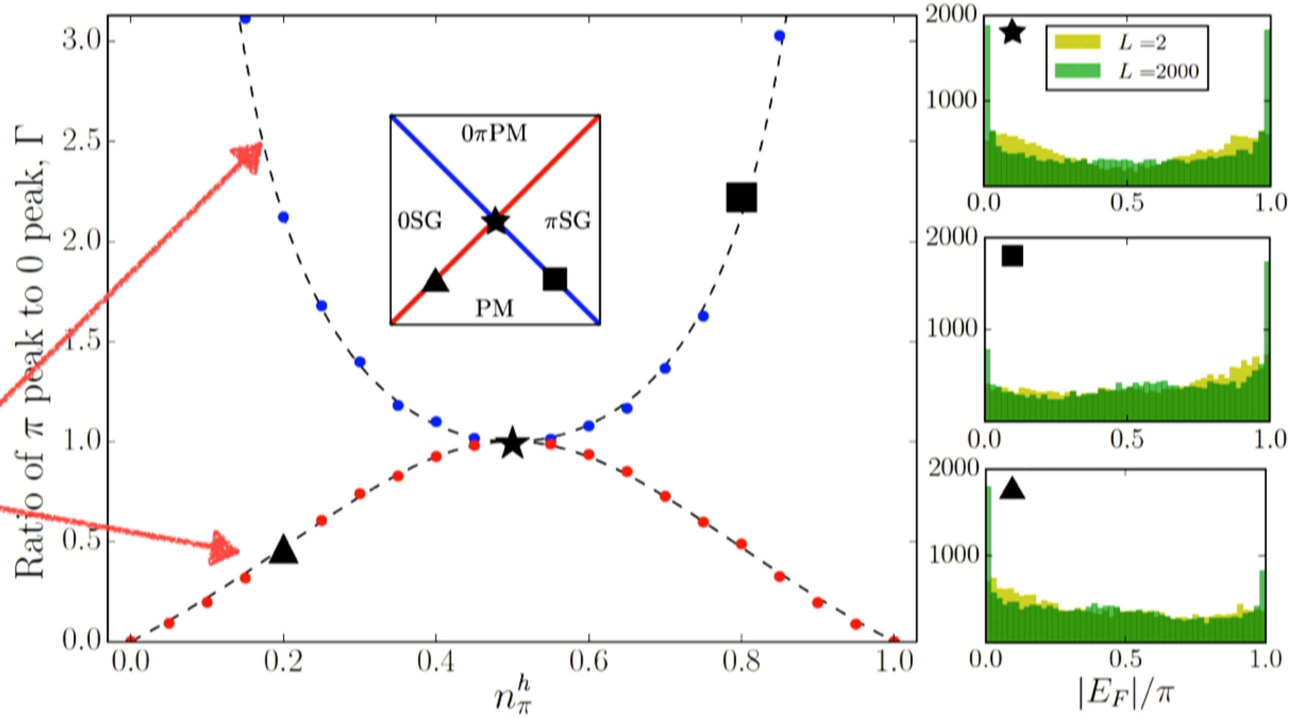
Extract full quasienergy distribution (high precision numerics)

DW counting:
analytical prediction

$$p_{DW} = 2n_\pi^h(1 - n_\pi^h)$$

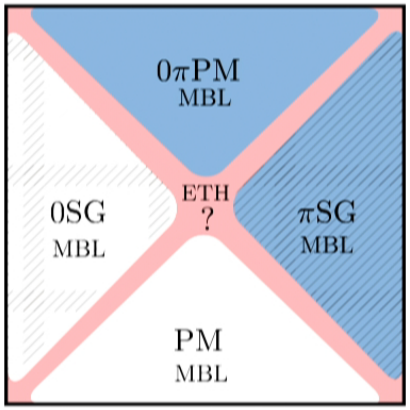
(prob of DW)

Good evidence for DW picture



Interactions

- Within the RSRG: Interactions within each species are RG irrelevant (flow to 0 more quickly than hopping terms) → won't change exponents
- Interactions between species also RG irrelevant [disordered XYZ - Slagle et al 2016]
- Could lead to thermalization (via long-ranged resonances - are these relevant?)

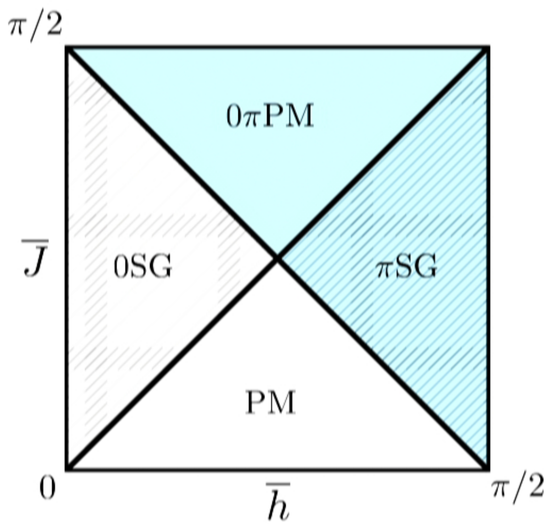


Observables

Critical Ising satisfies: $\overline{\langle Z_i(t) Z_i(0) \rangle} \sim \frac{1}{\log^{2-\varphi} t}$ [Vosk, Altman '14]

Spin autocorrelation spectral function: $C(\Omega, t) = \int_0^\infty dt' e^{-i\omega t'} \overline{\langle Z_i(t+t') Z_i(t') \rangle}$

[Yao et al '17]



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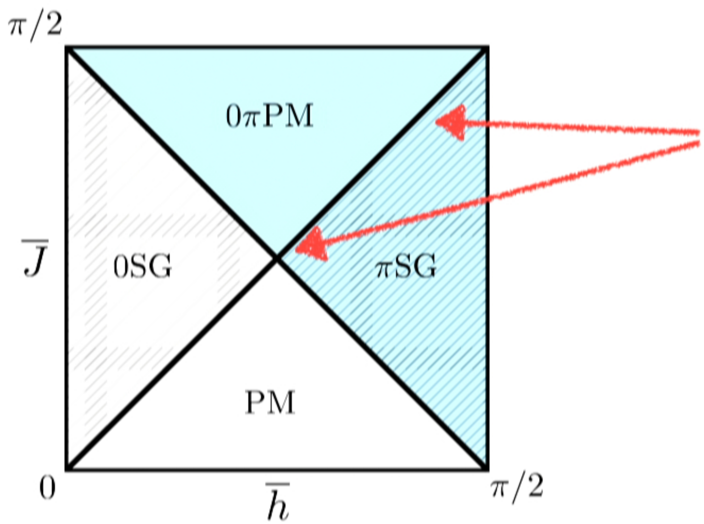
[Yao et al '17]



$C(0, t) \sim \frac{1}{\log^{2-\varphi} t}$

Conclusions

- We have introduced a strong-disorder RG for Floquet systems
- Periodic driving allows for new types of criticality (e.g. multicriticality)
- Intuitive “domain wall” argument extendable to other 1D topological phases
- Next steps: long range interactions, parafermions/clock models, sinusoidal driving

WB, MK, SAP and RV, *PNAS* 115 (38) 9491-9496 (2018)
WB, MK, SAP and RV, *PRB* 98,174203 (2018)