Title: SU(2) gauge theory on digital quantum computers

Speakers:

Series: Perimeter Institute Quantum Discussions

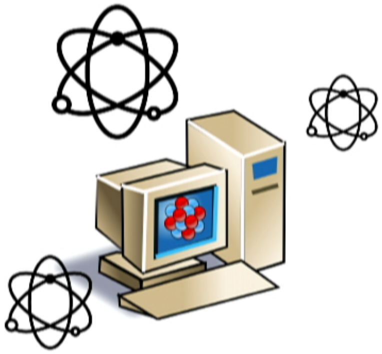
Date: November 14, 2019 - 11:00 AM

URL: http://pirsa.org/19110115

Abstract:

Results from the first digital quantum simulation of an SU(2) gauge theory are presented. This was done by analytically constructing gauge-invariant states and implementing a Trotterized time evolution operator for that basis on superconducting hardware. By using error mitigation techniques, electric energy measurements could be reliably extracted following one Trotter-Suzuki time step. This work is a small but important step toward determining what field-theoretic calculations will be possible using near-term devices.

SU(2) gauge theory on digital quantum computers



Jesse Stryker
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Work done w/ Natalie Klco (INT)
& Martin Savage (INT)

based on arXiv:1908.06935

Perimeter Institute Seminar
2019/11/14

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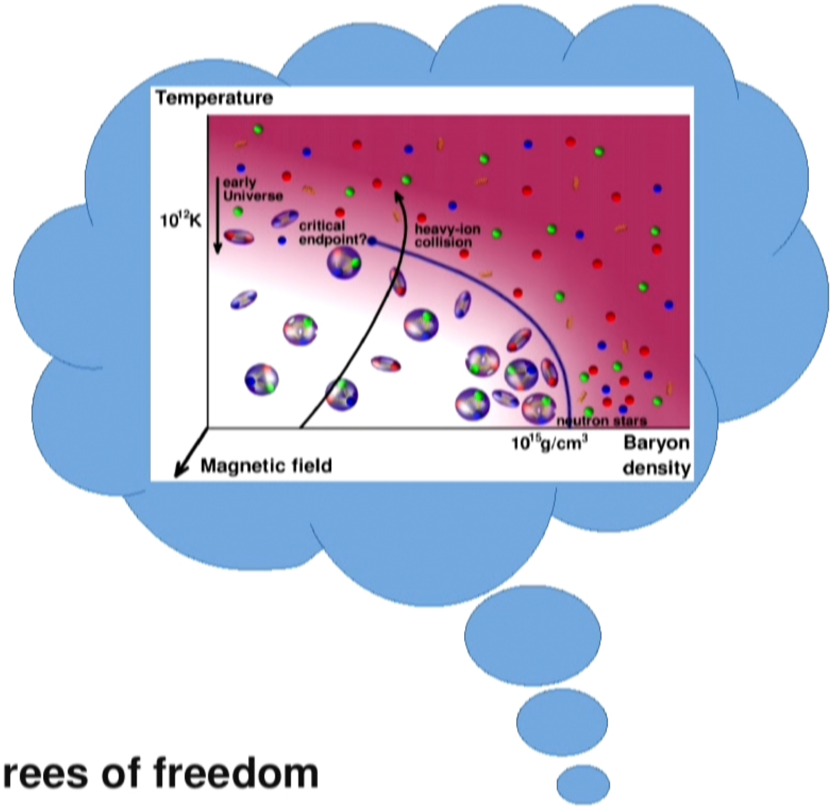
Big picture

Physics targets:

- **Simulation of quantum chromodynamics (QCD)**
  - Hadronization
  - Microscopic understanding of nuclear interactions
- **Complete phase diagram of QCD**
- **Nuclear equation of state**



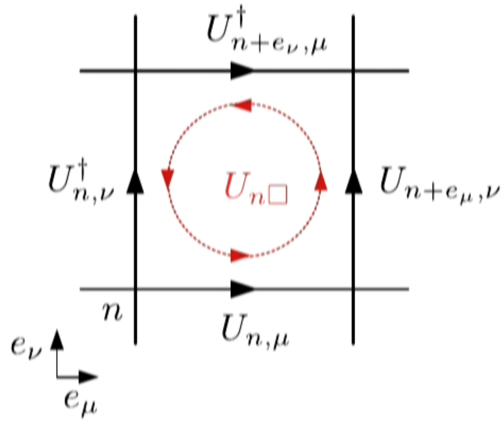
How to make these predictions?

- **Non-perturbative problems**
- **Numerically simulate QCD degrees of freedom**

Conjectured phase diagram credit: G. Endrődi J.Phys.Conf.Ser. 503 (2014) 012009

Traditional lattice ingredients

$x^\mu \to a n^\mu$



$$Z = \int \prod_{n,\mu} [\mathcal{D}U_{n,\mu}] e^{-S[U]}$$

Monte Carlo on this

Wilson gauge action

"link operators" $U_{n,\mu}$ in gauge group G

$$S_W = -\beta \sum_{n,\mu} \mathrm{tr}(\underbrace{U_{n,\mu} U_{n+e_\mu,\nu} U^\dagger_{n+e_\nu,\mu} U^\dagger_{n,\nu}}_{U_\square \text{ "plaquette" operator}} + U^\dagger_\square)$$

for non-Abelian

Relation to continuum gauge fields:

$$U_{n,\mu} = e^{iaA_\mu(x)} \qquad S_W = a^4 \sum_n F^\alpha_{\mu\nu} F^{\mu\nu}_\alpha + \cdots$$

Fermionic matter:

- Grassmann integrals done analytically → "Fermion determinant"

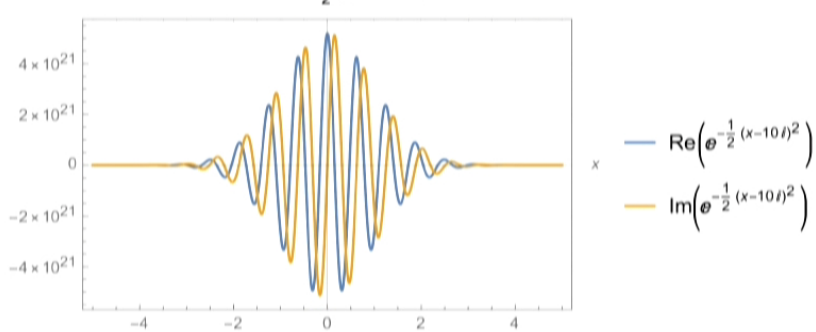
Traditional lattice

Euclidean path integral Monte Carlo

- **Great for static, equilibrium properties**
- **Real-time dynamics? Nonzero density? Topological term?**

-$S[U]$ generically complex-valued
→ "Sign problems"

Oscillations of a Gaussian: $S = \frac{1}{2}(x - 10\,i)^2$



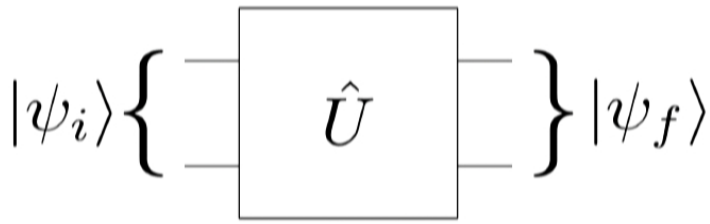
$$\int dx\, e^{-\frac{1}{2}(x-10i)^2} = \sqrt{2\pi} \ll 10^{21}$$

$$\langle e^{i\theta} \rangle = e^{-50}$$

Exponentially hard to sample oscillating path integral

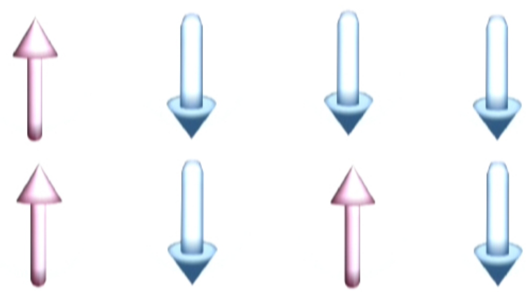
Classical problems.. quantum solutions?

Digital quantum computers:



Unitary gates: $e^{-it\hat{H}}$ with your favorite Hamiltonian

- Want to simulate non-perturbative gauge theory
  - → Gauge theory on the lattice
  - → Hamiltonian lattice gauge theory



General problem: How to map the Hilbert space $\mathcal{H}$ and $\hat{H}$ on to the qubits & gates?

Some past QFT simulation progress

Science

Quantum Computation of Scattering in Scalar Quantum Field Theories

Stephen P. Jordan, Keith S. M. Lee, and John Preskill

Quantum Algorithms for Quantum Field Theories
Stephen P. Jordan, Keith S. M. Lee and John Preskill

PHYSICAL REVIEW X 7, 041046 (2017)

Efficient Basis Formulation for (1+1)-Dimensional SU(2) Lattice Gauge Theory: Spectral Calculations with Matrix Product States

Mari Carmen Bañuls, Krzysztof Cichy, J. Ignacio Cirac, Karl Jansen, and Stefan Kühn

PHYSICAL REVIEW D 98, 074503 (2018)

PHYSICAL REVIEW A 99, 052335 (2019)

Digitization of scalar fields for quantum computing

Phase transitions in Z_n gauge models: Towards quantum simulations of the Schwinger-Weyl QED

Ultracold quantum gases and lattice systems: quantum simulation of lattice gauge theories

Uwe-Jens Wiese

Works meant to be representative… not exhaustive

PHYSICAL REVIEW LETTERS 121, 110504 (2018)

PRL 114, 070502 (2015) PHYSICAL REVIEW LETTERS

Fermion-Fermion Scattering in Quantum Field Theory with Superconducting Circuits

L. García-Álvarez, J. Casanova, A. Mezzacapo, I. L. Egusquiza, L. Lamata, G. Romero, and E. Solano

Electron-Phonon Systems on a Universal Quantum Computer

Quantum simulations of lattice gauge theories using ultracold atoms in optical lattices

Erez Zohar, J Ignacio Cirac and Benni Reznik

PRL 115, 240502 (2015) PHYSICAL REVIEW LETTERS

Non-Abelian SU(2) Lattice Gauge Theories in Superconducting Circuits

Real-time dynamics of lattice gauge theories with a few-qubit quantum computer

Esteban A. Martinez, Christine A. Muschik, Philipp Schindler, Daniel Nigg, Alexander Erhard, Markus Heyl, Philipp Hauke, Marcello Dalmonte, Thomas Monz, Peter Zoller & Rainer Blatt

PHYSICAL REVIEW A 92, 063825 (2015)

PHYSICAL REVIEW X 6, 011023 (2016)

Quantum simulation of quantum field theory using continuous variables

Real-Time Dynamics in U(1) Lattice Gauge Theories with Tensor Networks

T. Pichler, M. Dalmonte, E. Rico, P. Zoller, and S. Montangero

Gluon Field Digitization for Quantum Computers

Henry Lamm, Scott Lawrence, and Neill C. Warrington

First Abelian gauge theory digital simulation

Lattice gauge theories simulations in the quantum information era

M. Dalmonte and S. Montangero

Towards analog quantum simulations of lattice gauge theories with trapped ions

Simulating lattice gauge theories on a quantum computer

Tim Byrnes

Yoshihisa Yamamoto

PHYSICAL REVIEW A 100, 012320 (2019)

Simulations of subatomic many-body physics on a quantum frequency processor

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Talk outline

- Hamiltonian SU(2) lattice gauge theory
- Plaquette ladder
- Mapping to qubits
- Hardware results from IBM Tokyo
- Summary & future directions

Hamiltonian lattice gauge theory

Canonical quantization, temporal gauge

$U_{n,i} \to \hat{U}_{n,i}$ link variables: **matrices** of **operators**

Gauge transformations: $\hat{U}_{n,i} \to \Omega_n \hat{U}_{n,i} \Omega^\dagger_{n+e_i}$

$$[E^a_{L/R}, E^b_{L/R}] = i f^{abc} E^c_{L/R}$$
$$[E^a_R, U] = +UT^a$$
$$[E^a_L, U] = -T^a U$$

Left, right electric fields to generate left, right rotations.

$$\hat{H}_E = \frac{g^2}{2} \sum_{n,i} \hat{E}^a_{n,i} \hat{E}^a_{n,i} \qquad \hat{H}_B = -\sum_n \frac{1}{2g^2} \left(\hat{\Box}_n + \hat{\Box}^\dagger_n \right)$$

$$\hat{\Box}_n \equiv \mathrm{tr}(\hat{U}_{n,\Box})$$

gauge invariant Casimir

Hilbert space

coordinate-like basis

More info: Zohar & Burrello, PRD 91, 054506 (2015)

$$\hat{U}_{ij}\,|g\rangle = D_{ij}(g)\,|g\rangle \quad g \in G$$

momentum-like basis

$$E_L^3\,|j,m,m'\rangle = -m\,|j,m,m'\rangle$$
$$E_R^3\,|j,m,m'\rangle = m'\,|j,m,m'\rangle$$
$$E_L^a E_L^a\,|j,m,m'\rangle = E_R^a E_R^a\,|j,m,m'\rangle = j(j+1)\,|j,m,m'\rangle$$

$$\langle g|j,m,m'\rangle = \sqrt{\frac{d_j}{|G|}}\,D^{(j)}_{m,m'}(g)$$

group element state

irrep state

Plus Gauss law constraints

Hilbert space

More info: Zohar & Burrello, PRD 91, 054506 (2015)

coordinate-like basis

$$\hat{U}_{ij}\,|g\rangle = D_{ij}(g)\,|g\rangle \qquad g \in G$$

momentum-like basis

$$E^3_L\,|j,m,m'\rangle = -m\,|j,m,m'\rangle$$

$$E^3_R\,|j,m,m'\rangle = m'\,|j,m,m'\rangle$$

$$E^a_L E^a_L\,|j,m,m'\rangle = E^a_R E^a_R\,|j,m,m'\rangle = j(j+1)\,|j,m,m'\rangle$$

$$\langle g|j,m,m'\rangle = \sqrt{\frac{d_j}{|G|}}\,D^{(j)}_{m,m'}(g)$$

group element state

irrep state

Plus Gauss law constraints

$$\hat{\mathcal{G}}^a = \nabla \cdot \mathbf{E}^a - \rho^a = 0$$

lattice discrete "divergence"

$|j\ m\ m'\rangle$ in practice

- Truncation not ideal for qubit registers

$$|j, m, m'\rangle\,, \quad 0 \le j \le J \qquad d_j = (2j+1)^2$$

$$\Rightarrow \dim(\mathcal{H}) = (8/3)(J+1/2)(J+3/4)(J+1) \neq 2^n$$

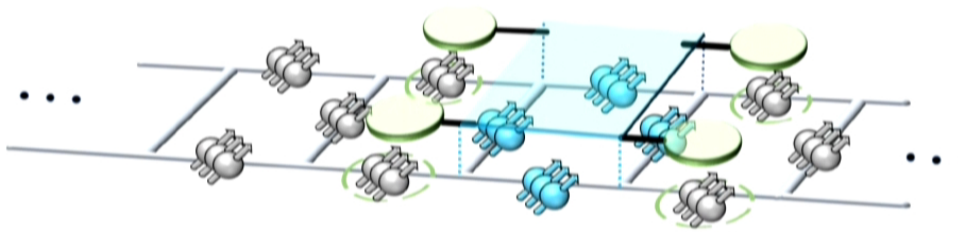
- Non-commuting constraints on superpositions

$$[\mathcal{G}^a, \mathcal{G}^b] \neq 0 \qquad (\mathcal{G}^a \equiv \nabla \cdot \mathbf{E}^a - \rho^a)$$

- Far too many d.o.f.s carried around

actual photograph of quantum programmer reacting to |jmm'>

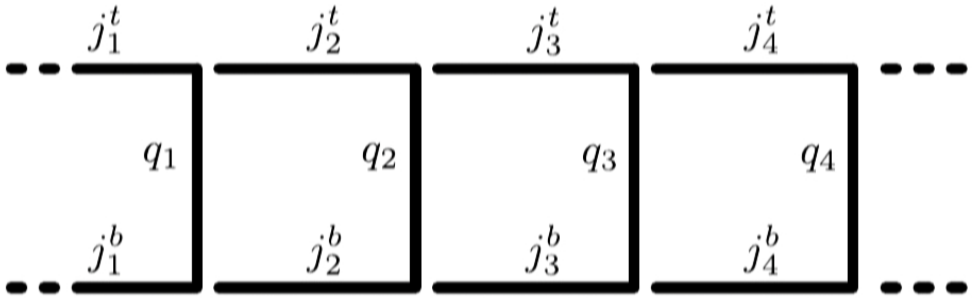
System choice

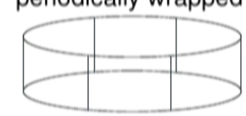


We consider a periodic **string of plaquettes** or “ladder”

- ✓ “1d,” but has H_B
- ✓ 3-pt vertices ↔ unique singlet combination per triplet of j’s
- ✓ Arbitrary length

Gauge singlet basis





Fully gauge invariant state of lattice with definite link angular momenta:

$$|\chi\rangle = \mathcal{N} \sum_{\{m\}} \prod_{i=1}^{L} \langle j_i^t, m_{i,R}^t, j_{i+1}^t, m_{i+1,L}^t | q_i, m_{q_i}^t \rangle$$

} CG's to form singlets at "top" vertices

$$\langle j_i^b, m_{i,R}^b, j_{i+1}^b, m_{i+1,L}^b | q_i, m_{q_i}^b \rangle$$

} CG's to form singlets at "bottom" vertices

$$|j_i^t, m_{i,L}^t, m_{i,R}^t\rangle \otimes |j_i^b, m_{i,L}^b, m_{i,R}^b\rangle \otimes |q_i, m_{q_i}^t, m_{q_i}^b\rangle$$

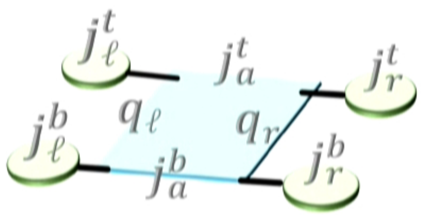
} Kets going around each "staple"

- Just using angular momentum addition (Clebsch-Gordan coefficients) to form singlets

Matrix elements of H

- **Non-diagonal elements derive from link operators in H_B : ***

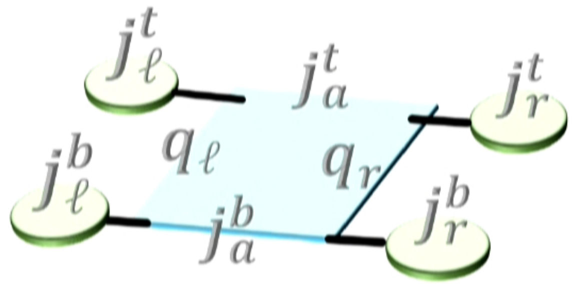
$$\hat{U}_{\alpha\beta}|j,a,b\rangle = \sum_{\oplus J} \sqrt{\frac{\dim(j)}{\dim(J)}}\ |J, a+\alpha, b+\beta\rangle$$
$$\times\ \langle j, a, \frac{1}{2}, \alpha | J, a+\alpha\rangle\langle j, b, \frac{1}{2}, \beta | J, b+\beta\rangle$$



each link op going round a plaquette "adds" ½-unit of angular momentum: $J = j \pm 1/2$

- **This is all the info needed to compute matrix $||H||$ w.r.t. singlet states**

Plaquette operator & gauge-variant completion

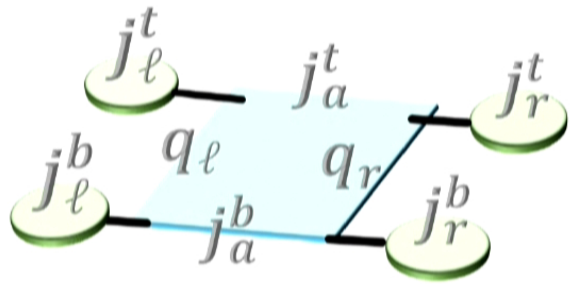


- With singlet basis, matrix elements of $\hat{\square}$ depend on plaquette's j's, as well as adjacent j's
- Still have disallowed states
  - Action of plaquette op on disallowed space is arbitrary

    → "Gauge-variant completion" (GVC): Only worry about getting correct matrix elements between allowed states

Specific truncation

$\Lambda_j = ½$



$\langle j_{\ell f}\ q_{\ell f}\ j_{af}\ q_{rf}\ j_{rf} \vert \hat{\Box}^{(1/2)} \vert j_{\ell i}\ q_{\ell i}\ j_{ai}\ q_{ri}\ j_{ri}\rangle$	
$\langle 00000 \vert \hat{\Box}^{(1/2)} \vert 0\tfrac{1}{2}\tfrac{1}{2}\tfrac{1}{2}0\rangle,\ \langle 0\tfrac{1}{2}\tfrac{1}{2}\tfrac{1}{2}0 \vert \hat{\Box}^{(1/2)} \vert 00000\rangle$	1
$\langle 000\tfrac{1}{2}\tfrac{1}{2} \vert \hat{\Box}^{(1/2)} \vert 0\tfrac{1}{2}\tfrac{1}{2}0\tfrac{1}{2}\rangle,\ \langle 0\tfrac{1}{2}\tfrac{1}{2}0\tfrac{1}{2} \vert \hat{\Box}^{(1/2)} \vert 000\tfrac{1}{2}\tfrac{1}{2}\rangle$	$\frac{1}{2}$
$\langle \tfrac{1}{2}\tfrac{1}{2}000 \vert \hat{\Box}^{(1/2)} \vert \tfrac{1}{2}0\tfrac{1}{2}\tfrac{1}{2}0\rangle,\ \langle \tfrac{1}{2}0\tfrac{1}{2}\tfrac{1}{2}0 \vert \hat{\Box}^{(1/2)} \vert \tfrac{1}{2}\tfrac{1}{2}000\rangle$	$\frac{1}{2}$
$\langle \tfrac{1}{2}0\tfrac{1}{2}0\tfrac{1}{2} \vert \hat{\Box}^{(1/2)} \vert \tfrac{1}{2}\tfrac{1}{2}0\tfrac{1}{2}\tfrac{1}{2}\rangle,\ \langle \tfrac{1}{2}\tfrac{1}{2}0\tfrac{1}{2}\tfrac{1}{2} \vert \hat{\Box}^{(1/2)} \vert \tfrac{1}{2}0\tfrac{1}{2}0\tfrac{1}{2}\rangle$	$\frac{1}{4}$

Relevant, nonzero matrix elements of the $\Lambda_j = 1/2$ truncated plaquette operator with $j_\ell^t = j_\ell^b \equiv j_\ell$

$$\hat{\Box}^{(1/2)} = \Pi_0 XXX\Pi_0 + \frac{1}{2}\Pi_0 XXX\Pi_1 + \frac{1}{2}\Pi_1 XXX\Pi_0 + \frac{1}{4}\Pi_1 XXX\Pi_1$$

$$|j=0\rangle \to |0\rangle\,, \quad |j=1/2\rangle \to |1\rangle$$

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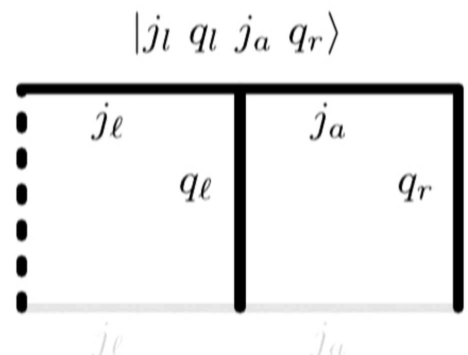
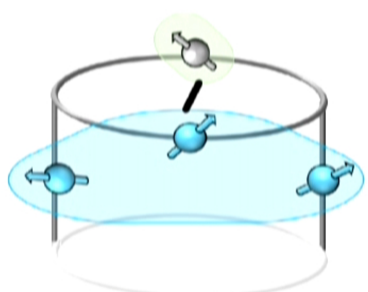
Specific truncation and volume

For our simulation:
- Cutoff Λ_j = ½
- Length L=2

\+ Simplifications
= Four 'active' links



Four qubits represent state



$$\hat{\square}^{(1/2)} = \Pi_0 XXX\Pi_0 + \frac{1}{2}\Pi_0 XXX\Pi_1 + \frac{1}{2}\Pi_1 XXX\Pi_0 + \frac{1}{4}\Pi_1 XXX\Pi_1$$

→GVC of plaquette operator:

$$\hat{\square}^{(1/2)} = \Pi_0 XXX + \frac{1}{4}\Pi_1 XXX$$

Trotter-Suzuki time evolution

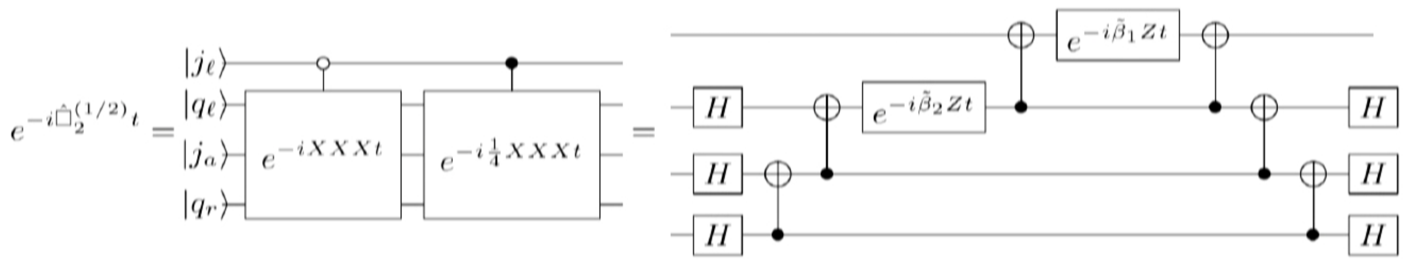
Time evolution operator replaced by Trotter-Suzuki approximation

$$e^{-i\ \Delta t(H_E+H_B)} \simeq e^{-i\ \Delta t H_E} e^{-i\ \Delta t \Box_1/(2g^2)} e^{-i\ \Delta t \Box_2/(2g^2)}$$

Try: *t* spread over one Trotter step, two Trotter steps, …
starting from strong-coupling vacuum (all j=0)

Circuit for a plaquette evolution

$$\tilde{\beta}_2 = (5/8)t, \quad \tilde{\beta}_1 = (3/8)t,$$



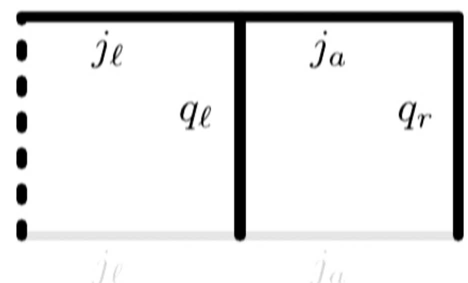
In this case:

- Circuit doesn't introduce further systematics
- Trotterization respects gauge constraints

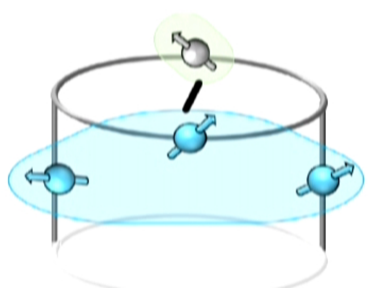
Specific truncation and volume

For our simulation:
- Cutoff $\Lambda_j = ½$
- Length L=2

\+ Simplifications
= Four 'active' links



Four qubits represent state



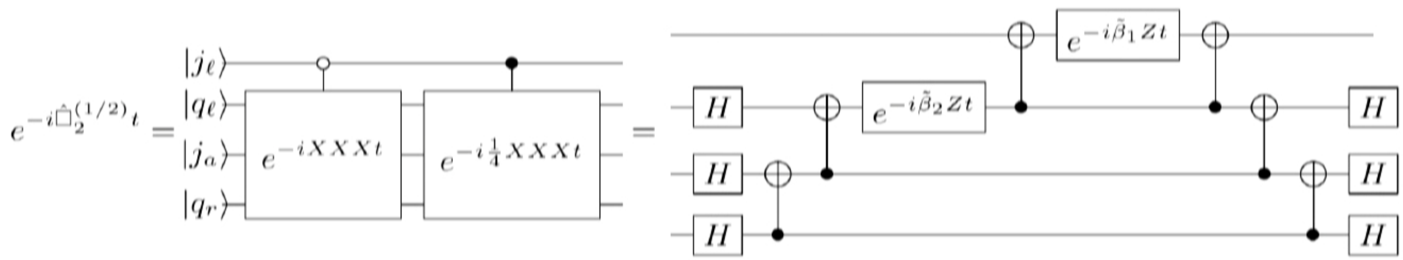
$$\hat{\Box}^{(1/2)} = \Pi_0 XXX\Pi_0 + \frac{1}{2}\Pi_0 XXX\Pi_1 + \frac{1}{2}\Pi_1 XXX\Pi_0 + \frac{1}{4}\Pi_1 XXX\Pi_1$$

→GVC of plaquette operator:

$$\hat{\Box}^{(1/2)} = \Pi_0 XXX + \frac{1}{4}\Pi_1 XXX$$

Circuit for a plaquette evolution

$\tilde{\beta}_2 = (5/8)t, \quad \tilde{\beta}_1 = (3/8)t,$



In this case:

- Circuit doesn't introduce further systematics
- Trotterization respects gauge constraints

Data processing

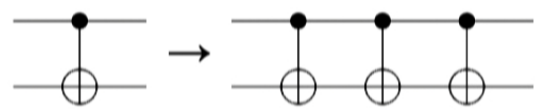
From IBM: Probabilities measured in computational basis

1) Constrained inversion → pre-measurement probabilities

- Needed because of measurement errors

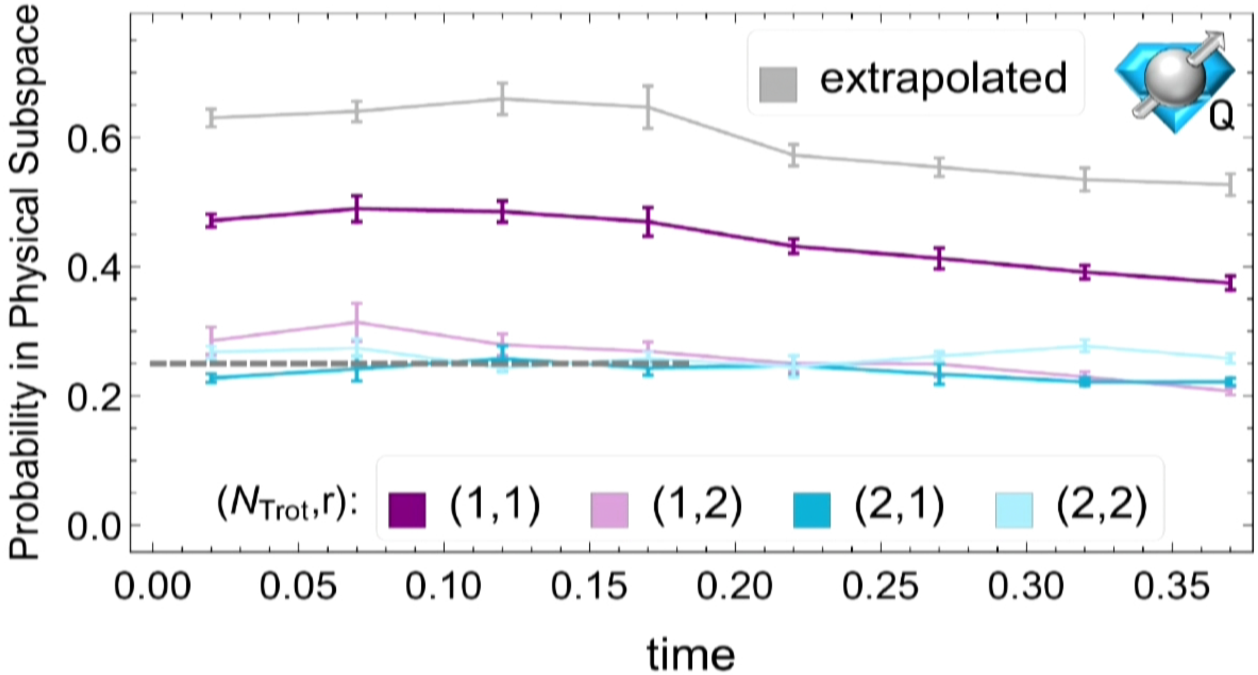
2) Run simulation with superfluous CNOT pairs inserted

- $(\text{CNOT})^2 = 1$, but introduces extra noise



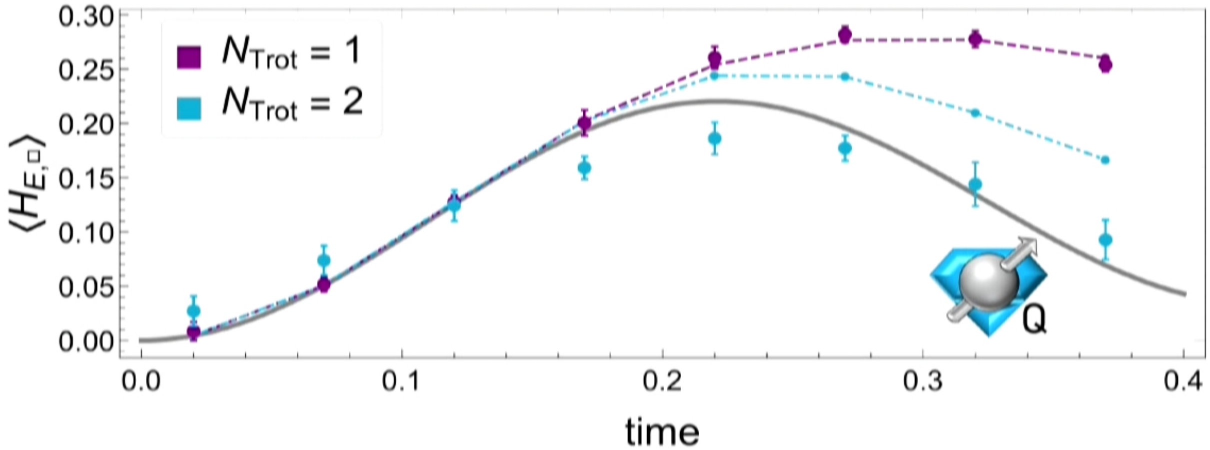
3) Extrapolate pre-measurement probabilities to zero CNOT noise

Probability extrapolation



- Errors into disallowed space mitigated for $N_{\text{Trot}} = 1$
- Coherence is lost for $N_{\text{Trot}} > 1$

Test observable: Plaquette electric energy



- Measure: Electric energy encircling one plaquette
  - Using extrapolated probability densities
- Compare to ideal Trotterized simulation outcome
- $N_{\text{Trot}} = 1$ gets it right within uncertainties!

Generalizations, future directions

- Higher cutoff
  - All links now active
  - More interesting GVC
- Higher dimensions
  - 3-point vertices important
  - More qubits, more gates, more noise

    → Maybe not today, but soon?
- SU(3)
  - Schwinger bosons (I. Raychowdhury et al.) may be helpful for computing reduced matrix elements

FIN

Thank you for your attention!

Questions?

Helpful conversations with: D. Kaplan, I. Raychowdhury, E. Zohar
JRS was supported by DOE Grant No. DE-FG02-00ER41132, and by the National Science Foundation Graduate Research Fellowship under Grant No.1256082.

Summary of hardware results

- First simulation of a truncated SU(2) system done on existing IBM hardware
- Used gauge theory constraints + NISQ-era tricks to mitigate subset of errors
- Low enough circuit depth → Can extract an observable

IBM Tokyo Q20 specs

Qubit Count	Qubit Connectivity			T1 (µsec)			T2 (µsec)		
	Min	Max	Ave	Min	Max	Ave	Min	Max	Ave
20	2	6	3.9	42.2	148.5	84.3	24.3	78.4	49.6

1-Qubit Gate Fidelity			2-Qubit Gate Fidelity			Readout Fidelity		
Min	Max	Ave	Min	Max	Ave	Min	Max	Ave
99.39%	99.94%	99.80%	92.88%	98.53%	97.16%	N/A	N/A	91.72%