Title: Angular momentum flux in Einstein-Maxwell theory

Speakers: Beatrice Bonga

Collection: Emmy Noether Workshop: The Structure of Quantum Space Time

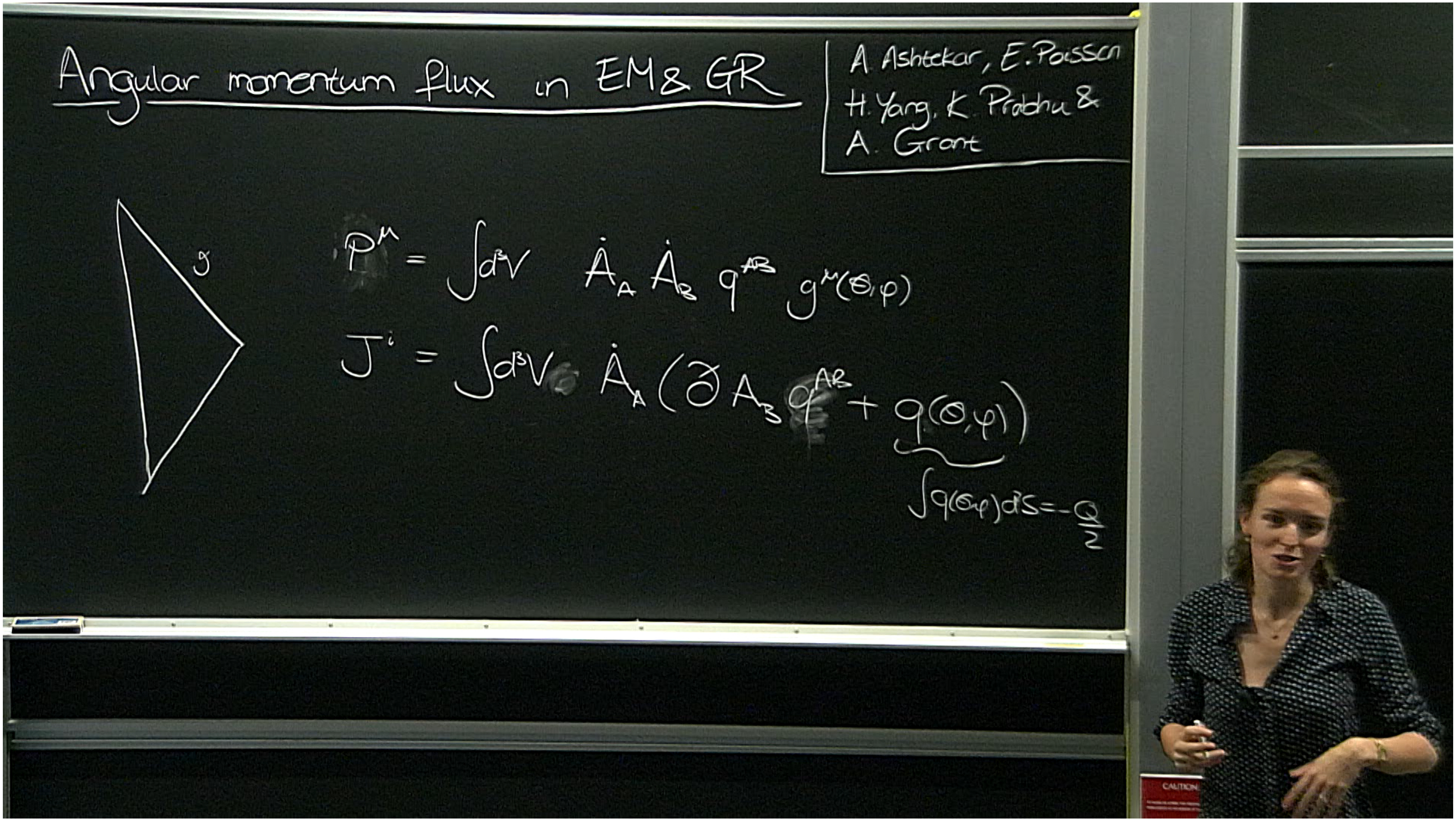
Date: November 19, 2019 - 11:40 AM

URL: http://pirsa.org/19110092

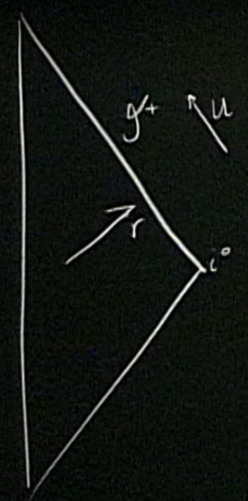
Abstract: There are three natural currents for Maxwell theory on a non-dynamical background: the stress, Noether and canonical current. Their associated fluxes across null infinity differ by boundary terms for asymptotically flat spacetimes. These boundary terms do not only quantitatively change the behavior of the flux associated with an asymptotic Lorentz symmetry, but also qualitatively: the stress flux contains both radiative and Coulombic information, whereas Noether and canonical ones are purely radiative.
While all methods are equally valid and have their own range of usefulness, it is reasonable to ask if one definition is more natural than the other. In order to answer this question, we turn to general relativity. With Maxwell theory coupled to gravity, we use the Wald-Zoupas formalism to obtain an expression for the flux of angular momentum and find that it is purely radiative. When the gravitational field is ``frozen", the Wald-Zoupas flux reduces to the Noether flux.

Angular momentum flux in EM & GR

A. Ashtekar, E. Poisson
H. Yang, K. Prabhu &
A. Grant


Angular momentum flux in EM & GR
A. Ashtekar, E. Poisson
H. Yang, K. Prabhu &
A. Grant
$P^\mu = \int d^3V \; \dot{A}_A \dot{A}_B q^{AB} g^\mu(\theta,\varphi)$
$J^i = \int d^3V \; \dot{A}_A (\partial A_B q^{AB} + q(\theta,\varphi))$
$\int q(\theta,\varphi) d^2S = -\frac{Q}{2}$
CAUTION

Bondi$^+$ formalism



$$ds^2 = -UV\,du^2 - 2U\,du\,dr + \gamma_{AB}(r\,d\theta^A + W^A du)(r\,d\theta^B + W^B du)$$

$$U(u,r,\theta,\varphi) = \boxed{1} + \frac{B(u,\theta,\varphi)}{r} + O\left(\frac{1}{r^2}\right)$$

$$V = \boxed{1} - \frac{2\boxed{M(u,\theta,\varphi)}}{r} + \frac{N}{r^2} + \cdots$$

$$W^A = \frac{A^A}{r} + \frac{B^A}{r^2} + \cdots$$

$$\gamma_{AB} = \boxed{\Omega_{AB}} + \frac{f_{AB}}{r} + \frac{1}{4}\frac{f_{CD}f^{CD}}{r^2}\Omega_{AB} + \cdots$$

$$\dot{J}^{ab} = -\dot{J}^{ab}$$

$$\dot{J}^{ab} = \int \underbrace{\dot{j}^{ab}} \, d^2\Omega$$

$$= -\frac{1}{4\pi} r \, \Omega^{[a} A^{b]} - \frac{1}{8\pi} \Omega^{[a} \left(3B^{b]} - f^{b]}{}_{c} A^{c} \right) + O\left(\frac{1}{r}\right)$$

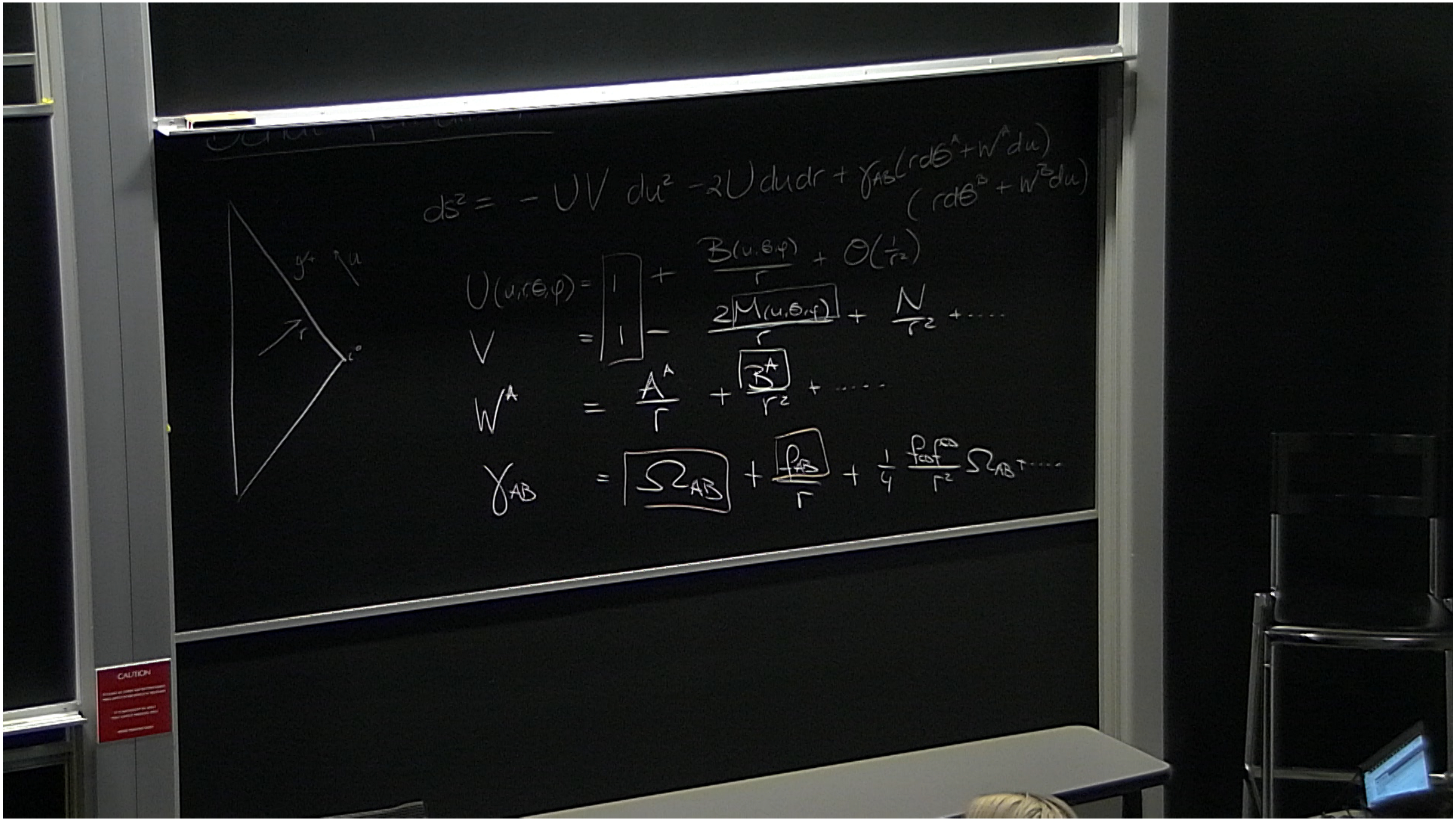
$$A \longrightarrow h$$

$$q \longrightarrow M$$

$$\frac{1}{32\pi}\dot{f}_{AB}\dot{f}^{AB}$$

$$\dot{E} = -\int \dot{P}\, d^2\Omega$$

$$E = \int \left[\frac{1}{4\pi} M - \frac{1}{32\pi} D_A D_B f^{AB}\right] d^2\Omega$$



$ds^2 = -UV\,du^2 - 2U\,du\,dr + \gamma_{AB}(r\,d\theta^A + W^A du)(r\,d\theta^B + W^B du)$
$U(u,r,\theta,\varphi) = \boxed{1} + \frac{B(u,\theta,\varphi)}{r} + O(\frac{1}{r^2})$
$V = \boxed{1} - \frac{2\boxed{M(u,\theta,\varphi)}}{r} + \frac{N}{r^2} + \cdots$
$W^A = \frac{A^A}{r} + \frac{\boxed{B^A}}{r^2} + \cdots$
$\gamma_{AB} = \boxed{\Omega_{AB}} + \frac{\boxed{f_{AB}}}{r} + \frac{1}{4}\frac{f_{CD}f^{CD}}{r^2}\Omega_{AB} + \cdots$
$\mathscr{I}^+$
u
r
i^0

$$\int q(\theta,\varphi)\, d^2S = -\frac{Q}{2}$$

$$\underline{\mathcal{L}} = \underline{\epsilon}\left(-\frac{1}{16\pi}F^2\right)$$

$$\Rightarrow \delta\underline{\mathcal{L}} = \underline{\epsilon}\left(\underbrace{\frac{1}{4\pi}(\nabla_b F^{ba})}_{=\text{eom}}\delta A_a - \underbrace{\frac{1}{4\pi}\nabla_b\left(\overbrace{F^{ba}\delta A_a}\right)}_{=d\underline{\Theta}}\right)$$

1. Symplectic current

$$\underline{\omega}(\delta_1 A, \delta_2 A) = \delta_1 \underline{\Theta}(\delta_2 A) - [1 \leftrightarrow 2]$$
$$= -\frac{1}{4\pi} \epsilon_{dabc} \left(\delta_1 F^{de} \delta_2 A_e - 1 \leftrightarrow 2 \right)$$

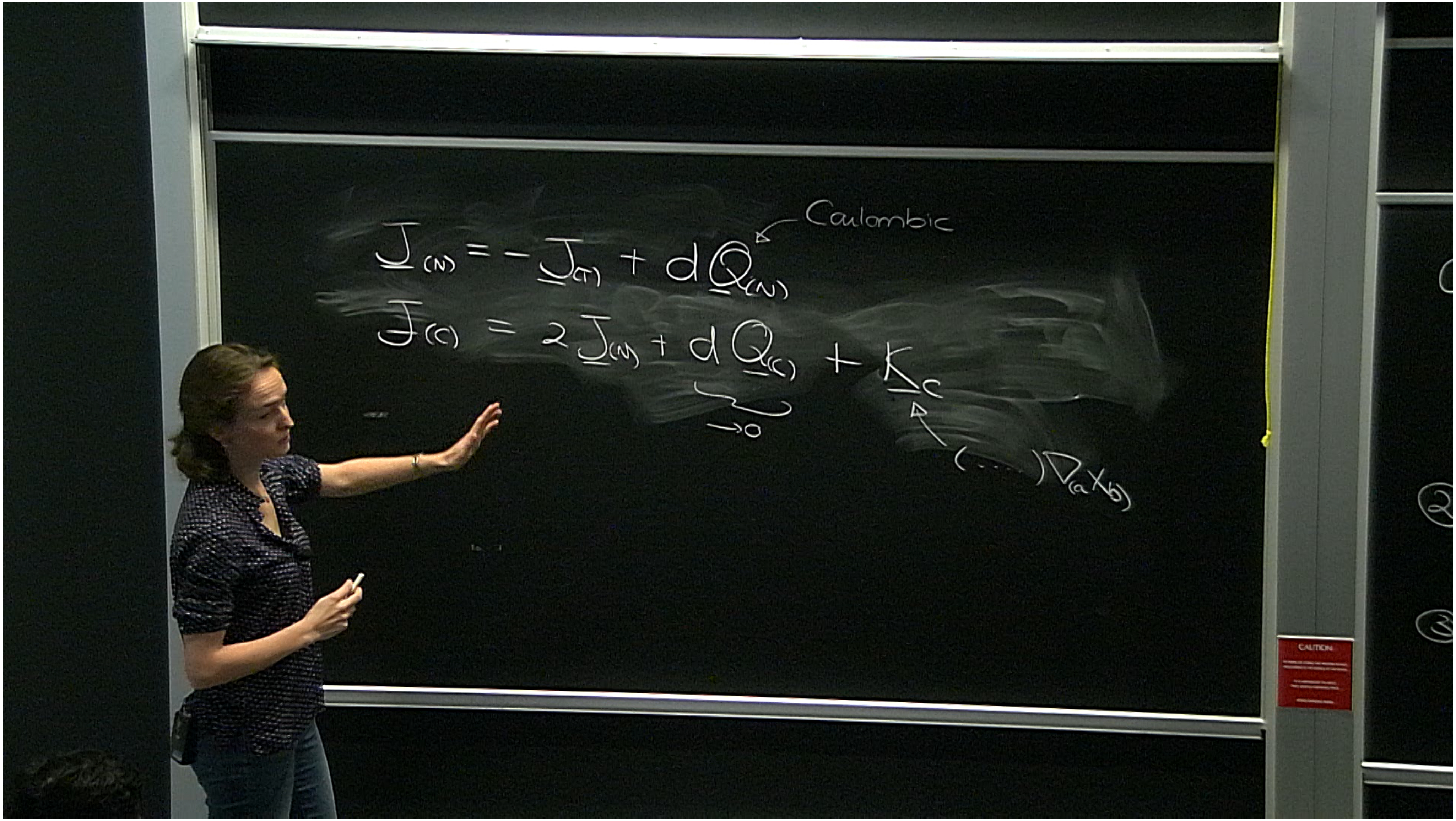
$$\Longrightarrow \underline{J}_{(C)} = \underline{\omega}(A, \delta_\xi A) =: \epsilon_{dabc} j^d_{(C)} \Rightarrow j^a_{(C)} = -\frac{1}{4\pi}\left[F^{ab}(\pounds_X A_b + \nabla_b \lambda) - g^{ac} g^{bd} A_b \pounds_X F_{cd} \right]$$

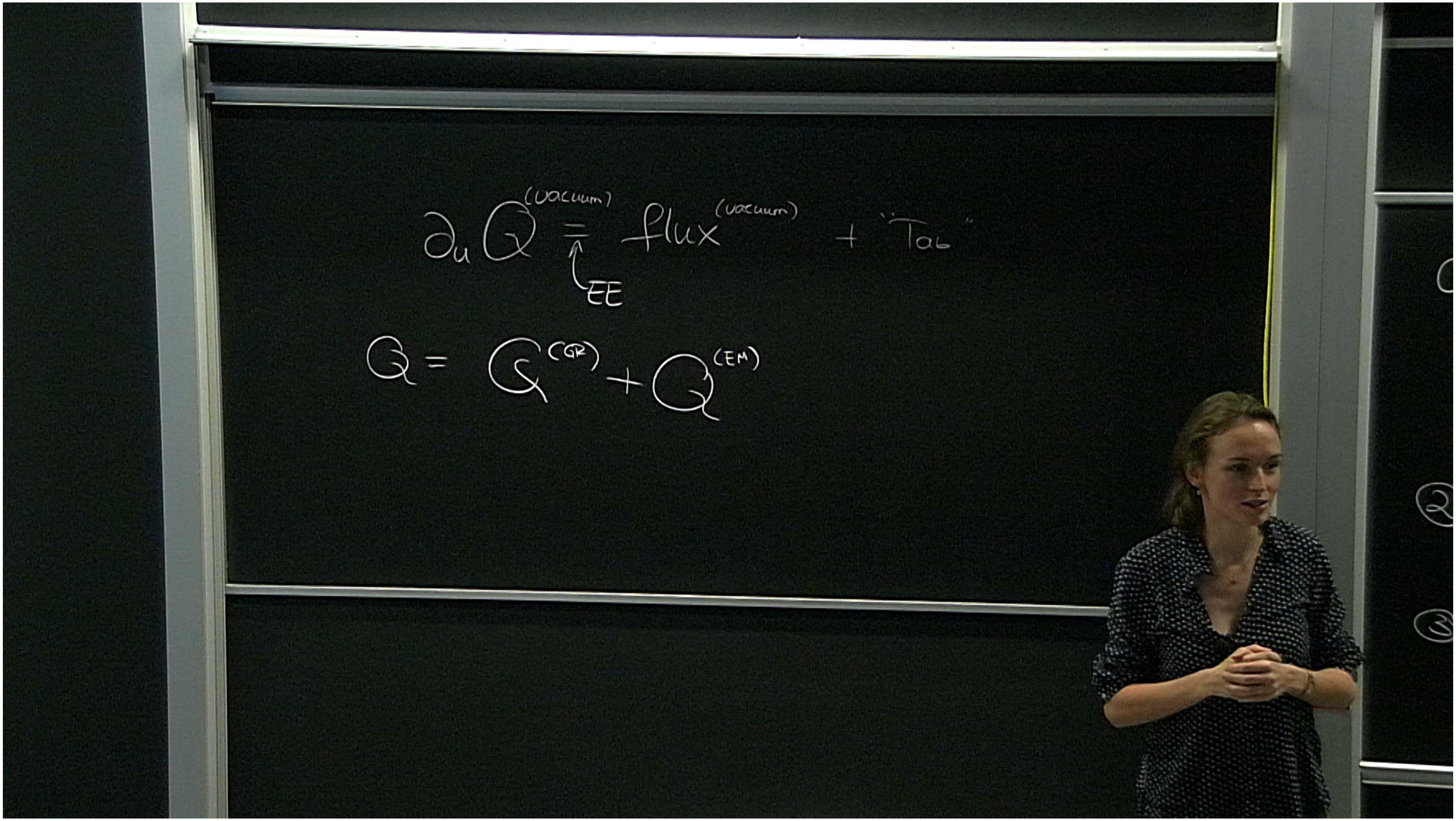
2. Noether current

$$\underline{J}_{(N)} = \underline{\Theta}(\delta_\xi A) - i_X \underline{L} \Rightarrow j^a_{(N)} = -\frac{1}{4\pi} F^{ab}(\pounds_X A_b + \nabla_b \lambda) + \frac{1}{16\pi} X^a F^2$$

3. Stress-energy current

$$j^a_{(T)} := T^{ab} X_b = \frac{1}{4\pi}\left(F^{ac} F_{bc} X^b - \frac{1}{4} X^a F^2 \right)$$


Coulombic
J(N) = - J(T) + d Q(N)
J(C) = 2 J(N) + d Q(C) + K_C
→0
(...) D(a X b)


$\partial_u Q^{(vacuum)} = flux^{(vacuum)} + "T_{ab}"$
EE
$Q = Q^{(GR)} + Q^{(EM)}$