Title: Magic resource theories and classical simulation

Speakers: Earl Campbell

Collection: Symmetry, Phases of Matter, and Resources in Quantum Computing

Date: November 26, 2019 - 4:45 PM

URL: http://pirsa.org/19110122

Abstract: I will review the stabiliser rank and associated pure state magic monotone, the extent, [Bravyi et. al 2019]. Then I will discuss several new magic monotones that can be regarded as a generalisation of the extent monotone to mixed states [Campbell et. al., in preparation]. My talk will outline several nice theorems we can prove about these monotones relate to each other and how they are related to the runtime of new classical simulation algorithms.


Earl Campbell
Sheffield
MAGIC MONOTONES AND CLASSICAL SIMULATION
MAINLY PRESENTING “IN PREPARATION” WORK WITH
EARL CAMPBELL, YINGKAI OUYANG - SHEFFIELD
HAKOP PASHAYAN - SYDNEY
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BIGGER STORY INCLUDES SEVERAL OTHER PAPERS WITH
DAVID GOSSET, SERGEY BRAVYI, MARK HOWARD, DAN BROWNE,
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# MAGIC MONOTONES AND CLASSICAL SIMULATION

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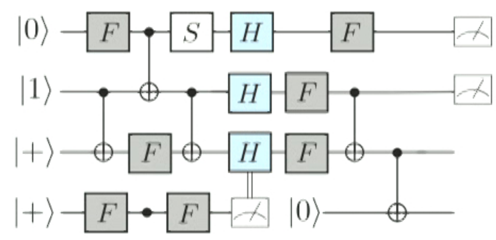
"We must find one magic monotone to rule them all" - *Not Spekkens*

**The stabiliser model of computation** allows for

- Preparation of stabiliser states.
- Clifford unitaries;
- Measurement of Pauli operators;
- Classical feedforward and adaptivity.




**The Gottesman-Knill theorem**

A stabiliser computation with n-qubit and t operations can be classically simulated using $O(n^2)$ bits* and $O(nt) + O(n^3)$ time.

*slight improvements possible using graph state representation of states.

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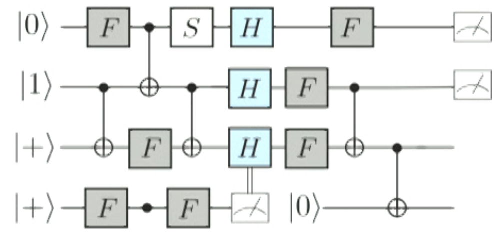


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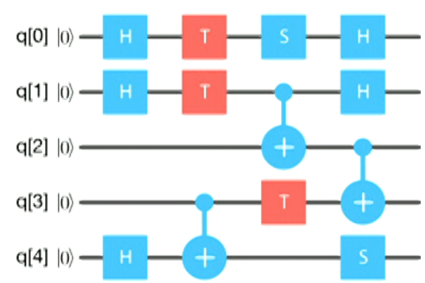
**Extended Gottesman-Knill**

Aaronson-Gottesman *Phys. Rev. A* 70, 052328 (2004)

Runtime $O(4^n)$ for ***n*** single-qubit nonCliffords

**Clifford gates = easy to simulate**

**NonClifford = difficult to simulate**

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What $\mathcal{E}$ can we classically efficiently simulate?

**[SC]** Seddon, Campbell
Proc. R. Soc. A 475 20190251 (2019)

Stabiliser operations
$\mathcal{SO}_n :=$ n-qubit Cliffords, Pauli measurements, etc... ✔

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Stabiliser operations

$\mathcal{SO}_n :=$ n-qubit Cliffords, Pauli measurements, etc...

Stabiliser preserving operations

$$\mathcal{SPO}_n := \{\mathcal{E} : \mathcal{E}(\mathrm{STAB}_n) \subseteq \mathrm{STAB}_n\}$$

e.g. Brandao, Gour, *Phys. Rev. Lett.* **115**, 070503 (2015)

**at least can't simulate** $\mathcal{E} \otimes \mathbb{I}$

Completely stabiliser preserving operations

$$\mathcal{CSPO}_n := \{\mathcal{E} : \mathcal{E} \otimes \mathcal{I}_n \in \mathcal{SPO}_{2n}\}$$

Also easy to characterise using Choi-isomorphism compare to $\mathcal{SO}_n$

# Further extensions

## Quasiprobability methods

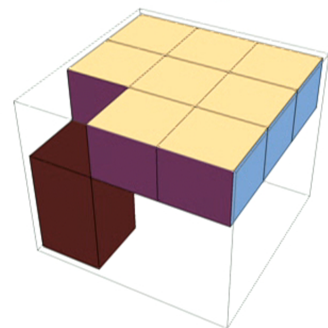
**[PWB]** Pashayan-Wallman-Bartlett,
*Phys. Rev. Lett.* 115, 070501 (2015).

**[RBDOB]** Raussendorf-Browne-Delfosse
-Okay-BermejoVega
*Phys. Rev. A* 95, 052334

**[RBTOZ]** Raussendorf-BermejoVega-
Tyhurst-Okay-Zurel
arXiv:1905.05374

**[HW]** Howard-Campbell
*Phys. Rev. Lett.* 118 090501 (2017)

**Discrete Wigner function**



Elegant for qu*d*its (**d>2**),
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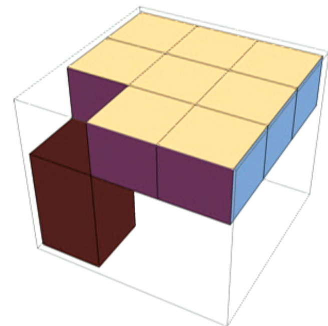

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$$\rho = \sum_i q_i |\psi_i\rangle\langle\psi_i|$$

$$\mathcal{R}(\rho) = \sum_i |q_i|$$ **Robustness of magic**

**Further extensions**

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**Pros:**
**Works for mixed states**
**Cons:**
**(comparatively) slow runtime, not multiplicative for qubits**

$$\mathcal{R}(\rho_1 \otimes \rho_2) \neq \mathcal{R}(\rho_1)\mathcal{R}(\rho_2)$$

# Further extensions

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## Stabiliser Rank methods

**[BSS]** Bravyi, Smith and Smolin
*Phys. Rev. X* 6, 021043 (2016)

**[BG]** Bravyi, Gosset
*Phys. Rev. Lett.* 116, 250501 (2016)

**[BBCCHG]** Bravyi, Browne, Calpin, Campbell, Gosset, Howard
*Quantum* 3 131 (2019)

**Pros:**
**Faster,**
**Typically multiplicative**
**Cons:**
**Only for pure states/circuits!**

**Further extensions**

## Quasiprobability methods

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Works for mixed states
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$\mathcal{R}$

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Faster,
Typically multiplicative
**Cons:**
Only for pure states/circuits!

$\chi \quad \xi$

**Connections?**

## New monotones / classical simulators

[COPRS] Campbell, Ouyang, Pashayan, Regula, Seddon - in preparation

*All the pros, none of the cons.*

$\Lambda^{+} \quad \Lambda \quad \Xi$

## Stabiliser rank [BBS]

minimum $\chi$ such that $|\Psi\rangle = \sum_{i=1}^{\chi(\Psi)} c_i |\Psi_i\rangle$

where $|\Psi_i\rangle$ are stabiliser states

## Good magic monotone

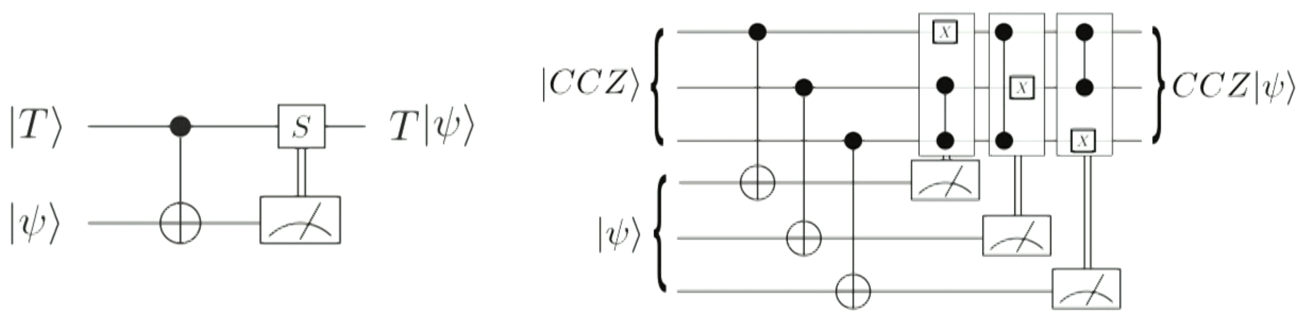
- **Monotonicity:** Nonincreasing under Clifford operations
- **Faithful** $\chi(\Psi) = 1$ if and only if $\Psi$ is a stabiliser state.
- **Submultiplicativity**

$$\chi(\psi \otimes \phi) \leq \chi(\psi)\chi(\phi)$$

*Bravyi, Smith and Smolin *Phys. Rev. X* **6**, 021043 (2016)

## Overview simulation method

**STEP 1:** replace all nonClifford gates with state injection gadgets




Let us use $|\psi\rangle$ as the collection of magic states needed.
and $K$ for the whole Clifford circuit (including postselections)

## Overview simulation method

**STEP 2:** find low rank decomposition

$$|\psi\rangle = \sum_{j=1}^{L} c_j |\psi_j\rangle \qquad \chi(\psi) \leq L$$

**STEP 3:** evolve decomposition

$$|\psi\rangle \rightarrow K|\psi\rangle$$

via $L$ Gottesman-Knill like updates

$$|\psi_j\rangle \rightarrow K|\psi_j\rangle$$

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$$|\psi_j\rangle \rightarrow K|\psi_j\rangle$$

**STEP 4:** measure

$$K|\psi\rangle \rightarrow \left(\frac{\mathbb{I} \pm Z_i}{2}\right) K|\psi\rangle$$

another $L$ Gottesman-Knill like updates

## Overview simulation method

**STEP 5:** calculate probabilities

$$P(\pm) = |\langle\psi^{\pm}|\psi^{\pm}\rangle|^2$$

where

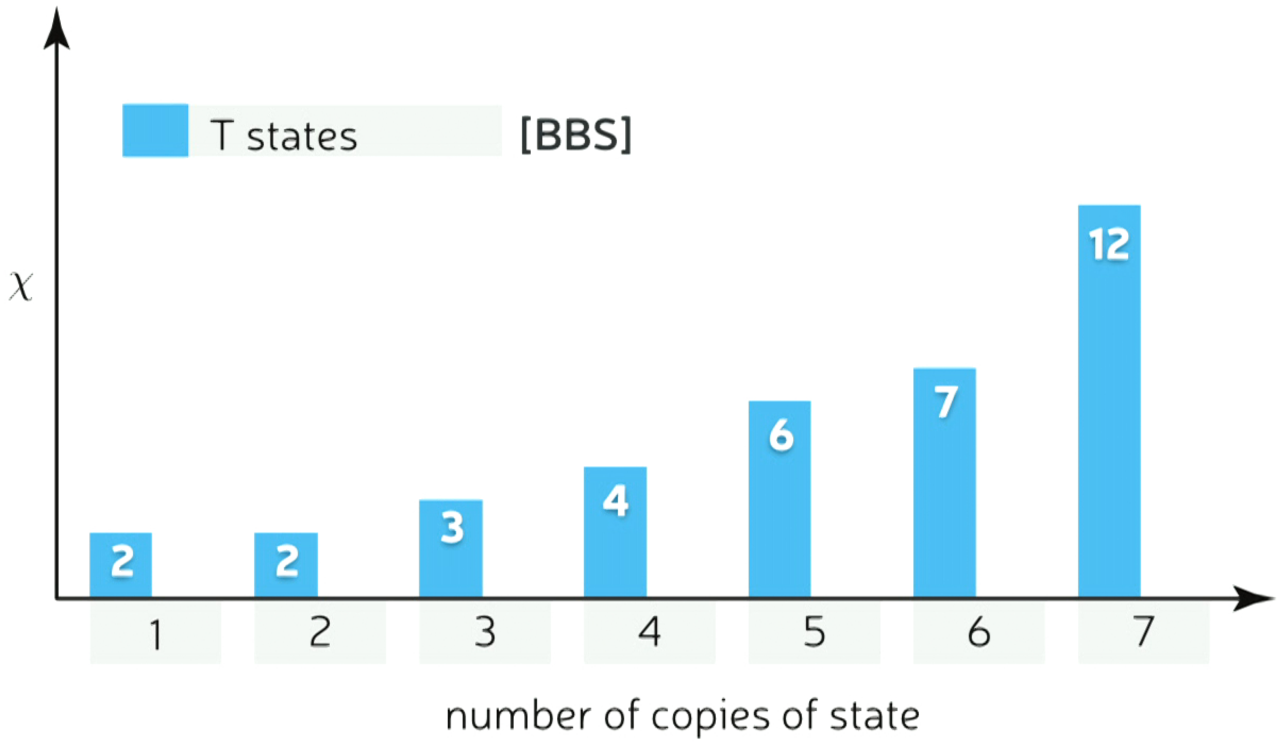
$$|\psi^{\pm}\rangle =\!\!\rightarrow \left(\frac{\mathbb{I} \pm Z_i}{2}\right) K|\psi\rangle$$

**Two methods**

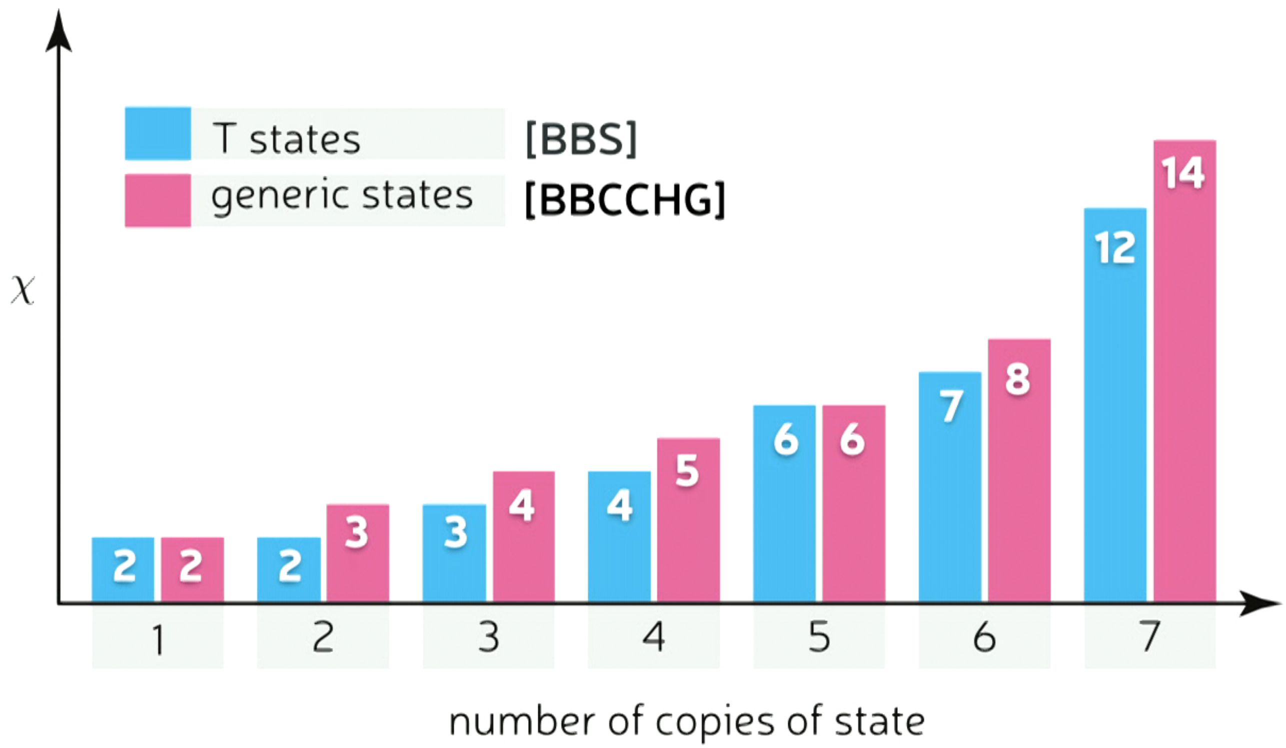
1. **direct calculation** $\sum_{i,j} c_i^* c_j \langle\Psi_i'|\Psi_j'\rangle$ $\text{runtime} = \chi(\Psi)^2$ **[BBS]**
2. **Fast norm est. (with multiplicative error)** $\text{runtime} = \chi(\Psi)$ **[BG]**

**Best known lower bound** $\chi(|T\rangle^{\otimes n}) = \Omega(\sqrt{n})$ [BBS]

**But** unless P=NP, then $\chi(|T\rangle^{\otimes n}) = \Omega(2^{\mathrm{poly}(n)})$



All single-qubit states behave same way,
related to stabiliser states forming a 3-design **[BBCCHG]**



## Approx stabiliser rank

$$\chi_\delta(\Psi) := \min\{\chi(\Psi_{\text{proxy}}); ||\Psi - \Psi_{\text{proxy}}|| \leq \delta\}||$$

enables simulation with $\delta$ error

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**Sparsification lemma [BBCCHG]**

if $|\Psi\rangle = \sum_j c_j |\phi_j\rangle$ ← *stabiliser states*

then $\chi_\delta(\Psi) \leq 1 + \frac{||c||_1^2}{\delta^2}$

Proof. by random construction,

sample $k = 1 + \frac{||c||_1^2}{\delta^2}$ terms, with prob $p_j = \frac{|c_j|}{\sum_j |c_j|}$ and compose.

Earl Campbell

Simulation overhead captured by $||c||_1^2$

Motivates

**$\xi$ Stabiliser extent**

$$\xi(\psi) := \min\{||c||_1^2 : |\psi\rangle = \sum_j c_j|\psi_j\rangle; \psi_j \in \mathrm{STAB}_n\}$$

A convex optimisation problem (solve in CVX)

**Prop 1. of [BBCCHG]**

Let $\{\psi_1, \psi_2, \ldots \psi_L\}$ be any set of states such that each state $\psi_j$ describes a system of at most three qubits. Then

$$\xi(\psi_1 \otimes \ldots \otimes \psi_j) = \prod \xi(\psi_j)$$

Open problem: is the above true in general?

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# Mixed state extent monotones

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## Ξ Convex-roof extension

$$\Xi(\rho) := \min\left\{\sum_j p_j \xi(\Psi_j) : \rho = \sum_j p_j |\Psi_j\rangle\langle\Psi_j|\right\}$$

## Λ Dyadic negativity

Primal $$\Lambda(\rho) := \min\left\{\|p\|_1 : \rho = \sum_j p_j |\psi_j^L\rangle\langle\psi_j^R|; \psi_j^L, \psi_j^R \in \mathrm{STAB}_n\right\}$$

Dual $$\Lambda(\rho) = \max\{\mathrm{Tr}[W\rho] : W \in \mathcal{W}\}$$

where $\mathcal{W} := \{W : |\langle\psi^L|W|\psi^R\rangle| \leq 1; \forall\psi^L, \psi^R \in \mathrm{STAB}_n\}$

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## $\Lambda^+$ Generalised robustness

Dual $$\Lambda^+(\rho) := \max\{\mathrm{Tr}[W\rho] : W \in \mathcal{W}; W \geq 0\}$$

**Extent:** for pure states only

$$\xi(\psi) := \min\{||c||_1^2 : |\psi\rangle = \sum_j c_j |\phi_j\rangle\}$$

The three sisters **[COPRS -** in prep**]**

$\Lambda^+$ Generalised robustness

$\Lambda$ Dyadic negativity

$\Xi$ Roof extension

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The three sisters **[COPRS - in prep]**

The odd one out!

$\Lambda^+$ Generalised robustness

$\Lambda$ Dyadic negativity

$\Xi$ Roof extension

$\mathcal{R}$ Robustness of Magic **[HC]**

Trivial to show $\Lambda^+[\rho] \leq \Lambda[\rho] \leq \Xi[\rho]$

Lem 1 [COPRS], see also Regula '17

For any **pure** state

$$\Lambda^+[|\psi\rangle\langle\psi|] = \Lambda[|\psi\rangle\langle\psi|] = \Xi[|\psi\rangle\langle\psi|] = \xi[\psi]$$

Thm. 5. [COPRS]

For any **product of 1-qubit states** $\rho = \bigotimes_{j=1}^{n} \rho_j$ we have

$$\Lambda^+[\rho] = \Lambda[\rho] = \Xi[\rho] = \prod_j \Lambda[\rho_j]$$

typically [HC]

$$\mathcal{R}[\rho_1 \otimes \rho_2] \neq \mathcal{R}[\rho_1]\mathcal{R}[\rho_2]$$

Extent useful for bounding conversion rates
of pure states. See e.g. many computed values

| $\|\psi\rangle$ | Best algo. (lower bound) [Ref.] $rn\|T\rangle \Rightarrow n\|\psi\rangle$ | Best algo. (upper bound) [Ref.] $n\|\psi\rangle \Rightarrow r'n\|T\rangle$ |
|---|---|---|
| $\|\sqrt{T}\rangle$ | 2.5 (1) [Fig. 7] | 0.25 (0.754933*) |
| $\|T\rangle$ | 1 (1) | 1 (1) |
| $\|CS\rangle = \|W_2\rangle$ | 3 (2.96818*) [Fig. 2] | 1 (2) [Fig. 2] |
| $\|CCS\rangle$ | 7 (4.53328*) [32] | 0.5 (3) [Prop. A.3] |
| $\|C^3S\rangle$ | 11 (4) [32] | 0.25 (3.82743*) [Prop. A.3] |
| $\|CCZ\rangle$ | 4 (3.63356*) [32] | 2 (3) [23] |
| $\|C^3Z\rangle$ | 6 (5.12122*) [32] | 1 (4) [Tab. 2] |
| $\|C^4Z\rangle$ | 12 (5) [32] | 0.5 (3.8233*) [Tab. 2] |
| $\|CCZ_{123,145}\rangle$ | 8 (5) [Tab. 2] | 2 (4.37739*) [Tab. 2] |
| $\|W_3\rangle$ | 4 (3.63356*) [Tab. 2] | 2 (3) [Tab. 2] |
| $\|W_4\rangle$ | 5 (4.99907*) [Fig. 4] | 3 (4) [Fig. 4] |
| $\|W_5\rangle$ | 6 (5.93637*) [Fig. 4] | 4 (5) [Fig. 4] |

The number (?*) are bounds from the extent
Beverland, Campbell, Howard, Kliuchnikov,
arXiv:1904.01124

Other interesting work
on state conversation:

inc. this meeting:
Fang, Liu
arXiv:1909.02540
(doesn't work for pure states)

## Equality Lemma [COPRS]

Let $|\omega\rangle$ be a witness so that $|\langle\phi|\omega\rangle|^2 \leq 1, \forall\phi \in \mathrm{STAB}$

Then define the set $S_\omega := \mathrm{ConvexHull}[\{\Psi : |\langle\Psi|\omega\rangle|^2 = \xi(\Psi)\}]$

For any $\rho \in S_\omega$ we have:

$$\Lambda^+[\rho] = \Lambda[\rho] = \Xi[\rho] = \mathrm{Tr}[|\omega\rangle\langle\omega|\rho]$$

## Ξ Convex-roof extension

$$\Xi(\rho) := \min \left\{ \sum_j p_j \xi(\Psi_j) : \rho = \sum_j p_j |\Psi_j\rangle\langle\Psi_j| \right\}$$

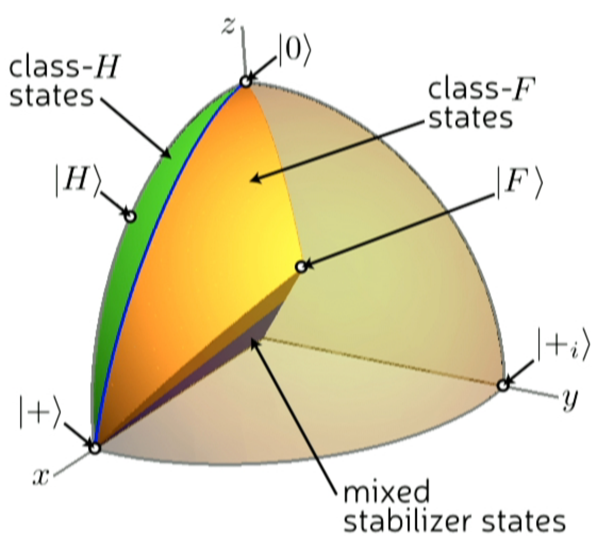
*if* $\rho = \sum_j p_j |\Psi_j\rangle\langle\Psi_j|$ *where* $\Psi_j \in S_\omega$

*then directly*

$$\Xi[\rho] \leq \sum_j p_j \xi[\Psi_j]$$

## Lemma 4 of [COPRS]

Paraphrased: every 1-qubit mixed states belongs to a suitable convex set such that we can use the previous Lemma.




**Class F:** uses the "face-Witness". Convex hull includes yellow states and 3 stabiliser states

**Class H:** more complex...., *On XZ plane* uses "Hadamard-witness" Convex hull includes whole XZ plane. *Off XZ plane* witness continuously changes from Hadamard to face.

## Upper bounds using decompositions (primal convex problem)

$$\xi(\otimes_j \Psi_j) \leq \prod_j \xi(\Psi_j)$$

## Lower bounds using witnesses (dual convex problem)

Any vector $\omega$ with $|\langle\phi|\omega\rangle| \leq 1, \forall\phi \in \mathrm{STAB}_n$ entails thats $|\langle\Psi|\omega\rangle|^2 \leq \xi(\Psi)$

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**Strong duality:** For every $|\Psi_j\rangle$ there exists a witness $|\omega_j^\star\rangle$ such that

$$|\langle\Psi_j|\omega_j^\star\rangle|^2 = \xi(\Psi_j)$$

Witness Lemma [BBCCHG]

Let $\{\omega_1^\star, \omega_2^\star, \ldots, \omega_m^\star\}$ be any set of vectors each in a Hilbert space of at most three qubits. Then

$$|\langle\phi|\omega_j^\star\rangle| \leq 1, \forall\phi \in \mathrm{STAB} \implies |\langle\phi|\Omega\rangle| \leq 1, \forall\phi \in \mathrm{STAB}$$

where $|\Omega\rangle := \otimes_j |\omega_j^\star\rangle$

# Simulation algorithms

**Three simulators:**

1) Convex roof simulator: sample and run stabiliser rank simulator (with fast norm), **expected** runtime $O(\Xi[\rho])$ many interesting cases this is possible with zero variance.

2) Dyadic quasiprobability, expectation estimator, runtime $O(\Lambda[\rho]^2)$

3) Generalised robustness: fast but noisy simulator, **error** $O(\Lambda^+[\rho])$

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## The expectation estimation problem

Want to estimate: $\mathrm{Tr}[P\mathcal{E}(\rho)]$
where $P$ is a Pauli
and $\mathcal{E}$ is a channel composed of stabiliser operations.

## Recall: The robustness of magic simulator

Given a quasiprobability distribution over **stabiliser states**

$$\rho = \sum_j p_j |\phi_j\rangle\langle\phi_j| \quad \text{where} \quad \mathcal{R}(\rho) \leq \sum_j |p_j|$$

i) sample $|\phi_j\rangle\langle\phi_j|$ with probability $p_j = |q_j|/||q||_1$

ii) compute and return $\mathrm{sign}(q_j)||q||_1 \mathrm{Tr}[P\mathcal{E}[|\phi_j\rangle\langle\phi_j|]]$

iii) Repeat and average $O(||q||_1^2)$ times

## The expectation estimation problem

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## Next: the dyadic negativity simulator

Given a quasiprobability distribution over **dyads**

$$\rho = \sum_{j} q_{j,k} |\phi_j\rangle\langle\phi_k| \quad \text{where} \quad \Lambda(\rho) \leq \sum_{j,k} |q_{j,k}|$$

i) sample $|\phi_j\rangle\langle\phi_k|$ with probability $p_{j,k} = |q_{j,k}|/||q||_1$

ii) compute and return $\mathrm{phase}(q_j)||q||_1 \mathrm{Tr}[P\mathcal{E}[|\phi_j\rangle\langle\phi_k|]]$

iii) Repeat and average $O(||q||_1^2)$ times

**Key observation: update can simulate the update of DYADS for all CSPOn**

- **Not possible using original stabiliser tableaux method,**
- **But stabiliser rank simulators track global phase ,**
- **Method does need some modification to deal with postselection and complex phases.**

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## Next: the dyadic negativity simulator

Given a quasiprobability distribution over **dyad**

$$\rho = \sum_j q_{j,k} |\phi_j\rangle\langle\phi_k| \quad \text{where} \quad \Lambda(\rho) \leq \sum_{j,k} |q_{j,k}|$$

i) sample $|\phi_j\rangle\langle\phi_k|$ with probability $p_{j,k} = |q_{j,k}|/||q||_1$

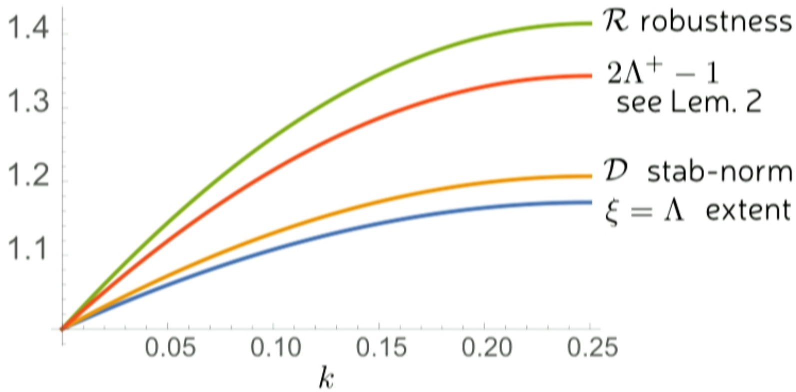
ii) compute and return $\mathrm{phase}(q_j)||q||_1 \mathrm{Tr}[P\mathcal{E}[|\phi_j\rangle\langle\phi_k|]]$

iii) Repeat and average $O(||q||_1^2)$ times

**Robustness of magic <u>much, much larger</u> than other monotones!**

Motivates looking at simulation algorithms based on new monotones

Magic monotones and lower bounds for the state $\cos(\pi k/2)|0\rangle + \sin(\pi k/2)|1\rangle$




The odd one out!

$\mathcal{R}$ Robustness of Magic **[HC]**

Lem 2 [COPRS]
bigness of robustness

$$\mathcal{R}[\rho] \geq 2\Lambda^+[\rho] - 1$$

## Open questions:

- When will the paper be finished?
- Are any monotones strictly multiplicative?
- Extensions to channel picture.
- Exponential lower bound on stabiliser rank.
- Can we dilute magic?

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**THANK YOU!**

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