Title: The Holographic Landscape of Symmetric Product Orbifolds

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Collection: Emmy Noether Workshop: The Structure of Quantum Space Time

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Abstract: I will discuss the application of Siegel paramodular forms to constructing new examples of holography. These forms are relevant to investigate the growth of coefficients in the elliptic genus of symmetric product orbifolds at large central charge. The main finding is that the landscape of symmetric product theories decomposes into two regions. In one region, the growth of the low energy states is Hagedorn, which indicates a stringy dual. In the other, the growth is much slower, and compatible with the spectrum of a supergravity theory on AdS_3. I will provide a simple diagnostic which places any symmetric product orbifold in either region. The examples I will present open a path to novel realizations of AdS_3/CFT_2.

The holographic landscape of symmetric product orbifolds

Alejandra Castro
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Emmy Noether Workshop, 2019

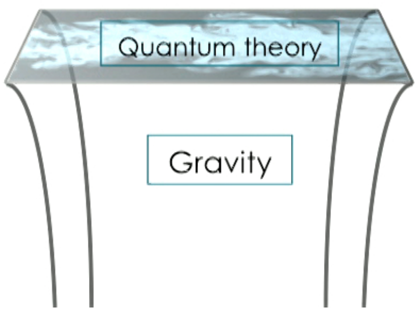
Based on
arXiv: 1611.04588 [hep-th]
arXiv: 1805.09336 [hep-th]
with Alex Belin, Joao Gomes and Christoph Keller

And
arXiv: 1910.05353 [hep-th]
arXiv: 1910.05342 [hep-th]
Alex Belin, Christoph Keller and Beatrix Mühlmann

Collaborators

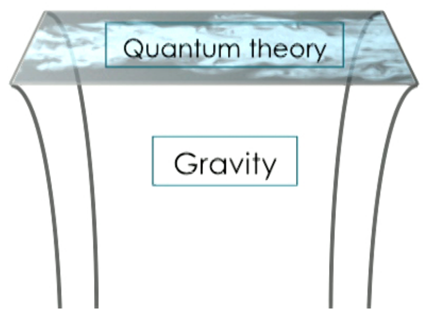
AdS/CFT provides a non-perturbative, UV-complete definition of quantum gravity in Anti-de Sitter space.

There are conditions on CFT such that it capture semi-classical gravitational features.

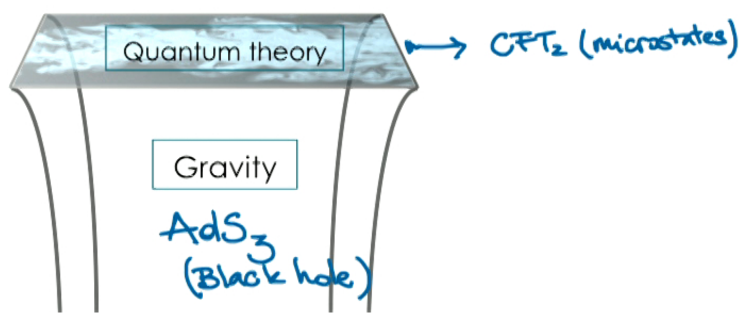


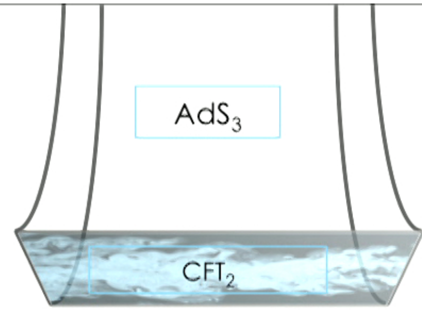
AdS/CFT provides a non-perturbative, UV-complete definition of quantum gravity in Anti-de Sitter space.

How to go about building CFTs with semi-classical gravitational features?



We will focus on the difficulties you encounter in AdS_3/CFT_2.
Not universal, but it illustrates the challenges.





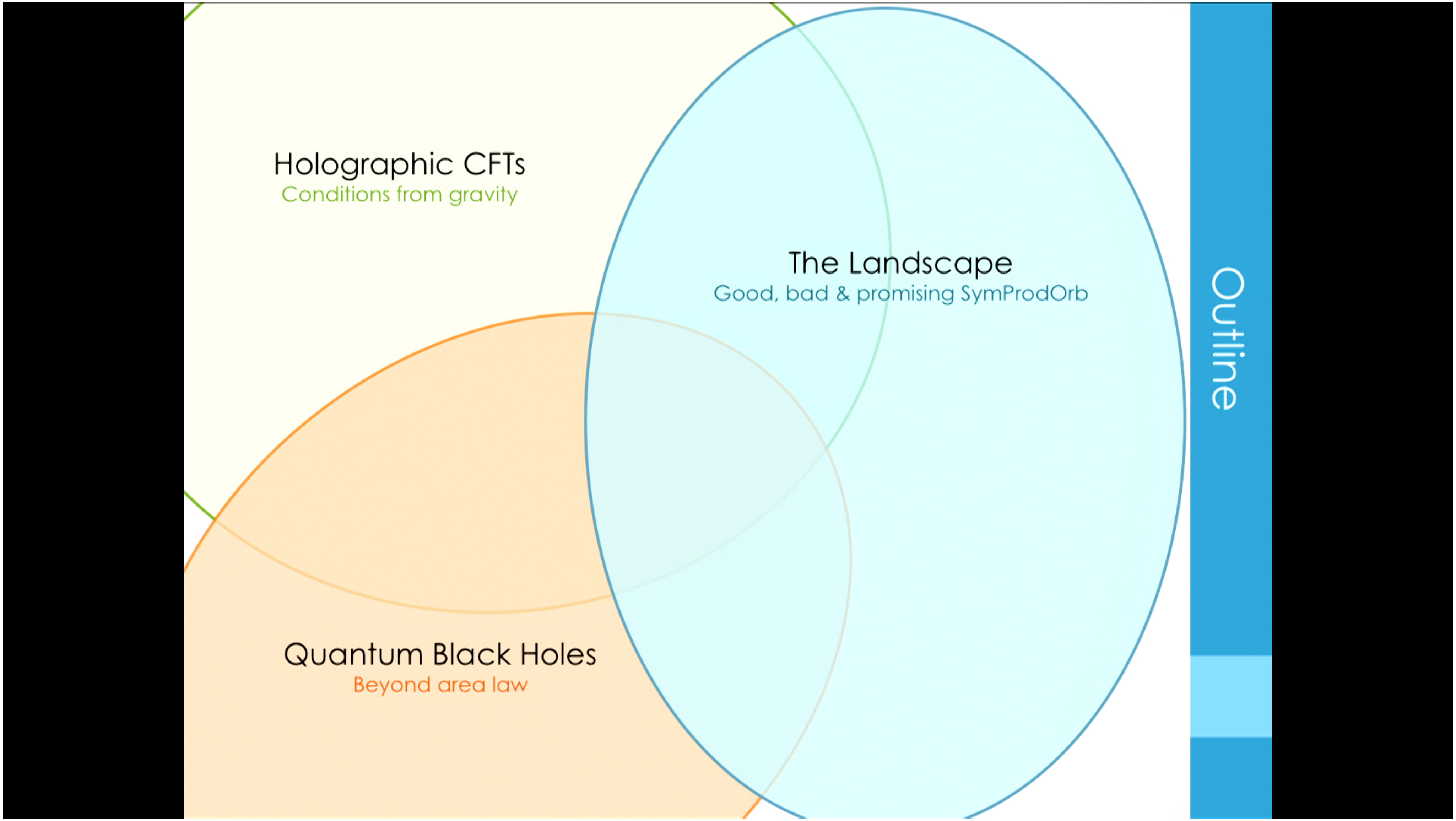
AdS_3/CFT_2

Supergravity description

Large central charge

STRATEGY

1. Inspiration from known examples in String Theory.
2. Exploit holography.
3. Exploit number theory: crafting suitable counting formulas.


Holographic CFTs
Conditions from gravity
The Landscape
Good, bad & promising SymProdOrb
Quantum Black Holes
Beyond area law
Outline

Holographic CFTs

Conditions from gravity

Necessary conditions on the spectrum of CFT_2

Strategy on describing the space of CFT_2

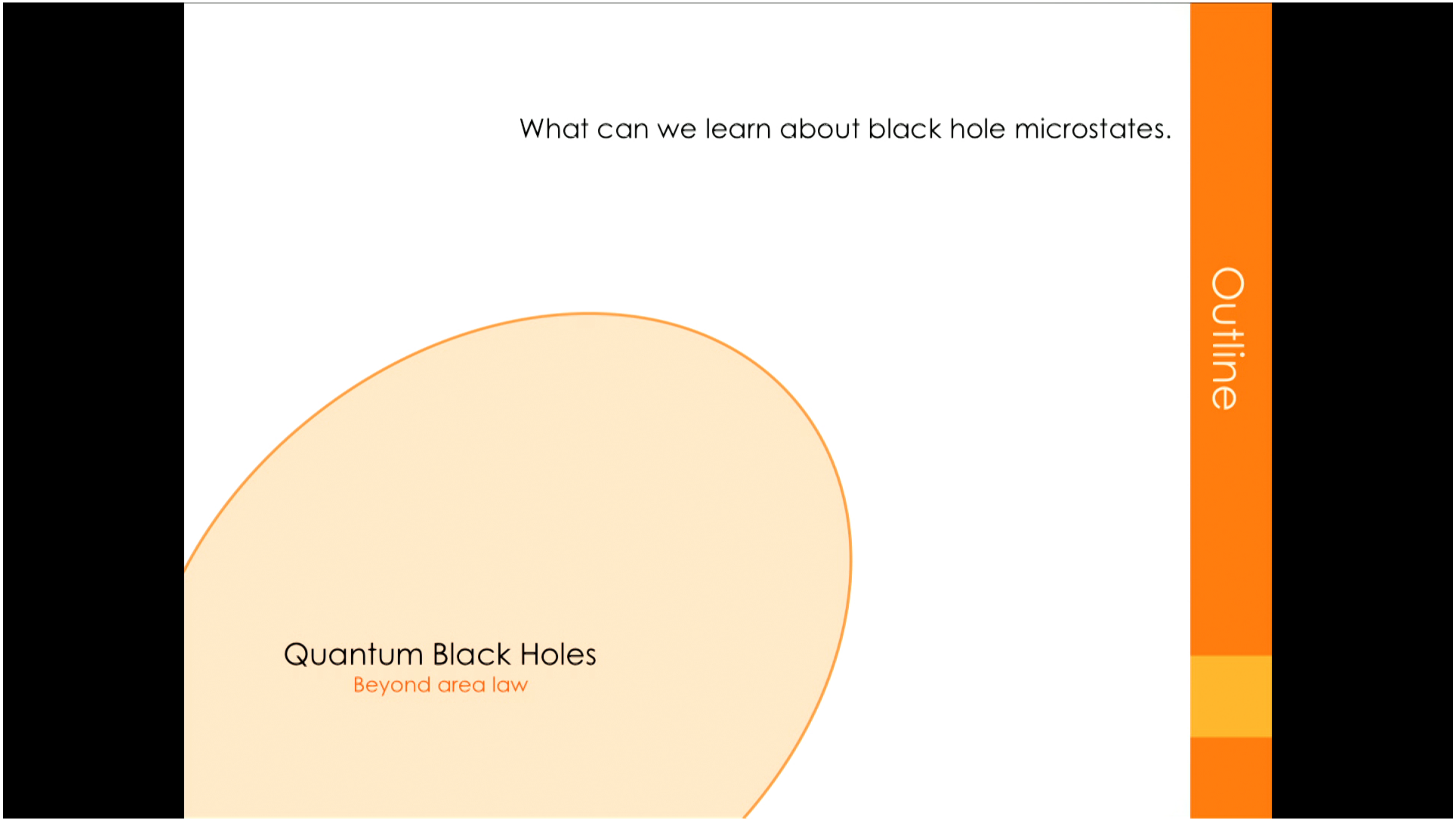
Outline

The Landscape

Good, bad & promising SymProdOrb

Outline

Implementing conditions.
Restrict the analysis to
supersymmetric states.
New results presented.


What can we learn about black hole microstates.
Quantum Black Holes
Beyond area law
Outline

Conditions from gravity

HOLOGRAPHIC CFTS

AdS_3 Gravity

The theory:

$$I_{3D} = \frac{1}{16\pi G_N}\int d^3x\sqrt{-g}\left(R + \frac{2}{\ell^2}\right) + \text{matter}$$

The spectrum:

1. Light States: Perturbative states
2. Heavy States: Black holes
3. Other stuff, e.g., multi-centered, conical defects (to be ignored today)

Universal entry in AdS_3/CFT_2: $$c = \frac{3\ell}{2G_N} \gg 1$$

Holographic CFT_2

We will impose two conditions

1. Black hole regime
2. Perturbative regime

Heavy states: BTZ Black Hole

Background:

$$ds^2 = -\frac{(r^2 - r_+^2)(r^2 - r_-^2)}{r^2} dt^2 + \frac{\ell^2 r^2}{(r^2 - r_+^2)(r^2 - r_-^2)} dr^2 + r^2 \left(d\phi + \frac{r_+ r_-}{r^2} dt \right)^2$$

+ U(1) CS field

Thermodynamics:

$$S_{BH} = \frac{A_H}{4G_N} \ , \quad M = \frac{r_+^2 + r_-^2}{8G_N \ell^2} \ , \quad J = \frac{r_+ r_-}{4G_N \ell}$$

Huge degeneracy!

Supersymmetry:

$$\ell M = J \ , \quad T_H = 0$$

+ quantization of U(1)

Holographic CFT$_2$

1. Black hole regime:

$$A_H \gg G_N$$

Very massive black hole with lots of entropy

$$S_{\rm BH} = \ln d(c, E)$$

symmetry

$$= 2\pi\sqrt{\frac{cE}{6}} + \cdots$$

[Strominger+Vafa; Strominger]

$$= \frac{A_{\rm H}}{4G} + \cdots$$

Holography

Holographic CFT_2

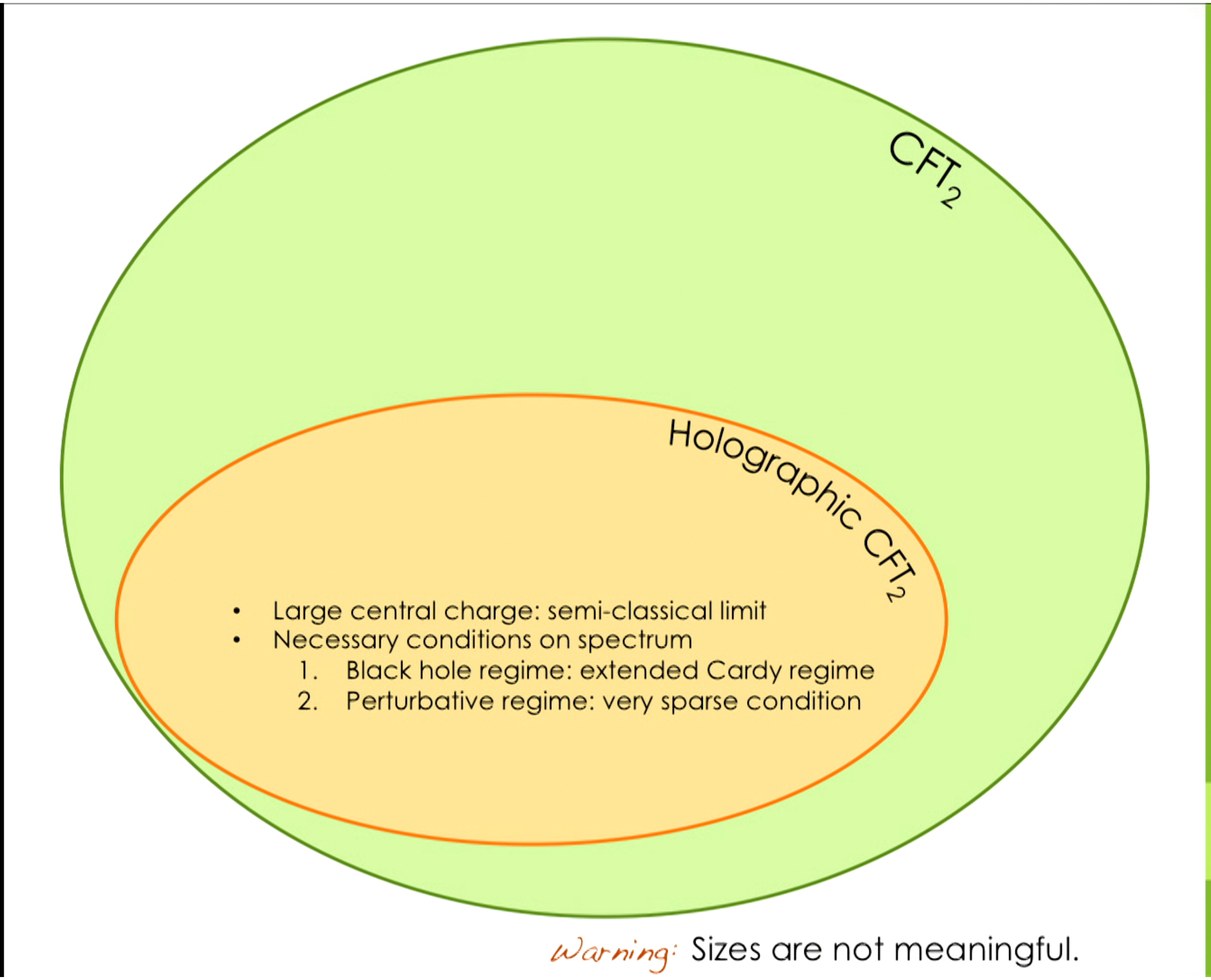
2. Perturbative regime:

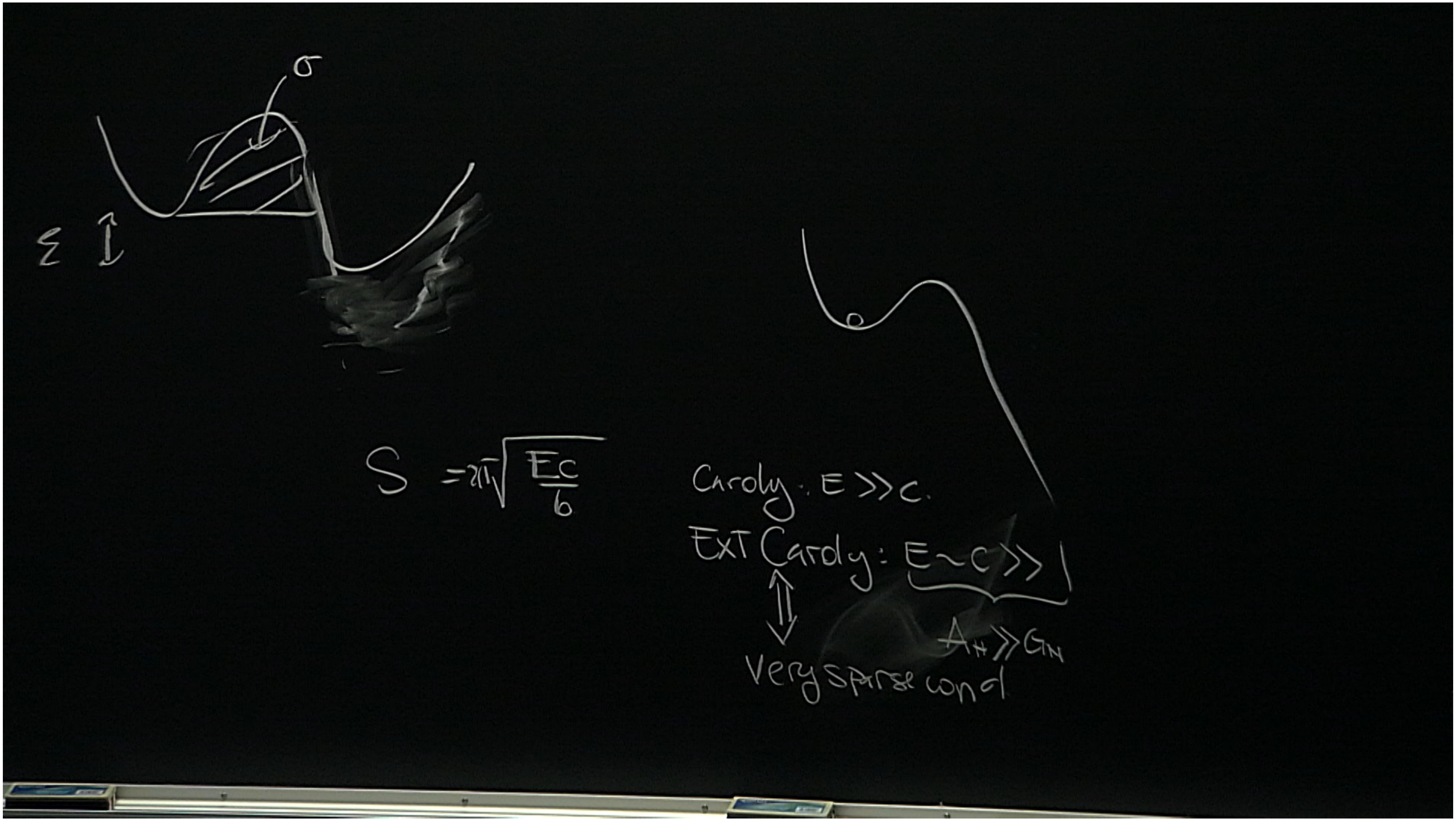
Light = Energy is O(1) in Planck units.
Perturbative excitations that do not form a black hole.

- Presence of Hawking-Page transition [Keller; Hartman, Keller, Stoica]
- Extended Cardy regime for BPS BHs [Benjamin, Cheng, Kachru, Moore, Paquette; Benjamin, Kachru, Keller, Paquette]

$$\ln d(E) \sim E^{\alpha} \qquad \alpha < 1$$

Very sparse spectrum


CFT$_2$
Holographic CFT$_2$
• Large central charge: semi-classical limit
• Necessary conditions on spectrum
1. Black hole regime: extended Cardy regime
2. Perturbative regime: very sparse condition
Warning: Sizes are not meaningful.

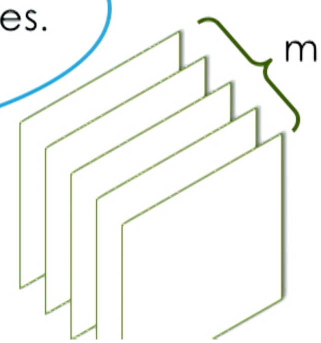

σ
ε
S = 2π √(Ec/6)
Cardy: E >> c.
EXT Cardy: E ~ c >> 1
A_H >> G_N
Very sparse cond

Strategy

We want CFTs with large central charge
We want control on the spectrum

Search within symmetric product theories.

$$\mathcal{C}_m = \mathcal{C}^{\otimes m}/S_m$$



Strategy

We want CFTs with large central charge
We want control on the spectrum

↓

Search within symmetric product theories.

$$\mathcal{C}_m = \mathcal{C}^{\otimes m}/S_m$$

$$\mathcal{Z}(\tau,\rho) = \sum_m Z(\tau;\mathcal{C}_m)\, p^m = \prod_{m>0,n\in\mathbb{Z}} \frac{1}{(1-p^m q^n)^{d(mn)}} = \sum_{m,n} d_m(n) p^m q^n$$

Strategy

We want CFTs with large central charge
We want control on the spectrum

↓

Search within symmetric product theories.

Big Obstruction: The partition function of symmetric product CFT_2 is stringy, i.e. Hagedorn like.

Strategy

We want CFTs with large central charge
We want control on the spectrum

↓

Search within symmetric product theories.

↓ *Only hope:* Add marginal deformation to make the spectrum sparse

Focus on quantities that are protected.
Those should have gravitational features.

BPS states in $SCFT_2$

Focus on protected quantities: the elliptic genera.

$$\chi(\tau, z) = \mathrm{tr}_{RR}\left((-1)^F q^{L_0 - \frac{c}{24}} y^{J_0} \bar{q}^{\bar{L}_0 - \frac{\bar{c}}{24}}\right)$$

The elliptic genera is related to a weak Jacobi form of index t.

Focus on symmetric product orbifolds: easy to get large values of c=6r.

$$\mathcal{Z}(\rho, \tau, z) = \sum_r \chi(\tau, z; \mathrm{Sym}^r(M))\, e^{2\pi i \rho\, t\, r}$$

Necessary condition on light states in NS sector as r → ∞ →

$$\ln d(E) \sim E^\alpha \qquad \alpha < 1$$

Note: The partition function of symmetric product CFT_2 has $\alpha = 1$. The elliptic genus can display cancellations that capture the spectrum away from the symmetric product point.

$$y^{-1}+10+y+(10y^{-2}-64y^{-1}+108-64y+10y^2)q+(y^{-3}+108y^{-2}-513y^{-1}+808-513y+108y^2+y^3)q^2+\dots$$

BPS states in SCFT$_2$

Focus on protected quantities: the elliptic genera.

$$\chi(\tau, z) = \mathrm{tr}_{RR}\left((-1)^F q^{L_0 - \frac{c}{24}} y^{J_0} \bar{q}^{\bar{L}_0 - \frac{\bar{c}}{24}}\right)$$

The elliptic genera is related to a weak Jacobi form of index t.

Focus on symmetric product orbifolds: easy to get large values of c=6r.

$$\mathcal{Z}(\rho, \tau, z) = \sum_r \chi(\tau, z; \mathrm{Sym}^r(M))\, e^{2\pi i \rho\, t\, r}$$

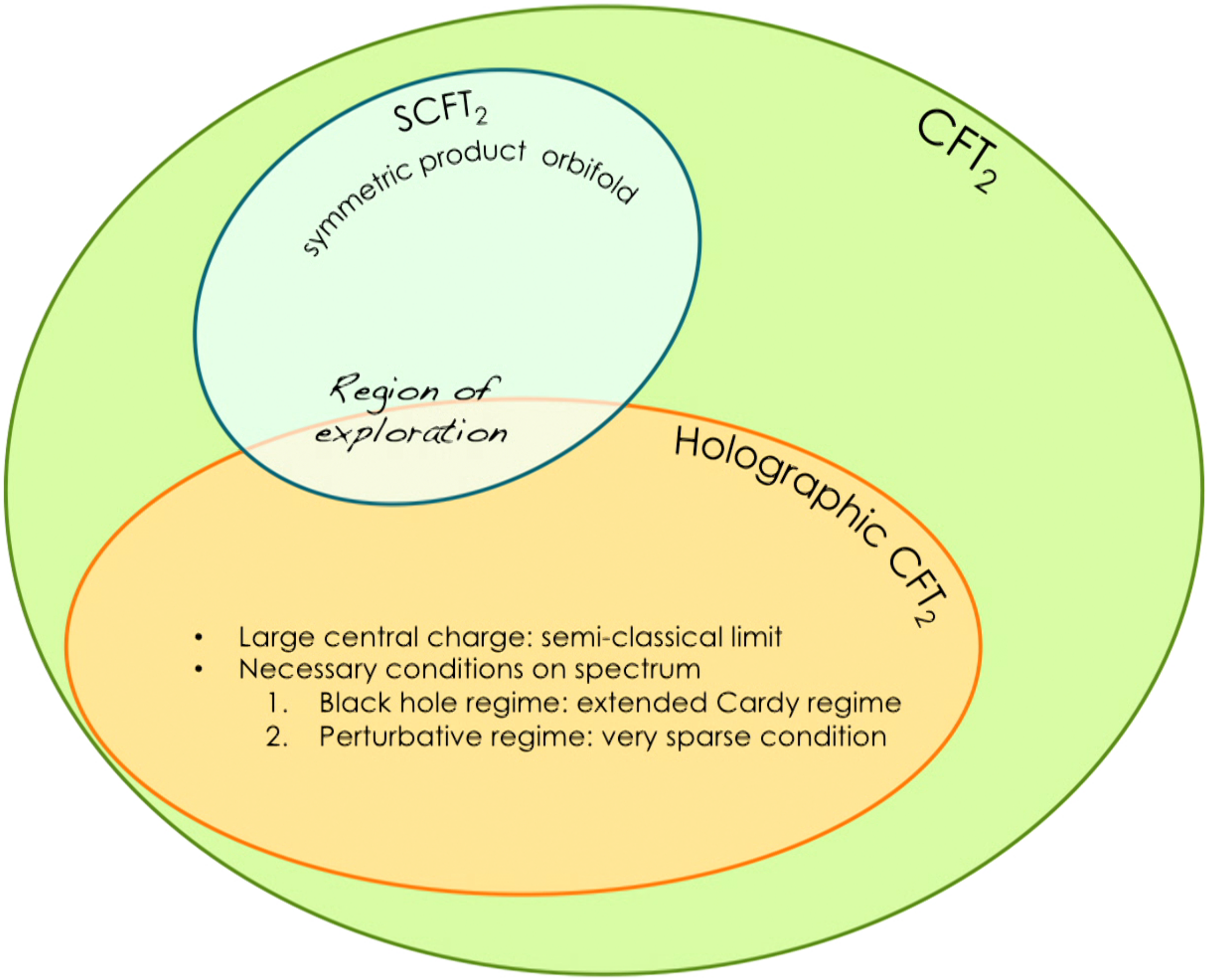
Necessary condition on light states in NS sector as r → ∞ →

$$\ln d(E) \sim E^{\alpha} \qquad \alpha < 1$$

Spoiler!

We can tell you unambiguously which wJFs are holographic, i.e α<1.
New examples are unveiled.

$y^{-1}+10+y+(10y^{-2}-64y^{-1}+108-64y+10y^2)q+(y^{-3}+108y^{-2}-513y^{-1}+808-513y+108y^2+y^3)q^2+\dots$



Warning: Sizes are not meaningful.

Good, bad & promising Symmetric Product Orbifolds

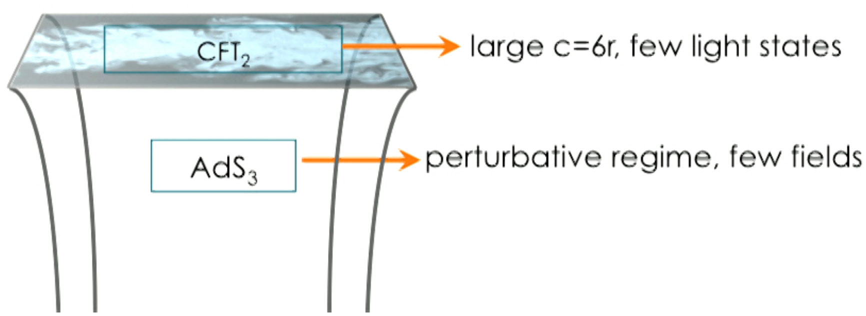
THE LANDSCAPE

The class of objects we are interested are of the form:

$$\mathcal{Z}(\rho,\tau,z) = \sum_r \chi(\tau,z;\mathrm{Sym}^r(M))\, e^{2\pi i\rho\, t\, r} = \prod_{\substack{n,l,r\in\mathbb{Z}\\ r>0}} (1-q^n y^l p^{tr})^{-c(nr,l)}$$

Task:

- to characterise the BPS spectrum in these theories.
- focus on the large r limit

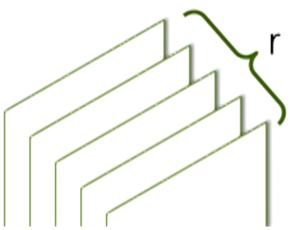


Our procedure in a few steps:

1. Select one seed theory: select a Jacobi form.
2. Perform symmetric product orbifold: increases c
3. Relate generating function to Siegel paramodular form: gives the mathematical control to extract spectrum.
4. Build a modular form that capture the light part of the CFT spectrum

$$\chi_{\mathrm{NS},\infty} \equiv \sum_{h,l} d_{\infty}^{\mathrm{NS}}(h,l) q^h y^l = \prod_{\substack{h \geq 0, l \in \mathbb{Z} \\ (h,l) \neq (0,0)}} \frac{1}{(1 - q^h y^l)^{f(h,l)}}$$

Generating functional for light (perturbative) states at infinite r



Promising Examples

a.k.a. SMFs that meet the two necessary conditions on the BPS spectrum

$$\ln d_{\infty}^{\mathrm{NS}}(h) \sim h^{\alpha} \qquad \alpha < 1$$

Very sparse spectrum

$$\chi_{\mathrm{NS},\infty} \equiv \sum_{h,l} d_{\infty}^{\mathrm{NS}}(h,l) q^h y^l = \prod_{\substack{h \geq 0, l \in \mathbb{Z} \\ (h,l) \neq (0,0)}} \frac{1}{(1 - q^h y^l)^{f(h,l)}}$$

Promising Examples

We found a very simple condition. Given a seed Jacobi form

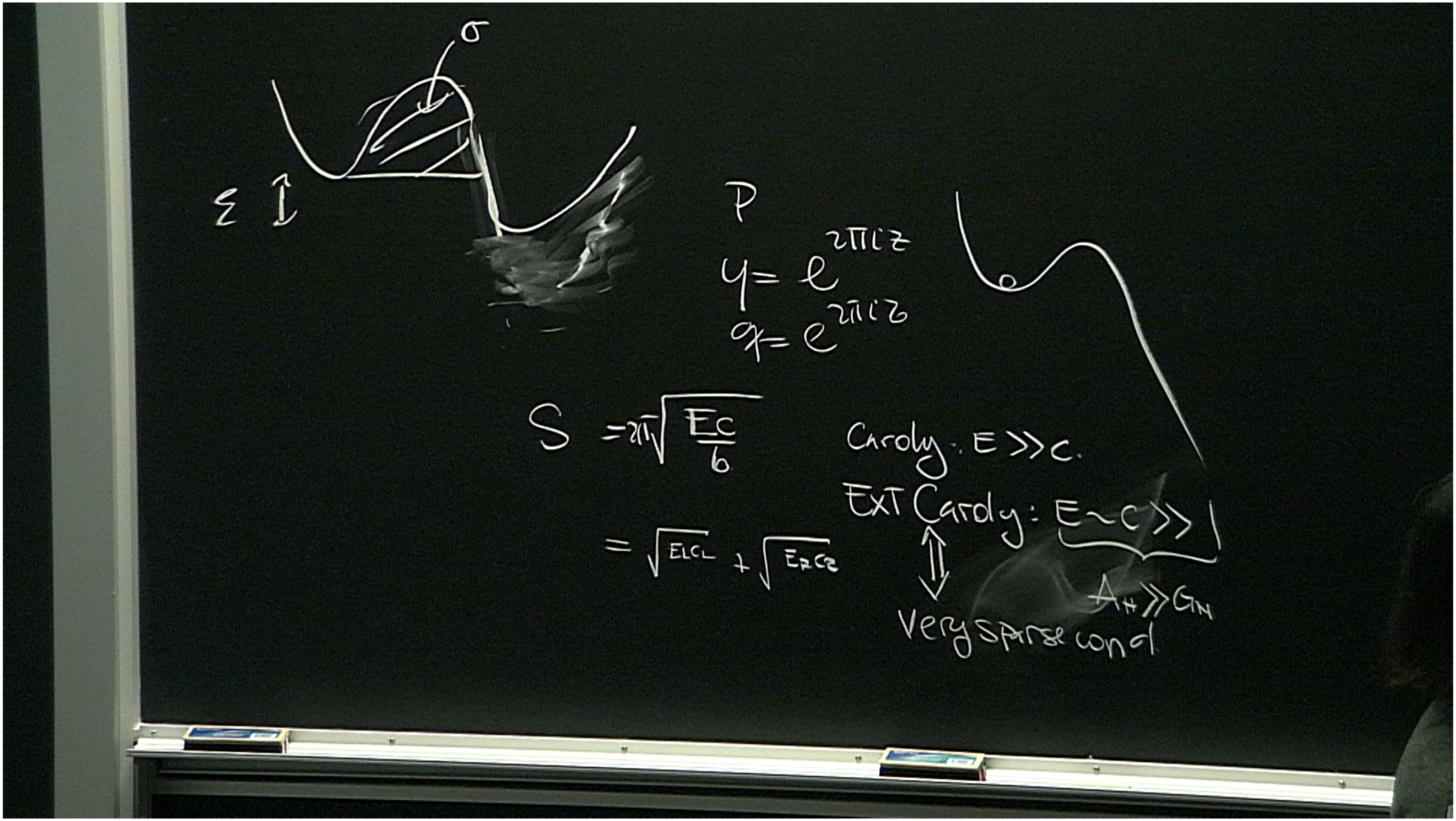
$$\phi_{0,t}(\tau, z) = q^0 y^{-b} + \cdots$$

Demand positivity of

$$\alpha = \max_{j=0,\ldots,b-1} \left(-\frac{t}{b^2} j \left(j - \frac{bl}{t} \right) - n \right)$$

Then f(h,l) is sum of Kronecker functions.
Necessary condition is $b^2 \leq t$

$$\chi_{\mathrm{NS},\infty} \equiv \sum_{h,l} d_{\infty}^{\mathrm{NS}}(h,l) q^h y^l = \prod_{\substack{h \geq 0, l \in \mathbb{Z} \\ (h,l) \neq (0,0)}} \frac{1}{(1 - q^h y^l)^{f(h,l)}}$$


σ
E
P
y= e^{2πiz}
q= e^{2πiτ}
S =2π√(Ec/6)
= √(ELcL) + √(ERcR)
Cardy: E >> c.
Ext Cardy: E ~ c >> 1
A_H >> G_N
Very sparse cond

Promising Examples

t	b	dim	t	b	dim	t	b	dim	t	b	dim
1	1	1	8	1	0	12	1	0	16	1	0
2	1	1	8	2	2	12	2	2	16	2	1
3	1	1	9	1	0	12	3	3	16	3	2
4	1	1	9	2	1	13	1	0	16	4	4
4	2	2	9	3	3	13	2	0	17	1	0
5	1	0	10	1	0	13	3	1	17	2	0
5	2	1	10	2	1	14	1	0	17	3	0
6	1	1	10	3	2	14	2	0	17	4	2
6	2	2	11	1	0	14	3	1	18	1	0
7	1	0	11	2	0	15	1	0	18	2	0
7	2	1	11	3	1	15	2	1	18	3	3
						15	3	2	18	4	3

Today

We can easily diagnose if a Symmetric Product Orbifold does the holographic trick.

$y^{-1}+10+y+(10y^{-2}-64y^{-1}+108-64y+10y^2)q+(y^{-3}+108y^{-2}-513y^{-1}+808-513y+108y^2+y^3)q^2+\ldots$

Promising Examples

t	b	dim	t	b	dim	t	b	dim	t	b	dim
1	1	1	8	1	0	12	1	0	16	1	0
2	1	1	8	2	2	12	2	2	16	2	1
3	1	1	9	1	0	12	3	3	16	3	2
4	1	1	9	2	1	13	1	0	16	4	4
4	2	2	9	3	3	13	2	0	17	1	0
5	1	0	10	1	0	13	3	1	17	2	0
5	2	1	10	2	1	14	1	0	17	3	0
6	1	1	10	3	2	14	2	0	17	4	2
6	2	2	11	1	0	14	3	1	18	1	0
7	1	0	11	2	0	15	1	0	18	2	0
7	2	1	11	3	1	15	2	1	18	3	3
						15	3	2	18	4	3

Work in Progress

Give a CFT and supergravity description.

What is the physical is the physical interpretation of the mathematical conditions?

$y^{-1}+10+y+(10y^{-2}-64y^{-1}+108-64y+10y^2)q+(y^{-3}+108y^{-2}-513y^{-1}+808-513y+108y^2+y^3)q^2+\ldots$

Good Examples

a.k.a. SMFs that meet the two necessary conditions on the BPS spectrum, and we know the symm prod CFT_2 and AdS_3 supergravity theory.

1. Igusa Cusp form

$$\Phi_{10}(\Omega) = \text{Exp-Lift}(2\phi_{0,1}) \qquad \text{with} \qquad \phi_{0,1} = \frac{1}{2}\chi(\tau, z; K3)$$

$$\frac{1}{\Phi_{10}(\Omega)} = \frac{\mathcal{Z}(\Omega)}{p\,\phi_{10,1}(\tau, z)} \xrightarrow{\text{light states}} \ln d(E) \sim E^{1/2}$$

- ¼ BPS dyons in N=4 D=4 string theory [DVV]
- Quantum black hole [Sen, Dabholkar, Murthy, Gomes, …]
- $AdS_3 \times S^3 \times K3$ supergravity spectrum matches N=(4,4) SCFT [de Boer]
- CHL generalizations [David, Jatkar, Sen; Paquette, Volpato, Zimet]

Good Examples

a.k.a. SMFs that meet the two necessary conditions on the BPS spectrum, and we know the symm prod CFT_2 and AdS_3 supergravity theory.

2. t=4 paramodular form

Seed

$$\phi_{0,1}(\tau, 2z) + 4(1+a)\phi_{0,4}(\tau, z) = y^{\pm 2} + 4(1+a)y^{\pm 1} + 14 + 4a + \mathcal{O}(q)$$

Pole structure

Humbert surfaces are $H_4(2)$ and $H_1(1)$

Perturbative regime

$$\ln d(E) \sim E^{1/2}$$

Holography

$\mathrm{AdS}_3 \times (S^3 \times \mathbb{T}^4)/G \longleftrightarrow$ N=(2,2) SCFTs [Datta, Eberhardt, Gaberdiel]

$y^{-1}+10+y+(10y^{-2}-64y^{-1}+108-64y+10y^2)q+(y^{-3}+108y^{-2}-513y^{-1}+808-513y+108y^2+y^3)q^2+\dots$

Beyond area law

QUANTUM BLACK HOLES

Black holes & SMFs

The task is to extract the physics content of Fourier coefficients for large (asymptotic) values of the charges.

$$d(m,n,l) = \oint_{p=0} \frac{dp}{2\pi i p} \oint_{q=0} \frac{dq}{2\pi i q} \oint_{y=0} \frac{dy}{2\pi i y} \frac{1}{\Phi_k(\Omega)} p^{-m} q^{-n} y^{-l}$$

↓

$$\Phi_k(\rho,\tau,z) = \Phi_k(t^{-1}\tau, t\rho, z)$$

Black holes & SMFs

The task is to extract the physics content of Fourier coefficients for large (asymptotic) values of the charges.

$$d(m,n,l) = \oint_{p=0} \frac{dp}{2\pi i p} \oint_{q=0} \frac{dq}{2\pi i q} \oint_{y=0} \frac{dy}{2\pi i y} \frac{1}{\Phi_k(\Omega)} p^{-m} q^{-n} y^{-l}$$

Asymptotic behavior of degeneracy

$$S_{\mathrm{BH}} = \ln d(c,E) = 2\pi\sqrt{\frac{cE}{6}} + \cdots = \frac{A_{\mathrm{H}}}{4G} + \cdots$$

symmetry → $d(E,c) = d(t\,E, t^{-1}c)$

Holography

Exchange symmetry gives us (almost) automatically an extended Cardy regime.

Quantum corrections

For lack of time, focus on logarithmic correction

$$S_{\rm BH} = \ln d(c,E,J) = \frac{A_H}{4G} + \#\ln(A_H/G) + \cdots$$

Scaling regime	$\tau^*_{1,2}$	A_H^2	$\ln\Lambda$
I. $E \gg 1$ $E \sim \Lambda^2,\ c \sim O(1),\ J \sim \Lambda$	Λ	Λ^2	$-(k+2)$
II. $E \sim c \gg 1$ $E \sim \Lambda,\ c \sim \Lambda,\ J \sim \Lambda$	Λ^0	Λ^2	$m_{1,1} - 2$
III. $c \gg E \gg 1$ $E \sim \Lambda,\ c \sim \Lambda^2,\ J \sim \Lambda^{3/2}$	$\Lambda^{-1/2}$	Λ^3	$m_{1,1} - 3 - \frac{k}{2}$

Cardy regime. (row I)

Regime relevant for BPS BHs in 4D & 5D [Sen et al] (rows II–III)

To be explored

1. Logarithmic corrections

$$S_{\rm BH} = \ln d(c, E, J) = \frac{A_H}{4G} + \# \ln(A_H/G) + \cdots$$

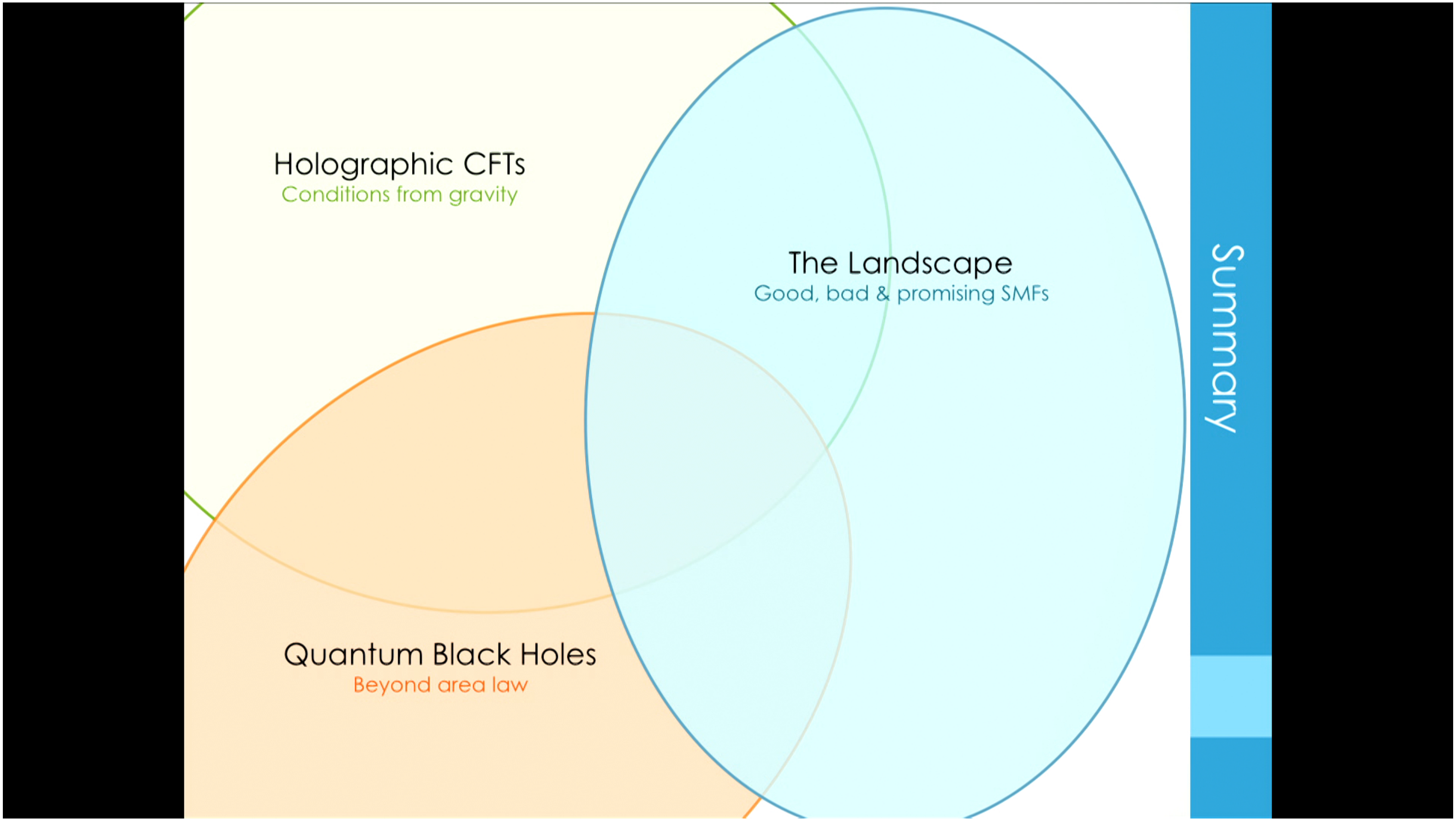
Universality of this correction is being tested in gravity.
Less is known microscopically.

[Charles, Larsen; AC, Godet, Larsen, Zeng]
[Mukhametzhanov, Zhiboedov]

2. Mock modular forms

Decompose counting formula as quantum degeneracies of single-centered and multicentered configurations.

[Dabholkar, Murthy, Zagier]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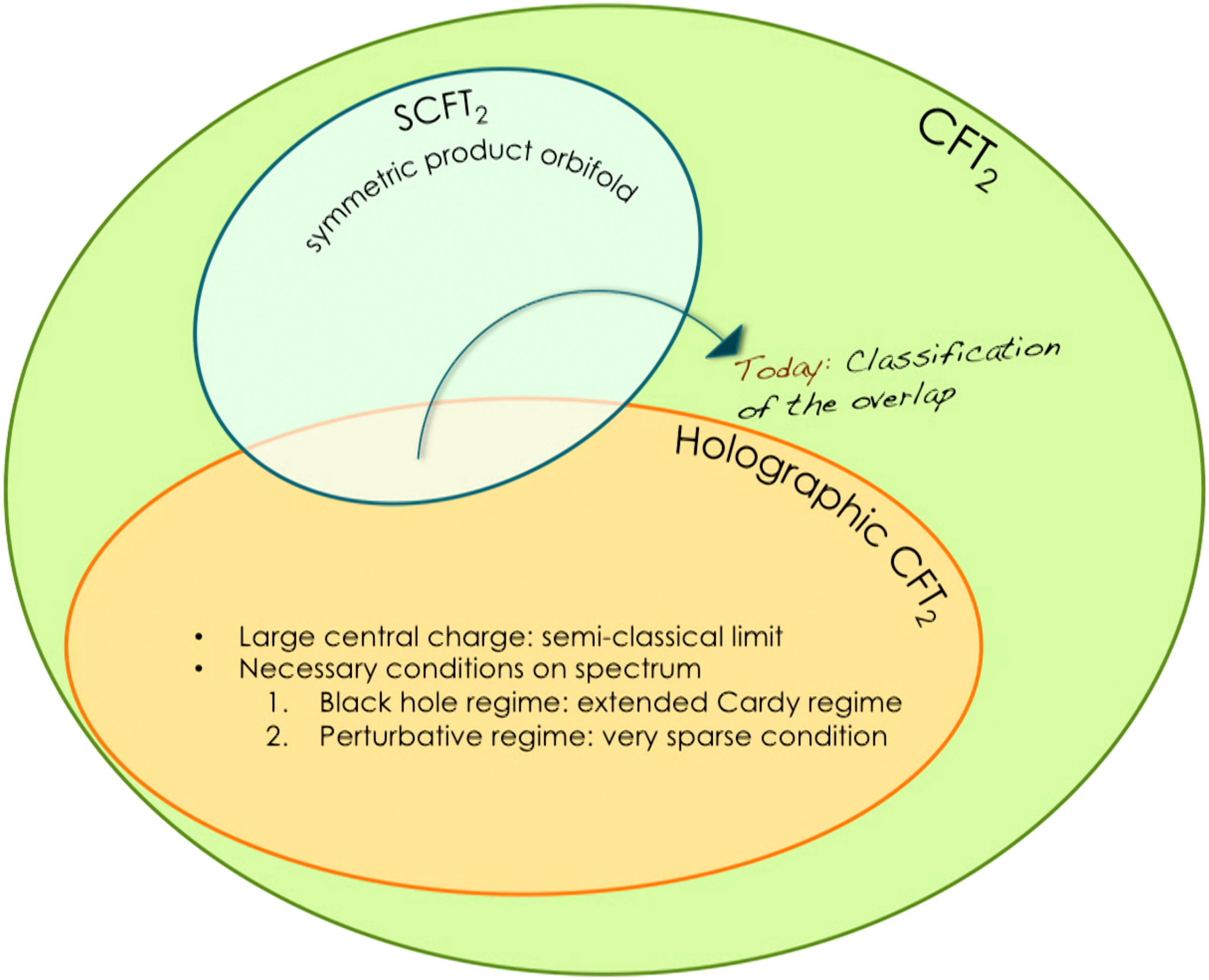

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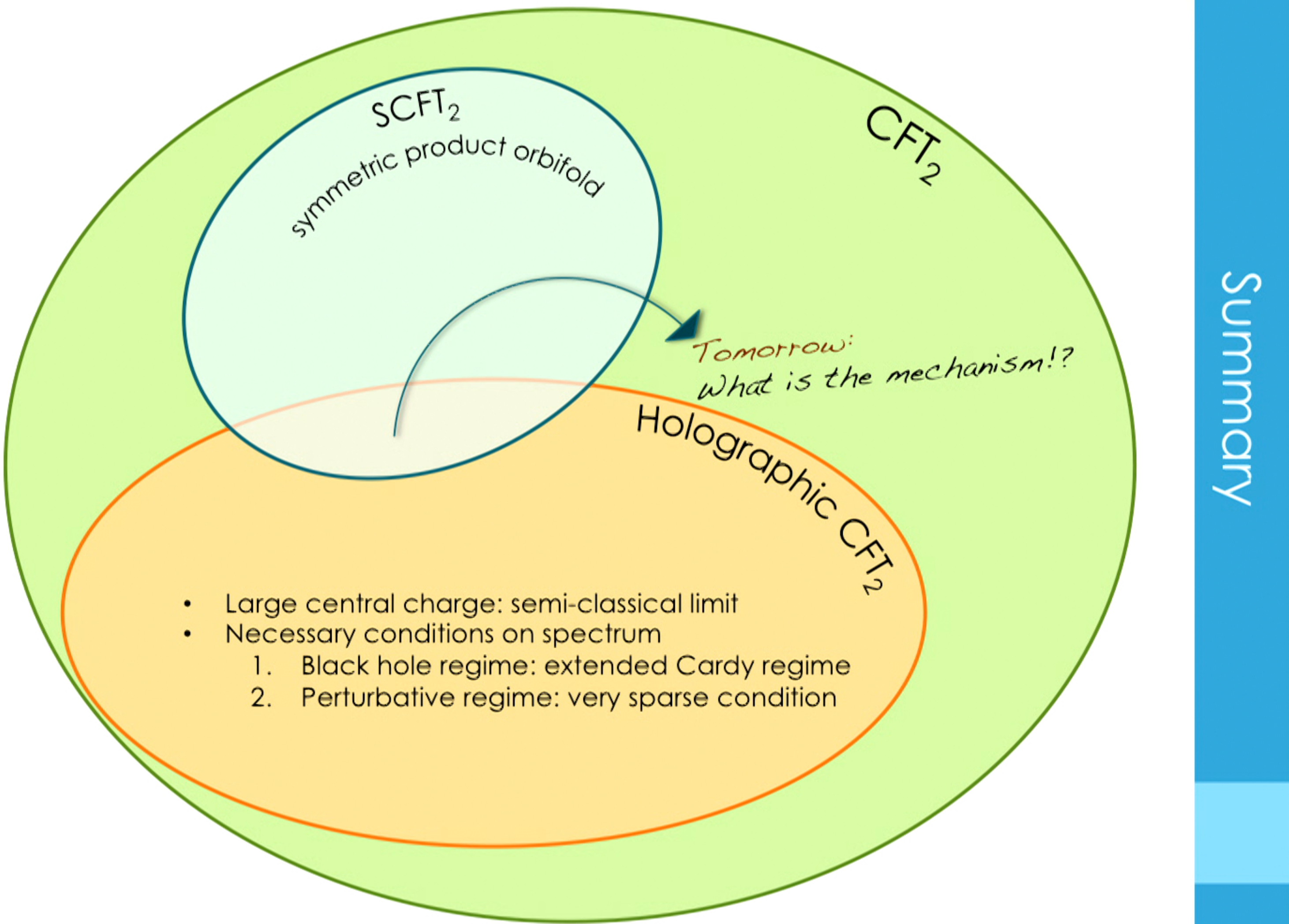

Warning: Sizes are not meaningful.

THANK YOU!

$$y^{-1}+2+y-(2y^{-3}+2y^{-2}-2y^{-1}+4-2y+2y^{2}+2y^{3})q+(y^{-5}-2y^{-4}-6y^{-3}-4y^{-2}+5y^{-1}+12+5y-4y^{2}-6y^{3}-2y^{4}+y^{5})q^{2}+\dots$$

$$y^{-1}+1+y-(y^{-4}+y^{-3}-y^{-1}-2-y+y^{3}+y^{4})q+(-y^{-5}-2y^{-4}-2y^{-3}+3y^{-1}+4+3y-2y^{3}-2y^{4}-y^{5})q^{2}+\dots$$

$$y^{-1}+10+y+(10y^{-2}-64y^{-1}+108-64y+10y^{2})q+(y^{-3}+108y^{-2}-513y^{-1}+808-513y+108y^{2}+y^{3})q^{2}+\dots$$

$$y^{-1}+4+y+(y^{-3}-8y^{-2}-y^{-1}+16-y+8y^{2}+y^{3})q+(4y^{-4}-y^{-3}-32y^{-2}+y^{-1}+56+y+32y^{2}-y^{3}+4y^{4})q^{2}+\dots$$

$$y^{-1}+y+(-y^{-5}+y^{-1}+y-y^{5})q+(-y^{-7}-y^{-5}+2y^{-1}+2y-y^{5}-y^{7})q^{2}+\dots$$