Title: A possible causality-condition for causal sets: persistence of zero

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A possible causality-condition for causal sets: persistence of zero

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Perimeter, November 2019

Today I want to go out on a limb and tell you about an idea that's only a few days old, but has the potential to answer a long-standing question in causal set theory — How to find a quantal dynamics which generalizes the well-established Classical Sequential Growth (CSG) models.

Just as General Relativity flows naturally from general covariance (GC) and locality, the CSG models flow from two basic principles called *Discrete General Covariance* (or label-invariance) and *Bell Causality*.

The former principle, that of label-invariance, generalizes naturally to the quantum case, but Bell Causality does not; and it has been a long standing problem to find some other causality-condition to replace it. I will propose that a certain principle of "persistence of zero" (PoZ) could be what we are looking for.

PoZ is formulated within the framework of Quantum Measure Theory (QMT), which in turn is a simple generalization of the path-integral framework. Thus my talk will be in three parts: first QMT, then PoZ, and finally a proposal for what Poz could mean in application to causal sets.

QUANTUM MEASURE THEORY (QMT)

There are different ways to think about QMT but I will try to present it in a way that will allow the condition of PoZ to be defined most simply. The basic concepts needed for this purpose are **event** and **quantum-measure**.

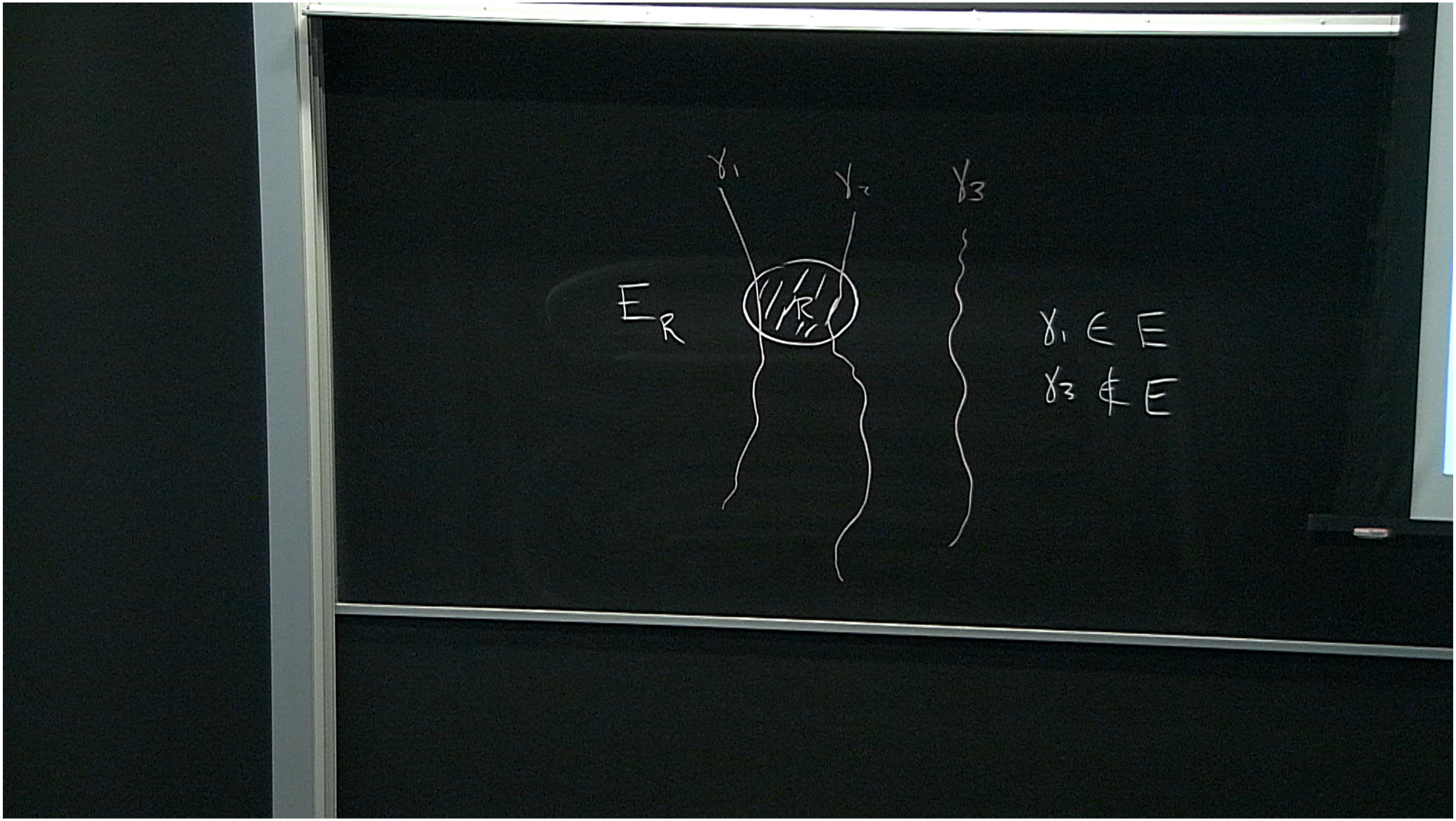
First what is an event *E*? It is something that can happen (but might or might not happen in reality.) What this means concretely depends on the theory one is considering.

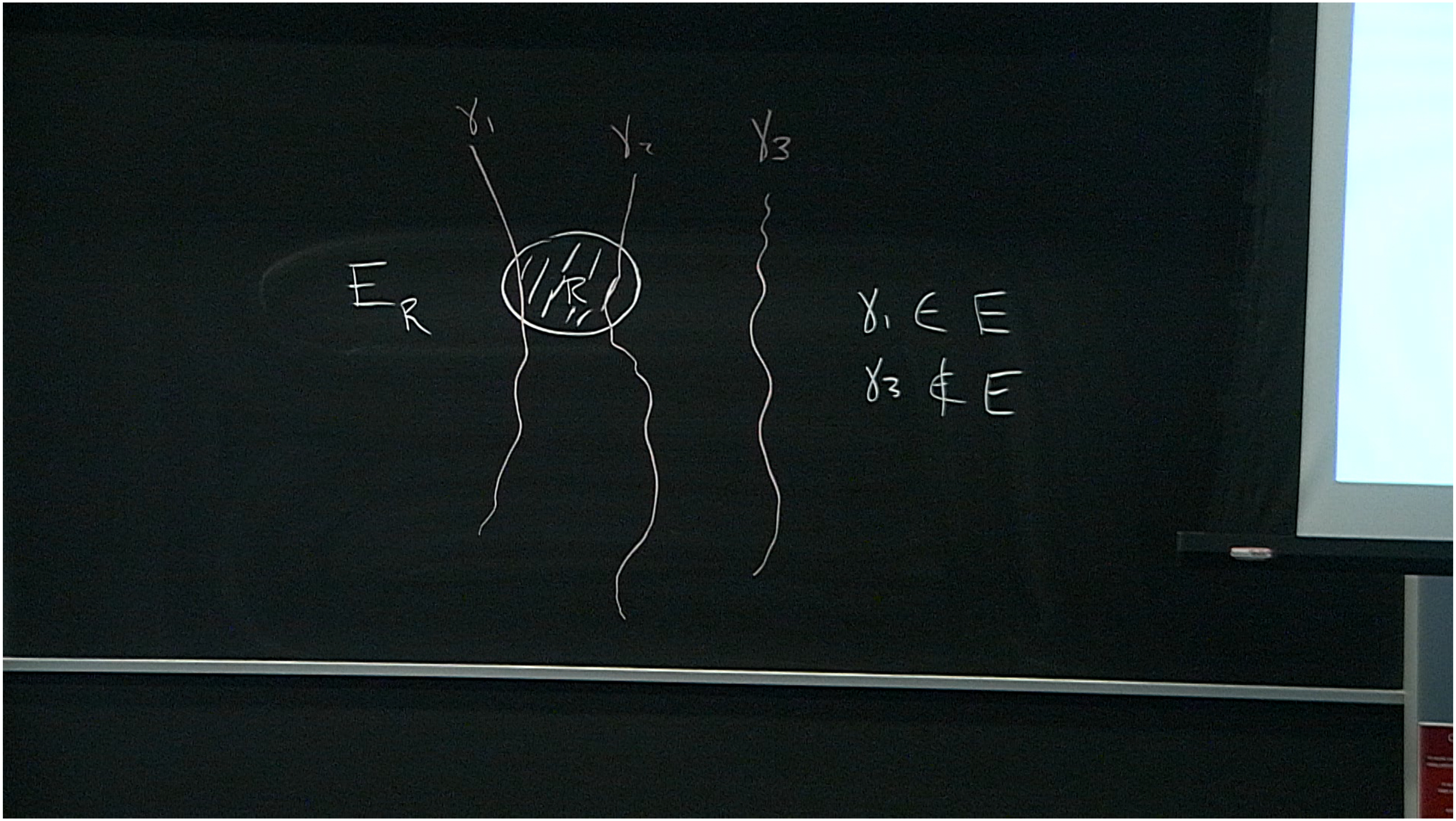
For example, in the theory of a single particle, *E* might be the event that the particle encounters a certain spacetime region *R*, as in the picture [PICTURE].

In cosmology E might be the event that the cosmos will expand to 100 Gpc and after 30 billion years, start to contract down to a singularity.

In a theory of causet growth, *E* could be the event that someday an element will be born such that every other element is either its ancestor or will be its descendent. Such an element is called a “post”, and it is the causet counterpart of a cosmic bounce. Or *E* could be something much simpler, like the event that the first 5 elements to be born form a chain.

Finally if the system in question is a laboratory-instrument like a photon-detector, E might be the event that the detector “clicked” (or equally that it remained silent).


γ1
γ2
γ3
E_R
R
γ1 ∈ E
γ3 ∉ E



γ1
γ2
γ3
E_R
R
γ1 ∈ E
γ3 ∉ E

It is clear that an event E can happen in many ways. Each of these ways is one of the histories referred to in the phrase "sum over histories". Thus we can identify an event with some subset of the space Ω of all conceivable histories. (In our first example, E would be the set of worldlines that meet the region R).

Now given two events A and B we can define the event $A \& B$, which — classically speaking — happens iff *both* A and B happen. In terms of subsets of Ω, $A \& B$ is more precisely the intersection $A \cap B$.

So far time has played no role, in what I've been saying, but when we come to PoZ the combination $A \,\&then\, B$ will be important. In writing it we imply that B is temporally later than A.

The second concept we need is that of the **measure** of an event, which generalzes the classical concept of probability-measure. Actually a given event E has two different measures, one derived from the other.

The first measure is a measure in the usual sense (it is additive), except that its value is a vector in a Hilbert space, rather than just a real number. I will write the measure of E in this sense as $|E\rangle$, using a Dirac-like notation.

The second measure of E is

$$\mu(E) = \langle E|E\rangle$$

which is the square of the first. It is a positive real number, but unlike $|E\rangle$, it is *not* additive. Instead it satisfies a certain 3-slit sum rule that expresses the absence of higher-order interference in Quantum Mechanics.

When E is an instrument-event, one can interpret $\mu(E)$ as the probability that E actually happens (e.g. the probability that the detector clicks). This makes it clear that the formula $\mu(E) = \langle E|E\rangle$ is an expression of Born's rule.

We now have what we need to define PoZ, but first let me illustrate how $|E\rangle$ would be defined in ordinary quantum mechanics for a *unitarily evolving closed* system with initial *"pure state"* given by $|\psi_0\rangle$.

In that case,

$$|E\rangle = U_E|\psi_0\rangle$$

where U_E is the evolution-operator computed by a path-integral with the history γ restricted to belong to the event E:

$$\int_{\gamma \in \mathbf{E}} d\gamma \; e^{i\,S(\gamma)}$$

Thus $|E\rangle$ in this case belongs to the usual Hilbert space of the theory. More generally though, e.g. for non-unitary evolution, or for an initial density-matrix, $|E\rangle$ will lie in some other Hilbert space.

PERSISTENCE of ZERO (PoZ)

Consider the setup in the picture [PICTURE]. There is a source that emits a particle (say a neutron) and there are "slits" as shown. Let A_1 be the event that the particle passes through the first slit, B the event that it passes through the slit named B, etc. Then for example $A_1 \,\&\, B$ is the event that it passes through both these slits.

For us, the interesting case is that where B lies in a dark band so that $\mu(B) = 0$. We have then

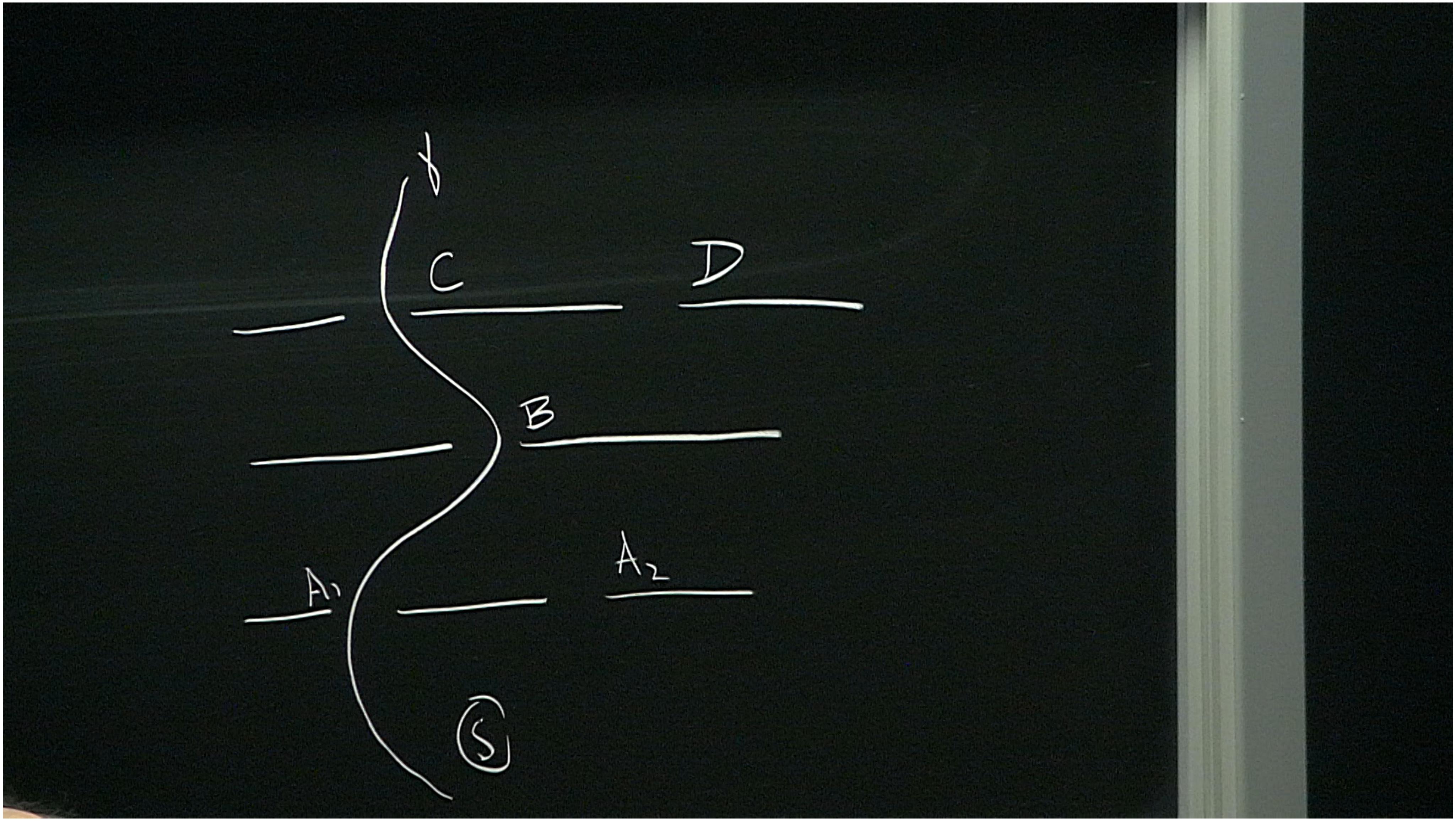
$$\mu(B) = 0 \implies \langle B|B\rangle = 0 \implies |B\rangle = 0$$

Of course just because $|B\rangle = 0$ that does not mean that $|B \,\&\, A_1\rangle = 0$ also, because there is **interference**.

(It means only that $|B \,\&\, A_1\rangle + |B \,\&\, A_2\rangle = |B\rangle = 0$.)

On the other hand $\mu(B) = 0$ **does** imply that $\mu(B \,\&\, D) = 0$. If the particle cannot make it through B it cannot proceed to D either.

What is the difference? It is evidently than A_1 and A_2 are earlier than B whereas C and D are later.


γ
C
D
B
A_1
A_2
S

We can say this another way.

Classically $\mu(A) = 0 \implies \mu(A \& B) = 0$, no matter what event B may be,

because any subset of an event of measure 0 also has measure 0.

Quantally this is no longer true in general, because of interference. Nevertheless it **is** true if B is **later than** A so that we can write $A \& B$ as $A \& then\ B$.

In other words,

$$\mu(A) = 0 \implies \mu(A \& then\ B) = 0$$

This condition is a special case of PoZ, but it is not the full condition.

To advance to the full PoZ condition let's first write our special case equivalently as

$$|A\rangle = 0 \implies |A \,\&\, then\, B\rangle = 0$$

This says that if a certain vector vanishes then so also do various other vectors. But there are many ways to produce a null vector besides starting with a null event. Because the vectors of the form $|E\rangle$ are in general far from being independent, there will be linear relations like

$$\sum_j \lambda_j \,|A_j\rangle = 0$$

Then PoZ requires also that

$$\sum_j \lambda_j \,|A_j \,\&\, then\ B\rangle = 0$$

for any event B subsequent (or spacelike) to all the A_j.

Remark. One usually introduces the "decoherence functional" defined for two events, A and B, by $D(A,B) = \langle A|B\rangle$. It can be computed directly from a Double Path Integral. In terms of D, the condition $\sum_j \lambda_j |A_j\rangle = 0$ is equivalent to $\sum_{ij} D(A_i, A_j)\bar{\lambda}_i \lambda_j = 0$

From Bell Causality together with a condition of label-invariance, one can derive a family of classically stochastic growth-laws known as Classical Sequential Growth (CSG) models.

Some of these models produce a kind of cartoon version of a “Tolman-Boltzmann” cosmos that undergoes cycles of contraction and re-expansion to bigger and bigger sizes. But they can’t really be expected to produce manifold-like causets because classically there is no provision for the kind of constructive interference that leads to smooth classical trajectories in the path integral.

Now back to causal sets and their growth. To carry PoZ over to this setting will be straightforward if we can figure out what it means for one causet-event to be earlier than another causet-event.

To see why this might be difficult, consider the following table:

theory	**causal structure**
QFT	fixed in advance
CSG	dynamical but definite
QSG/QG	dynamical and indefinite

Every theory of Quantum Gravity will face this same problem (unless gravity remains classical).

But now I think there is an idea that's worth trying (and that seems specific to causal sets).

What we call “sequential growth” is a birth-process, where each new element has as parents a specific subset of the elements born previously. For example [PICTURE]:

The labels indicate order of birth, and thus can serve a kind of time-parameter (which is exactly what happens in CSG). One must be careful not to conflict with GC (for example $2 \leftrightarrow 3$ changes nothing physical), but this can be done just as in CSG.

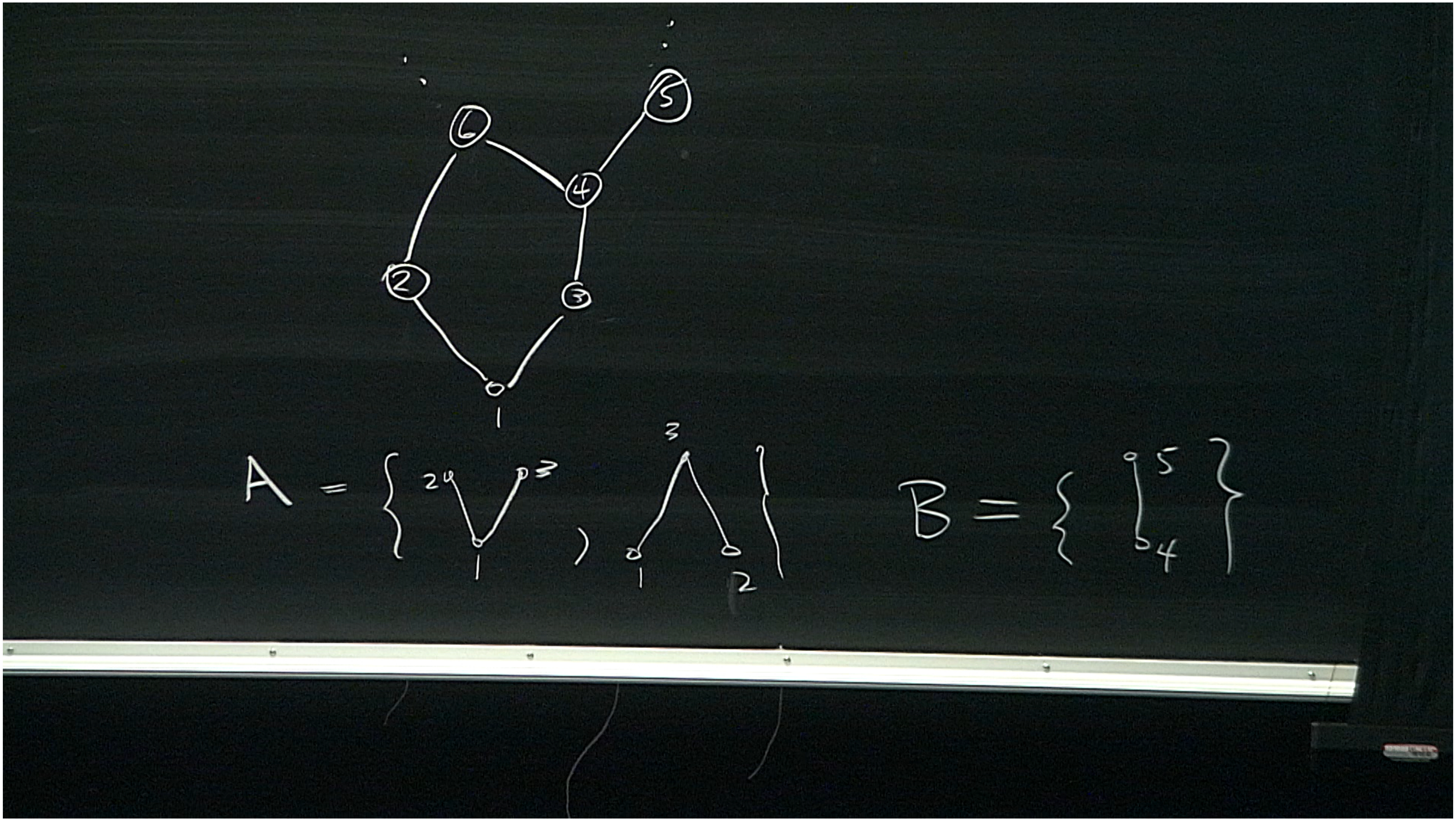
The idea then is to use birth-order to define earlier and later for the purposes of PoZ

For example the event A = “elements 1-3 form either a ‘V’ or a ‘Λ’” would be regarded as earlier than the event B = “elements 4-5 form a 2-chain”

[PICTURE]

Since B would belong to a later “time” than A, the event A &then B would be meaningful and could occur in a PoZ equation.

(A possible worry: For any given label n, the events before n, taken together with those after n, do not generate the whole event-algebra.)


A =
B =

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This at any rate is the proposal.

The question now is whether PoZ defined this way is on one hand *permissive enough* to allow for non-trivial quantum measures μ, but at the same time *strict enough* that the resulting family of measures μ is tractable — and of course, physically relevant.

Thank You!