Title: Quantum reference frames for space and time

Speakers: Flaminia Giacomini

Collection: Emmy Noether Workshop: The Structure of Quantum Space Time

Date: November 18, 2019 - 2:50 PM

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Abstract: In physics, every observation is made with respect to a frame of reference. Although reference frames are usually not considered as degrees of freedom, in all practical situations it is a physical system which constitutes a reference frame. Can a quantum system be considered as a reference frame and, if so, which description would it give of the world? In the first part of my talk, I will introduce a general method to quantise reference frame transformations within a Galilean-relativistic setting, which generalises the usual reference frame transformation to a â€œsuperposition of coordinate transformationsâ€•. We describe states, measurement, and dynamical evolution in different quantum reference frames, without appealing to an external, absolute reference frame, and find that entanglement and superposition are frame-dependent features. The transformation also leads to a generalisation of the notion of covariance of dynamical physical laws. In the second part of my talk, I will show how these ideas can be used to operationally define the localization of events with respect to quantum clocks, each of which identifies a â€œtime reference frame". In particular, I will consider clocks that i) are quantum mechanical, and ii) interact, gravitationally or otherwise, with other quantum systems. We find that, when gravitational effects are important, the time localisability of events becomes a relative concept, depending on the time reference frame. We discuss the physical significance of "jumping" onto a time reference frame with respect to which specific events are localised, in the context of indefinite causal structures arising from the interplay between quantum mechanics and gravity.

QUANTUM REFERENCE FRAMES FOR SPACE AND TIME

Flaminia Giacomini

Perimeter Institute for Theoretical Physics

Joint work with: A. Belenchia, Č. Brukner, E. Castro Ruiz

F. Giacomini, E. Castro Ruiz, Č. Brukner, arXiv:1712.07207, 2017
published in *Nat. Commun.* **10**(494), 2019
E. Castro Ruiz, F. Giacomini, A. Belenchia, Č. Brukner, arXiv1908.10165, 2019

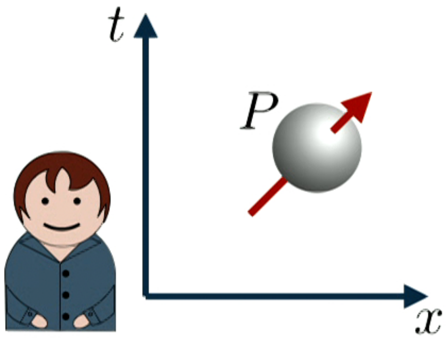
Emmy Noether Workshop:
The structure of quantum space time

Waterloo, 18 November 2019

What is a reference frame?

Reference frames are abstract entities, used to fix the point of view from which observations are carried out.

The laws of physics are the same regardless of the choice of the reference frame.
(Principle of covariance)



Translation $\hat{U}_T = e^{\frac{i}{\hbar} X_0 \hat{p}}$

Galilean boost $\hat{U}_B = e^{\frac{i}{\hbar} v \hat{G}}$ $\hat{G} = \hat{p}t - m\hat{x}$

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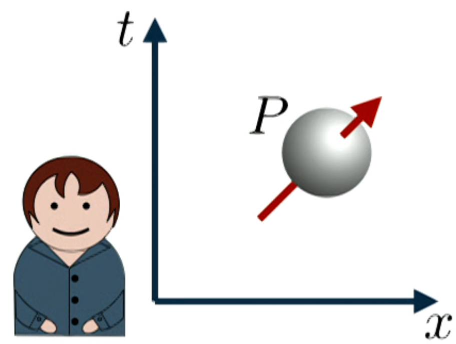

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⋮

The reference frame enters the transformation as a **parameter**.

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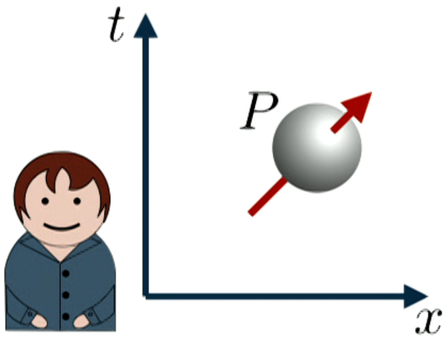

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⋮	

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Covariance of physical laws

$$\hat{H}' = \hat{U}\hat{H}\hat{U}^\dagger + i\hbar \frac{d\hat{U}}{dt}\hat{U}^\dagger$$

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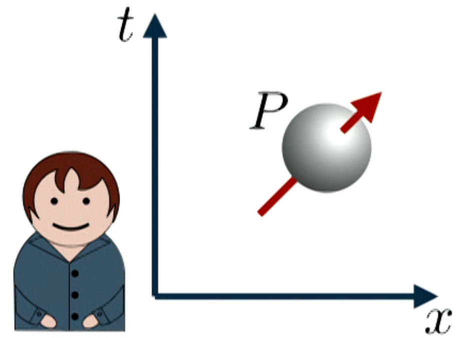

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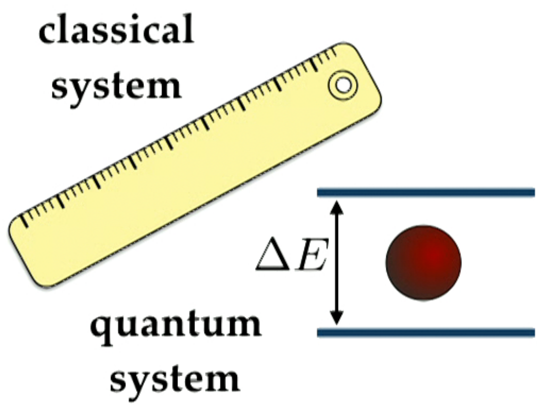
Symmetry

$$\hat{H}' = \hat{H}$$

What is a reference frame?

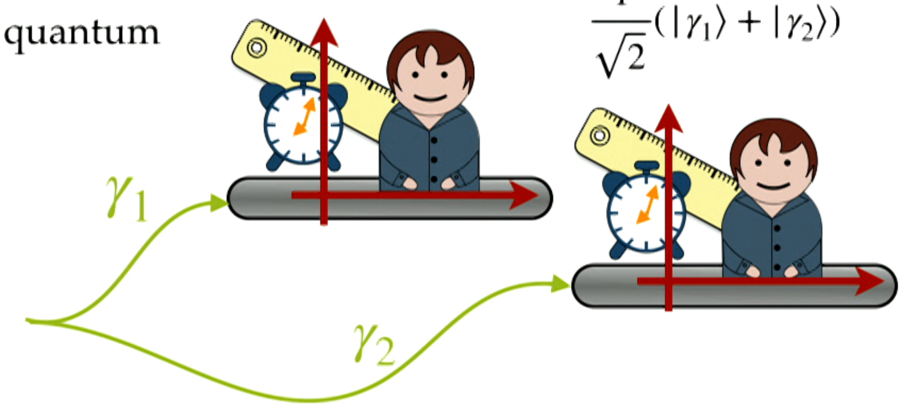


Every observation is carried out by means of a physical system…

… and taking as reference a physical object.

Physical systems are ultimately quantum

$$\frac{1}{\sqrt{2}}(|\gamma_1\rangle + |\gamma_2\rangle)$$



Can we associate a reference frame to a quantum particle, whose state is in a quantum superposition (in some basis)?

Quantum reference frames

Disclaimer:
Does not describe spacetime fuzziness, classical reference frames which are in a quantum relationship

Outline

QUANTUM REFERENCE FRAMES FOR SPACE

- Overview of the formalism
- Results
  - Frame dependence of entanglement and superposition
  - Extension of the covariance of quantum mechanics

F. Giacomini, E. Castro Ruiz, Č. Brukner, Nat. Commun. 10(494), 2019, arXiv:1712.07207, 2017

QUANTUM REFERENCE FRAMES FOR TIME

- Motivation
- Formalism
- Superposition of causal orders

E. Castro Ruiz, F. Giacomini, A. Belenchia, Č. Brukner, arXiv1908.10165, 2019

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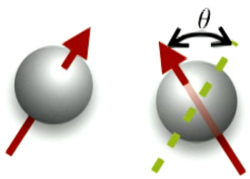
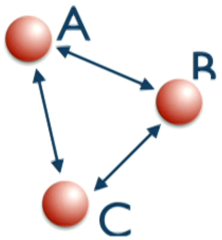
QUANTUM REFERENCE FRAMES FOR SPACE

No absolute space



Relational approach: only **relative** quantities are considered.

No need of an absolute reference frame.



FG, E. Castro Ruiz, C. Brukner, Nat Commun. (2019) arXiv:1712.07207
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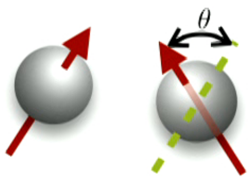

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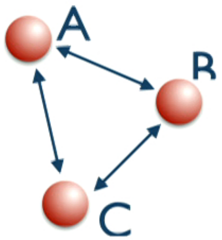

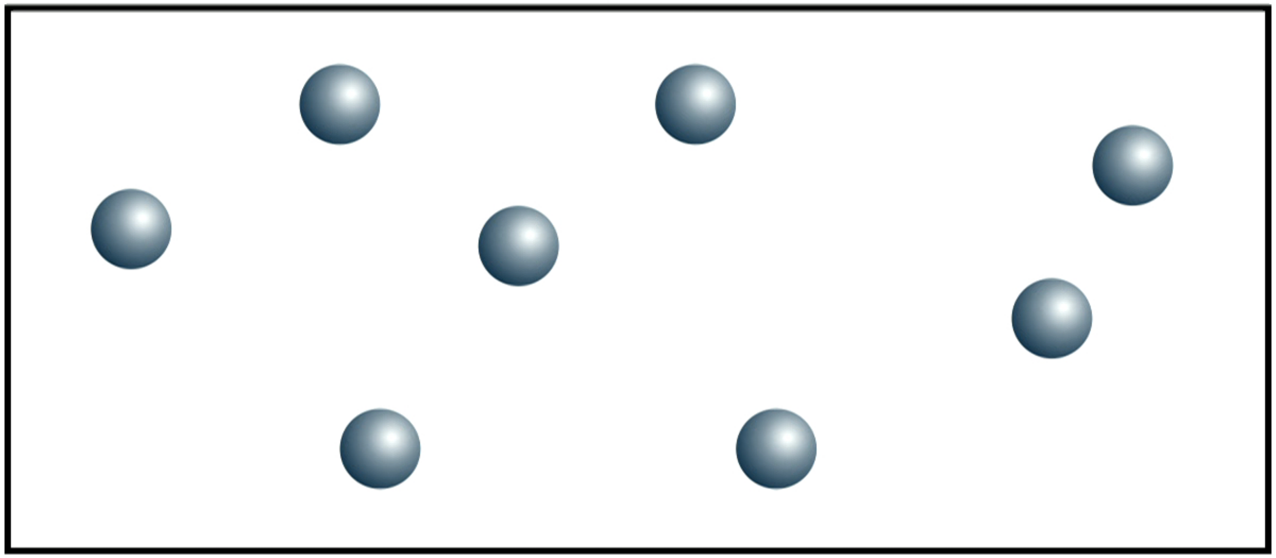

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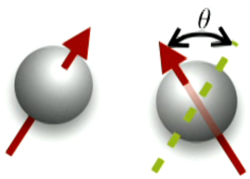

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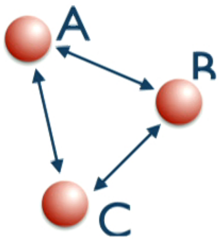

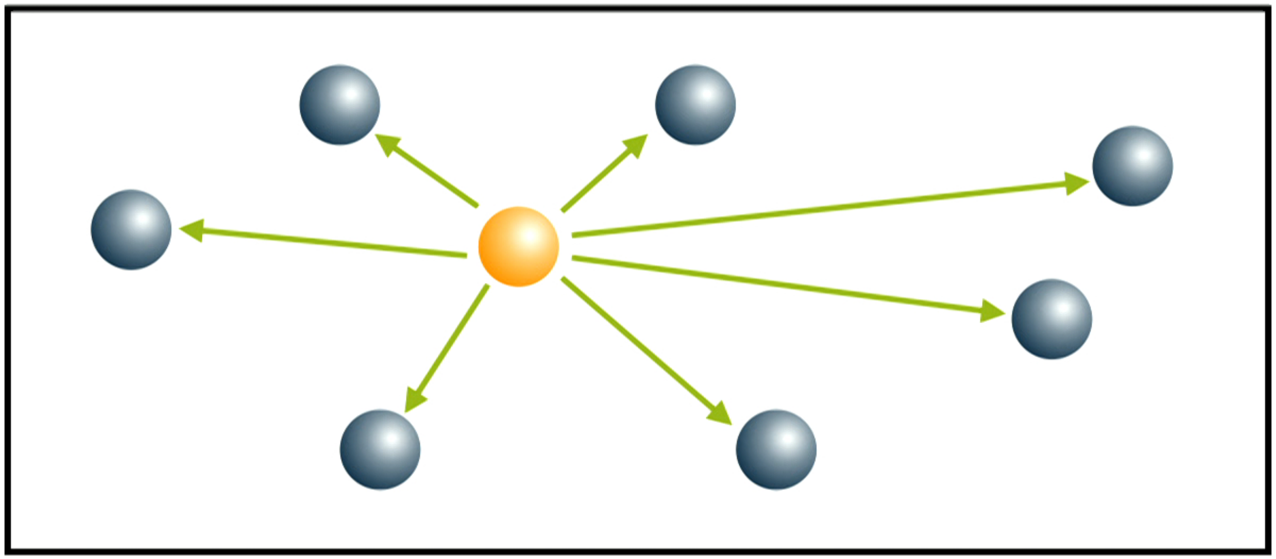

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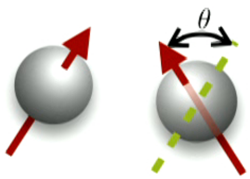

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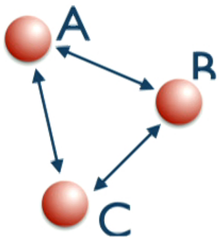

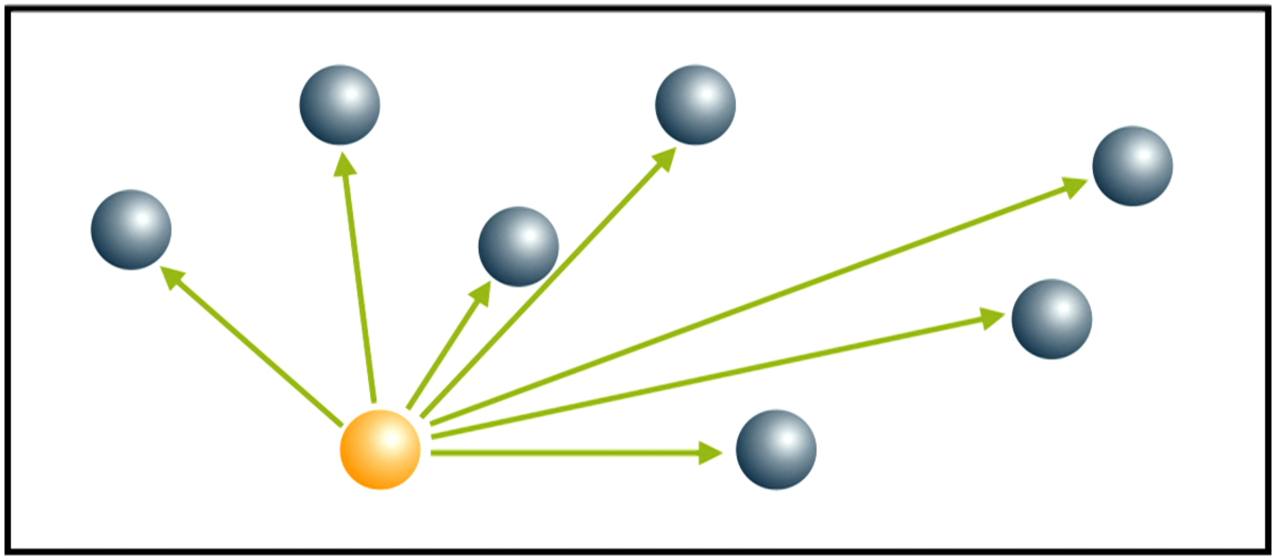

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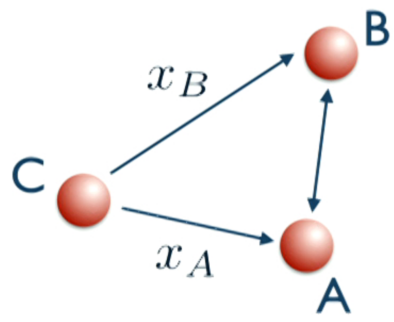
Quantum reference frames

Transformation to relative coordinates

$x_A \mapsto -q_C$

$x_B \mapsto q_B - q_C$



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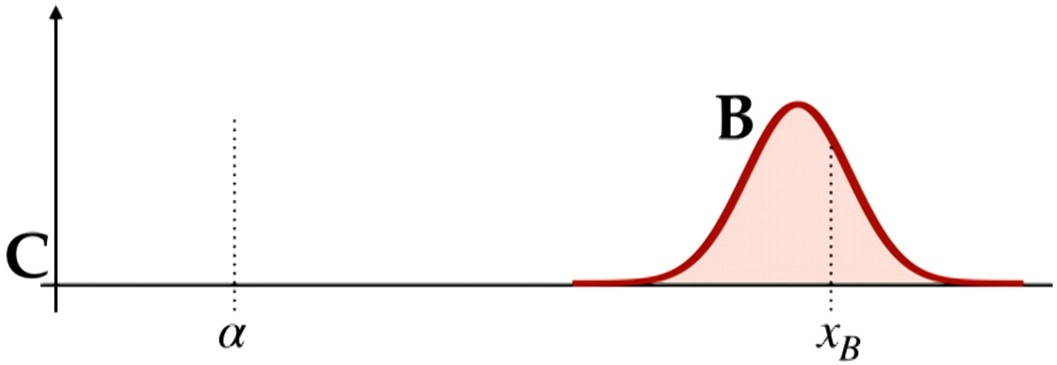

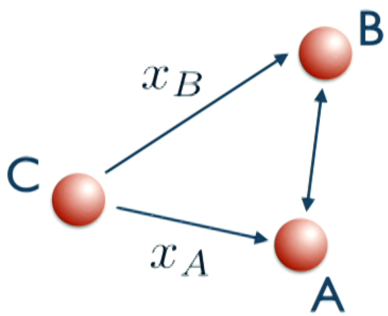

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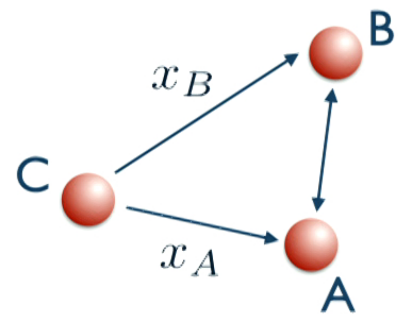

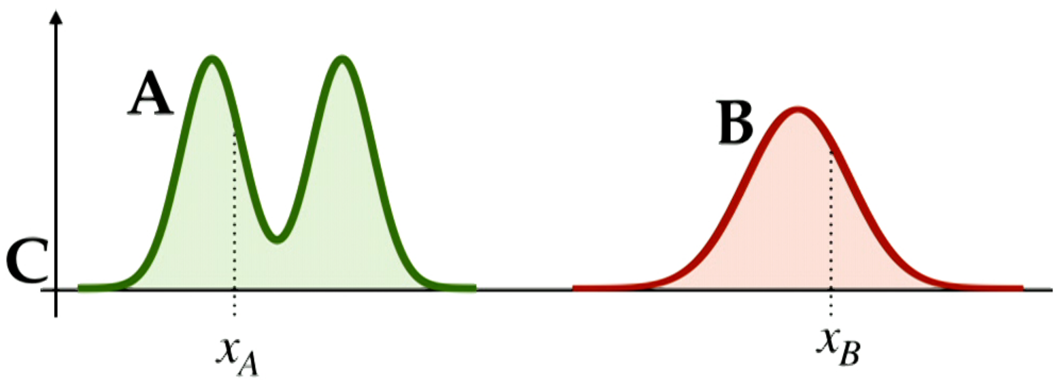

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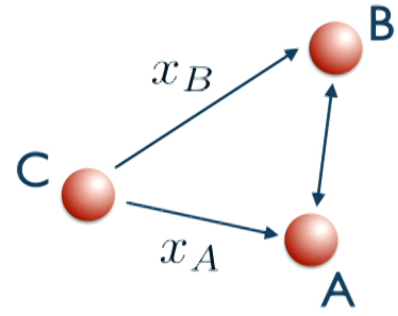

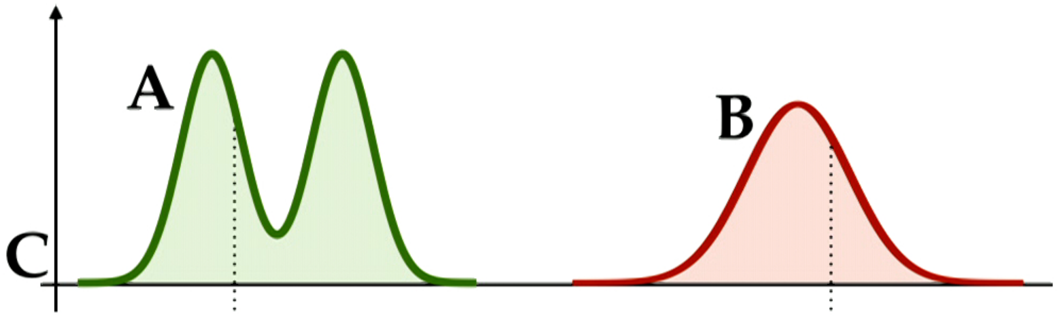

x_A x_B

$e^{\frac{i}{\hbar}\hat{x}_A\hat{p}_B}|\phi\rangle_A|\psi\rangle_B$

$$\hat{S}_x = \mathscr{P}_{AC}e^{\frac{i}{\hbar}\hat{x}_A\hat{p}_B}$$

$$\rho_{BC}^{(A)} = \hat{S}_x\rho_{AB}^{(C)}\hat{S}_x^\dagger$$

$\mathscr{P}_{AC}\hat{x}_A\mathscr{P}_{AC}^\dagger = -\hat{q}_C$

parity-swap operator

FC, E. Castro Ruiz, C. Brukner, Nat Commun. (2019)
arXiv:1712.07207

Relative states

$$\hat{S}_x = \mathscr{P}_{AC} e^{\frac{i}{\hbar}\hat{x}_A \hat{p}_B}$$

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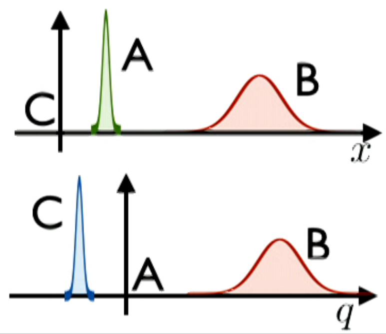
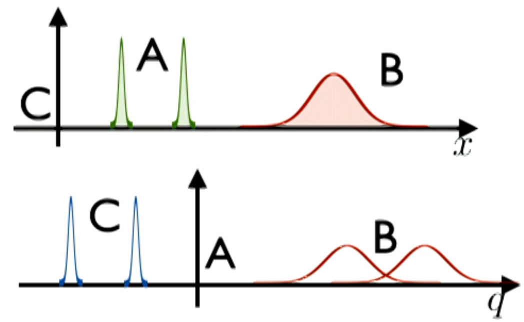
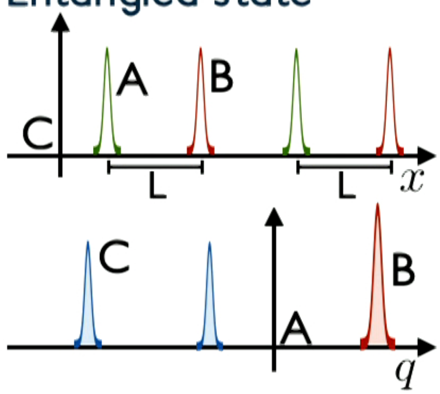
Localised state of A



Product state and spatial superposition

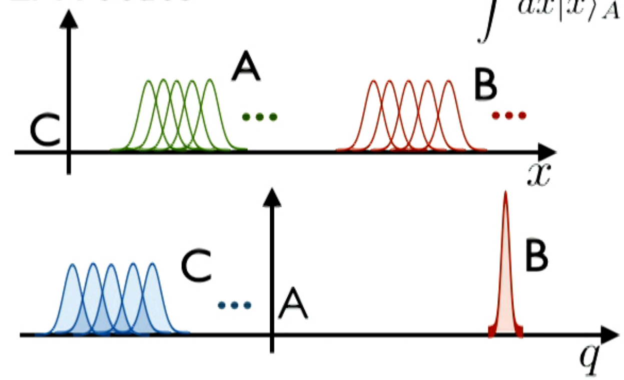


Entangled state



EPR state

$$\int dx |x\rangle_A |x + X\rangle_B$$



A: new reference frame;
B: quantum system; C: old reference frame

Extended covariance

Schrödinger equation in C's reference frame

$$i\hbar \frac{d\rho_{AB}^{(C)}}{dt} = \left[H_{AB}^{(C)}, \rho_{AB}^{(C)}(t) \right]$$

A: new reference frame
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FG, E. Castro Ruiz, C. Brukner,
Nat Commun. (2019)arXiv:1712.07207

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To change to the frame of A we apply the transformation $\hat{S}$

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To change to the frame of A we apply the transformation $\hat{S}$

$$i\hbar\frac{d\rho_{BC}^{(A)}}{dt} = \left[H_{BC}^{(A)}, \rho_{BC}^{(A)}(t)\right]$$

$$\hat{H}_{BC}^{(A)} = \hat{S}\hat{H}_{AB}^{(C)}\hat{S}^{\dagger} + i\hbar\frac{d\hat{S}}{dt}\hat{S}^{\dagger}$$

$$\hat{\rho}_{BC}^{(A)} = \hat{S}\hat{\rho}_{AB}^{(C)}\hat{S}^{\dagger}$$

The evolution in the new reference frame is unitary.

We define an extended symmetry transformation as:

$$\hat{S}\hat{H}\left(\{m_i, \hat{x}_i, \hat{p}_i\}_{i=A,B}\right)\hat{S}^{\dagger} + i\hbar\frac{d\hat{S}}{dt}\hat{S}^{\dagger} = \hat{H}\left(\{m_i, \hat{x}_i, \hat{p}_i\}_{i=B,C}\right)$$

FG, E. Castro Ruiz, C. Brukner,
Nat Commun. (2019)arXiv:1712.07207

Superposition of translations

The new QRF is described by system A at time 0.

$$|\Psi_0\rangle_{AB} = \frac{1}{\sqrt{2}}\left(|x_1\rangle_A + |x_2\rangle_A\right)|\phi_0\rangle_B \qquad \hat{H}_{AB}^{(C)} = \frac{\hat{p}_A^2}{2m_A} + \frac{\hat{p}_B^2}{2m_B}$$

We want to jump
to the QRF of A

$$\hat{S}_T = \exp\left(-\frac{i}{\hbar}\frac{\hat{\pi}_C^2}{2m_C}t\right)\hat{\mathcal{P}}_{AC}^{(x)}\exp\left(\frac{i}{\hbar}\hat{x}_A\hat{p}_B\right)\exp\left(\frac{i}{\hbar}\frac{\hat{p}_A^2}{2m_A}t\right)$$

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$\hat{S}_x$ translation to a reference frame which is frozen in time.

$$\hat{H}^{(A)}_{BC} = \frac{\hat{\pi}_B^2}{2m_B} + \frac{\hat{\pi}_C^2}{2m_C}$$

The free hamiltonian is symmetric under generalised translations.

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QUANTUM REFERENCE FRAMES FOR TIME

A simple clock model

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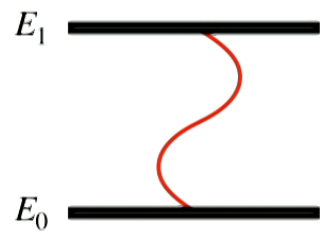


$$\frac{1}{\sqrt{2}}(|E_0\rangle + |E_1\rangle)$$

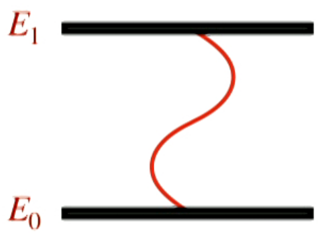
$$H_C = E_0|E_0\rangle\langle E_0| + E_1|E_1\rangle\langle E_1|$$

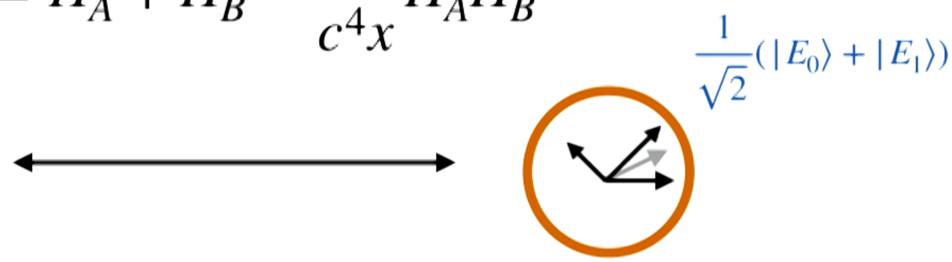
$$t_\perp = \frac{\pi\hbar}{(E_1 - E_0)}$$

13

Gravitating clocks lead to a non-classical spacetime

$$H = H_A + H_B - \frac{G}{c^4 x} H_A H_B$$





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E.Castro Ruiz, FG, C Brukner, *PNAS* (2017)

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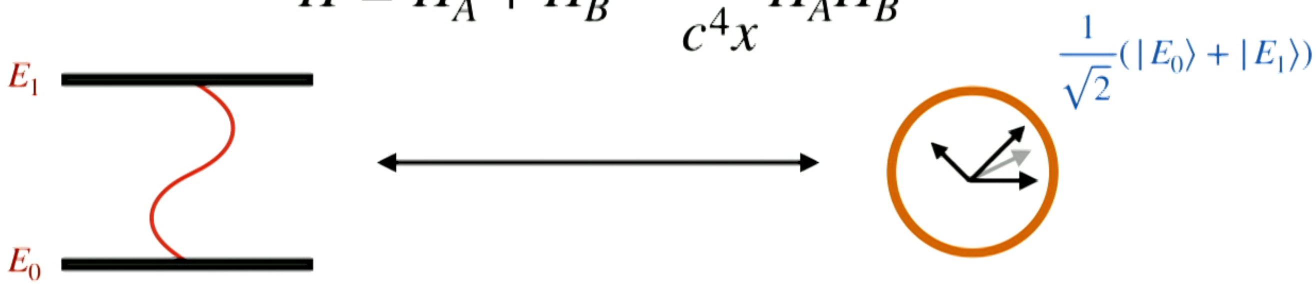

$$t_\perp = \frac{\pi\hbar}{(E_1 - E_0)}$$

$$\Delta t = \frac{G(E_1 - E_0)}{c^4 x} t$$

$$t_\perp \Delta t = \frac{\pi\hbar G t}{c^4 x}$$

E.Castro Ruiz, FG, C Brukner, *PNAS* (2017)

QM with no time parameter?

Option 1: Far-away observer

E.Castro Ruiz, FG, C Brukner, *PNAS* (2017)

15

QM with no time parameter?

Option 1: Far-away observer

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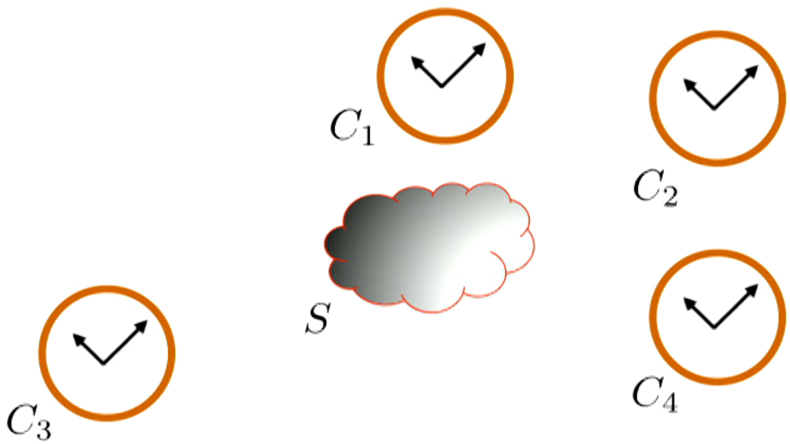
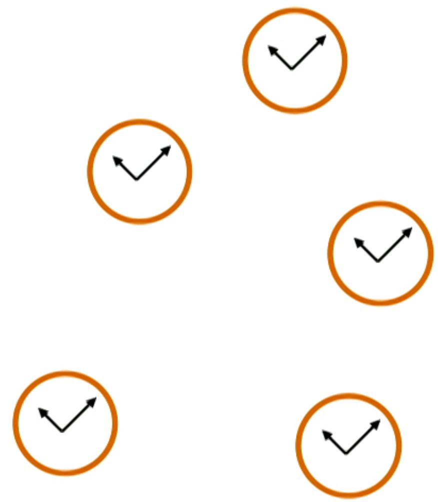
Option 2: Reference frames for time evolution (this talk)



Can we "stand" on different clocks and describe quantum dynamics from their point of view?

Timeless quantum mechanics



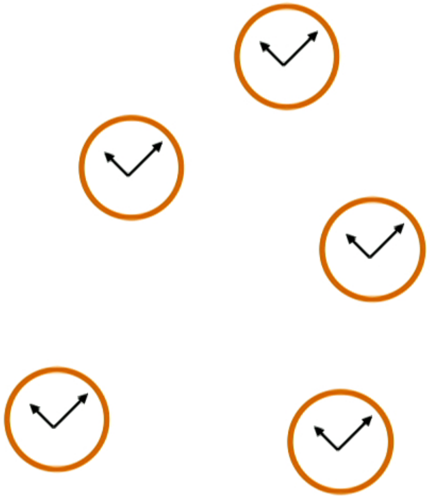
D. Page, W. Wootters, PRD (1983)
M. Reisenberger, C. Rovelli, PRD (2002)

Timeless quantum mechanics

$$\hat{C}|\Psi\rangle_{ph} = 0 \qquad \hat{C} = \sum_{k=1}^{N} \hat{H}_k + \sum_{j<k} \lambda_{jk} \hat{H}_j \hat{H}_k$$

$$|\Psi\rangle_{ph} \propto \int d\alpha e^{\frac{i}{\hbar}\hat{C}\alpha} |\phi\rangle$$

$$\lambda_{jk} = -\frac{G}{c^4 x_{jk}}$$



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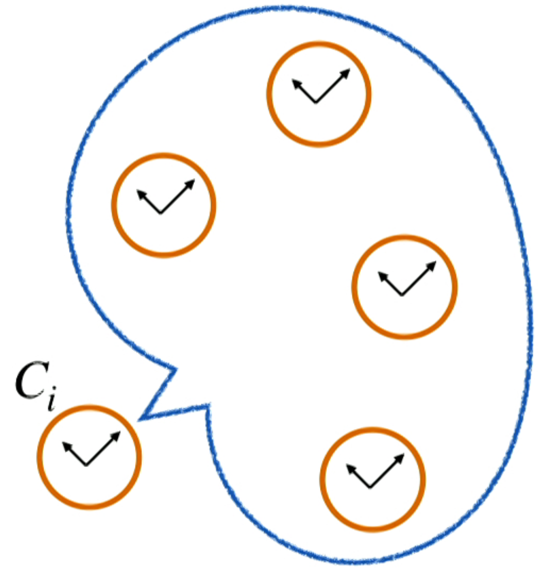
$$|\Psi\rangle_{ph} \propto \int d\alpha e^{\frac{i}{\hbar}\hat{C}\alpha} |\phi\rangle$$

$$\lambda_{jk} = -\frac{G}{c^4 x_{jk}}$$

Perspective of clock i

$${}_i\langle t_i|\Psi\rangle_{Ph} = |\psi(t_i)\rangle^{(i)}$$



D. Page, W. Wootters, PRD (1983)
M. Reisenberger, C. Rovelli, PRD (2002)

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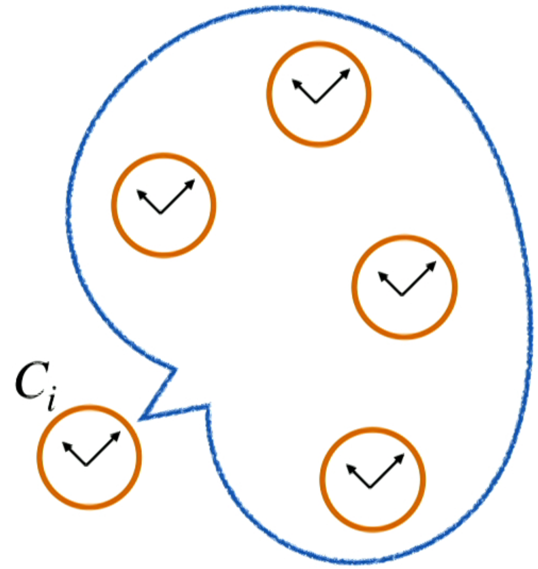
Perspective of clock i

$${}_i\langle t_i|\Psi\rangle_{Ph} = |\psi(t_i)\rangle^{(i)}$$

$$i\hbar\left(1 + \sum_{k\neq i}\lambda_{ik}\hat{H}_k\right)\frac{d|\psi(t_i)\rangle^{(i)}}{dt_i} = \left(\sum_{k\neq i}\hat{H}_k + \sum_{j<k}\lambda_{jk}\hat{H}_j\hat{H}_k\right)|\psi(t_i)\rangle^{(i)}$$



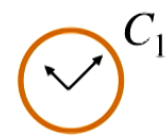
$\lambda_{ik} \to 0$ Clock hamiltonian from far-away observer

D. Page, W. Wootters, PRD (1983)
M. Reisenberger, C. Rovelli, PRD (2002)

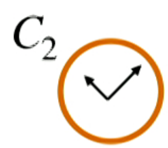
Introducing the measurement

$[\hat{C}, \hat{O}] = 0$

Non-evolving quantities?
Restriction of observables?



Solution: “Purify” the measurement



F Hellmann, M Mondragon, A Perez, C Rovelli PRD (2007)
V Giovannetti, S Lloyd, L Maccone, PRD (2015)

17

Introducing the measurement

$$[\hat{C}, \hat{O}] = 0$$

Non-evolving quantities?
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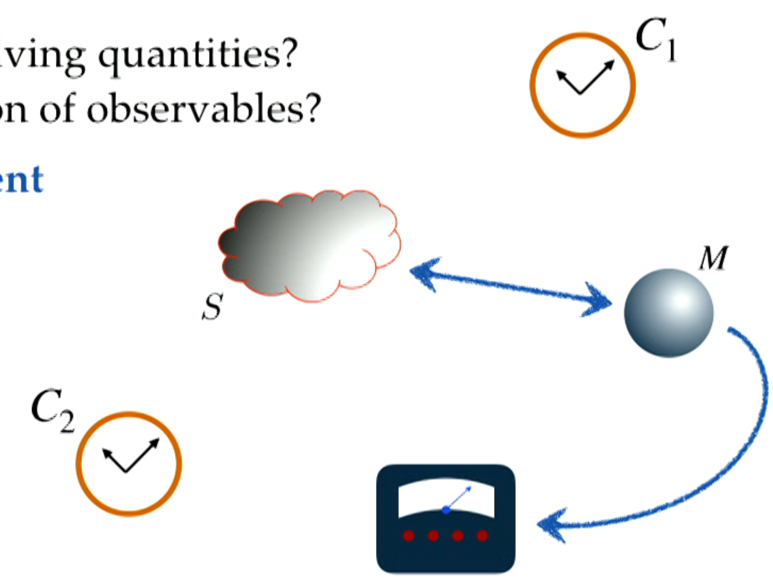


Solution: "Purify" the measurement

Clocks 1 and 2 

System S 

Ancilla M 

Previous Hamiltonian

$$\hat{C} = \hat{H}_1 + \hat{H}_2 + \hat{H}_S + \lambda \hat{H}_1 \hat{H}_2 + (1 + \lambda \hat{H}_1) \sum_i \delta(\hat{T}_2 - t_i) \hat{K}_i^{MS}$$

Time dilation factor due to clock 1

Time of measurement controlled by clock 2

Observable on S and M

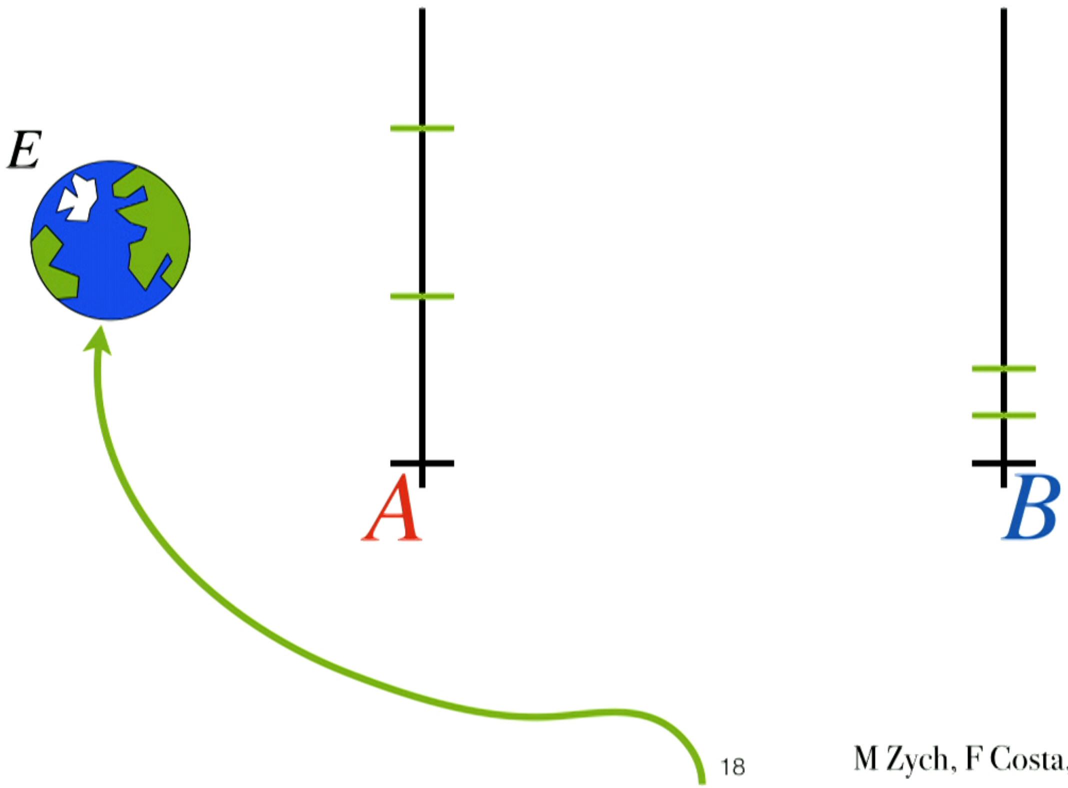
F Hellmann, M Mondragon, A Perez, C Rovelli PRD (2007)
V Giovannetti, S Lloyd, L Maccone, PRD (2015)

The gravitational switch

18

M Zych, F Costa, I Pikovski, C Brukner (2017)

The gravitational switch



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M Zych, F Costa, I Pikovski, C Brukner (2017)

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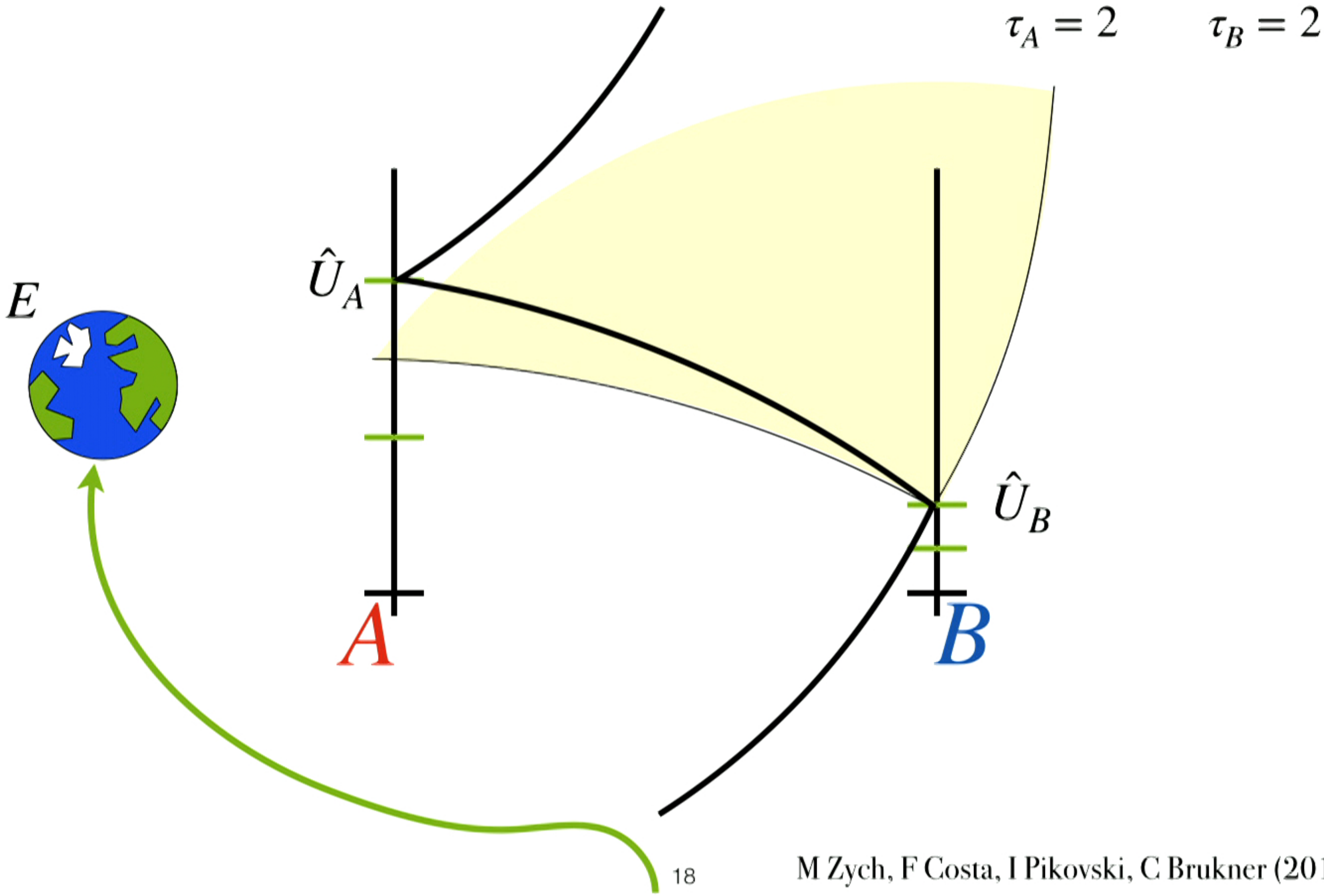

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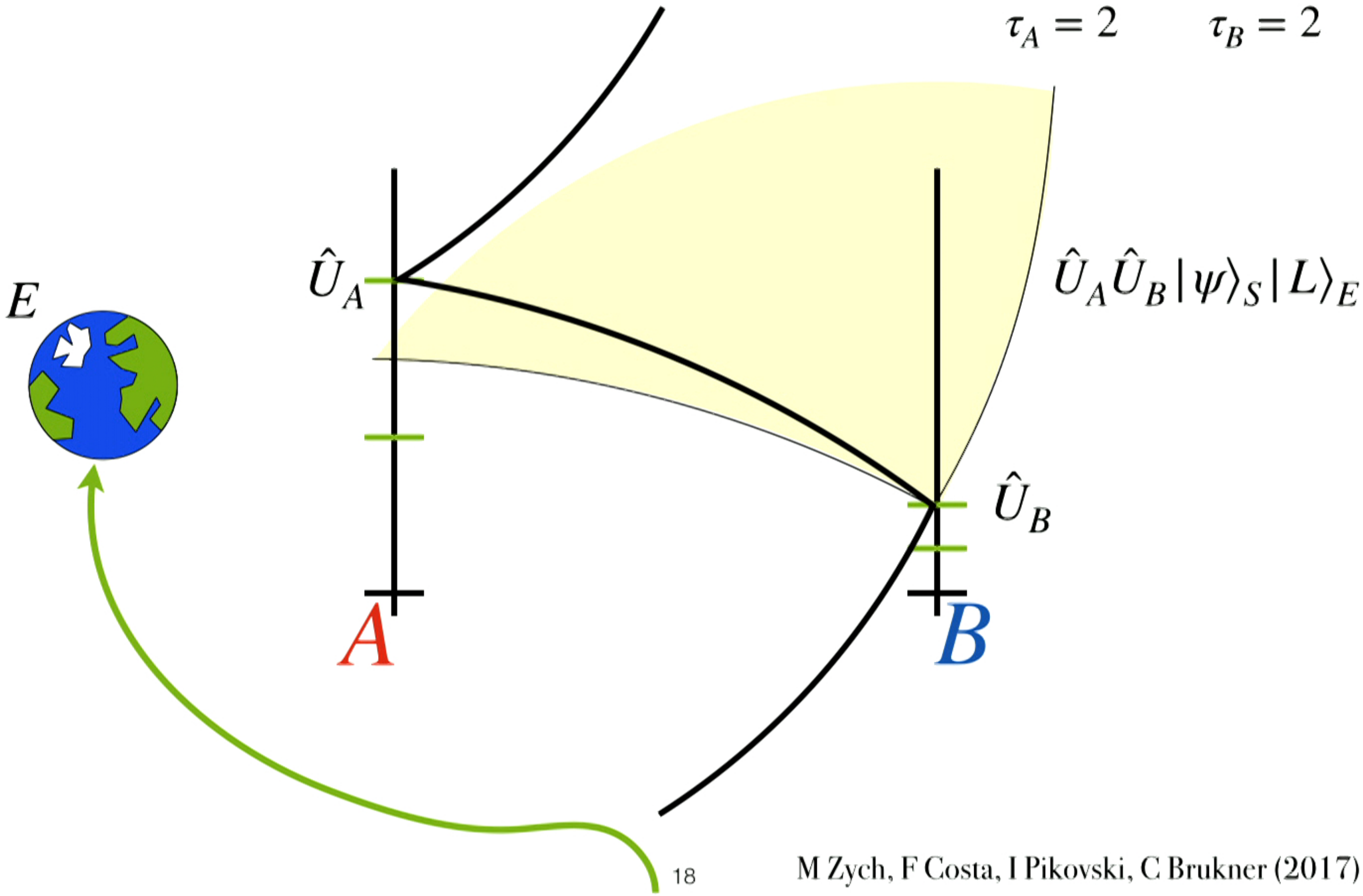

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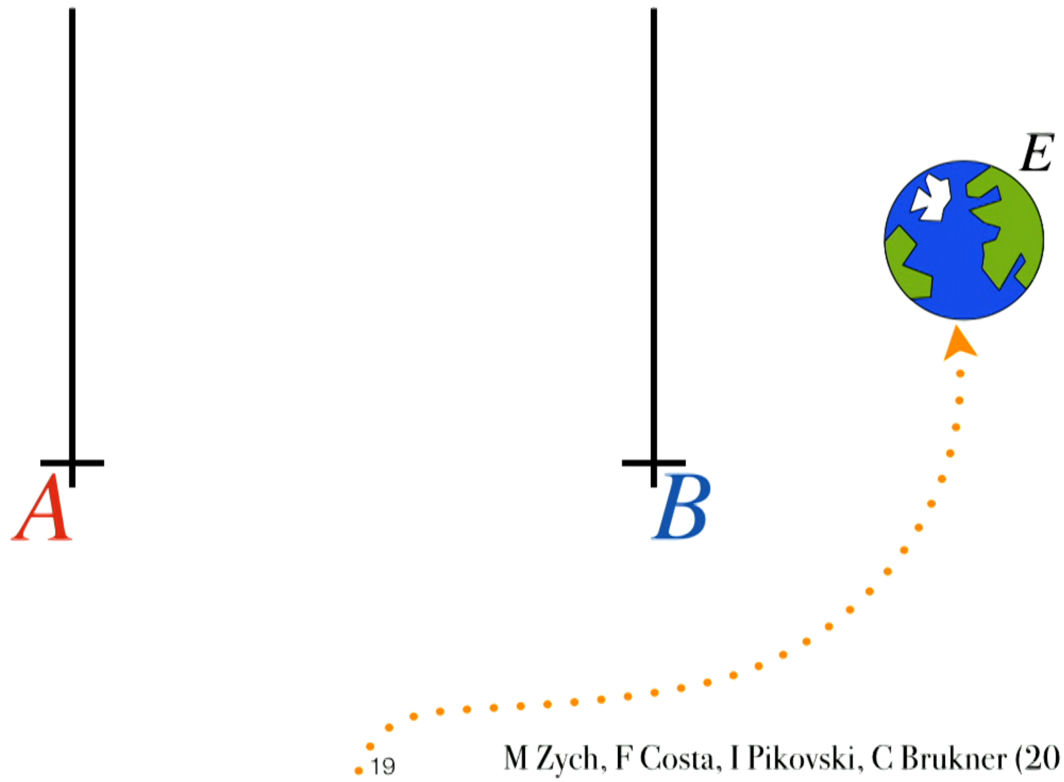
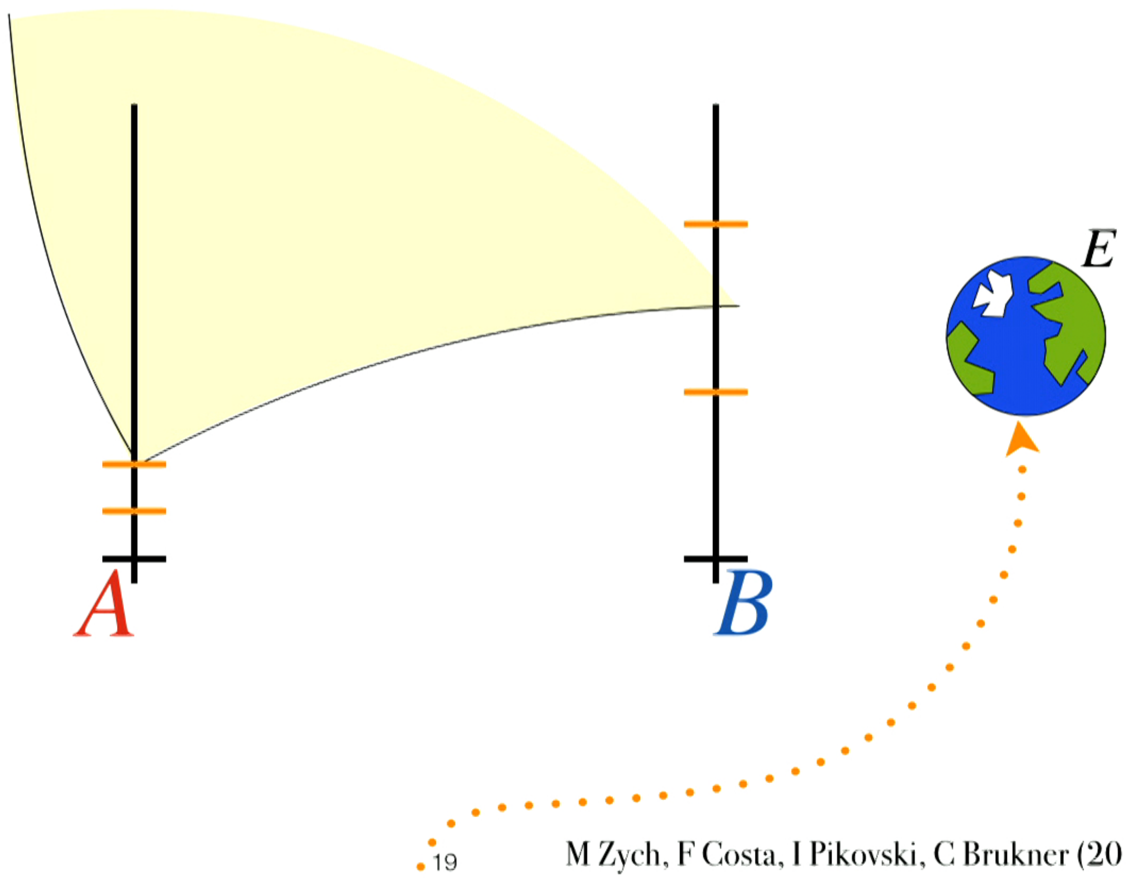


19

M Zych, F Costa, I Pikovski, C Brukner (2017)

The gravitational switch

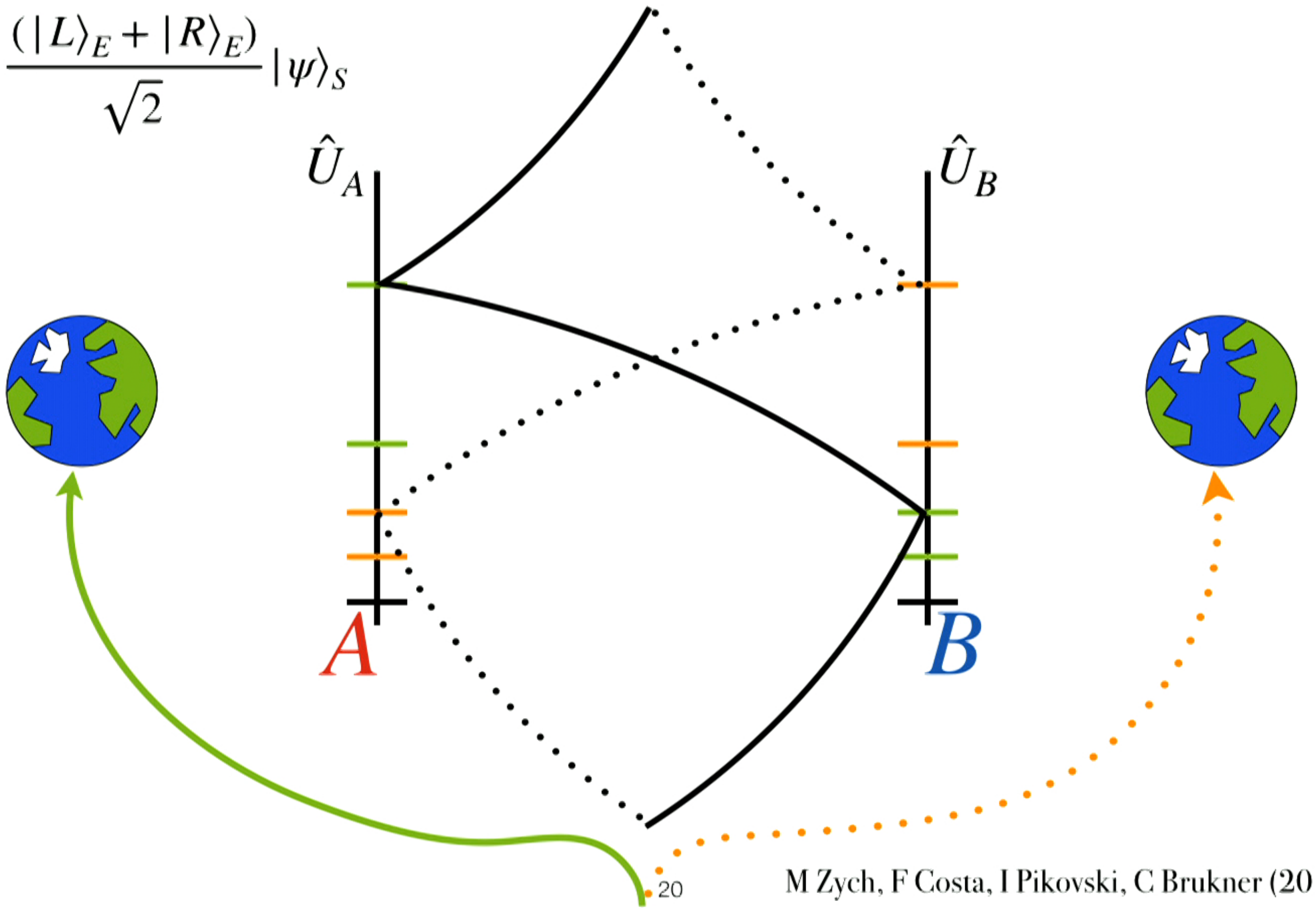


19

M Zych, F Costa, I Pikovski, C Brukner (2017)

The gravitational switch

$$\frac{(|L\rangle_E + |R\rangle_E)}{\sqrt{2}} |\psi\rangle_S$$



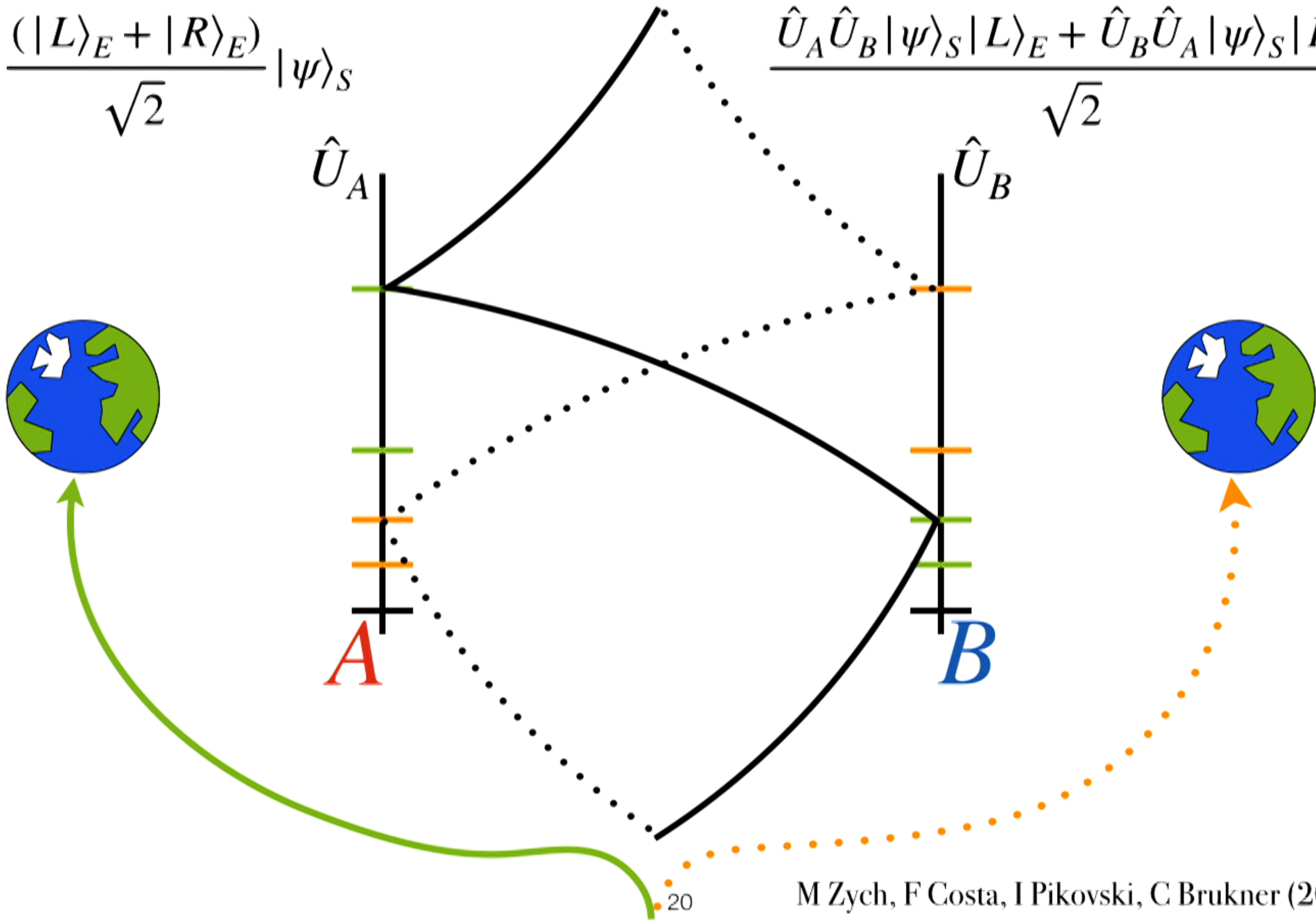
20

M Zych, F Costa, I Pikovski, C Brukner (2017)

The gravitational switch

$$\frac{(|L\rangle_E + |R\rangle_E)}{\sqrt{2}} |\psi\rangle_S$$

$$\frac{\hat{U}_A \hat{U}_B |\psi\rangle_S |L\rangle_E + \hat{U}_B \hat{U}_A |\psi\rangle_S |R\rangle_E}{\sqrt{2}}$$



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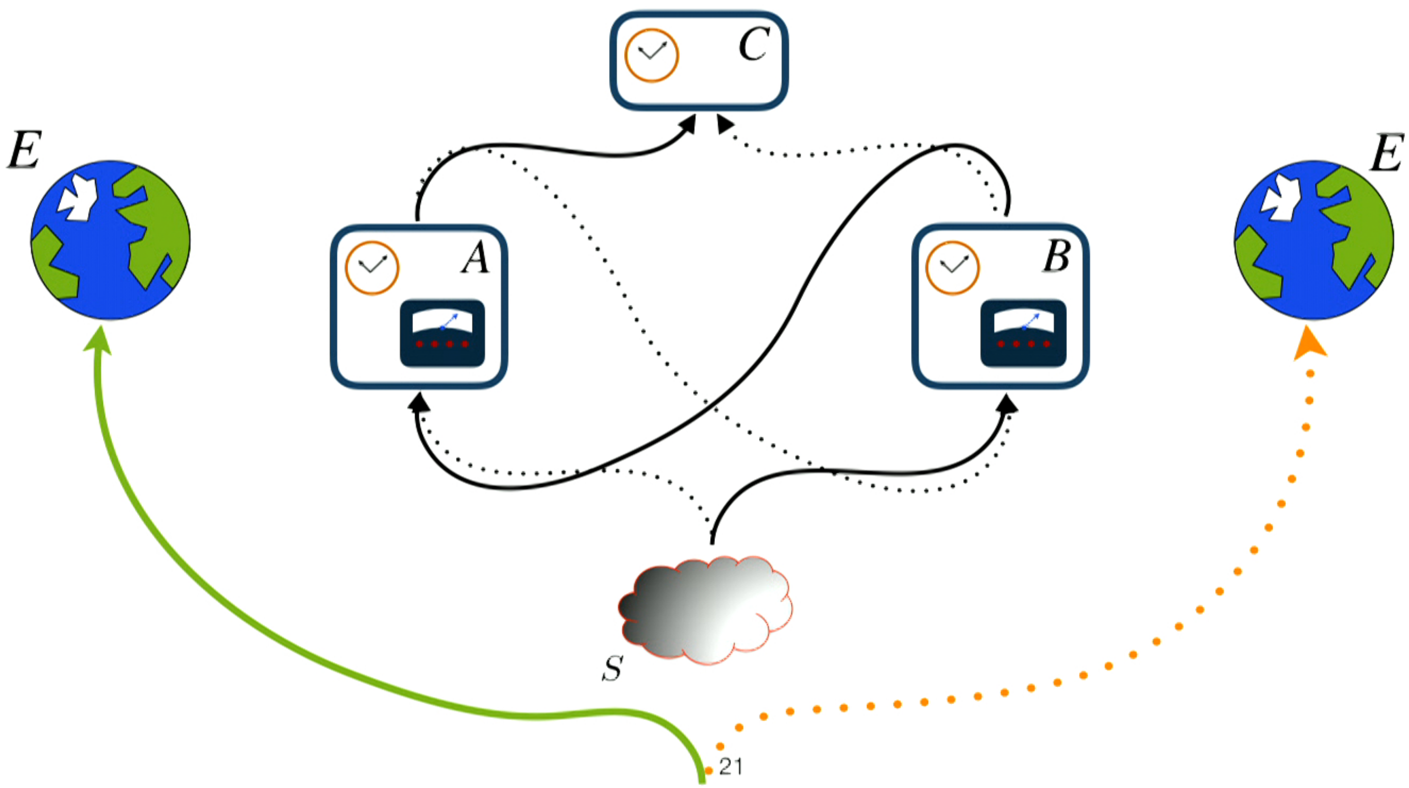
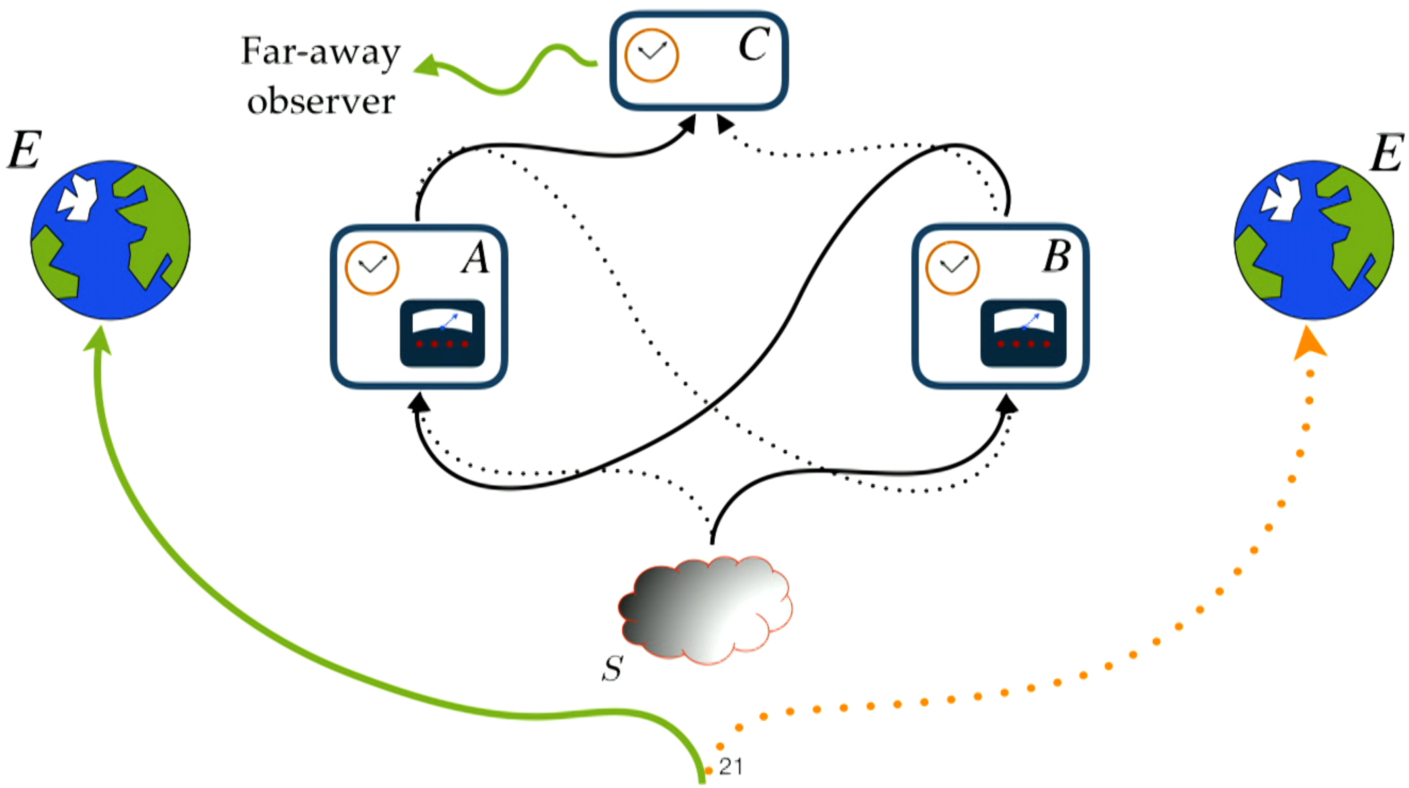
M Zych, F Costa, I Pikovski, C Brukner (2017)

Relative localisation of events



Relative localisation of events

$$\hat{C} = \sum_{i=A,B,C} \hat{H}_i(1+\hat{\phi}_i) + \sum_{i=A,B} \delta(\hat{T}_i - t^*)\hat{K}_i^S(1+\hat{\phi}_i) \qquad \hat{\phi}_i = -\frac{GM_E}{c^2\hat{x}_i}$$



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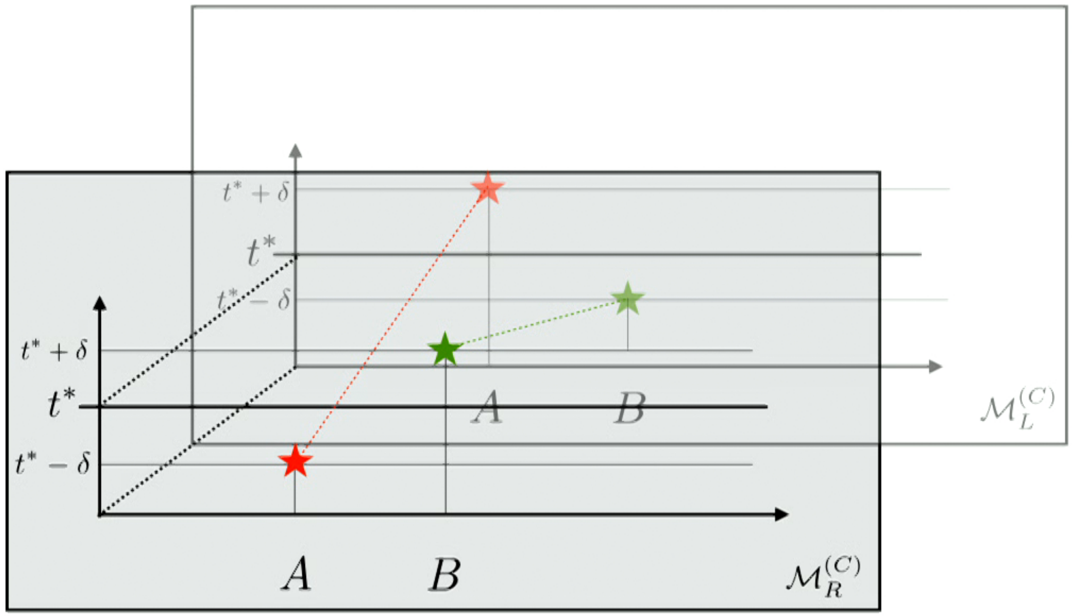
From C's point of view



Summary

Operational and relational formalism for quantum reference frames
for space and time.

For space:
Frame-dependence of entanglement and superposition
Generalisation of covariance
Generalisation of the weak equivalence principle
Operational definition of the rest frame of a quantum system (relativistic spin)

For time:
Hamiltonian for interacting clocks (with gravitational time dilation)
Superposition of causal orders
Relativity of interactions

ALESSIO

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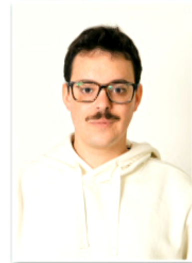

ESTEBAN

THANK YOU

Useful literature:

F. Giacomini, E. Castro Ruiz, Č. Brukner, arXiv:1712.07207, 2017
published in *Nat. Commun.* **10**(494), 2019
A. Vanrietvelde, P. A. Höhn, F. Giacomini, E. Castro Ruiz, arXiv:1809.00556, 2018
A. Vanrietvelde, P. A. Höhn, F. Giacomini, arXiv:1809.05093, 2018
F. Giacomini, E. Castro Ruiz, Č. Brukner, *Phys. Rev. Lett.* **123(9)** (090404), 2019
E. Castro Ruiz, F. Giacomini, A. Belenchia, Č. Brukner, arXiv1908.10165, 2019

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CoQuS ComplexQuantumSystems