

Title: Spacetime Foam and the Cosmological Constant

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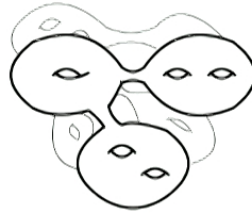
URL: <http://pirsa.org/19110107>

Abstract: I describe a radical proposal for the cosmological constant problem: perhaps  $\Lambda$  really is very large, but is "hidden" in Planck-scale fluctuations of geometry and topology. I show that an enormous set of initial data describe a universe with such a hidden cosmological constant at an initial time. The question of whether this structure is preserved under time evolution is still open, but I provide some evidence that it may be. I close with a discussion of open questions that might lead to further insight (or perhaps kill the idea).

# Spacetime Foam

Wheeler (1950s): near Planck scale,  $\delta g_{ab} \sim 1$

⇒ “foamlike” structure that may be invisible at large scales



“...it is essential to allow for fluctuations in the metric and gravitational interactions in any proper treatment of the compensation problem—the problem of compensation of ‘infinite’ energies that is so central to the physics of fields and particles.”

## The “Old” Cosmological Constant Problem

Gravity is universal  $\Rightarrow$  gravity should couple to vacuum energy  
 $\Rightarrow$  expect a large cosmological constant

Standard effective field theory arguments:

$$\Lambda \sim \sum \frac{\ell_p^2}{\lambda_i^4} \times \text{logs} \sim \begin{cases} 100 \text{ cm}^{-2} & \text{for Standard Model} \\ 10^{66} \text{ cm}^{-2} & \text{if new physics up to Planck mass} \end{cases}$$

Too large by a factor of at least  $10^{58}$  (maybe  $10^{122}$ )

Desperate measures:

- extraordinarily fine tuned cancellation
- ad hoc revision of field equations to decouple vacuum energy
- anthropic arguments

## If $\Lambda$ really were very large, would we know it?

Obviously yes...

...but what if curvature is hidden in spacetime foam?

Easy to imagine for Riemannian space:



Harder for spacetime:

$\Lambda \Rightarrow$  exponential expansion

Can have expanding and contracting regions: solutions depend on  $\pm\sqrt{|\Lambda|}$

Can we move beyond hand-waving?

## The initial value formulation

Spatial slice  $\Sigma$  characterized by

- intrinsic metric  $g_{ij}$
- extrinsic curvature  $K^i_j = \sigma^i_j + \frac{1}{3}\delta^i_j K$

Physically,  $K$  is local expansion/Hubble constant,  $\sigma$  is local shear

Constraints:

$$R + K^2 - K^i_j K^j_i - 2\Lambda = 0$$

$$D_i(K^i_j - \delta^i_j K) = 0$$

Time reversal symmetry:

If  $(g_{ij}, K^k_\ell)$  satisfies constraints, so does  $(g_{ij}, -K^k_\ell)$

## Gluing initial data

Results of Chrusciel, Isenberg, and Pollack:

Two manifolds  $\Sigma_1$  and  $\Sigma_2$  with initial data  $(g_1, K_1)$  and  $(g_2, K_2)$  can be “glued” to form a manifold  $\Sigma_1 \# \Sigma_2$  with the same initial data outside arbitrarily small neighborhoods of the gluing

(Technical caveat: not too much symmetry in gluing region)



In particular:

Glue manifold  $\Sigma$  with data  $(g, K)$  to identical manifold  $\bar{\Sigma}$  with data  $(g, -K)$

Then  $\Sigma \# \bar{\Sigma}$  has isometry with  $(g, K) \rightarrow (g, -K)$

$\Rightarrow \langle K^i_j \rangle = 0$  for any sensible definition of average

## Generalization

Start with random collection of manifolds  $\Sigma_1, \Sigma_2, \dots, \Sigma_N$   
with initial data  $(g_\alpha, K_\alpha)$

Glue:  $\tilde{\Sigma} = \Sigma_1 \# \Sigma_2 \# \dots \# \Sigma_N$

For each  $\Sigma_\alpha$ , data  $(g, K)$  and  $(g, -K)$  are equally likely  
 $\Rightarrow \langle K^i_j \rangle \sim 0$  for a large enough collection\*

May also need  $\langle \frac{\partial_i N}{N} \rangle = 0$ : also generically true

Large class of initial data describes spacetimes  
whose average expansion and shear vanish

\*Note: 1 cubic centimeter  $\sim 10^{100}$  Planck volumes

## How typical is this?

We don't know. . .

- For fixed topology, glued structure is fairly special  
(“necks” typically contain marginally trapped surfaces)
- For topological fluctuations, connected sum decomposition always exists  
(and in three dimensions is basically unique)
- Physically, local fluctuations should arguably have arbitrary sign of  $K$



### What about curvature?

$$\text{Constraints: } \langle R \rangle = 2\langle \sigma^2 \rangle + 2\Lambda - \frac{2}{3}\langle K^2 \rangle$$

So if  $\langle K^2 \rangle$  is large (for  $\Lambda > 0$ )

or  $\langle \sigma^2 \rangle$  is large (for  $\Lambda < 0$ )

average curvature can also be small

Note: in the connected sum

factors typically have negative curvature, necks have positive curvature

Is small  $\langle R \rangle$  preferred by quantum gravity? Or by time evolution?

## Evolution

Two naive expectations:

- Expanding regions grow, contracting regions shrink  
so expanding regions should dominate
- But nothing special about our initial time slice:  
quantum fluctuations should reproduce foamy structure

Note (Unruh & Wang): if  $\Lambda < 0$ , local regions oscillate  
 $\Rightarrow$  no long-term domination by expanding regions  
but must continue through (classical) singularities

## **Evolution is hard:**

- Evolution of averages is ambiguous
  - How does region change with time?
  - How does measure change with time?
  - What “time” do you use for evolution?
- Evolution is probably highly nonclassical
  - Evolution of glued manifolds  $\Rightarrow$  singularities
  - If complicated structure is generated by quantum fluctuations, it may be reproduced in time

**Preliminary question:**

**is there *any* classical evolution that preserves this structure?**

Evolution equations

$$\mathcal{L}_n g_{ij} = 2g_{ik} K^k_j$$

$$\mathcal{L}_n K^i_j = -R^i_j - K K^i_j + \Lambda \delta^i_j + \frac{D^i D_j N}{N}$$

Lapse function  $N$ :

- Can be chosen arbitrarily (gauge choice)
- True observables are independent of choice

But physics sometimes picks a “preferred” choice for us:

- Homogeneity and isotropy are not statements about “our Universe,” but about “our space with a particular lapse function” (“there exists a lapse function such that. . .”)

Choose volume average:  $\langle X \rangle_{\mathcal{U}} = \int_{\mathcal{U}} X \sqrt{g} d^3x / \text{Vol}(\mathcal{U})$

$$\Rightarrow \frac{d}{dt} \langle K \rangle = \frac{1}{\text{Vol}(\mathcal{U})} \int_{\mathcal{U}} N \left( \Lambda + \frac{2}{3} K^2 - 2\sigma^2 \right) \sqrt{g} d^3x$$

Can choose  $N$  such that  $d\langle K \rangle/dt$  vanishes

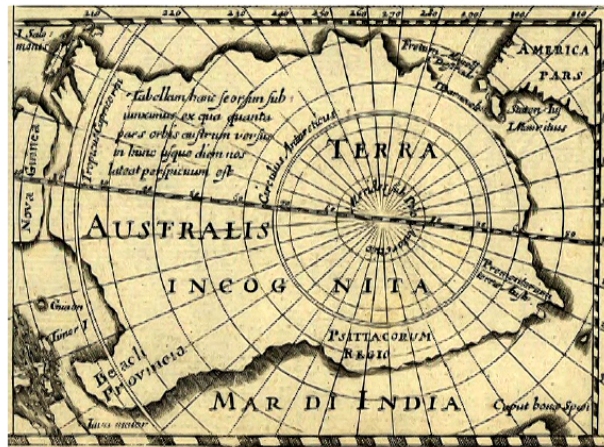
Higher derivatives of  $\langle K \rangle$  contain higher derivatives of  $N$

Probably slicing exists with  $\langle K \rangle = 0 = \left\langle \frac{\partial_i N}{N} \right\rangle =$  near initial surface

Like homogeneity and isotropy: “there exists a lapse function. . .”

Is it “preferred”?

## Quantum evolution?



Does “spacetime foam” really occur?

One hopeful sign: may only need constraints

But may not be able to avoid “problem of time”

# How much geometry is in a truncated spectral triple?

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Plan for this talk:

- Spectral triples
- Fuzzy spaces
- Truncated triples
- Painting a picture

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21st November 2019



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