Title: Symmetry-protected topologically ordered phases for measurement-based quantum computation

Speakers: Akimasa Miyake

Collection: Symmetry, Phases of Matter, and Resources in Quantum Computing

Date: November 26, 2019 - 10:15 AM

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Abstract: Measurement-based quantum computation (MBQC) is a computational scheme to simulate spacetime dynamics on the network of entanglement using local measurements and classical communication. The pursuit of a broad class of useful entanglement encountered a concept of symmetry-protected topologically ordered (SPTO) phases in condensed matter physics. A natural question is "What kinds of SPTO ground states can be used for universal MBQC in a similar fashion to the 2D cluster state?" 2D SPTO states are classified not only by global on-site symmetries but also by subsystem symmetries, which are fine-grained symmetries dependent on the lattice geometry. Recently, all ground states within SPTO cluster phases on the square and hexagonal lattices have been shown to be universal, based on the presence of subsystem symmetries and associated structures of quantum cellular automata. Motivated by this observation, we analyze the computational capability of SPTO cluster phases on all vertex-translative 2D Archimedean lattices. We show that there are four different "fundamental" subsystem symmetries, called here ribbon, cone, fractal, and 1-form symmetries, for cluster phases, and the former three types one-to-one correspond to three classes of Clifford quantum cellular automata. We conclude that nine out of the eleven Archimedean lattices support universal cluster phases protected by one of the former three symmetries, while the remaining lattices with the 1-form symmetry have a different capability related to error correction.

Matter, Symmetry, Resource Workshop @ PI, November 26 (2019)

Symmetry-protected topologically ordered phases for measurement-based quantum computation

Akimasa Miyake

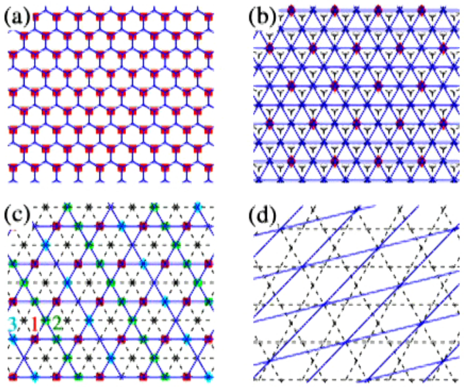
[1] CQuIC, University of New Mexico

Center for Quantum Information and Control

(Incomplete) zoo of universal entanglement for MBQC

Resource question: what is an entanglement resource useful for computation?

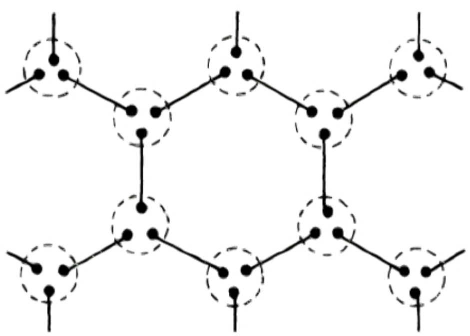
graph states, connection to graph theory



Raussendorf, Briegel, 2001

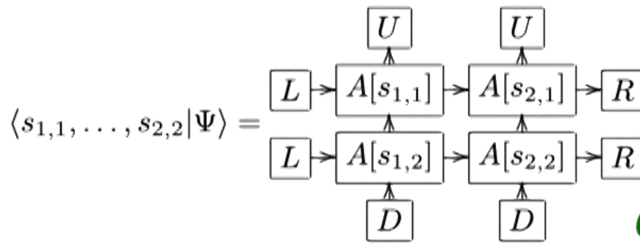
Van den Nest, Miyake, Duer, Briegel, 2006 <- entanglement monotones

AKLT states by two-body Hamiltonian (with exp. decaying two-point correlation)



Wei, Affleck, Raussendorf, 2011; Miyake 2011

tensor network states



Gross, Eisert, 2007

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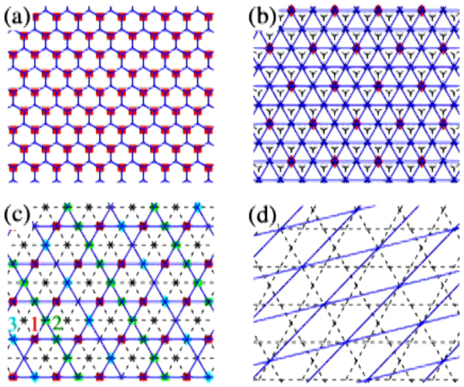

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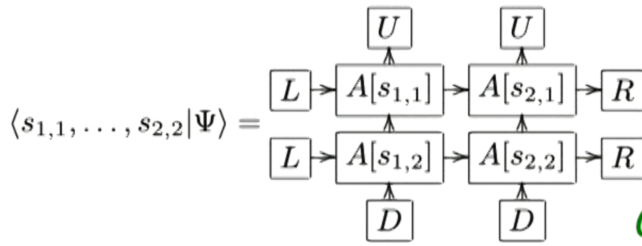

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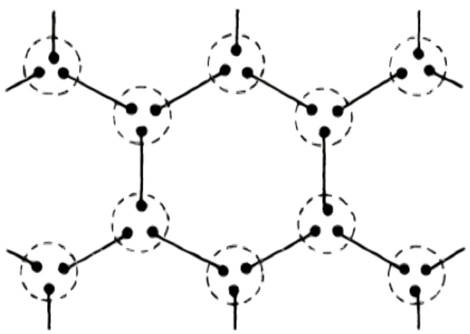
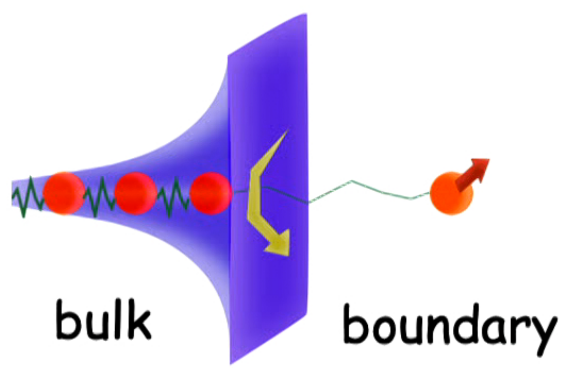


Wei, Affleck, Raussendorf, 2011; Miyake 2011

symmetry-protected topologically ordered (SPTO) phase



Miyake 2010;
Else, Schwarz, Bartlett, Doherty 2012;
Miller, Miyake, 2015;
Stephen et al., 2017;
Raussendorf et al., 2019 ← newly for 2D

Motivation of the talk

Nature isn't classical, and seems to have an own way to make computation robust

computation with quantum error correction

robust against a few arbitrary local errors under assumption of locality of errors and Markovianity

stabilizer subspace by discretization of Pauli errors, given non-Clifford resources like magic states

epsilon neighborhood of pure-state computation and very low uncertainty/entropy

Real QC devices may encounter global crosstalk (e.g., ion traps) and 1/f noise (e.g., transmons). -> complementary approach

computation in a computational phase of matter

robust against ”symmetric” errors (similar to decoherence free subspace)

ground states in a phase, whose macroscopic features are common. ground states are not stabilizer states

gate sequence is insensitive to states, and only overhead differs. Any mixed state by mixture of ground states works as well.

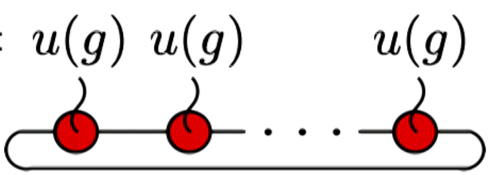
Progress for universal MBQC, but no fault-tolerance results yet.

Symmetry-protected topological order

SPTO by spin-1/2 systems on a lattice

- Symmetry
  - Family of states with a common symmetry
  - Generally consider finite abelian groups G such as copies of $\mathbb{Z}_2$.
- Global symmetries act in an "on-site" manner.

$S(g) =$



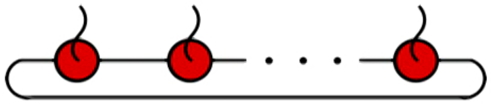
$u(g)$ = Onsite representation

$S(g)$ = Global representation

- X-type symmetries of cluster states define SPTO.
  - Want to do MBQC in a way that is compatible with symmetry.
  - Mainly restrict to X- measurements. (non-network MBQC without Z-measurement)

Symmetry-protected topological order

- Topological
  - For periodic boundary conditions the ground state is unique

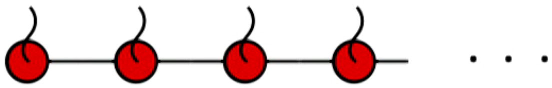


$$H = -\sum_{j=1}^{N} Z_{j-1} X_j Z_{j+1}$$

*Local Hamiltonian terms give complete set of commuting observables.

invariant under global X's ($\mathbb{Z}_2$), or even-site X's and odd-site X's ($\mathbb{Z}_2 \times \mathbb{Z}_2$)

  - Degeneracy of ground states occurs for open boundary (fractionalized edge states)



$$H = -\sum_{j=2}^{N-1} Z_{j-1} X_j Z_{j+1}$$

$$X^L = X_1 Z_2 \simeq \otimes_j X_{2j+1}$$
$$Z^L = Z_1 \simeq \otimes_j X_{2j}$$

bulk-boundary correspondence in MBQC:
universality at boundary in terms of bulk symmetric entanglement

Symmetry-protected topological order

- Topological considerations of symmetry

  - For 1D SPTO the symmetry is represented projectively at the boundary

Linear Representation → $u(g)u(h) = u(gh)$

Projective Representation → $V(g)V(h) = \omega(g,h)V(gh)$

where $\omega(g,h) \in U(1)$

$\mathbb{Z}_2 \times \mathbb{Z}_2$

Linear Rep. → $xy = z$

Projective Rep. → $XY = iZ$

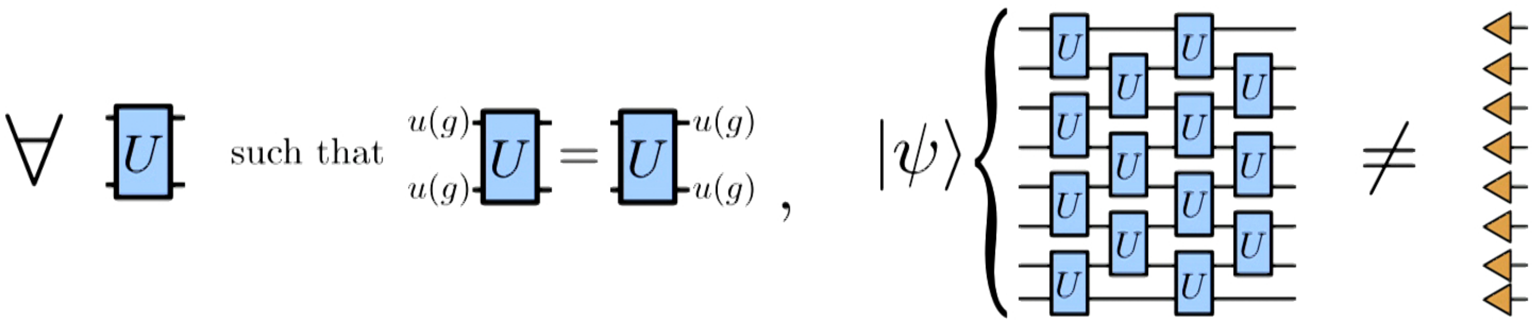
  - The only non-trivial projective representation of $\mathbb{Z}_2 \times \mathbb{Z}_2$ is equivalent to the Pauli matrices.
  - For each copy of $\mathbb{Z}_2 \times \mathbb{Z}_2$ we get a qubit degree of freedom at the edge!

Chen, Gu, Wen, PRB 83, 035107 (2011).
Schuch, Pérez-García, Cirac, PRB 84, 165139 (2011).
Pollmann, Berg, Turner, Oshikawa, PRB 85, 075125 (2012).

Phase of symmetry-protected topological order

- Symmetry-protected
  - No symmetry respecting perturbation can lift the degeneracy.
  - Ground states in same phase related by a symmetric local unitary (SLU).

constant-depth local quantum circuit



- No intrinsic topological order!

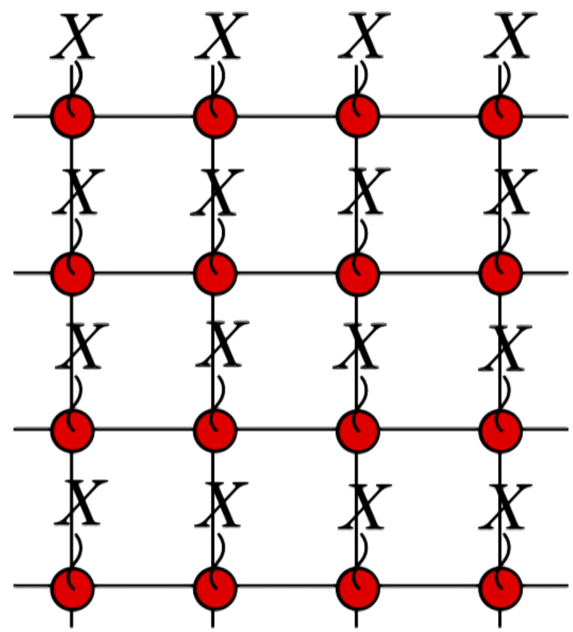
Chen, Gu, Wen, PRB 82, 1555138 (2010).

Symmetries of 2D cluster states

- Cluster states have many fancy symmetries
- Biggest symmetry group is full stabilizer group.

$$\mathcal{S} = \langle \{ X_v \bigotimes_{l \in \mathcal{N}(v)} Z_l \ |\forall v \} \rangle$$

- Smallest is global $\mathbb{Z}_2$ symmetry.
  - Apply all stabilizers!
  - One symmetry generator

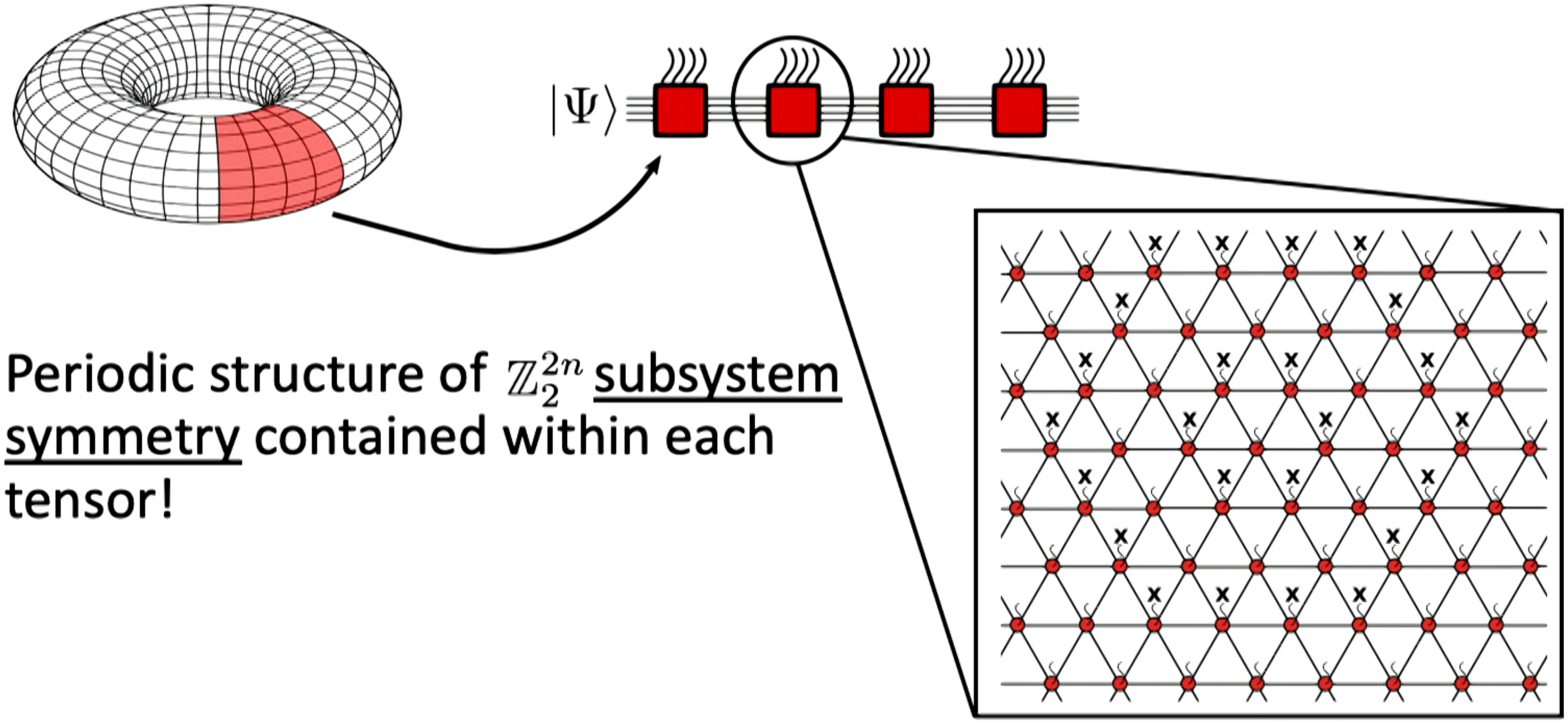


*Note for odd degree lattice the action is a global Y.

From 2D to quasi-1D

NEW: Raussendorf, Okay, Wang, Stephen, Nautrup, PRL 122, 090501 (2019)

- Embed a 2D lattice cluster state on a torus and group together an $n \times n$ block of sites.



- Periodic structure of $\mathbb{Z}_2^{2n}$ <u>subsystem symmetry</u> contained within each tensor!

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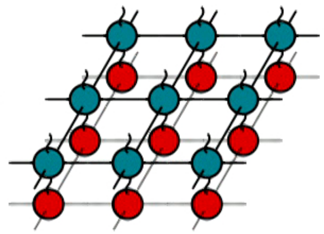
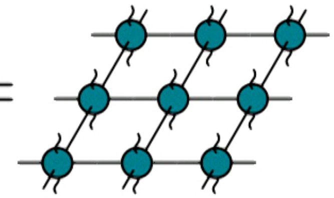
MBQC with the phase

early works in Haldane phase
Miyake, PRL 105, 040501 (2010)
Bartlett, Brennen, Miyake, Renes, PRL 105, 110502 (2010)

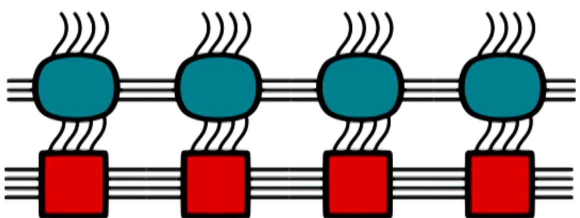
- Consider all states connected by an SLU to the cluster state (RG fixed point state).

$|\phi\rangle = U_\phi|\psi_C\rangle =$ 

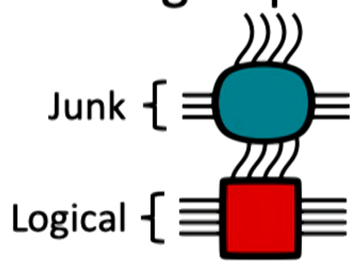
$U_\phi =$ 

polynomial parameters

- In the quasi-1D picture (by contracting a spatial direction),



One period of QCA cycles



$$= \sum_{\mathbf{j}} C(\mathbf{j}) \otimes B(\mathbf{j}) \otimes |\mathbf{j}\rangle$$

Measurement in superposition basis will couple subsystems

2D SPTO phase of cluster states by sublattice symmetry

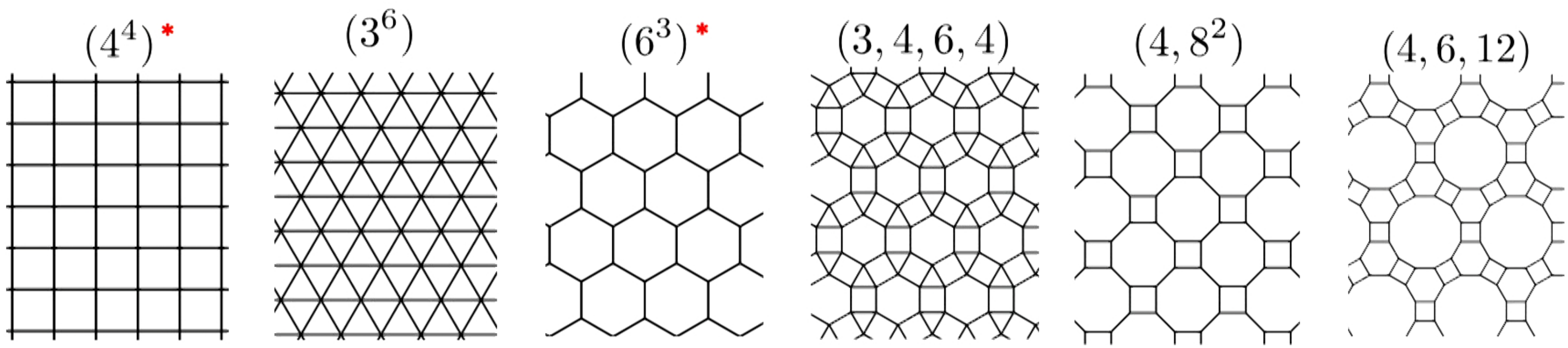
arXiv:1907.13279

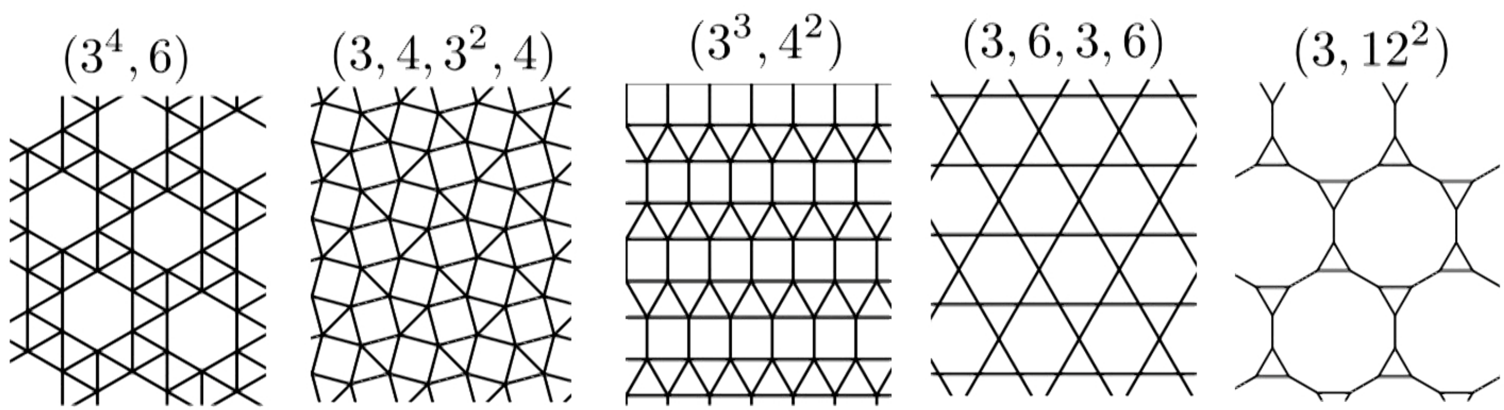
working with

Austin Daniel
Rafael Alexander

When 2D SPTOs meet lattice geometry

Archimedean lattices: 11 vertex translative lattices in 2D (each vertex locally looks the same).





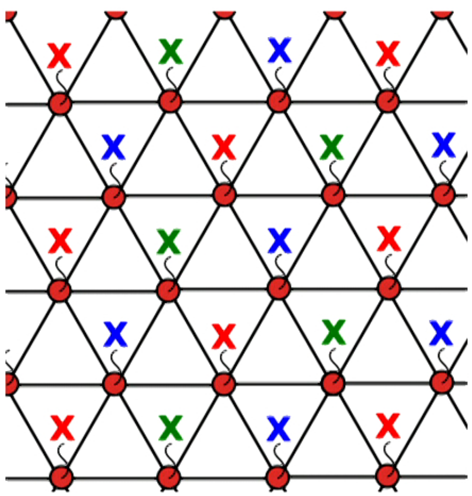
*Previously known

Raussendorf et al.,
PRL 122, 090501 (2019)
Devakul, Williamson
PRA 98, 022332 (2018)

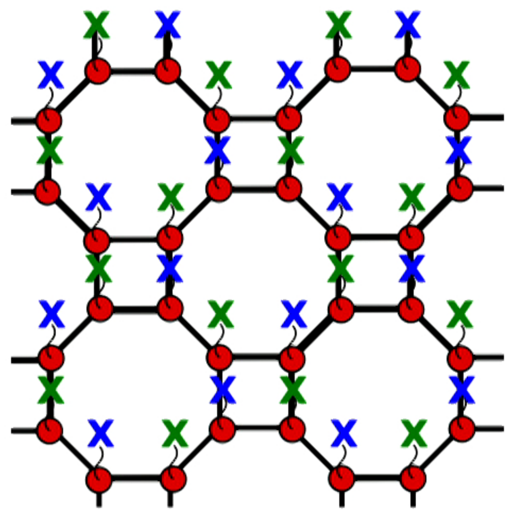
Symmetries of cluster states

- 1/k-Fractional symmetry.
  - On a k-colorable graph, apply stabilizers on each vertex of a given color.

$\mathbb{Z}_2 \times \mathbb{Z}_2 \times \mathbb{Z}_2$



$\mathbb{Z}_2 \times \mathbb{Z}_2$



k-colorable

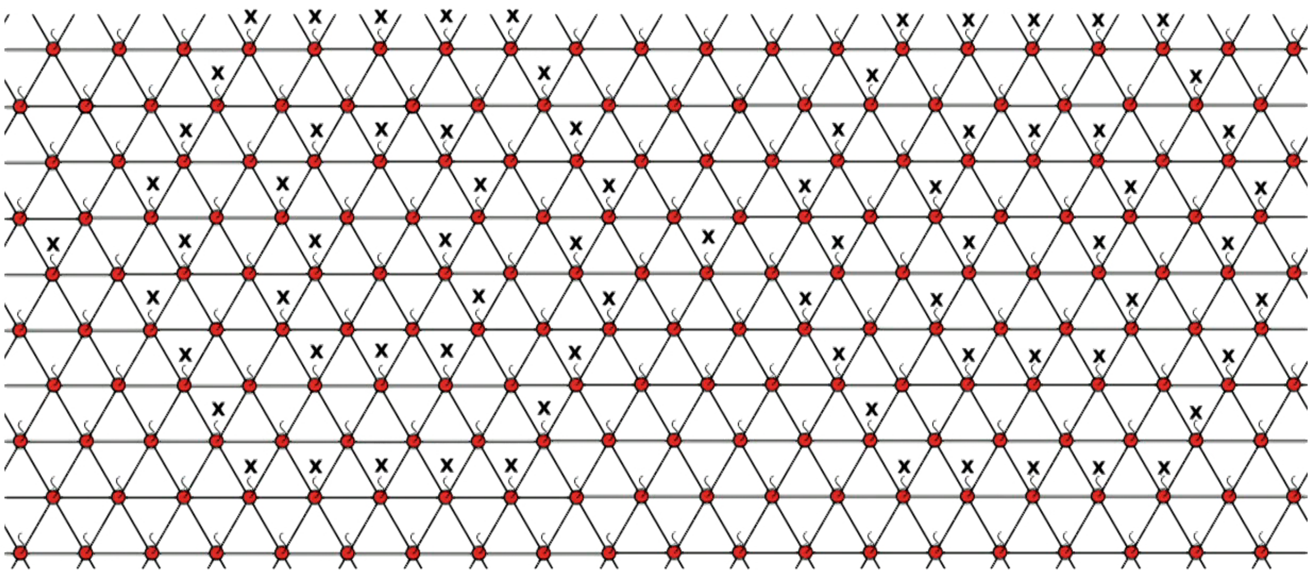
$\Downarrow$

$\mathbb{Z}_2^k$

Symmetries of cluster states

- Subsystem symmetry
  - Apply stabilizer on some site. Try to add as few more to cancel all Z's.
  - Periodic structure for periodic boundaries.
  - There are three fundamental symmetries: Ribbon, Cone, Fractal

circumference n (space)



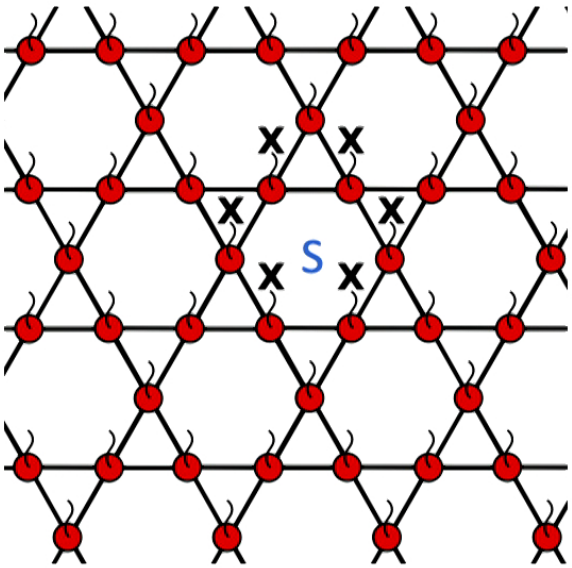
N (time) > n

$$\mathbb{Z}_2^{2n}$$

*Symmetry group is subextensively large.

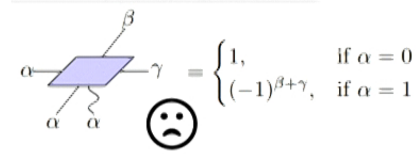
Symmetries of cluster states

- 1-form symmetry.
  - Closed loops of X - operators.
  - Deformable = Product of two loops is a bigger loop

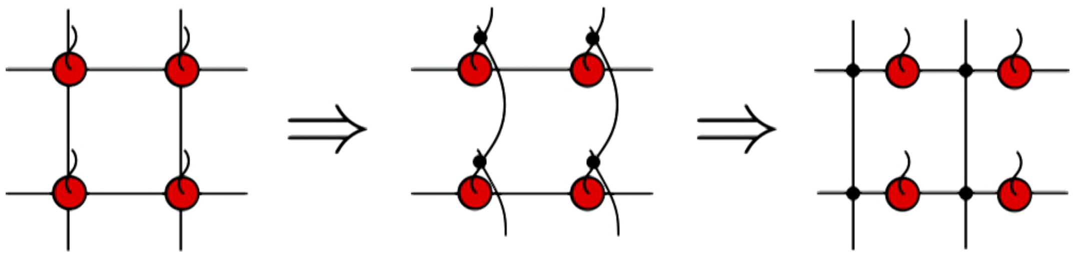


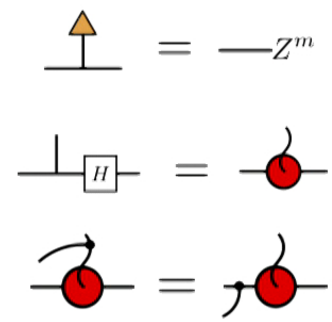
Convenient tensor networks for 2D cluster states

As opposed to…

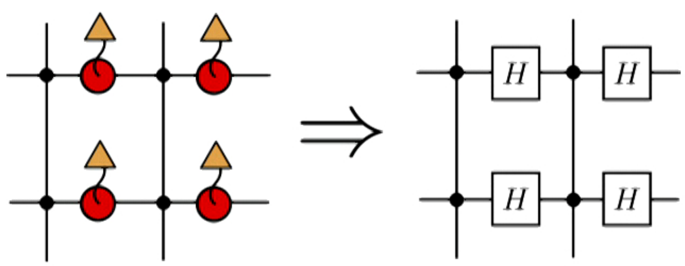


- Think of 2D cluster state as coupled 1D cluster states.





- X- basis measurements turn the TN into a Clifford quantum cellular automaton.



Raussendorf, Okay, Wang, Stephen, Nautrup, PRL 122, 090501 (2019)

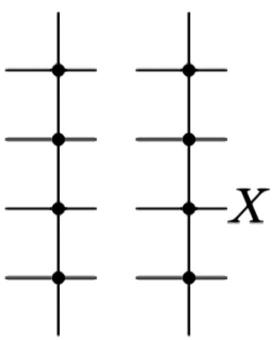
Quantum Cellular Automata

- Clifford QCA can be classified into 3 types.

Guetschow et al., JMP 51, 015203 (2010)
Stephen et al., 1806.08780

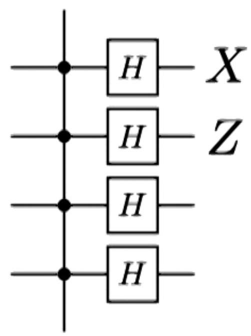
Periodic

- Period is constant and independent of system size n



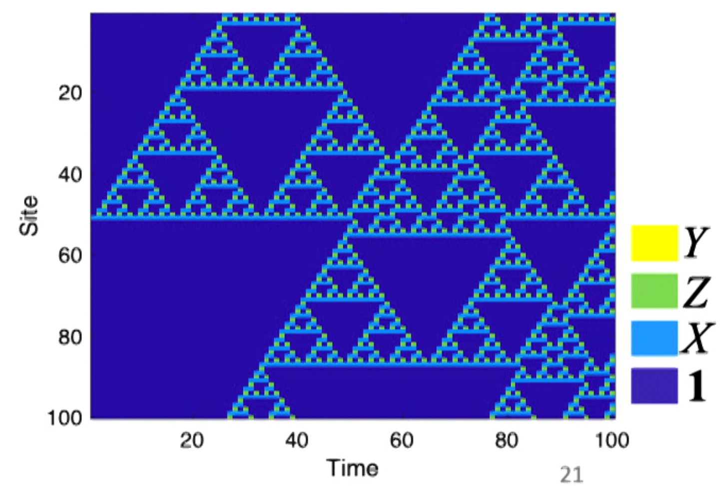
Glider

- Supports gliders (eigenoperators up to translation)
- Period linear in system size n



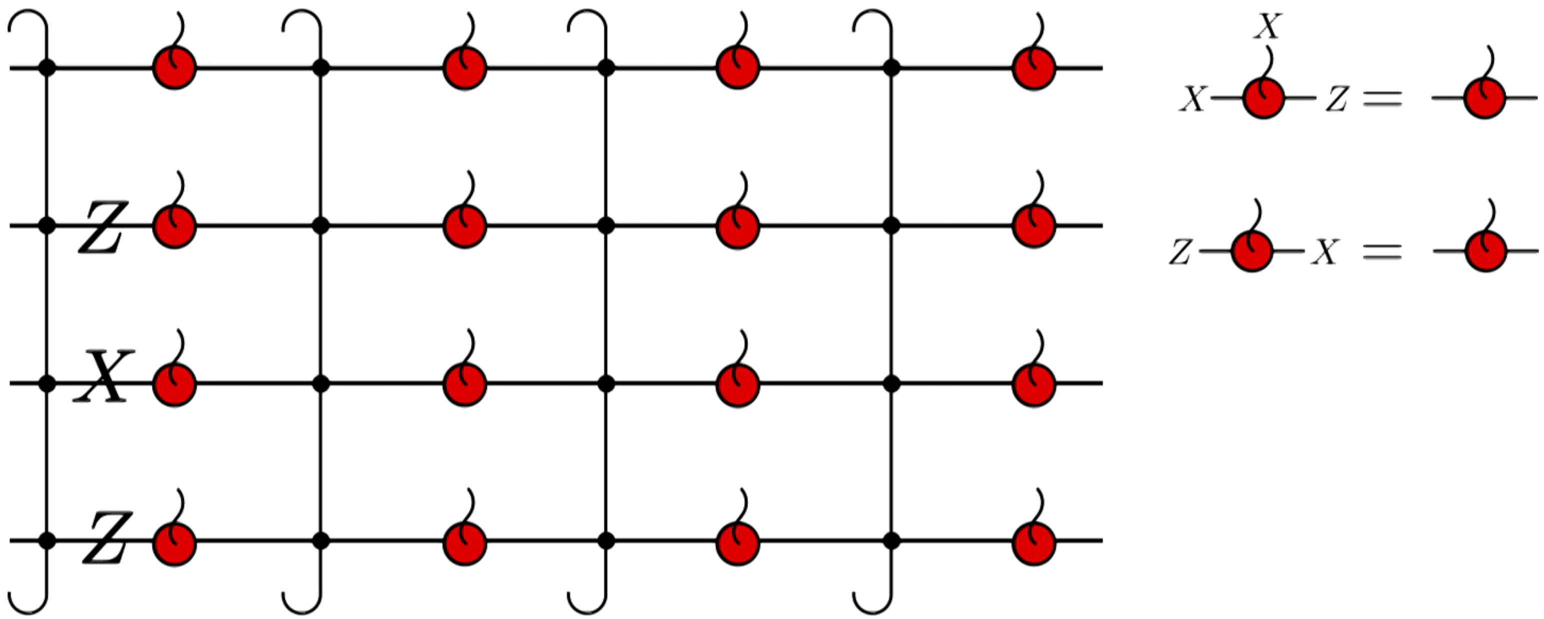
Fractal

- Operator support is fractal
- Period varies wildly



QCA and subsystem symmetries

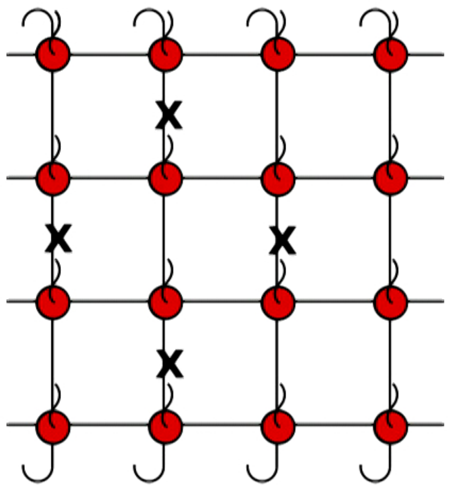
- There is a 1-1 correspondence between QCA evolution and subsystem symmetries of cluster phases.



QCA and subsystem symmetries

subsystem symmetry of a cluster phase guarantees a <u>common</u> Clifford QCA structure, regardless of states.

- 2n generators of Pauli group = 2n real-space symmetry generators.
- Subsystem symmetries define a SPT phase, cluster phase, universal for MBQC.

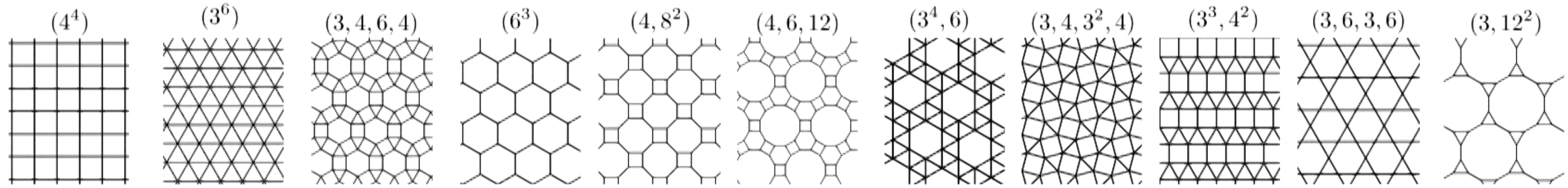


2D cluster SPTO phase by subsystem symmetry

Daniel, Alexander, Miyake, arXiv:1907.13279

All ground states (which are not necessarily stabilizer states) in 2D cluster phase on a 2D Archimedean lattice with Cone/Fractal subsystem symmetry are universal for MBQC.

- Archimedean lattices are vertex translative (each vertex locally looks the same).



Real space symmetry	Real space symmetry group	Virtual space QCA structure	Computational phase	Lattices
Ribbon	$\mathbb{Z}_2^{2n}$	Periodic	Yes	Rectangular*
Cone	$\mathbb{Z}_2^{2n}$	Glider	Yes	(4^4)*, (3^6), $(3,4,6,4)$
Fractal	$\mathbb{Z}_2^{2n}$	Fractal	Yes	(6^3)*, $(4,8^2)$, $(4,6,12)$, $(3^4,6)$, $(3,4,3^2,4)$, $(3^3,4^2)$
1 - Form	$\mathbb{Z}_2^{\mathcal{O}(nN)}$	No	No	$(3,6,3,6)$, $(3,12^2)$
$\frac{1}{k}$Fractional	$\mathbb{Z}_2^k$	-	-	All

*Previously known

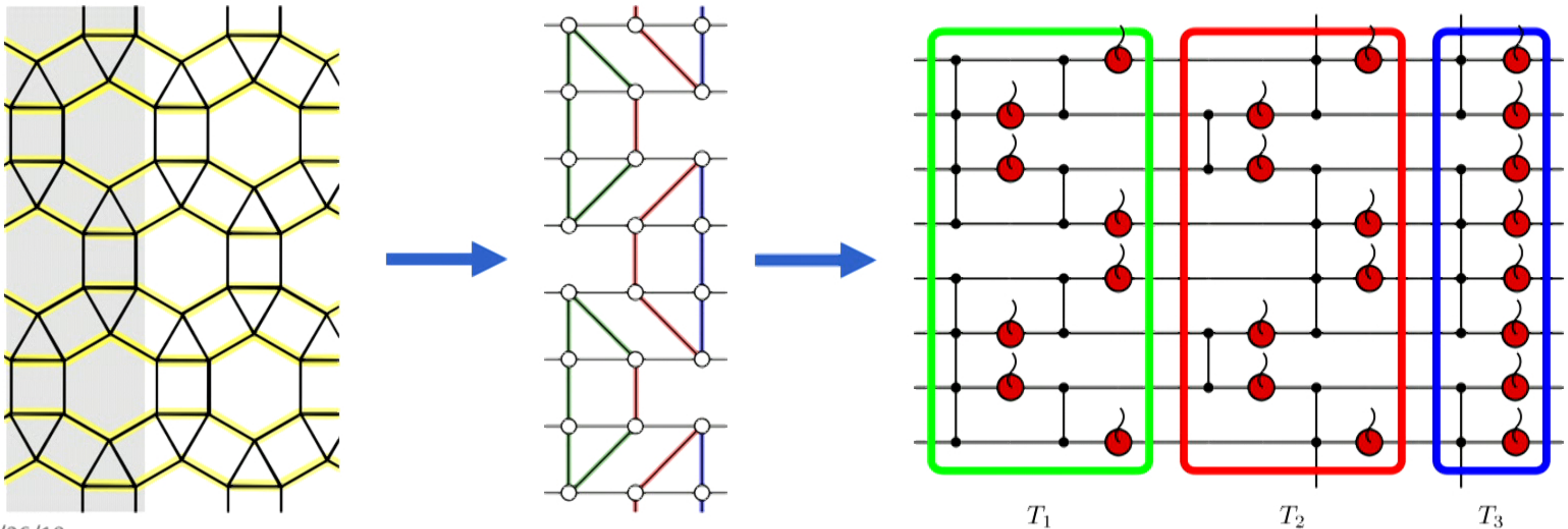
Raussendorf et al., PRL 122, 090501 (2019)
Devakul, Williamson PRA 98, 022332 (2018)
Stephen et al., 1806.08780

Lattices supporting glider QCA

Note: line-like symmetry does not provide glider QCA in general.

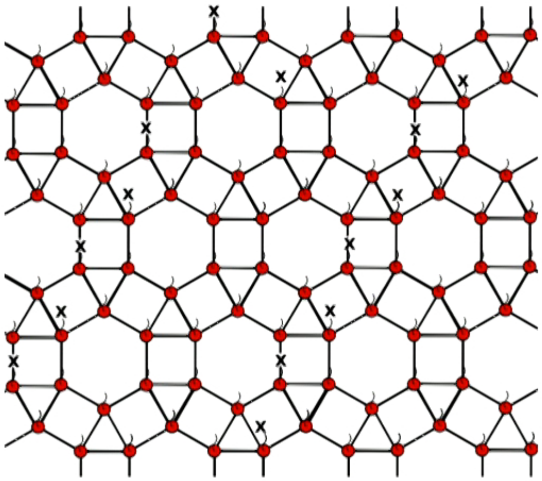
i.e., conjecture of Stephen et al., 1806.08780 is false

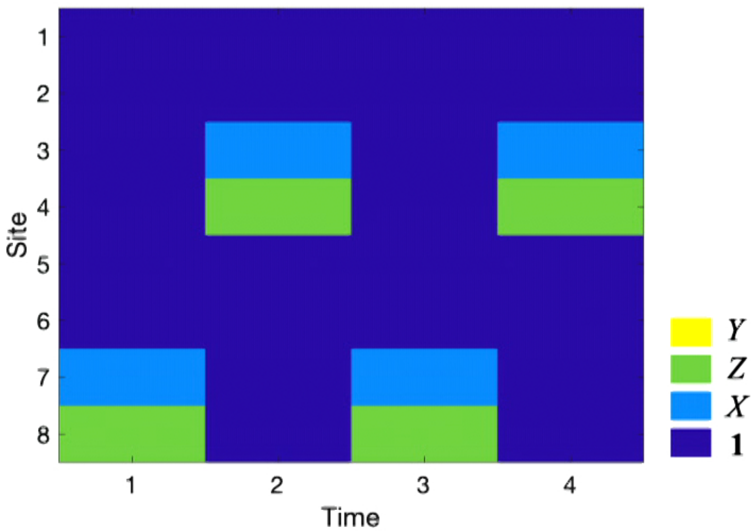
- Consider the $(3,4,6,4)$ lattice. We first construct tensor network description.



Lattices supporting glider QCA

- Gliders are operators whose support is translated by the QCA.



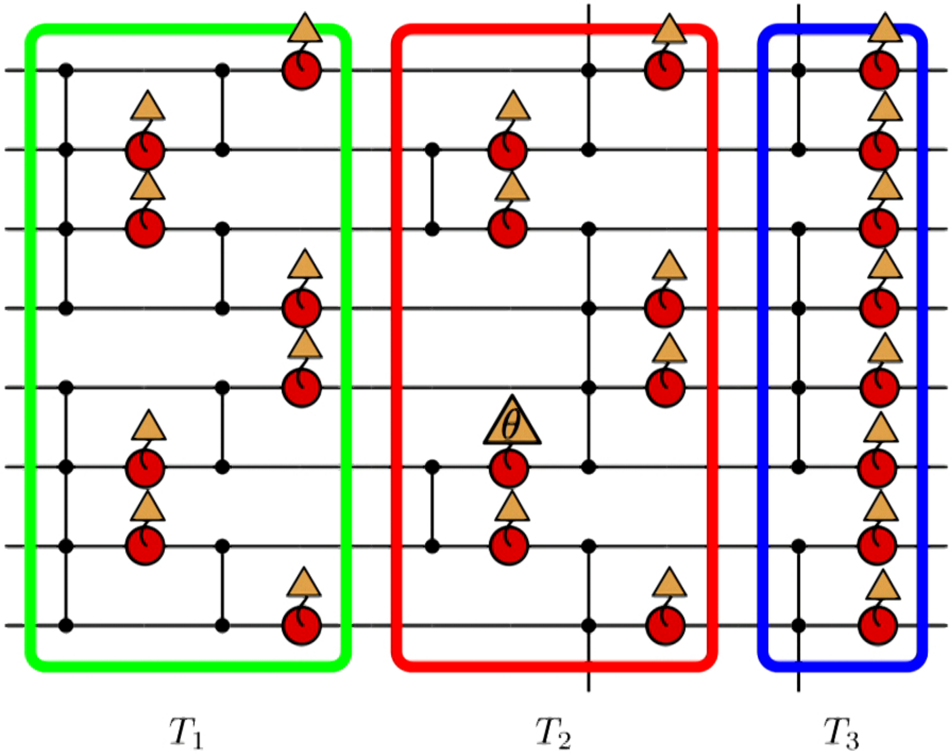


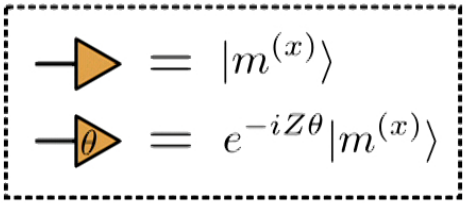
11/26/19

Note: line-like symmetry does not provide glider QCA in general.

Lattices supporting glider QCA

- **Universal gates achieved via measurement in (X,Y)-plane.**





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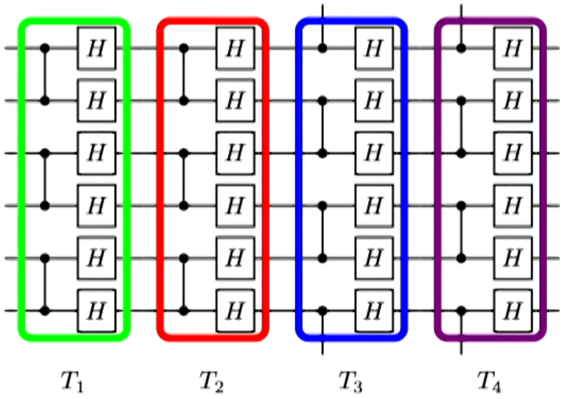
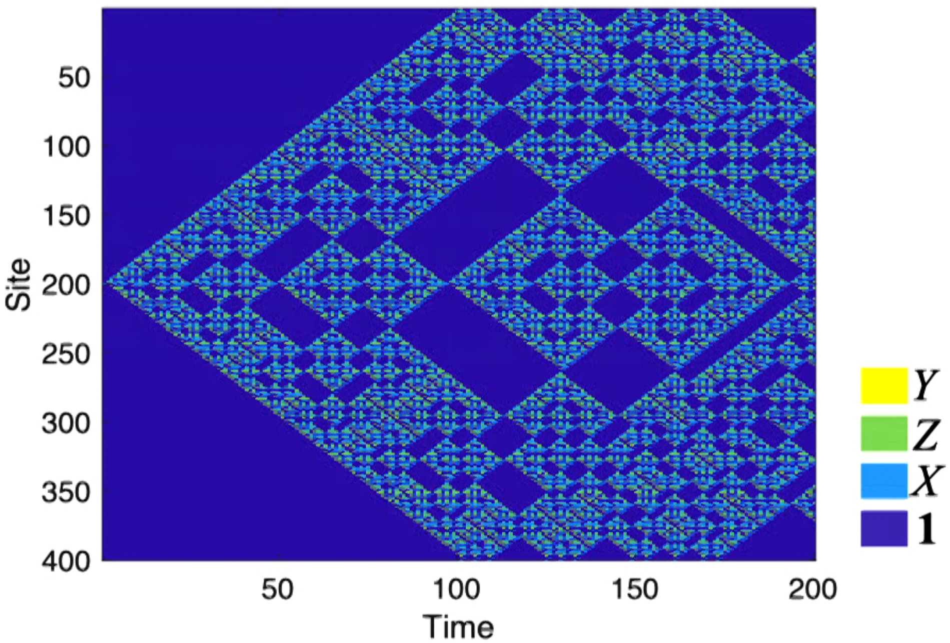
Lattices supporting fractal QCA

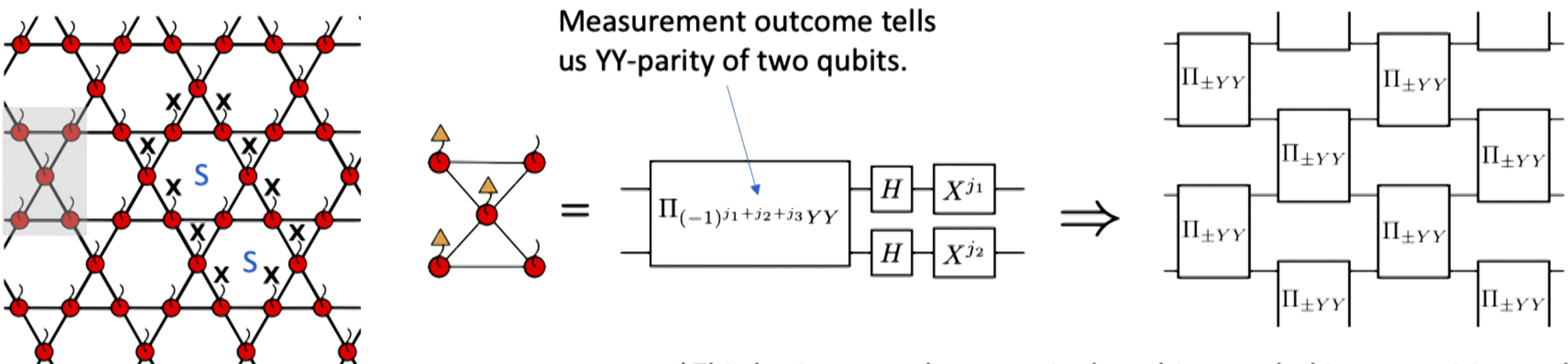
- Fractal QCA are characterized by operators supported on a fractal subset of space-time points.





Lattices with no QCA structure

- Lattices with 1-form symmetries ("gauge condition") prevent unital QCA.
  - 1-form symmetries are closed loops of operators
  - Symmetry operators are deformable as multiplying two together gives a larger loop.
  - Pauli-X measurements at the edge implement YY parity measurements.

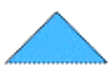


*This lattice can teleport a single qubit encoded in a repetition code.

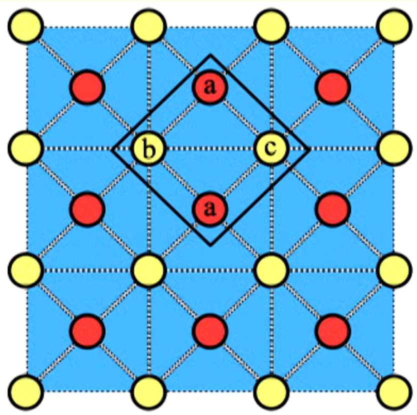
higher-form SPTOs in 2D

Miller and Miyake, Nature Quantum Information 2, 16036 (2016)

Discovery of new universal entanglement for MBQC using only single-qubit Pauli measurements, in terms of 2D higher-form SPTO.



3-body = CCZ



	2D Cluster	Union Jack	3-coycle, 3-colorable
SPTO	1D	2D	2D
Universal?	✓	✓	✓
Pauli Universal?	✗	✓	✓

$$S_{UJ}^{(i)} = X^{(i)} \bigotimes_{(j,k)\in \mathrm{tri}(i)} CZ^{(j,k)},$$

→ general 3-body diagonal gates with 3-cocyle

Renormalization fixed-point states on 3-colorable lattice are universal entanglement for MBQC if and only if 2D SPTO is nontrivial.

Miller and Miyake, Phys. Rev. Lett. **120**, 170503 (2018)

Summary and Outlook

Summary: MBQC with symmetry without fine-tuning

All ground states (which are not necessarily stabilizer states) in 2D cluster phase on a 2D Archimedean lattice with Cone/Fractal subsystem symmetry are universal for MBQC.

Real space symmetry	Real space symmetry group	Virtual space QCA structure	Computational phase	Lattices
Ribbon	$\mathbb{Z}_2^{2n}$	Periodic	Yes	Rectangular
Cone	$\mathbb{Z}_2^{2n}$	Glider	Yes	(4^4), (3^6), $(3,4,6,4)$
Fractal	$\mathbb{Z}_2^{2n}$	Fractal	Yes	(6^3), $(4,8^2)$, $(4,6,12)$, $(3^4,6)$, $(3,4,3^2,4)$, $(3^3,4^2)$
1 - Form	$\mathbb{Z}_2^{\mathcal{O}(nN)}$	No	No	$(3,6,3,6)$, $(3,12^2)$
$\frac{1}{k}$Fractional	$\mathbb{Z}_2^k$	-	-	All

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Outlook for fault-tolerant cluster-phase computation:

- Combination of cone and 1-form symmetries in 3D cluster state
  -> state-independent protocol for infinitesimal rotations is not protected.
- universal set of CCZ and H available by higher-form SPTOs