Title: A resource theory of nonclassicality in Bell scenarios

Speakers: Robert Spekkens

Collection: Symmetry, Phases of Matter, and Resources in Quantum Computing

Date: November 26, 2019 - 11:30 AM

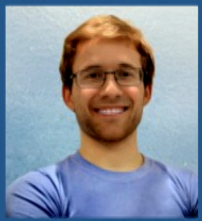
URL: http://pirsa.org/19110120

Abstract: We take a resource-theoretic approach to the problem of quantifying nonclassicality in Bell scenarios. The resources are conceptualized as probabilistic processes from the setting variables to the outcome variables which have a particular causal structure, namely, one wherein the wings are only connected by a common cause. The distinction between classical and nonclassical is then defined in terms of whether or not a classical causal model can explain the correlations. The relative nonclassicality of such resources is quantified by considering their interconvertibility relative to the set of operations that can be implemented using a classical common cause (which correspond to local operations and shared randomness). Among other results, we show that the information contained in the degrees of violation of facet-defining Bell inequalities is not sufficient for quantifying nonclassicality, even though it is sufficient for witnessing nonclassicality. In addition to providing new insights on Bell nonclassicality, our work sets the stage for quantifying nonclassicality in more general causal networks and thus also for a resource-theoretic account of nonclassicality in computational settings. (Joint work with Elie Wolfe, David Schmid, Ana Belen Sainz, and Ravi Kunjwal)

The Resource Theory of Nonclassicality in Bell scenarios

Robert Spekkens
Perimeter Institute

joint work with:
Elie Wolfe, David Schmid, Ana Belén Sainz, and Ravi Kunjwal

Symmetry, Phases of Matter, and Resources for Quantum Computing
Perimeter, Nov 26, 2019

Uncontroversial: computational advantages over classical theories arise from a resource of nonclassicality

No consensus: What is the right way to conceptualize the notion of nonclassicality that is at play

The view taken in this work: the relevant notion of nonclassicality is the one witnessed by the failure to explain operational statistics within a classical ontological framework for describing causal structure and Bayesian inferences

The view taken in this work: the relevant notion of nonclassicality is the one witnessed by the failure to explain operational statistics within a classical ontological framework for describing causal structure and Bayesian inferences

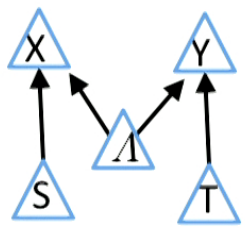
Local causality and **generalized-noncontextuality** are natural features of such classical explanations. Their failure, therefore, is an example of the relevant type of nonclassicality

A good starting point:

A resource theory of nonclassicality in prepare-and-measure scenarios
(i.e., of “contextuality”)

Causal Model

Causal Structure



Causal-Statistical Parameters

P_S

P_T

P_Λ

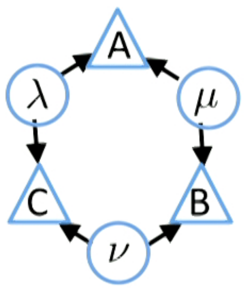
$P_{X|\Lambda S}$

$P_{Y|\Lambda T}$

$$P_{XYST\Lambda} = P_{X|S\Lambda}P_{Y|T\Lambda}P_\Lambda P_S P_T$$

Causal Model

Causal Structure



Causal-Statistical Parameters

$\mathrm{P}_{A|\mu\lambda}$

$\mathrm{P}_{B|\mu\nu}$

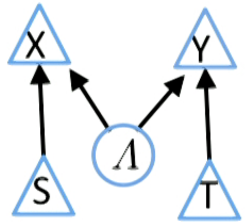
$\mathrm{P}_{C|\nu\lambda}$

P_{μ}

P_{ν}

P_{λ}

$$\mathrm{P}_{ABC} = \sum_{\mu\nu\lambda} \mathrm{P}_{A|\mu\lambda}\mathrm{P}_{B|\mu\nu}\mathrm{P}_{C|\nu\lambda}\mathrm{P}_{\mu}\mathrm{P}_{\nu}\mathrm{P}_{\lambda}$$



P_Λ

$P_{X|\Lambda S}$

$P_{Y|\Lambda T}$

$$P_{XY|ST} = \sum_\Lambda P_{X|S\Lambda} P_{Y|T\Lambda} P_\Lambda$$

Conditional independence relations (equality constraints)

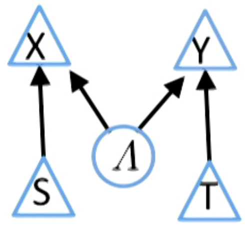
$$X \perp T|S$$

$$Y \perp S|T$$

Bell inequality constraint

$$\frac{1}{4}\sum_{x=y} P_{XY|ST}(xy|00) + \frac{1}{4}\sum_{x=y} P_{XY|ST}(xy|01)$$
$$\frac{1}{4}\sum_{x=y} P_{XY|ST}(xy|10) + \frac{1}{4}\sum_{x\neq y} P_{XY|ST}(xy|11) \leq \frac{3}{4}$$

J.S. Bell, Physics 1, 195 (1964)
Clauser, Horne, Shimony and Holte, Phys. Rev. Lett.23, 880 (1967)

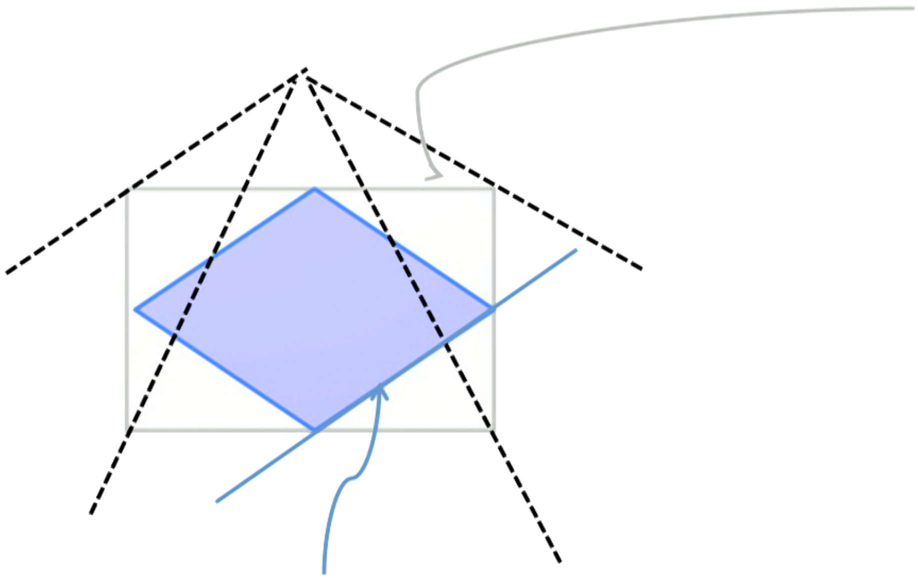


P_Λ

$P_{X|\Lambda S}$

$P_{Y|\Lambda T}$

$$P_{XY|ST} = \sum_\Lambda P_{X|S\Lambda} P_{Y|T\Lambda} P_\Lambda$$



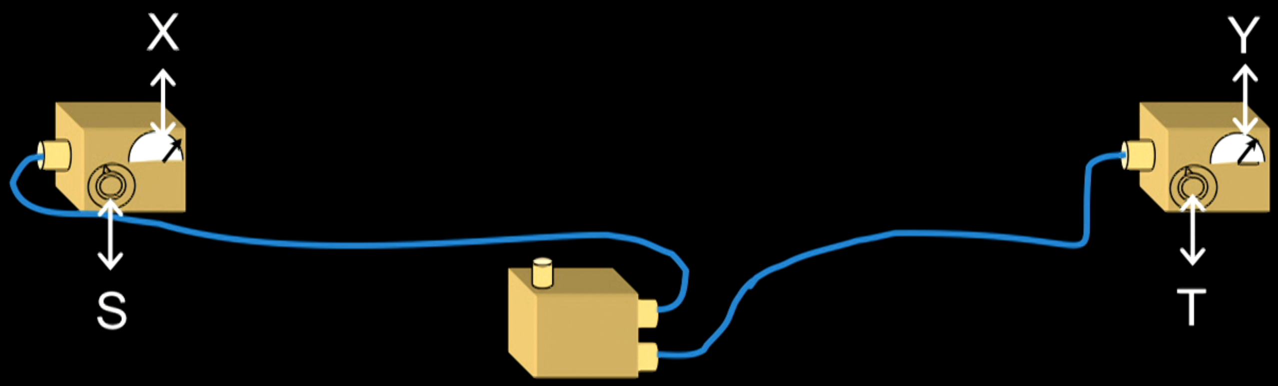
Equality constraints

$$X \perp T|S$$

$$Y \perp S|T$$

Inequality constraint

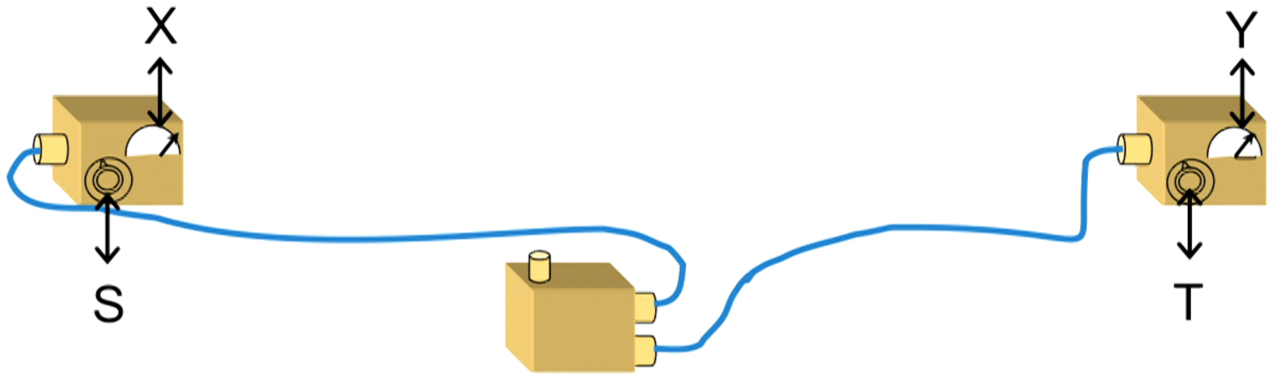
$$\frac{1}{4}\sum_{x=y} P_{XY|ST}(xy|00) + \frac{1}{4}\sum_{x=y} P_{XY|ST}(xy|01)$$
$$\frac{1}{4}\sum_{x=y} P_{XY|ST}(xy|10) + \frac{1}{4}\sum_{x\neq y} P_{XY|ST}(xy|11) \leq \frac{3}{4}$$



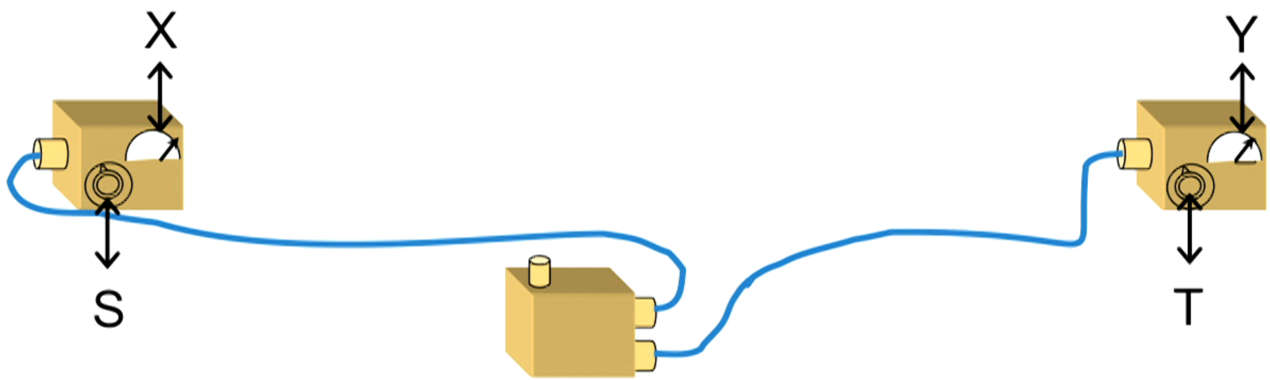
P(X,Y|S,T)

	X=0, Y=0	X=0, Y=1	X=1, Y=0	X=1, Y=1
S=0, T=0	0.427	0.073	0.073	0.427
S=0, T=1	0.427	0.073	0.073	0.427
S=1, T=0	0.427	0.073	0.073	0.427
S=1, T=1	0.073	0.427	0.427	0.073

Quantumly realizable example of correlations that *violate* the Bell inequalities

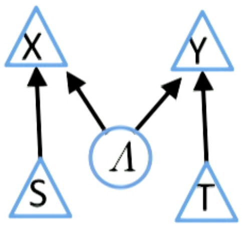


Relativity theory → no causal influence between the wings



But we still need to provide a causal explanation of the experimental statistics

Classical Causal Models



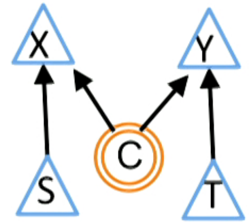
P_Λ

$P_{X|\Lambda S}$

$P_{Y|\Lambda T}$

$$P_{XY|ST} = \sum_\Lambda P_{X|S\Lambda} P_{Y|T\Lambda} P_\Lambda$$

Quantum Causal Models



ρ_C

$\rho_{X|SC}$

$\rho_{Y|TC}$

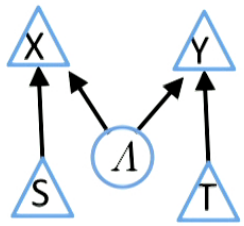
where

$$[\rho_{X|SC}, \rho_{Y|TC}] = 0$$

$$\rho_{XY|ST} = \mathrm{Tr}_C(\rho_{X|SC}\rho_{Y|TC}\rho_C)$$

Leifer, RWS, Phys. Rev. A 88, 052130 (2013)
Allen, Barrett, Horsman, Lee, RWS, Phys. Rev. X 7, 031021 (2017)

Classical Causal Models



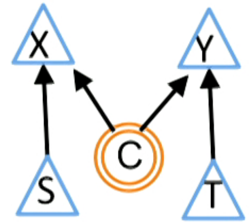
P_Λ

$P_{X|\Lambda S}$

$P_{Y|\Lambda T}$

$$P_{XY|ST} = \sum_\Lambda P_{X|S\Lambda} P_{Y|T\Lambda} P_\Lambda$$

Quantum Causal Models



ρ_C

$\rho_{X|SC}$

$\rho_{Y|TC}$

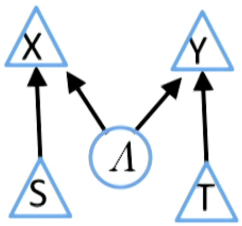
where

$$[\rho_{X|SC}, \rho_{Y|TC}] = 0$$

$$\rho_{XY|ST} = \mathrm{Tr}_C(\rho_{X|SC}\rho_{Y|TC}\rho_C)$$

Leifer, RWS, Phys. Rev. A 88, 052130 (2013)
Allen, Barrett, Horsman, Lee, RWS, Phys. Rev. X 7, 031021 (2017)

Classical Causal Models



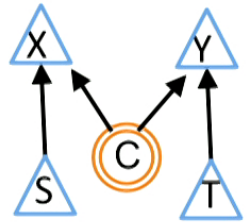
P_{Λ}
$P_{X|\Lambda S}$
$P_{Y|\Lambda T}$

$$P_{XY|ST} = \sum_{\Lambda} P_{X|S\Lambda} P_{Y|T\Lambda} P_{\Lambda}$$

$$P_{XY|ST}(xy|st) = \sum_{\lambda} P_{X|S\Lambda}(x|s\lambda) P_{Y|T\Lambda}(y|t\lambda) P_{\Lambda}(\lambda)$$

Satisfies the Bell inequalities

Quantum Causal Models



ρ_C
$\rho_{X|SC}$
$\rho_{Y|TC}$
where
$[\rho_{X|SC}, \rho_{Y|TC}] = 0$

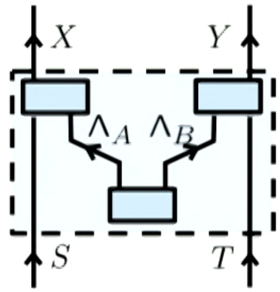
$$\rho_{XY|ST} = \mathrm{Tr}_C(\rho_{X|SC} \rho_{Y|TC} \rho_C)$$

$$P_{XY|ST}(xy|st) = \langle x|\langle y|\langle s|\langle t|\rho_{XY|ST}|x\rangle|y\rangle|s\rangle|t\rangle$$

Violates the Bell inequalities

Leifer, RWS, Phys. Rev. A 88, 052130 (2013)
Allen, Barrett, Horsman, Lee, RWS, Phys. Rev. X 7, 031021 (2017)

Classical Causal Models



$P_{\Lambda_A \Lambda_B}$

$P_{X|\Lambda_A S}$

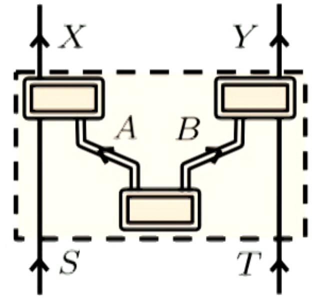
$P_{Y|\Lambda_B T}$

$$P_{XY|ST} = \sum_{\Lambda_A \Lambda_B} P_{X|S\Lambda_A} P_{Y|T\Lambda_B} P_{\Lambda_A \Lambda_B}$$

$$P_{XY|ST}(xy|st) = \sum_{\lambda_A \lambda_B} P_{X|S\Lambda_A}(x|s\lambda_A) P_{Y|T\Lambda_B}(y|t\lambda_B) P_{\Lambda_A\Lambda_B}(\lambda_A \lambda_B)$$

Satisfies the Bell inequalities

Quantum Causal Models



ρ_{AB}

$\rho_{X|SA}$

$\rho_{Y|TB}$

$$\rho_{XY|ST} = \mathrm{Tr}_{AB}((\rho_{X|SA} \otimes \rho_{Y|TB})\rho_{AB})$$

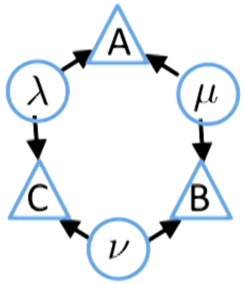
$$P_{XY|ST}(xy|st) = \langle x|\langle y|\langle s|\langle t|\rho_{XY|ST}|x\rangle|y\rangle|s\rangle|t\rangle$$

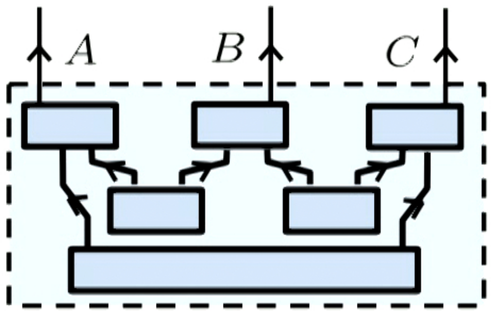
Violates the Bell inequalities

Inequalities describing compatibility with a classical causal model for causal structures different from that of the Bell scenario

Pearl, UAI proceedings (1995)
Janzing and Beth, IJQI 4, 347 (2006)
Steudel and Ay, arXiv:1010:5720
Fritz, New J. Phys. 14, 103001 (2012)
Chaves and Fritz, PRA 85 (2012)
Branciard, Rosset, Gisin, Pironio, PRA 85, 3 (2012)
Henson, Lal, Pusey, New J. Phys. 16, 113043 (2014)
Chaves, Luft, Gross, New J. Phys. 16, 043001 (2014)
Wolfe, Fritz, RWS, J. Causal Inference 7 (2019)
Fraser and Wolfe, Phys. Rev A 98, 022113 (2018)
Chaves, Carvacho, Agresti, Di Giulio, Aolita, Giacomini, and Sciarrino, Nat. Phys. 14, 291 (2018)

Classical Causal Models





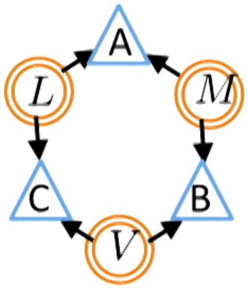
No Conditional independence relations among observed variables

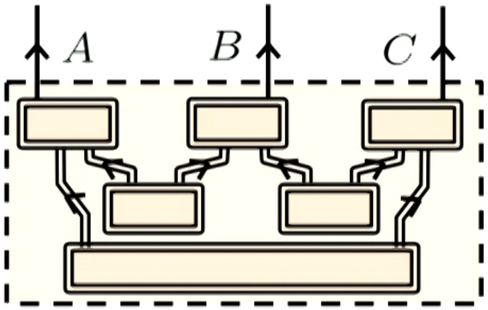
Bell-type inequality constraints:

$$\begin{aligned}
&+P_{A_lB_l}(11) - P_{A_lB_lC_lC_r}(1111) + P_{A_lB_l}(00)P_{C_lC_r}(11) + P_{C_lC_r}(01)P_{C_lC_r}(10)\\
&\quad -P_{C_lC_r}(11)P_{A_lA_rB_lB_rC_lC_r}(000000) - P_{C_lC_r}(11)P_{A_lA_rB_lB_rC_lC_r}(010100)\\
&\quad -P_{C_lC_r}(10)P_{A_lA_rB_lB_rC_lC_r}(001001) - P_{C_lC_r}(10)P_{A_lA_rB_lB_rC_lC_r}(011101)\\
&\quad -P_{C_lC_r}(01)P_{A_lA_rB_lB_rC_lC_r}(100110) - P_{C_lC_r}(01)P_{A_lA_rB_lB_rC_lC_r}(110010) \leq 0\\
&\quad +P_{C_lC_r}(00)P_{A_lA_rB_lB_rC_lC_r}(101111) + P_{C_lC_r}(00)P_{A_lA_rB_lB_rC_lC_r}(111011)
\end{aligned}$$

Wolfe, Fritz, RWS, J. Causal Inference 7 (2019)
Fraser and Wolfe, Phys. Rev A 98, 022113 (2018)

GPT Causal Models

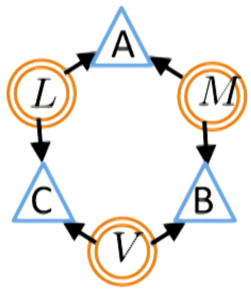


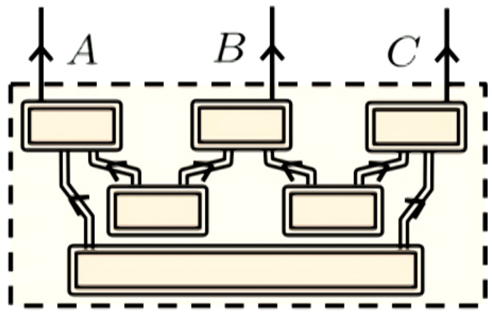


No Conditional independence relations among observed variables

Tsirelson-type inequalities can be violated

GPT Causal Models





No Conditional independence relations among observed variables

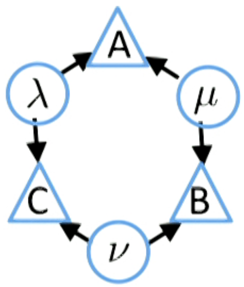
Tsirelson-type inequalities can be violated

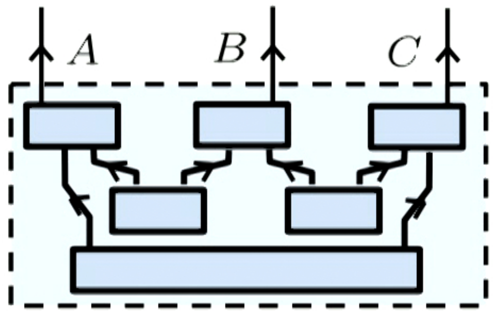
Theory-independent inequality constraints

$$I(A:B) + I(A:C) \leq H(A)$$

Henson, Lal, Pusey, New J. Phys. 16, 113043 (2014)
Wolfe, Fritz, RWS, J. Causal Inference 7 (2019)

Classical Causal Models





No Conditional independence relations among observed variables

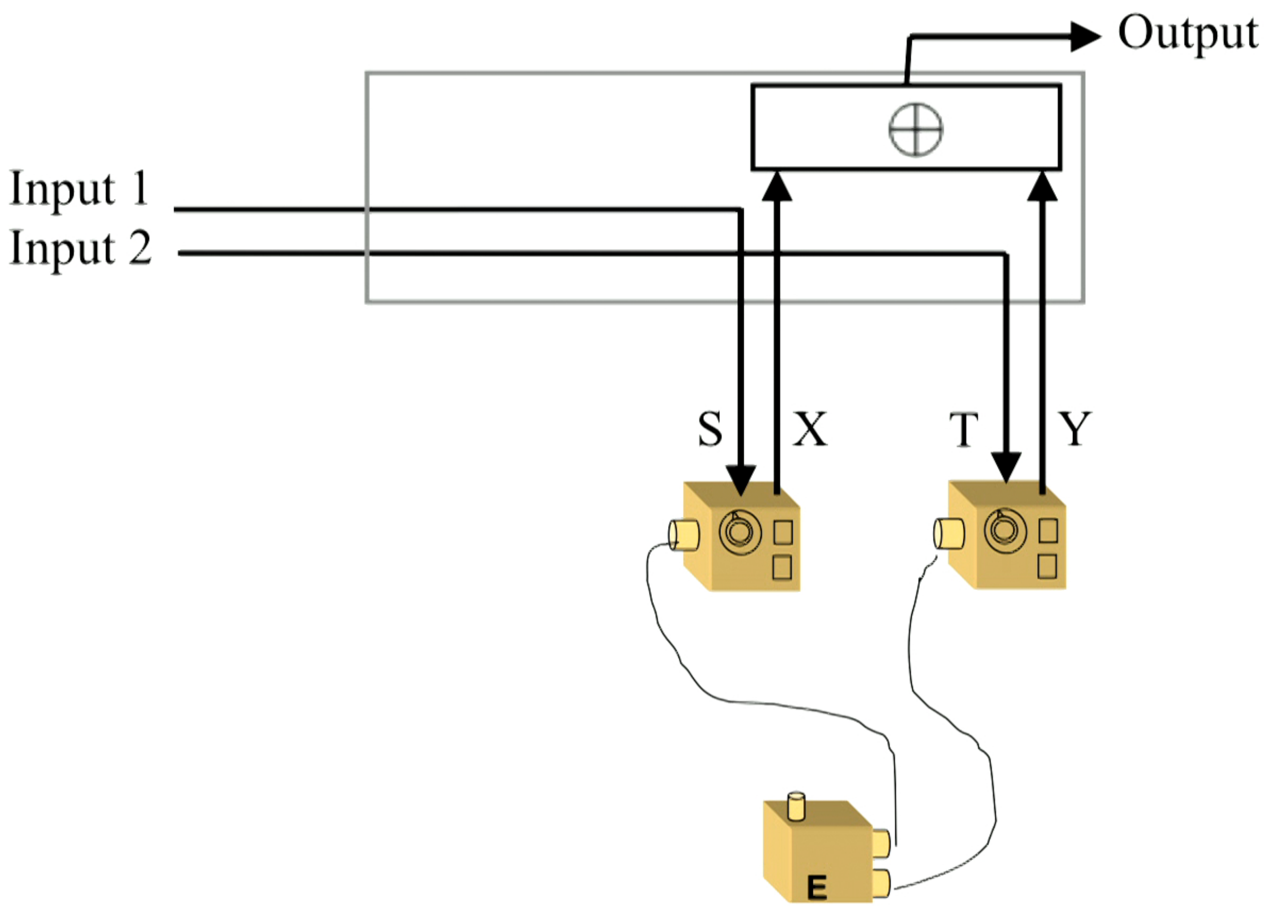
Bell-type inequality constraints:

$$
\begin{aligned}
&+P_{A_l B_l}(11) - P_{A_l B_l C_l C_r}(1111) + P_{A_l B_l}(00) P_{C_l C_r}(11) + P_{C_l C_r}(01) P_{C_l C_r}(10) \\
&\quad - P_{C_l C_r}(11) P_{A_l A_r B_l B_r C_l C_r}(000000) - P_{C_l C_r}(11) P_{A_l A_r B_l B_r C_l C_r}(010100) \\
&\quad - P_{C_l C_r}(10) P_{A_l A_r B_l B_r C_l C_r}(001001) - P_{C_l C_r}(10) P_{A_l A_r B_l B_r C_l C_r}(011101) \\
&\quad - P_{C_l C_r}(01) P_{A_l A_r B_l B_r C_l C_r}(100110) - P_{C_l C_r}(01) P_{A_l A_r B_l B_r C_l C_r}(110010) \leq 0 \\
&\quad + P_{C_l C_r}(00) P_{A_l A_r B_l B_r C_l C_r}(101111) + P_{C_l C_r}(00) P_{A_l A_r B_l B_r C_l C_r}(111011)
\end{aligned}
$$

Wolfe, Fritz, RWS, J. Causal Inference 7 (2019)
Fraser and Wolfe, Phys. Rev A 98, 022113 (2018)

Intrinsically quantum correlations as resources for computation

Browne and Anders, Phys. Rev. Lett. 102, 050502 (2009)
Raussendorf, Phys. Rev. A 88, 022322 (2013)



Proposal: A given computational architecture is doing something **intrinsically nonclassical** (though not necessarily computationally useful) if the operational statistics it generates cannot be explained by a classical causal model

e.g., computation in previous slide

Characterizing the boundary between classical and nonclassical is not enough:

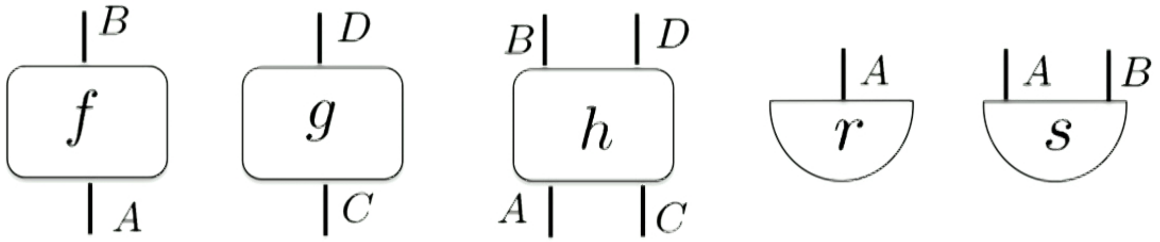
We also need to **quantify** the nonclassicality

Process theory T:

System types A, B, C,…
(including the trivial system)

Processes f, g, h, r, s, …

Closed under parallel and sequential composition



etc.

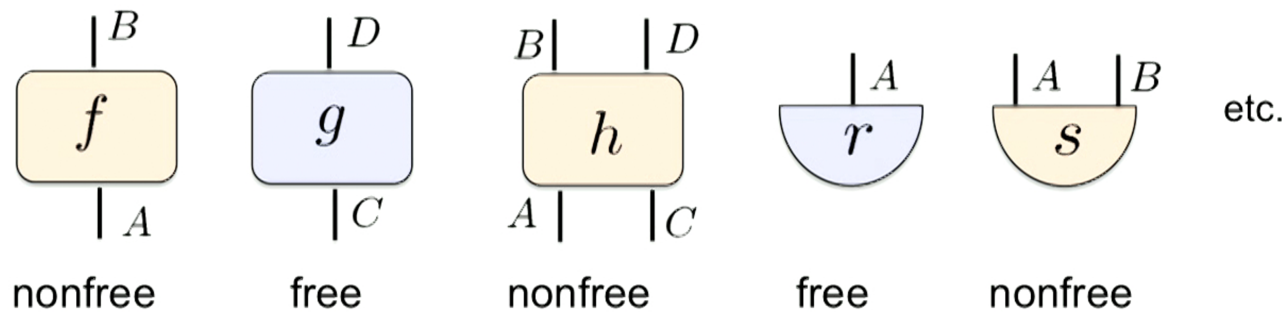
Coecke, Fritz, RWS, Information and Computation **250**, 59 (2016).

Process theory T:

System types A, B, C,…
(including the trivial system)

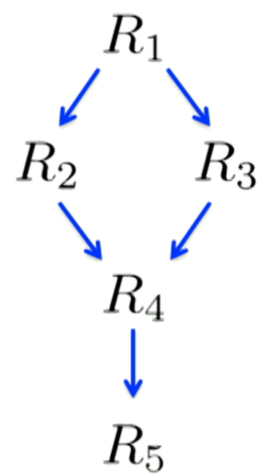
Processes f, g, h, r, s, …

Closed under parallel and sequential composition



Coecke, Fritz, RWS, Information and Computation **250**, 59 (2016).

Quotienting equivalences, one gets a partial order of resources



The nature of the pre-order teaches us about the resource

Properties of a pre-order

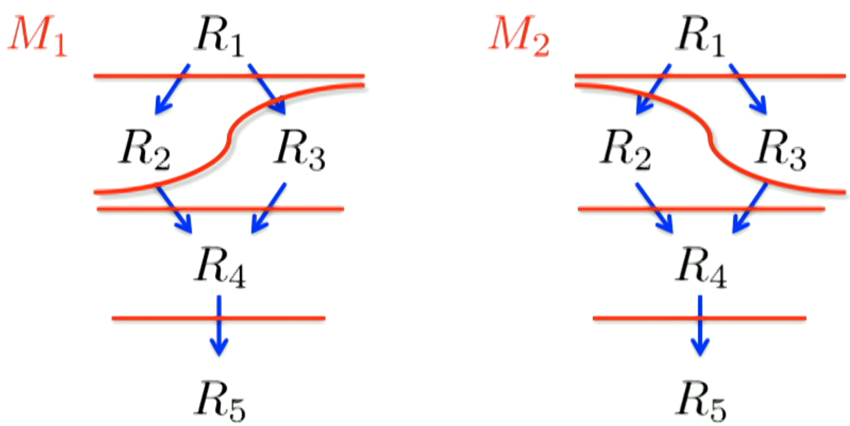
- Totally pre-ordered (no incomparable elements) or not
- weak (incomparability relation is transitive) or not
- Height (cardinality of the largest chain)
- Width (cardinality of the largest antichain)
- Locally finite (finite number of inequivalent elements between any two ordered elements) or not

Measures of a resource

Def'n: A function *M* from resources to the reals is a resource monotone if

$$\forall R_1, R_2 : R_1 \xrightarrow{free} R_2 \qquad \Rightarrow \qquad M(R_1) \geq M(R_2)$$

Equivalently, M must respect the partial order



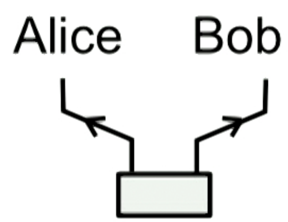
Yield and Cost constructions

Given any function *f* (monotone or not) together with a set ***S*** of resources

Yield construction
$$M[f\text{-yield}, \boldsymbol{S}](R) := \max_{R^\star \in \boldsymbol{S}} \{f(R^\star) \ \text{ s.t. } \ R \longmapsto R^\star\}$$

Cost construction
$$M[f\text{-cost}, \boldsymbol{S}](R) := \min_{R^\star \in \boldsymbol{S}} \{f(R^\star) \ \text{ s.t. } \ R^\star \longmapsto R\}$$

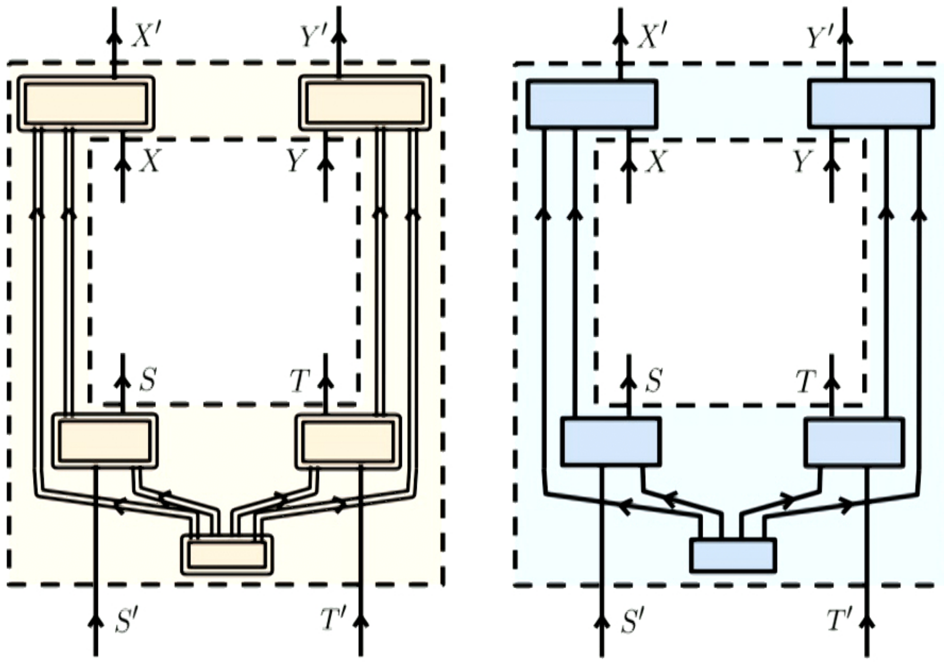
2-party network with a common cause structure



Enveloping theory:	quantum/GPT processes compatible with this causal structure
Free subtheory	classical processes compatible with this causal structure

Types of resources within a 2-party network with a common-cause structure

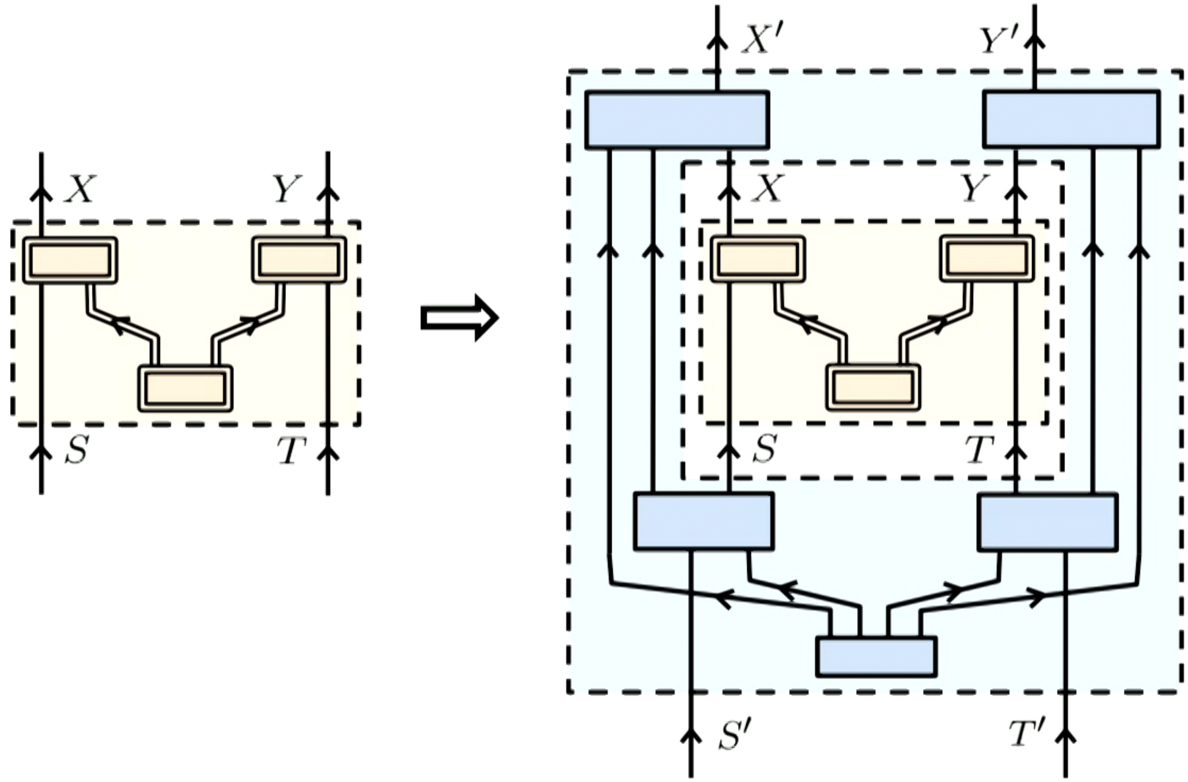
Box-to-box processing Resources:



Enveloping theory

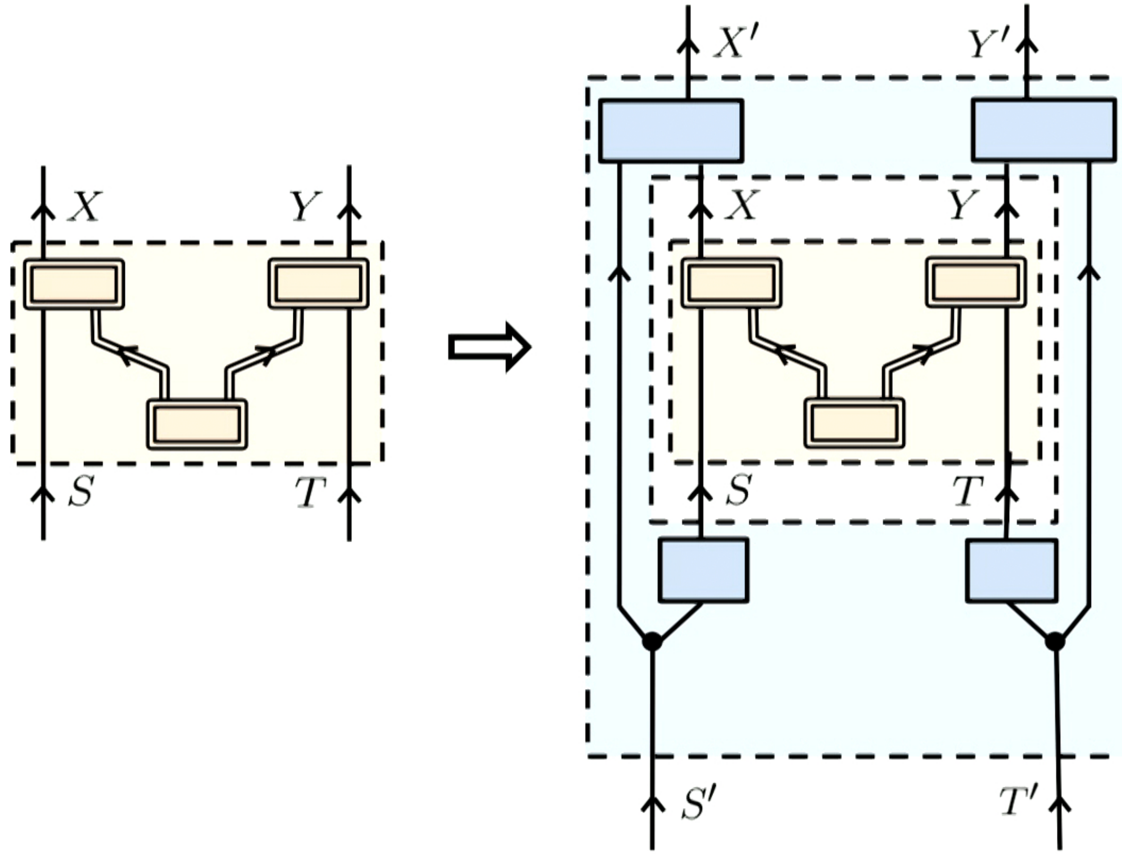
Free subtheory

Free operations taking a box to a box

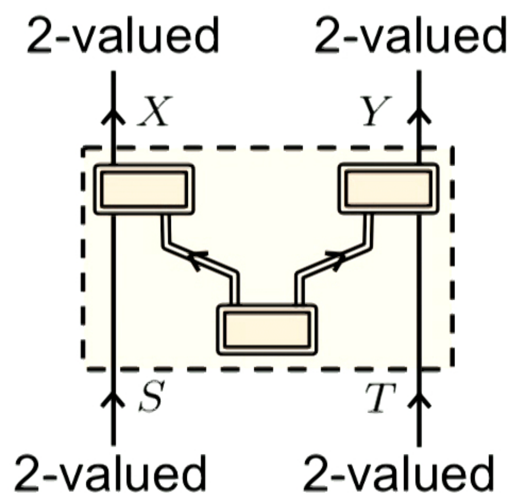


$$P_{XY|ST} \mapsto P_{X'Y'|S'T'}$$

Locally deterministic operations (LDO)



Boxes of type-$\binom{2\ 2}{2\ 2}$

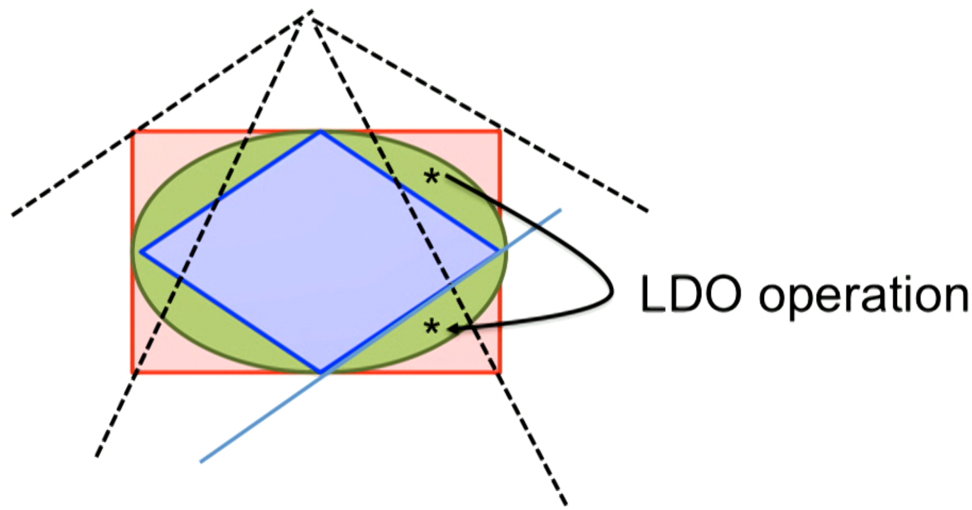


$\boldsymbol{S}^{G}_{\binom{2\ 2}{2\ 2}}$:= set of boxes of type-$\binom{2\ 2}{2\ 2}$ compatible with GPT causal model

A facet-defining Bell functional

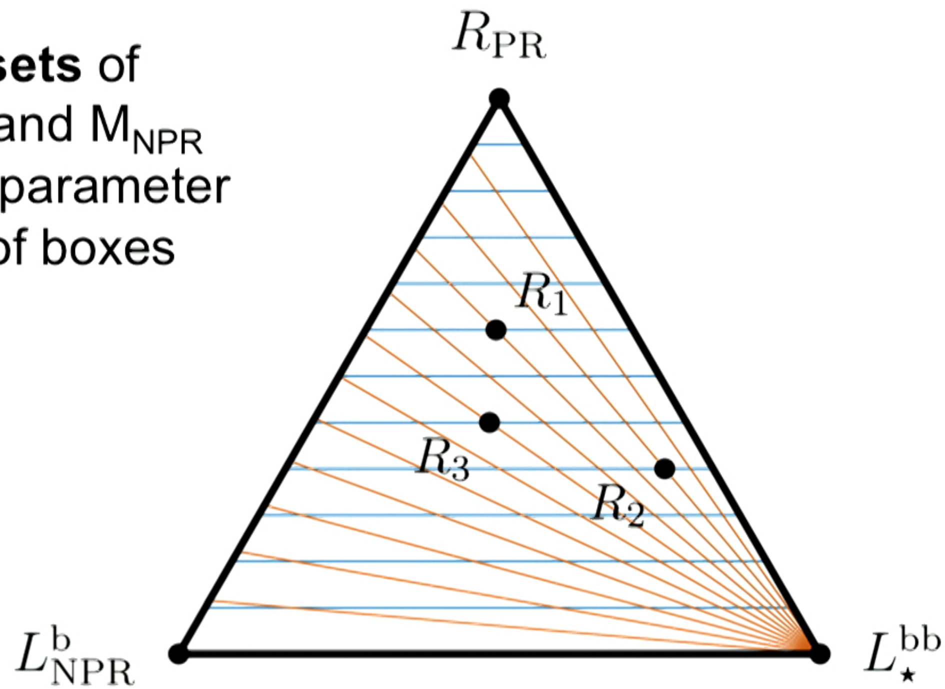
$$\mathrm{CHSH}(R) := \langle A_0 B_0 \rangle + \langle A_0 B_1 \rangle + \langle A_1 B_0 \rangle - \langle A_1 B_1 \rangle$$

where $\langle A_t B_s \rangle := \sum_{x,y \in \{0,1\}} (-1)^{(x \oplus y)} P_{XY|ST}(xy|st).$



Therefore, not a monotone!

Level sets of M_{CHSH} and M_{NPR} for a 2-parameter family of boxes



Question: In terms of quantifying resourcefulness of correlations for computation, is one of M_{CHSH} and M_{NPR} more relevant than the other?

Open problems

2-party with common cause (bipartite Bell)

- Determine which parameters of a box are necessary and sufficient to determine its equivalence class
- Find a complete set of monotones for boxes of type $\binom{2\ 2}{2\ 2}$
- Consider interconversion of boxes and states
- Consider asymptotic conversion, catalysis, etcetera

N-party with common cause (multipartite Bell)

- What new features emerge?

N-party with other causal structures (triangle, bilocality, etc.)

- What new features emerge? (note: nonconvex!)

Applications to understanding the type of nonclassicality associated to computational advantages?