Title: Central extension and black hole entropy

Speakers: Lin-Qing Chen

Collection: Emmy Noether Workshop: The Structure of Quantum Space Time

Date: November 22, 2019 - 11:40 AM

URL: http://pirsa.org/19110102

Abstract: Recent developments on asymptotic symmetries and soft modes have deepened our understanding of black hole entropy and the information paradox. The asymptotic symmetry charge algebra of certain classes of spacetimes could have a nontrivial central extension, which plays a crucial role in black hole physics. The Cardy formula of the asymptotic density of states of the dual CFT has been famously used to reproduce the Bekenstein-Hawking entropy formula. However, without assuming holography, it remains obscure from the point of view of gravity how such a constant on the gravitational phase space encodes the information about the density of black hole microstates, and what the gravitational degrees of freedom accounting for the black hole entropy truly are. I will discuss my ongoing efforts of understanding these questions in the covariant phase space formalism.

Central Extension and Black Hole Entropy

Lin-Qing Chen, OIST

Ongoing efforts with Slava Lysov, OIST

Emmy Noether Workshop: The structure of quantum spacetime Perimeter Institute 2019.11.22.

Outline

Deriving Bekenstein-Hawking entropy formula has been one of the first test ground for any theory of quantum gravity. I will discuss one particular approach and unfold my questions.

- Background and questions
- Covariant phase space
- Central extension
- Non-equivariance of charges
- Implication from diffeomorphism edge modes

- It was suggested 25 years ago that BH entropy is governed by the horizon symmetries. The present of horizon as a boundary makes the would-be-gauge d.o.f. to be new physical states. *[Carlip, 94]*
- The first example of using Cardy formula for computing BH entropy: *[Strominger, 97']*
  - For the class of black holes which near horizon geometry is locally AdS, but globally and topologically might not.
  - Derivation doesn't based on string theory
  - eg: BTZ $M = \frac{1}{l}(L_0 + \bar{L}_0) = \frac{r_+^2 + r_-^2}{8Gl^2}$

    $J = L_0 - \bar{L}_0 = \frac{r_+ r_-}{4Gl}$
  - Using micro-canonical Cardy formula for 2d CFT

$$S_{CFT} = ln\rho = 2\pi(\sqrt{\frac{c_R \Delta}{6}} + \sqrt{\frac{c_L \bar{\Delta}}{6}}) = \frac{2\pi r_+}{4G}$$

Extremal Kerr / CFT

[Starting from Guica, Hartman, Song, Strominger, 08′]

- Extreme Kerr $J = GM^2 \qquad r_+ = r_- = M$
- Near horizon limit of the throat geometry *[Bardean, Horowizz, 99′]*

Zoom in $\hat{r} \sim M$ region: $r = \dfrac{\hat{r} - M}{\lambda M}, \quad \lambda \to 0$ but hold ratio fixed

metric becomes:

$$ds^2 = (1 + cos^2\theta)J[\frac{dr^2}{r^2} + d\theta^2 - r^2dt^2 + (\frac{2sin\theta}{1 + cos^2\theta})^2(d\phi + rdt)^2]$$

fixed θ locally warped $AdS_3, \quad \phi \sim \phi + 2\pi$ identified

Finite temperature CFT $\quad T = 1/2\pi$

- Asymptotic symmetry vector fields for the near horizon limit of the throat geometry

$$\xi = \epsilon(\phi)\partial_\phi - r\epsilon'(\phi)\partial_r$$

- Charge algebra forms Virasoro:

$$\{L_m, L_n\} = (m-n)L_{m+n} + \frac{c}{12}m^3\delta_{m+n,0} \qquad c = 12J/\hbar$$

- Quotient of AdS3 related to finite temprature CFT

- Applying canonical Cardy formula:

$$S_{CFT} = \frac{\pi^2}{3}cT = 2\pi M r_+ = S_{ExKerr}$$

- Near extremal Kerr: The phase space of near-horizon region exhibits hidden conformal symmetries $\omega r \ll 1$

[Castro, Maloney, Strominger, 10' etc.]

- Generic Kerr

- Generic black hole: horizon preserving diffeomorphisms are enhanced to BMS_3 symmetry. Using a generalized Cardy-formula one can reproduce the Bekenstein-Hawking entropy!

[Carlip, 17' 19'etc.]

Questions

$$S_{CFT} = \frac{\pi^2}{3} cT$$

- How does central charge encode black hole entropy?
- What microscopic states are accounted for?

The usual answer:

Cardy formula is a generic property for 2-d CFT. The black hole microstates are the CFT states due to the holographic duality.

Questions

- How universal is such derivation?

- Since there is no central charge for odd dimension CFT. Does it imply dimensional reduction for 4-d generic black hole? Or, is there a universal mechanism that beyond what 2-d CFT could capture?

- From the gravitational phase space point of view, how to understand the appearance of central extension encodes the information?

- Let us see how does it arise generically......

Covariant phase space formalism

[Peierl 50s, Ashtekar, Witten, Crnkovic 80s, Wald, Lee, Iyer, Zoupas, Barnich, Compere, Harlow, Wu etc.]

- It is a formalism to study the d.o.f. of a gauge system, the Hamiltonian and asymptotic symmetries in general covariant way.
- Starting from the Lagrangian: $L(\phi, \partial_\mu \phi \ldots)$ spacetime d-form

$$\delta L = E\delta\phi + d\theta(\phi, \delta\phi)$$

symplectic potential $\theta(\phi, \delta\phi)$: spacetime d-1-form, field space 1-form

- Building the symplectic structure from the space of on-shell solutions:

$$\omega = \delta\theta(\phi, \delta\phi) = \delta_1\theta(\phi, \delta_2\phi) - \delta_2\theta(\phi, \delta_1\phi)$$

- Eg: QED $\omega = \delta A \wedge *\delta F$

- For local covariant theory:

Noether current: $j_\xi = \theta(\phi, \mathcal{L}_\xi \phi) - \iota_\xi L$

$$d\omega = 0 \quad \Big\downarrow \quad \delta_\xi L = \mathcal{L}_\xi L$$

$$dj_\xi = 0$$

- On shell, the Noether current is trivially conserved

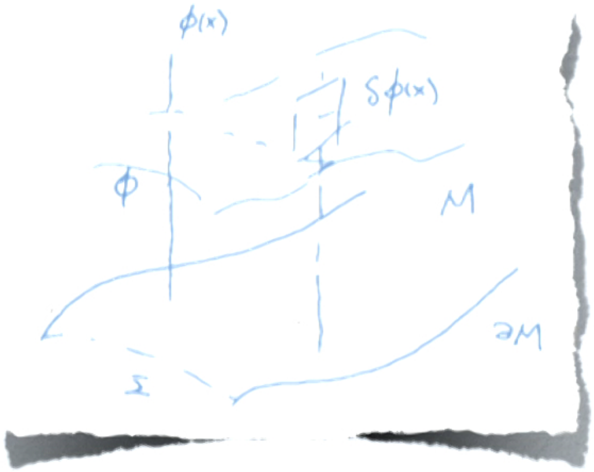
So locally $\exists Q_\xi^N$ $\qquad j_\xi = dQ_\xi^N(\phi) + C \cdot E$

(doesn't generate symmetry)

- A presymplectic form associated with a Cauchy surface

$$\Omega_\Sigma = \int_\Sigma \omega(\phi, \delta_1\phi, \delta_2\phi)$$

- Since $d\omega = 0$ $\quad \Omega_{\Sigma_1} = \Omega_{\Sigma_2}$ $\quad$ if $\quad \partial\Sigma_1 = \partial\Sigma_2$



- We need to map a spacetime vector field to phase space

$$\hat{\xi} \to \xi \qquad \xi = \int d^d x \, \pounds_{\hat{\xi}} \phi(x) \frac{\delta}{\delta \phi}$$

- The diffeomorphisms that are truly gauge -- they are degenerate direction of the presymplectic form

$$\iota_\xi \Omega_\Sigma(\phi, \delta_1 \phi, \delta_2 \phi) = \Omega_\Sigma(\phi, \delta_1 \phi, \pounds_\xi \phi) = 0$$

- Physical phase space is obtained by quotienting the subgroup of gauge transformation whose vector fields on the phase space corresponding to the degenerate direction of the presympectic form.

- The "large diffeomorphisms" that as physical symmetry acting on the phase space.

- To find the Hamiltonian which generates non-trival gauge transformations on the phase space:

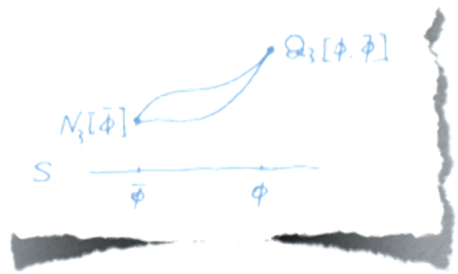
(if $\iota_\xi \tilde{\Omega}_\Sigma(\phi, \delta_1 \phi, \delta_2 \phi)$ is not exact: add Wald-Zoupus counter term)

if it is exact:

$$\delta \mathcal{Q}_\xi = \int_{\partial \Sigma} q_\xi[\delta\phi, \phi] \quad \text{where} \quad q_\xi[\delta\phi, \phi] = \delta Q^N_\xi - \iota_{\hat{\xi}} \theta(\delta\phi, \phi)$$

- The charge will be independent of the path:

$$\mathcal{Q}_\xi[\phi; \bar{\phi}] = \int_{\bar{\phi}}^{\phi} \int_{\partial\Sigma} q_\xi[\delta\phi, \phi] + N_\xi[\bar{\phi}]$$



A few important points

- Space of on-shell solutions
- Symplectic form from Lagrangian
- With boundary: trivial diffeomorphisms v.s. large diffeomorphisms
- Boundary symmetry charges
- Asymptotic symmetries: the boundary condition and fall-off conditions restrict on the space of solutions.

The central extension:

- When the charge is integrable,

$$\mathcal{Q}_\xi[g;\bar{g}] = \int_{\bar{g}}^{g} \int_{\partial\Sigma} q_\xi[\delta g, g] + N_\xi[\bar{g}]$$

- The charge algebra: $\{\mathcal{Q}_\xi, \mathcal{Q}_\eta\} = \mathcal{Q}_{[\xi,\eta]} + \mathcal{K}_{\xi,\eta}[\bar{g}]$

  where $\mathcal{K}_{\xi,\eta}[\bar{g}] = \int_{\partial\Sigma} q_\xi[\mathcal{L}_{\xi}\bar{g};\bar{g}] - N_{\xi,\eta}[\bar{g}]$

- Two cocycle: $\mathcal{K}_{[\xi,\eta],\sigma} + \mathcal{K}_{[\sigma,\xi],\eta} + \mathcal{K}_{[\eta,\sigma],\xi} = 0$

- It is a Casmir function on the phase space, doesn't generate flow.

Non-equivariance of charges

- After getting the physical phase space with non-degenerate symplectic form, symplectic geometry language can becomes very intuitive:
- Let us denote the action of the symmetry group (Lie group) G on the phase space $\mathcal{P}$ by Φ_g
- The Lie algebra element maps to a vector on the phase space

$$\xi \in \mathfrak{g} \rightarrow \xi_p \in T\mathcal{P}$$

under $\Phi_g^* \xi_p = (Ad_{g^{-1}} \xi)_p$

- The surface charge $\mathcal{Q}_\xi[\phi;\bar{\phi}]$ is essentially the "moment map":

$$\hat{\mathcal{Q}} : \mathcal{P} \to \mathfrak{g}^*$$

$$\forall \tilde{\xi} \in \mathfrak{g}, \quad -\delta\langle \hat{\mathcal{Q}}, \tilde{\xi} \rangle \;=\; \iota_\xi \Omega_\Sigma$$

- The existence of non-trivial $\mathcal{K}_{\xi,\eta}$

$$\begin{array}{ccc} \mathcal{P} & \xrightarrow{\hat{\mathcal{Q}}} & \mathfrak{g}^* \\ \Phi_g \downarrow & & \downarrow Ad^*_{g^{-1}} \\ \mathcal{P} & \xrightarrow{\hat{\mathcal{Q}}} & \mathfrak{g}^* \end{array}$$

Does not close!

$$\mathcal{Q}_\xi[\Phi_g \cdot \phi] \neq \mathcal{Q}_{Ad_{g^{-1}}\xi}[\phi]$$

$$\mathcal{Q}_\xi[\Phi_g \cdot \phi] - \mathcal{Q}_\xi[\phi] - (\mathcal{Q}_{Ad_{g^{-1}}\xi}[\phi] - \mathcal{Q}_\xi[\phi]) \neq 0$$

- The surface charge $\mathcal{Q}_\xi[\phi;\bar{\phi}]$ is essentially the "moment map":

$$\hat{\mathcal{Q}} : \mathcal{P} \to \mathfrak{g}^*$$
$$\forall \tilde{\xi} \in \mathfrak{g}, \quad -\delta\langle \hat{\mathcal{Q}}, \tilde{\xi} \rangle \;=\; \iota_\xi \Omega_\Sigma$$

- The existence of non-trivial $\mathcal{K}_{\xi,\eta}$

$$\begin{array}{ccc} \mathcal{P} & \xrightarrow{\hat{\mathcal{Q}}} & \mathfrak{g}^* \\ \Phi_g \downarrow & & \downarrow Ad^*_{g^{-1}} \\ \mathcal{P} & \xrightarrow{\hat{\mathcal{Q}}} & \mathfrak{g}^* \end{array}$$

Does not close!

$$\mathcal{Q}_\xi[\Phi_g \cdot \phi] \neq \mathcal{Q}_{Ad_{g^{-1}}\xi}[\phi]$$

$$\mathcal{Q}_\xi[\Phi_g \cdot \phi] - \mathcal{Q}_\xi[\phi] - (\mathcal{Q}_{Ad_{g^{-1}}\xi}[\phi] - \mathcal{Q}_\xi[\phi]) \neq 0$$

One sufficient condition that $\mathcal{K}_{\xi,\eta}$ is trivial:

$$\Omega_\Sigma = -\delta\Theta_\Sigma \quad \& \quad \mathcal{L}_\xi \Theta_\Sigma = 0$$

$$\begin{aligned}\{\mathcal{Q}_\xi, \mathcal{Q}_\eta\} &= \iota_\xi \iota_\eta \Omega = -\iota_\xi \iota_\eta \delta\Theta \\ &= -\iota_\xi \mathcal{L}_\eta \Theta + \iota_\xi \delta \iota_\eta \Theta = -\mathcal{L}_\xi \iota_\eta \Theta + \iota_\eta \mathcal{L}_\xi \Theta \\ &= -\iota_{[\eta,\xi]}\Theta = \mathcal{Q}_{[\eta,\xi]}\end{aligned}$$

- Hence we need to break at least one of the two conditions, to get the central term in the charge algebra....

$\pounds_\xi \Theta = g \neq 0$

- Some part of the non-invariance can be absorbed by adding a field-space exact 1-form to the symplectic potential , which is essentially a canonical transform on the phase space

$$\Theta \to \Theta + \delta\alpha$$

- For the rest of non-invariance, we can express absorb it in a_ξ

$$\iota_\xi \Theta = \mathcal{Q}_\xi + a_\xi$$

- Then the central extension could be expressed as

$$\mathcal{K}_{\xi,\eta} = a_{[\xi,\eta]} - \pounds_\xi a_\eta + \pounds_\eta a_\xi$$

- In the series work of the diffeomorphism edge modes

[Donnelly, Freidel, Speranza, Riello, Geiller etc.]

with presence of boundary, $\Theta_\Sigma(\phi, \delta\phi)$ is not gauge invariant

- The edge modes phase space is constructed from adding $\tilde{\Theta}_{\partial\Sigma}$ so that the total symplectic potential is gauge invariant

$$\mathcal{L}_\xi(\Theta + \tilde{\Theta}_{\partial\Sigma}) = 0$$

- Working in progress: the symplectic potential of the "edge modes" which corresponds to normal diffemorphisms (generated by vector fields normal to the boundary) cannot be fully localized on the boundary.

Discussions

- The obstruction of constructing line-bundle in the geometrical quantization
- Non-invariance of symplectic potential and the state degeneracy
- How does this picture of black hole entropy match to our understanding of entanglement entropy?
- How does the Cardy formula inspired derivation can be related to the discrete approaches of quantum gravity?
- Relation to the holographic renormalization?