Title: Symmetry, topology, and thermal stability

Speakers: Stephen Bartlett

Collection: Symmetry, Phases of Matter, and Resources in Quantum Computing

Date: November 26, 2019 - 9:30 AM

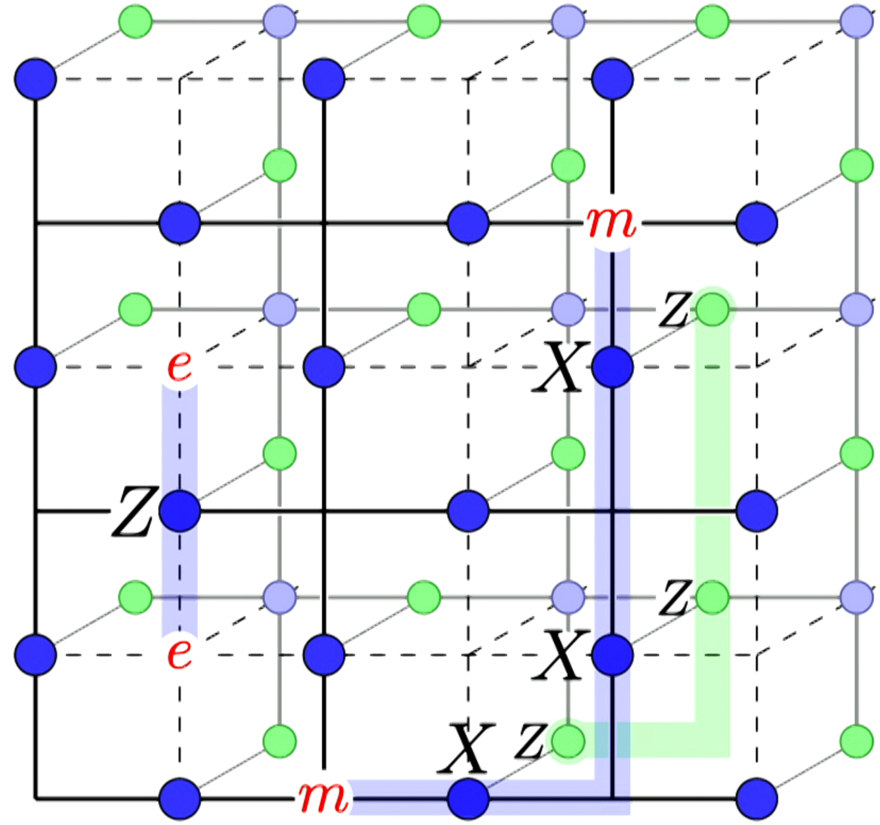
URL: http://pirsa.org/19110118

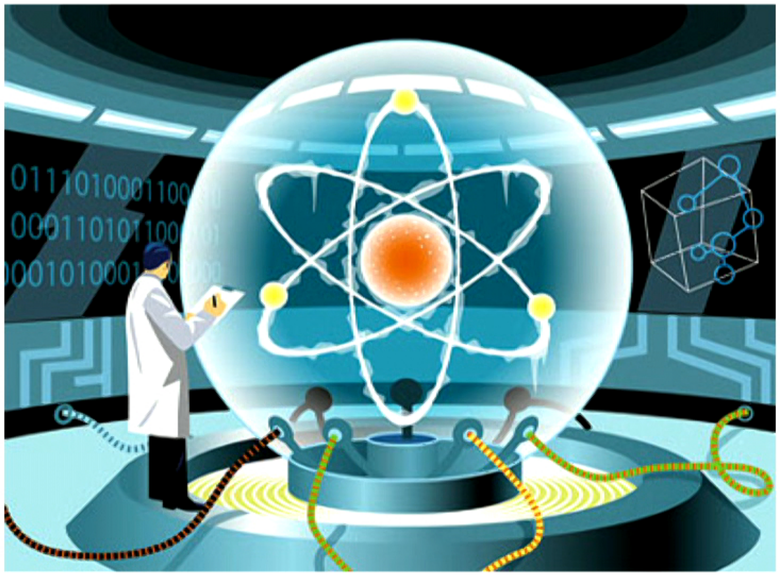
Abstract: The interplay of symmetry and topology in quantum many-body systems can lead to novel phases of matter, with applications in quantum memories and resources for quantum computing. While we understand the range of phenomena quite well in 2-d systems, there are many open questions for the 3-d case, in particular what kind of symmetries and topology can allow for thermal stability in 3-d models. Iâ€™ll present some of the results and open questions in this direction, using the 3-d toric code and the RBH models as examples.

Symmetry, topology, and thermal stability

Stephen Bartlett

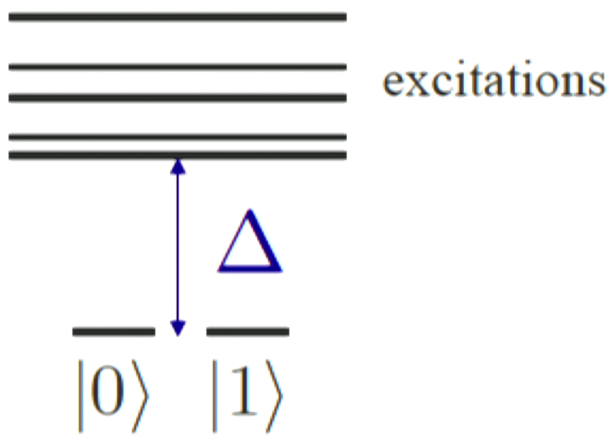
Joint work with Sam Roberts
arXiv:1805.01474





? =

The miracle of topological codes…



- Degenerate ground states allow for storage of quantum information
- No relaxation
- No dephasing (actually exponentially suppressed)
- Errors (excitations) to higher energy levels
  - suppressed by the gap
  - correctable if local
- High thresholds, nice q computing architectures

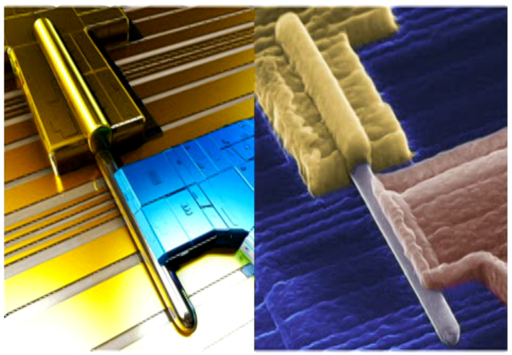
But: Topological order is a ground-state phenomenon
With (most) topological stabilizer codes, quantum information is **not stable** on its own
Need to constantly perform error correction

From topological order to symmetry-protected topological order

Symmetry-protected topological order

- Restricted form of topological order
- Robust to local perturbations that respect a symmetry

Majorana fermions in nanowires

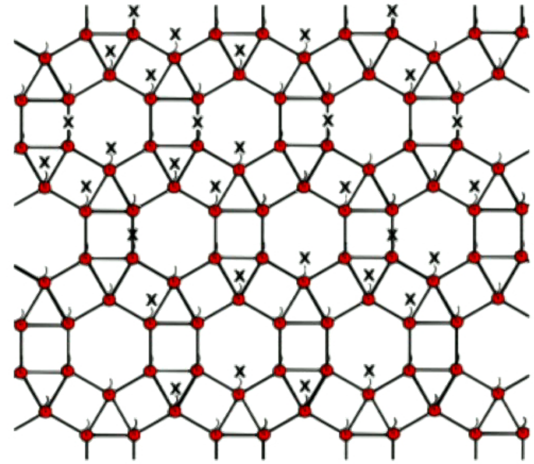


Signatures of Majorana Fermions in Hybrid Superconductor-Semiconductor Nanowire Devices

V. Mourik,[1*] K. Zuo,[1*] S. M. Frolov,[1] S. R. Plissard,[2] E. P. A. M. Bakkers,[1,2] L. P. Kouwenhoven[1†]

www.sciencemag.org SCIENCE VOL 336 25 MAY 2012

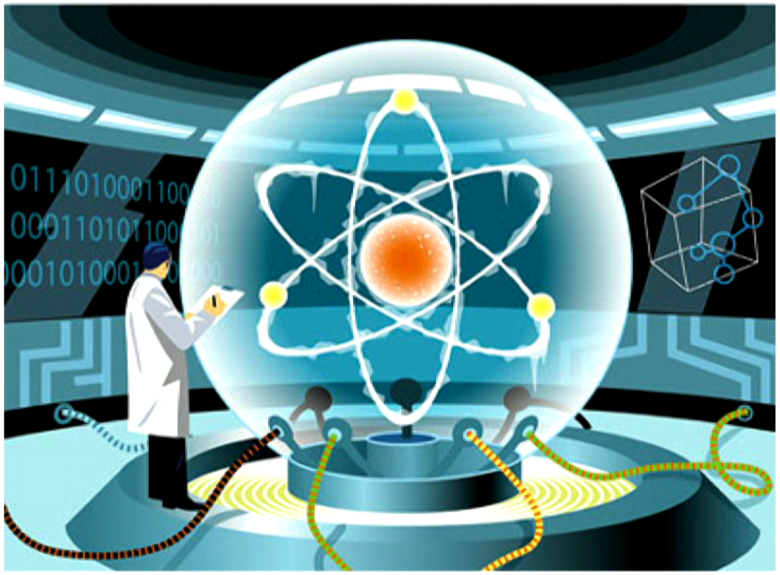
Topological insulators



Computational resources

SPT order also a ground state property

Schemes using SPT may also require active correction?



? =

Key question:
Can we find resourceful quantum orderings that are thermally stable?

The canonical example: Toric code in four dimensions

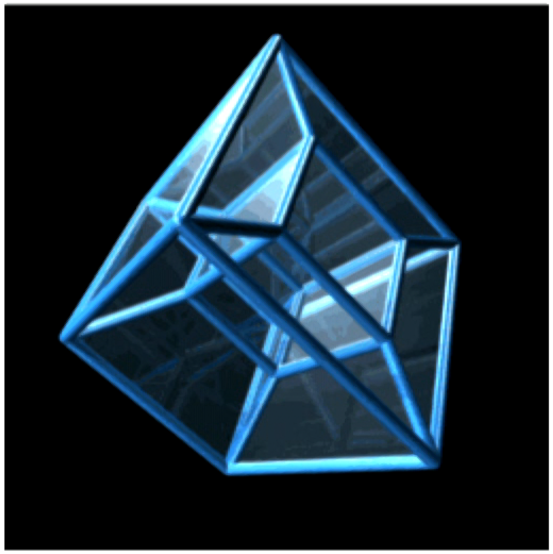
How good are you at picturing 4D?

- Topological: Degenerate ground space can protect quantum information

- Electric and magnetic 'charges' are not point-like, but loop-like extended objects with *tension*, meaning errors need increasing energy to grow

- Finite-temperature phase transition

- A self-correcting quantum memory



Dennis, Kitaev, Landahl, Preskill 2003

Problems

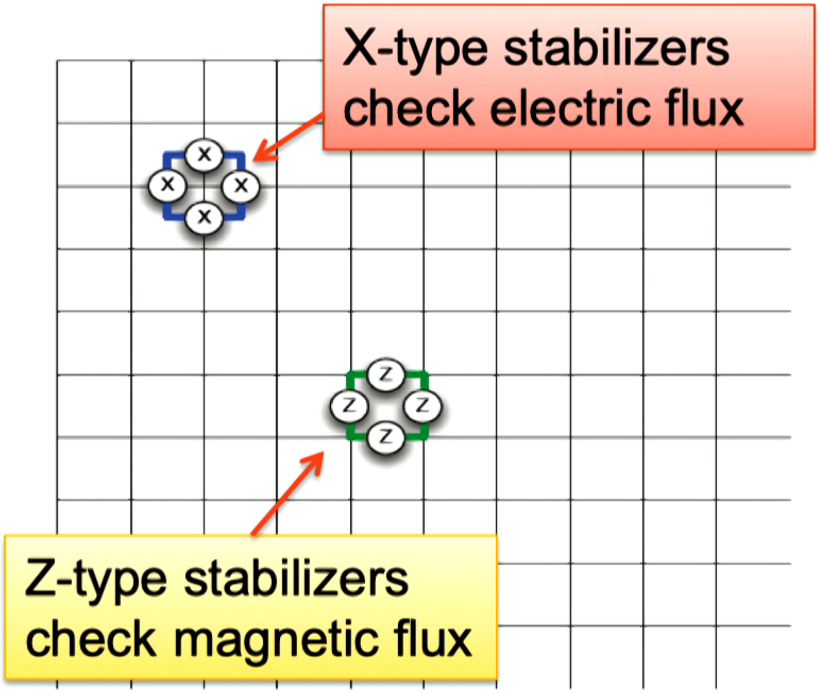
- Hastings (2011)
  - 2D topological order in commuting projector Hamiltonian models cannot persist at nonzero temperature

- Roberts, Yoshida, Kubica, Bartlett (2017)
  - SPT order protected by a global on-site symmetry, in any dimension, cannot persist at nonzero temperature

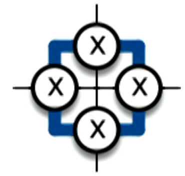
No-go theorems exist to be circumvented

Let’s play with symmetry and topology

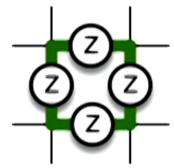
Let's do something silly…



- Take the standard 2-d toric code
- Enforce a symmetry:

$A_v =$ 

$= +1$ everywhere

$B_p =$ 

$= +1$ everywhere

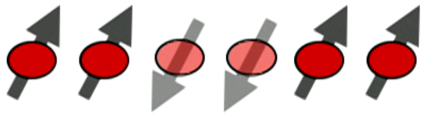
- No errors!
- The model is trivial; no excitations
- Logical information is protected but also fixed. Can't do logic gates with local moves

How do things work with *classical* ordering?

Ising spin magnets as classical memories

Each spin prefers to be aligned with its neighbour(s)

1-D chain:



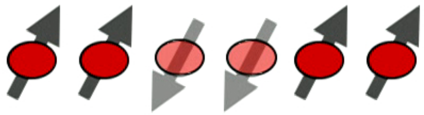
Excitations (domain walls) are point-like,
Deconfined
Errors can move freely and corrupt encoded information
No Curie temperature

How do things work with *classical* ordering?

Ising spin magnets as classical memories

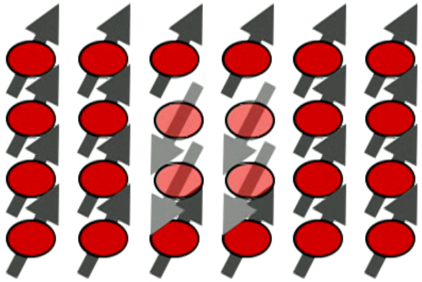
Each spin prefers to be aligned with its neighbour(s)

1-D chain:



Excitations (domain walls) are point-like,
Deconfined
Errors can move freely and corrupt encoded information
No Curie temperature

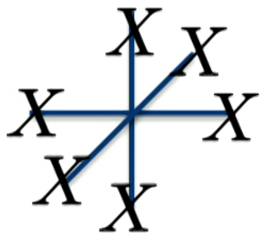
2-D lattice:



Excitation are string/loop-like,
Confined, with tension
Errors cost more energy to grow in size
Tc>0 Curie temperature

Silly idea #2: the 3D toric code

- Electric charges are point-like and deconfined
- Magnetic charges are loop-like and confined
- Enforce a symmetry:

$A_v =$ 

$= +1$ everywhere

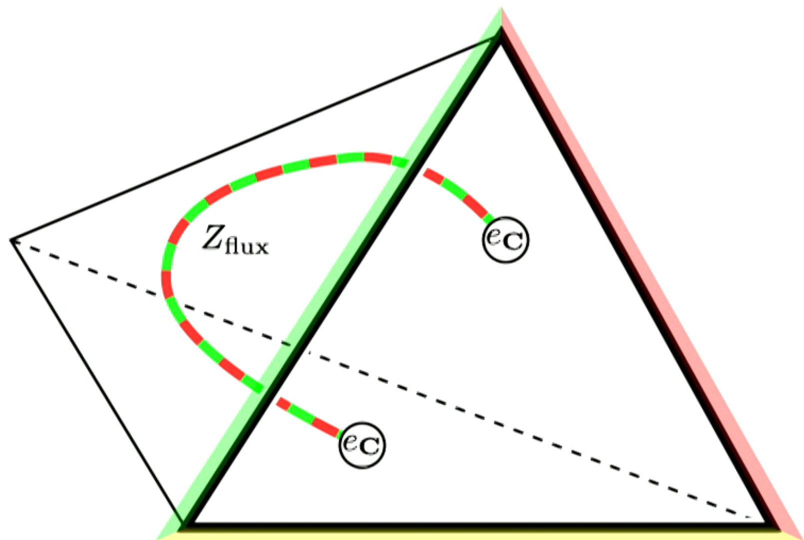
- Recover the Ising lattice gauge model – confinement – ‘Curie’ temperature
- Magnetic charges remain confined, electric charges are removed altogether
- Can’t do logical Z operation with local moves

- **Lesson:** don’t use symmetry to remove topological charge, use it to confine what was deconfined

Some observations about 3D topological codes…

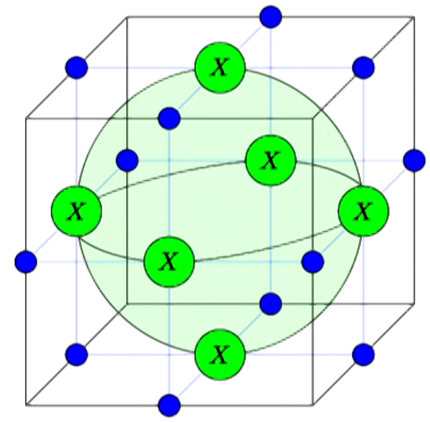
- 3D topological models encode quantum information on the boundaries
- Bulk excitations can be confined (string-like) but boundary excitations are deconfined
- If we can couple the boundary and bulk theories, we can have confinement of all excitations
- An exotic type of symmetry is needed:

  1-form symmetry



A little nugget from string theory!

- A new type of symmetry: **1-form symmetry**
- Imposes a Gauss-type law on topological charge
- Natural generalization of on-site (0-form) symmetry
- Global q-form symmetry acts as $U_g(\mathcal{M})$ on a closed q-codimension manifold $\mathcal{M}$
- Charged excitations have dimension q
- Symmetries impose conservation laws on higher-dimensional charged objects



Baez and Huerta (2010)
Kapustin and Thorngren (2013)
Kapustin and Seiberg (2014)
Giaotto, Kapustin, Seiberg, and Willett (2015)
Yoshida (2015)

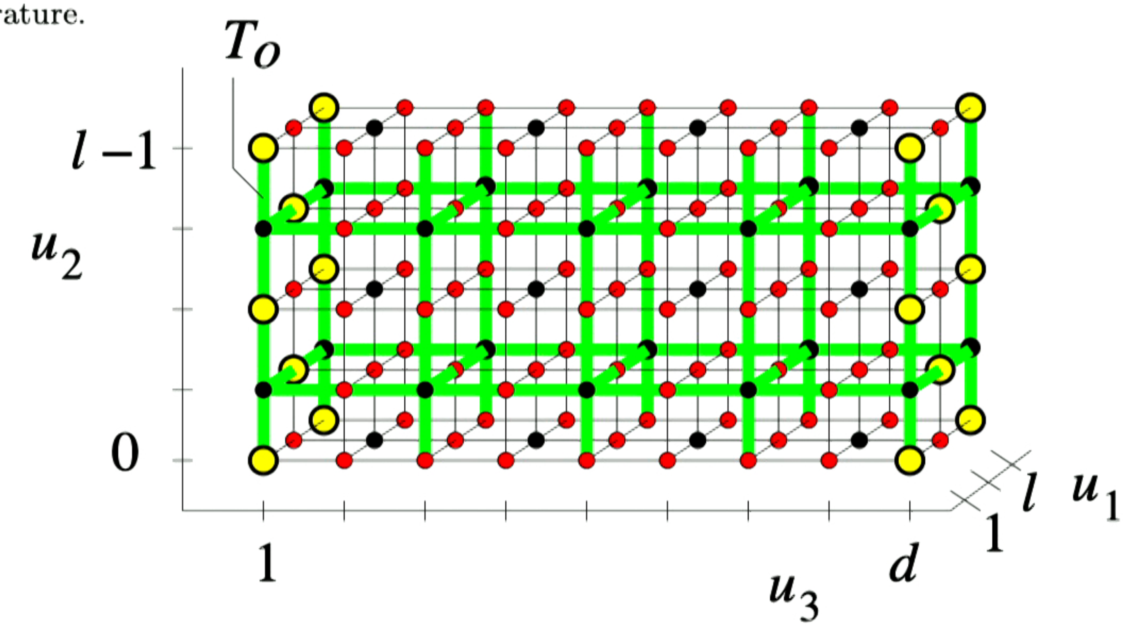
Long-range quantum entanglement in noisy cluster states

Robert Raussendorf, Sergey Bravyi and Jim Harrington[1]

[1] *California Institute of Technology, Institute for Quantum Information, Pasadena, CA 91125, USA*

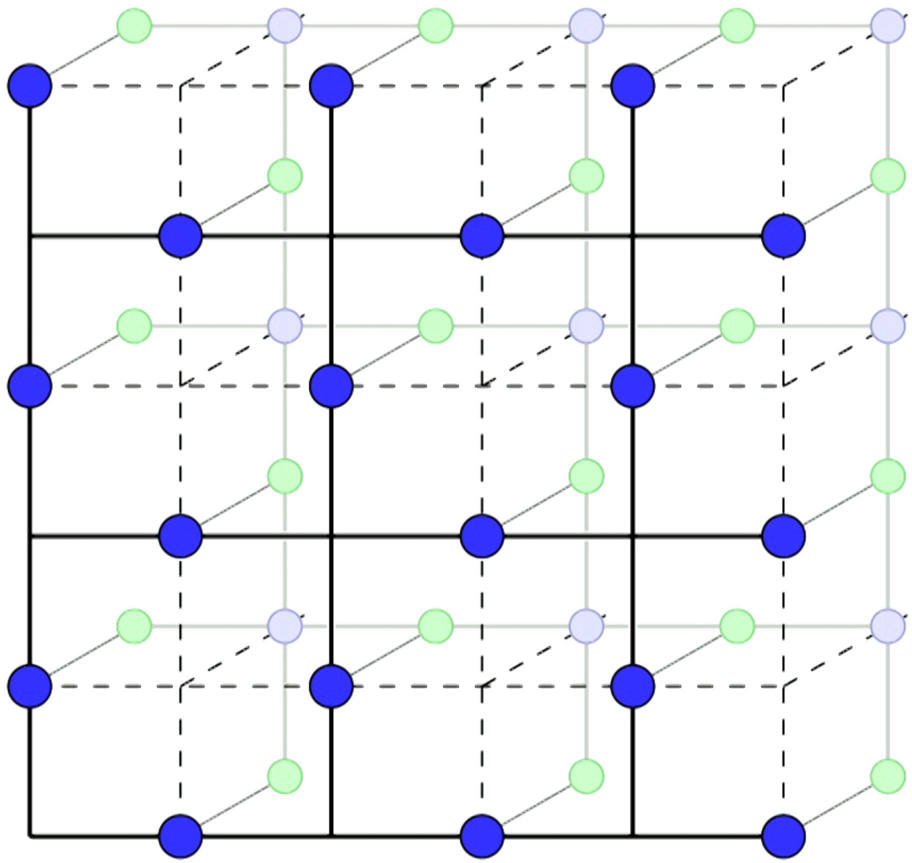
(Dated: February 9, 2008)

We describe a phase transition for long-range entanglement in a three-dimensional cluster state affected by noise. The partially decohered state is modeled by the thermal state of a short-range translation-invariant Hamiltonian. We find that the temperature at which the entanglement length changes from infinite to finite is nonzero. We give an upper and lower bound to this transition temperature.



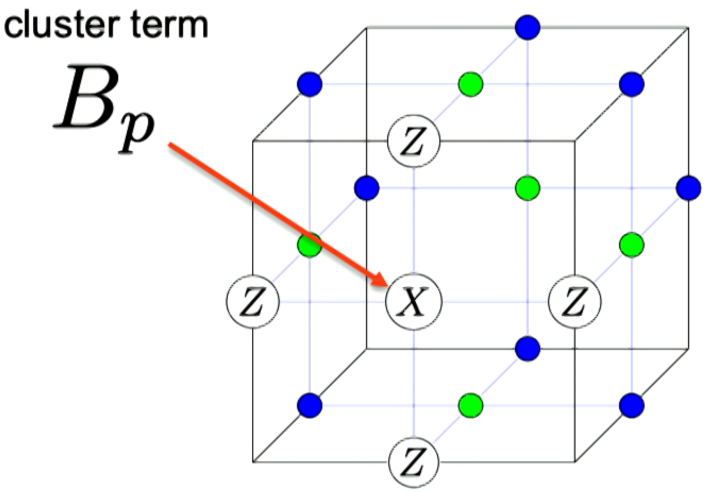
RBH model and its symmetries

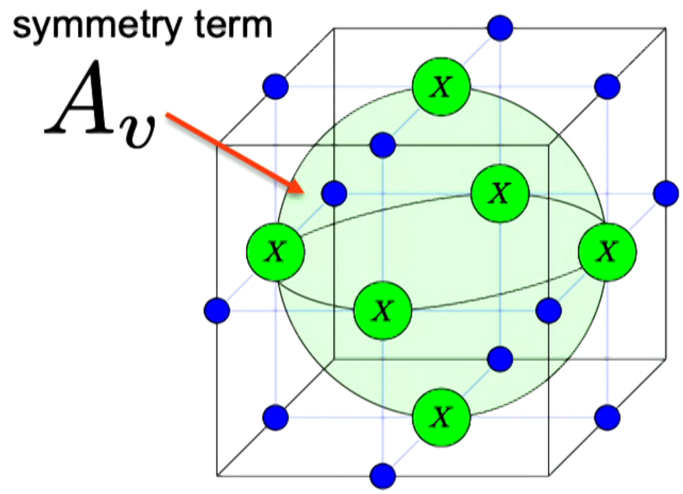
Primal qubits (blue)



Dual qubits (green)

RBH model and its symmetries



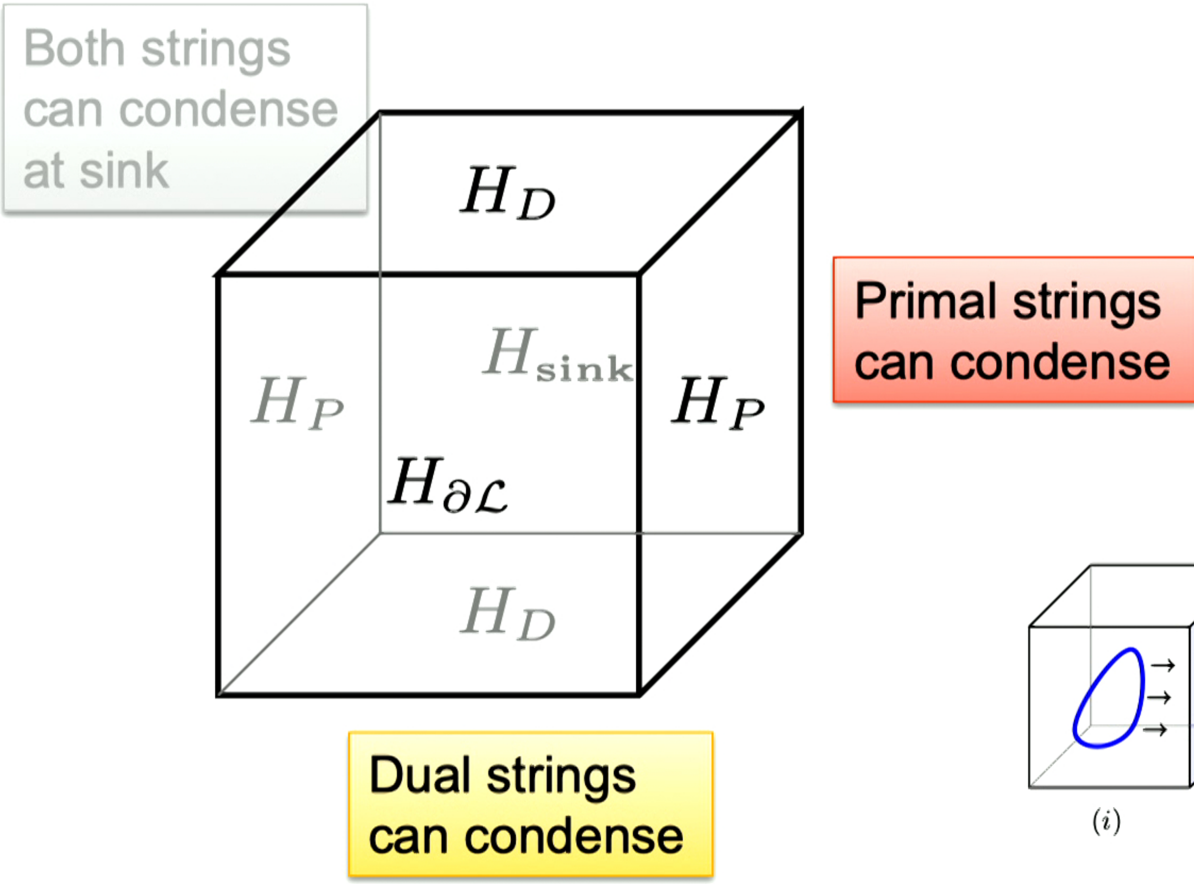


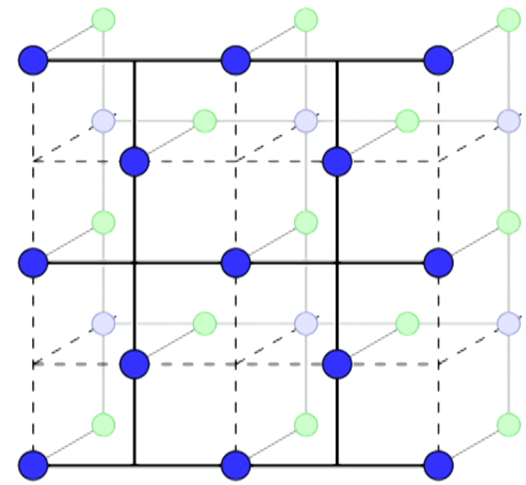
$$H_{\mathrm{RBH}} = -\sum_p B_p$$

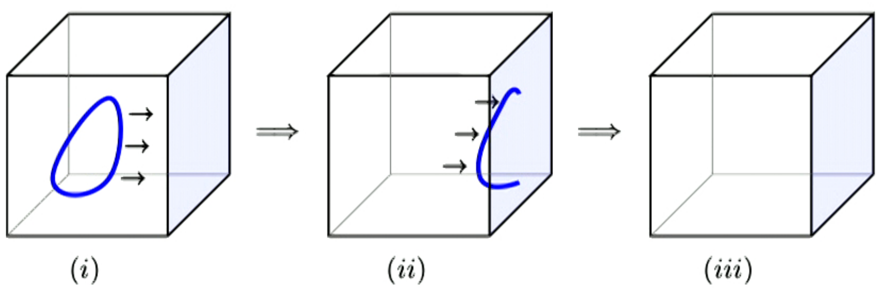
$$H_{\mathrm{triv}} = -\sum_p X_p$$

- For both models:
  - Bulk ground state is nondegenerate
  - Low energy excitations Z strings with tension (confinement)
  - Symmetry term forbids open strings

RBH model with boundaries



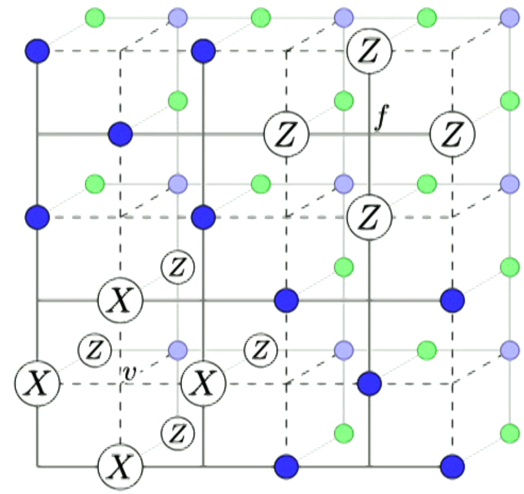


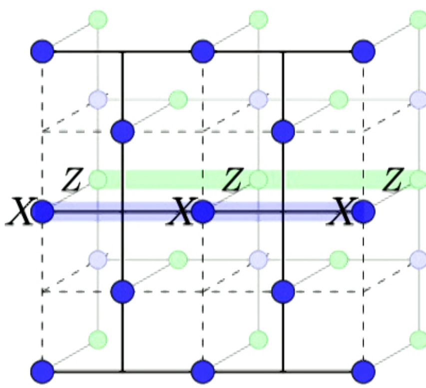


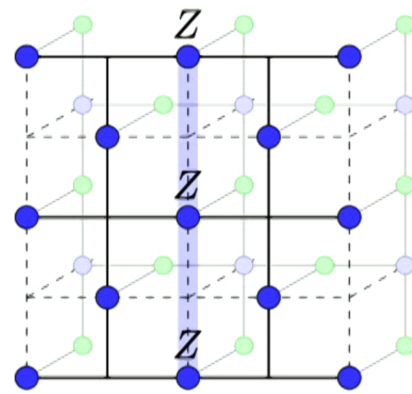
Boundary Hamiltonian

- Define a surface code Hamiltonian on the boundary degrees of freedom
  - Boundary ground state degeneracy (1 qubit)
  - Surface code has deconfined excitations

$$H_{\partial\mathcal{L}} = -\sum_v \tilde{A}_v - \sum_p \tilde{B}_p$$







Model gets more interesting if we include the symmetry…

A tale of two models…

- Bulk is trivial paramagnet

$$H_{\mathrm{triv}} = -\sum_p X_p$$

- 1-form symmetry confines excitations in the bulk
- Bulk ordering is SPT-trivial
- Boundary is topologically ordered
- Boundary remains deconfined, uncoupled from the bulk

No thermal stability of boundary order

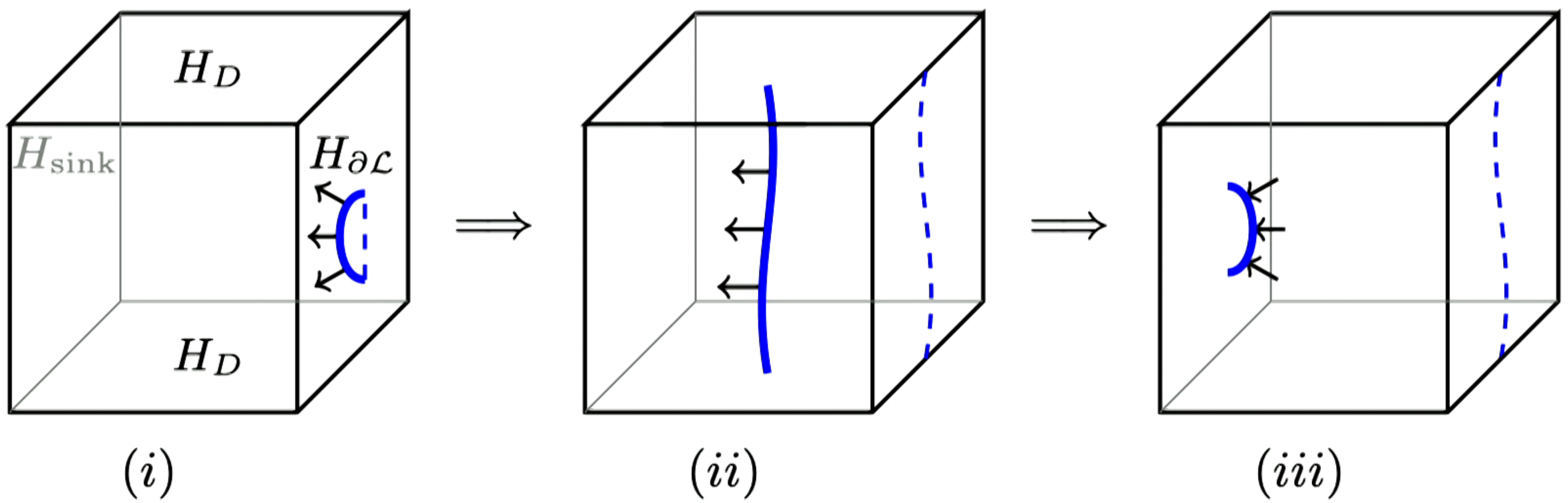
- Bulk is RBH

$$H_{\mathrm{RBH}} = -\sum_p B_p$$

- 1-form symmetry confines excitations in the bulk
- Bulk ordering is SPT-nontrivial
- Boundary is anomalous, SET ordered
- Boundary point-like excitations are coupled to bulk strings, becoming confined

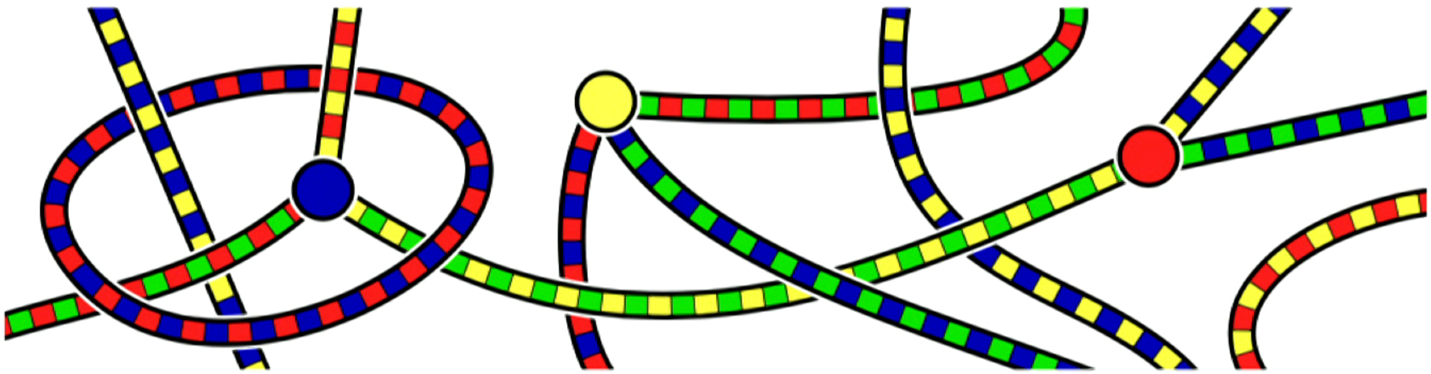
SPT ordered model is thermally stable!

Logical operations



Self-correction, symmetries, and emergence

- 1-form symmetric SPT phases in 3D can lead to thermal stability in interesting topologically ordered models
- Must go beyond on-site symmetries, to higher-form symmetries, for thermal stability, but are very strong symmetry constraints
- Can 1-form symmetries be emergent?



Gauge color code (Bombin, 2015, 2016)

WARNING! Wild speculation ahead

- Emergent symmetries
  - Topological models are low-energy effective theories, where symmetries such as topological charge conservation appear at long distances
  - We have a solid foundation for this in 2D, less so in higher dimensions

- Why would you think that 1-form symmetries could be emergent?
  - In higher-dim topological models, with extended excitations (i.e., loops), we still expect conservation laws for topological charge
  - Examples of this, such as gauge flux conservation in the gauge color code, manifest themselves as 1-form symmetries
  - Could higher-form symmetries have a defining role in higher-dimensional topological order?