Title: Quantum Spacetime from Lattice Gravity Ã la CDT

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Abstract: Causal Dynamical Triangulations (CDT) is a candidate theory for quantum gravity, formulated nonperturbatively as the scaling limit of a lattice theory in terms of triangulated spacetimes. An important feature of this approach is its elegant resolution of the problem of diffeomorphism symmetry in the full, background-free quantum theory. This has enabled the concrete computation of geometric observables in a highly nonperturbative, Planckian regime, an important step in putting quantum gravity on a quantitative footing, and understanding the structure of quantum spacetime. While the need to find quantum observables describing this regime is common to all approaches, CDT provides a concrete testing ground for implementation and measurements. In particular, a new notion of quantum Ricci curvature has opened a new window on the counterintuitive properties of quantum geometry.

Quantum Spacetime from Lattice Gravity à la CDT

Perimeter Institute,
18 Nov 2019

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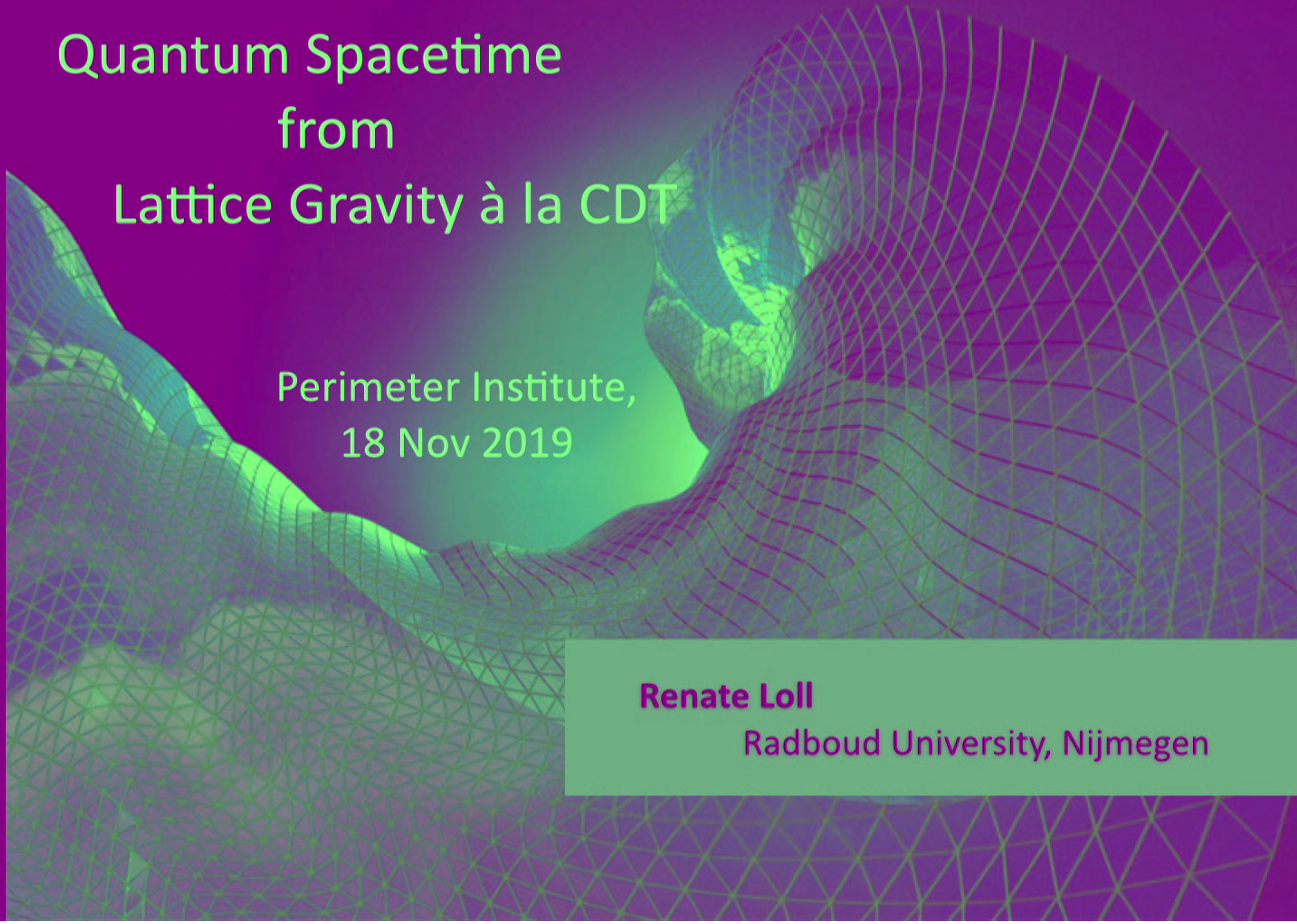


Preview

The context of my talk is the search for a ***theory of quantum gravity beyond perturbation theory*** and the ongoing research program of Causal Dynamical Triangulations (CDT) addressing the problem. In view of the workshop's theme and audience, I will try to take a broad perspective. I will describe some structural aspects, important problems and new insights this approach has helped bring to the fore, and which may well be relevant for quantum gravity generally.

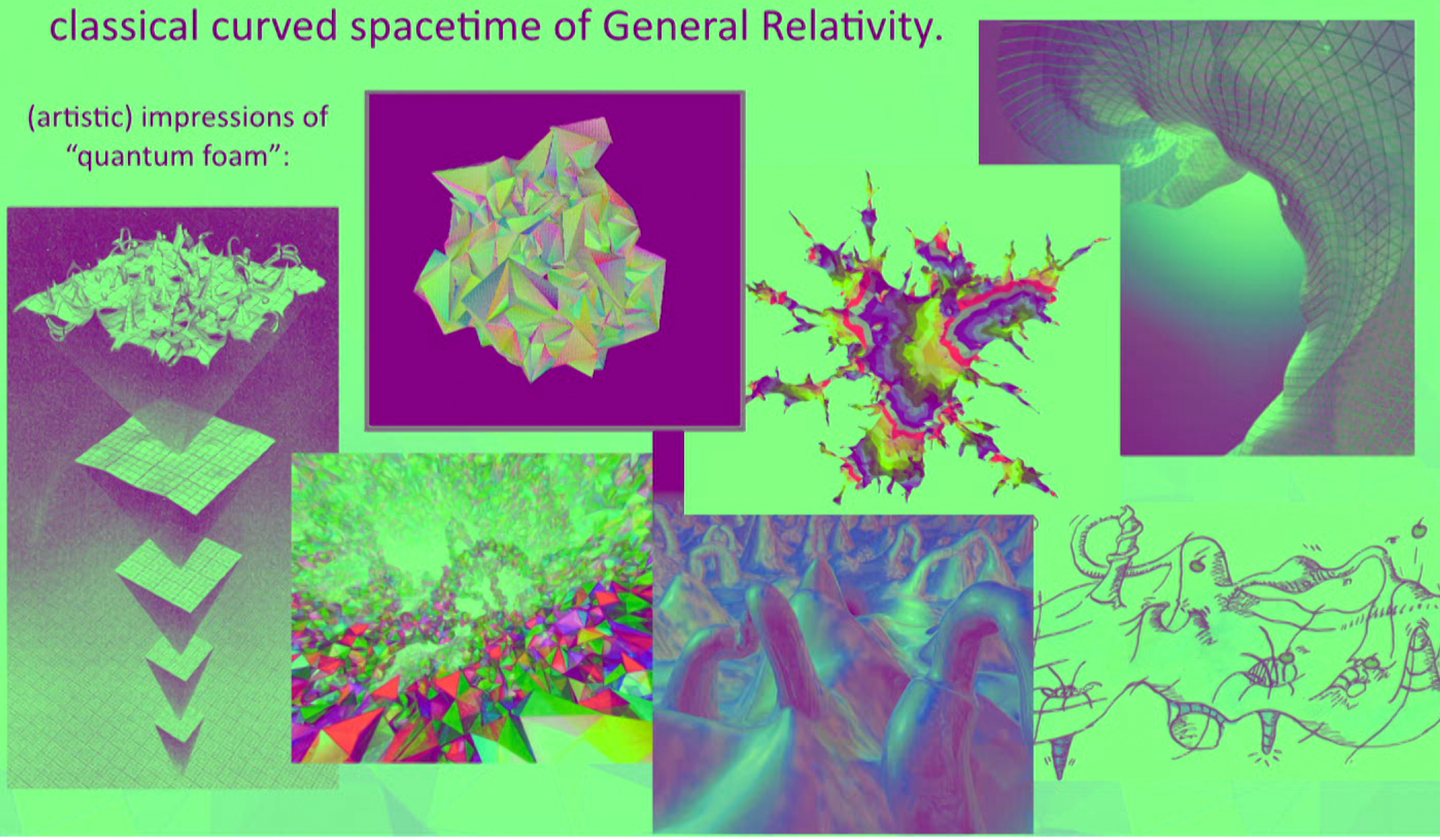
My presentation will discuss

- "quantum spacetime", whence and whither?
- "fundamental continuity" and the hidden power of Regge calculus
- symmetry and diffeomorphisms (homage to Noether)
- nonperturbative, scale-dependent observables

What *is* quantum spacetime?

A "spacetime" with quantum properties near the Planck scale $\sim \ell_{\mathrm{Pl}}$, which in a suitable macroscopic limit can be approximated by a classical curved spacetime of General Relativity.

(artistic) impressions of "quantum foam":



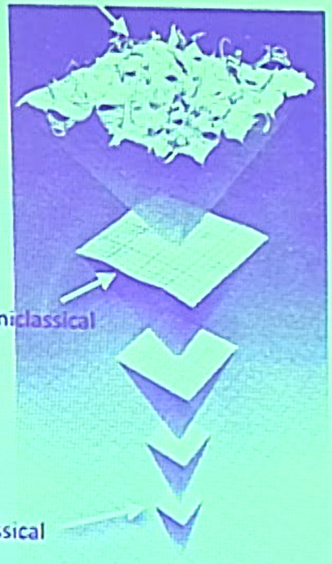
Quantum spacetime from quantum gravity

More specifically, a "quantum spacetime" implies the presence of ***dynamics*** and refers to a solution of suitable ***quantum*** equations of motion. Even more specifically, we are interested in the theory's vacuum, the "mother of all vacua". This requires **quantum gravity beyond perturbation theory**, with no single way of how to proceed.

> Apart from their choice of elementary *degrees of freedom* and a *dynamical principle*, different approaches to quantum gravity can be distinguished by how much *background structure* they use, e.g. whether metric, differentiable and manifold structure, topology, dimension etc. are fixed a priori or part of dynamics, and which *extra structures and assumptions* they use, e.g. additional symmetries, a choice of preferred "variables", extra dimensions, ...

CDT QG makes choices that are "simple, but not too simple".

Quantum spacetime: sense or nonsense?



zooming in on a piece of empty spacetime

For "quantum spacetime" or "quantum geometry" to have any relevance, we assume:

There is a quantitative description of physics at the Planck scale, despite $\ell_{Pl} \approx 10^{-35} m$.

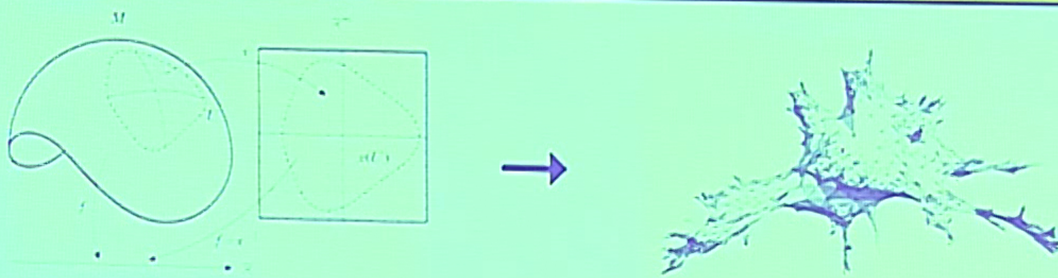
This Planckian physics has an associated phenomenology; quantum signatures of quantum spacetime exist beyond perturbation theory.

The underlying quantum gravity theory is "reasonably unique" (few tunable parameters). What ensures uniqueness or universality at ℓ_{Pl}?

Going far beyond just discarding smooth classical Lorentzian manifolds $(M,g_{\mu\nu})$, it is often argued that quantum gravity and spacetime near ℓ_{Pl} must be "**fundamentally discrete**" (operational meaning?).

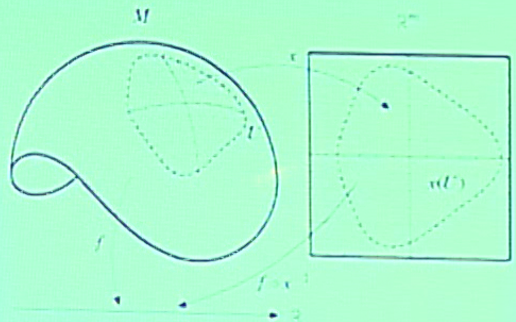
An alternative suggestion

Key to progress in nonperturbative QG is <u>fundamental continuity</u> ("continuous, but not smooth"), in the spirit of Regge calculus.



- nonsmoothness "standard" in quantum theory (cf. quantum particle)
- much less radical than fundamental discreteness, can still calculate
- Regge calculus, apart from being a convenient regularization of curved geometries in terms of piecewise flat spaces, does not use coordinates. While classically this is "just a feature", ***its full power becomes apparent in nonperturbative quantum gravity à la CDT.***

Spacetime geometry (textbook version)



differentiable manifold M and a coordinate chart

- Classically, differentiable manifolds M provide powerful and extremely convenient models of spacetime.
- geometric properties encoded in the Riemann curvature tensor $R^{\kappa}{}_{\lambda\mu\nu}(x)$

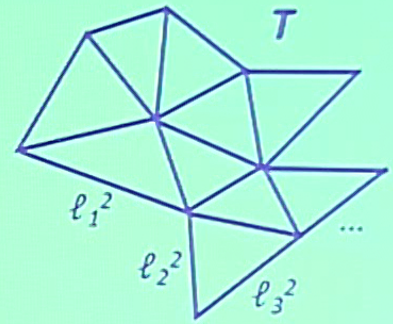
- However, this description comes with an enormous redundancy, the "freedom to choose coordinates" without affecting the physics.
- The "gauge" group of GR is the infinite-dim. group of coordinate transformations (diffeomorphisms) on M. **Two key challenges of quantum gravity are how to implement this symmetry and describe physics in terms of diffeomorphism-invariant *quantum observables*.**

Regge Calculus

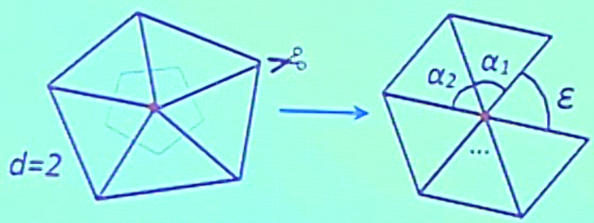
simplicial approximation of a curved manifold

$$(M, g_{\mu\nu}(x)) \rightarrow (T, \{\ell_i^2, i=1,\ldots,n\})$$

in terms of a 'triangulated' manifold T with (squared) edge length assignments ℓ_i^2



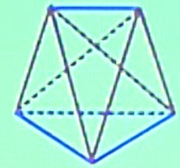
The geometry is encoded in the connectivity of the triangulation T and the edge lengths which fix the geometry of each flat triangular building block uniquely. In terms of these variables, one can describe ***"General Relativity without Coordinates"*** (T. Regge, 1961), with the gravitational action substituted, $S^{EH}[g_{\mu\nu}] \rightarrow S^{Regge}(T, \{\ell_i^2\})$.



simplicial manifolds carry singular curvature assignments in the form of "deficit angles" $\varepsilon = 2\pi - \sum \alpha_i$

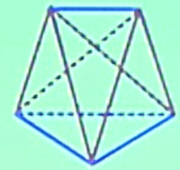
Gluing five equilateral triangles around a vertex generates a surface with Gaussian curvature (deficit angle ε) at the vertex.

Regge Calculus reloaded



- classical Regge calculus: try and implement in numerical evolution schemes (R. Sorkin et al., L. Brewin, A. Gentle, ...)
- semiclassical/linearized Regge calculus: recover correct degrees of freedom (M. Rocek & R. Williams, B. Dittrich et al., ...)
- N.B.: Regge calculus has no fundamental status, it is a (non-unique) approximation scheme, and convergence issues must be considered
- **very different application** in nonpert. 4D gravitational path integral:
  (a) "Quantum Regge Calculus": fix T, integrate over all ℓ_i^2 (B. Berg, H. Hamber & R. Williams, ...)
  (b) "Dynamical Triangulations": fix $\ell_i^2 = \pm a^2$, sum over all T of fixed topology (M. Agishtein & A. Migdal, J. Ambjørn & J. Jurkiewicz, ...)
- (a): no analytical calculations; residual gauge symmetry? measure? *not* Liouville gravity in 2D; most important: ***QRC is purely Euclidean***

Regge Calculus reloaded



4D simplex, building block of a spacetime in Regge calculus

- classical Regge calculus: try and implement in numerical evolution schemes (R. Sorkin et al., L. Brewin, A. Gentle, ...)
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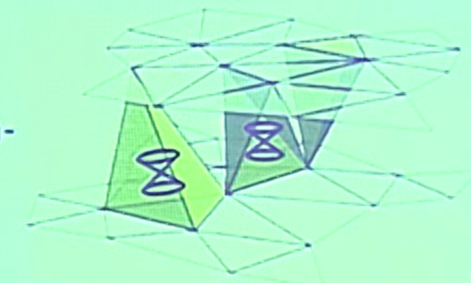
Introducing CDT Quantum Gravity

We have learned from Monte Carlo "experiments" that nonperturbative path integrals for 4D gravity based on statistical ensembles of (Euclidean or Wick-rotated) geometries suffer from very generic instabilities, which seem to prevent the dynamical generation of extended 4D geometry in a suitable classical limit.

The only known cure is that of **Causal Dynamical Triangulations (CDT)**, a manifestly diffeomorphism-invariant and background-independent path integral à la DT, whose gluing rules implement a well-defined *causal (light cone) structure*.



CDT configuration with local light cones

- no models are known where causal structure "emerges"
- causal structure not fixed, but quantum-fluctuates as part of geometry
- Euclidean path integral (probably) not good enough
- gravity 'on a lattice' does not imply breaking diffeomorphism invariance!

Quantum Gravity from CDT

The (formal, ill-defined) continuum gravitational path integral

$$Z(G_N,\Lambda) = \int_{\substack{\text{spacetimes}\\ g\in\mathcal{G}}} \mathcal{D}g\; e^{iS^{\rm EH}_{G_N,\Lambda}[g]}$$

Newton's constant; cosmological constant; Einstein-Hilbert action

("sum over histories")

is turned into a finite regularized sum over triangulated spacetimes,

$$Z(G_N,\Lambda) := \lim_{\substack{a\to 0\\ N\to\infty}} \sum_{\substack{\text{inequiv.}\\ \text{triangul.s}\\ T\in\mathcal{G}_{a,N}}} \frac{1}{C(T)}\; e^{iS^{\rm Regge}_{G_N,\Lambda}[T]}$$

UV cutoff; # building blocks; |Aut(T)|; bare, discretized EH action

whose continuum limits are investigated after an analytic continuation.
(N.B.: **no** residual gauge symmetry; obeys reflection positivity)

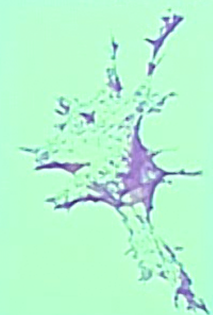
REVIEWS: J. Ambjørn, A. Görlich, J. Jurkiewicz & RL, Phys. Rep. 519 (2012) 127 [arXiv: 1203.3591]; **NEW**: RL, CQG 2019 [arXiv:1905.08669].

The Emergence of Classicality from Causal Dynamical Triangulations (CDT)

From pure quantum excitations, CDT generates a spacetime with semiclassical properties *dynamically*, without using a background metric.

obtaining a macroscopic universe with a *de Sitter shape,*

from a superposition of "wild" path integral histories:



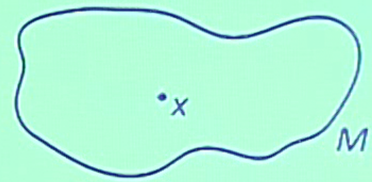
Other key results/properties:

- crucial role of causal structure
- nontrivial phase structure, with "classical" phases
- second-order phase transitions (unique)
- scale-dependent spacetime dimension ($2 \rightarrow 4$)
- applicability of renormalization group methods

Everything we have learned about "quantum spacetime" in CDT QG comes from measuring a few <u>quantum observables</u>.

The story of observables: classical

- Classical gravitational observables are diffeomorphism-invariant (and therefore usually nonlocal) quantities.
  For example, $g_{\mu\nu}(x)$ and $R(x)$ are **not** observables while $\int_M d^4x\sqrt{g}\,R(x)$ is.



- In continuum approaches to quantum gravity, implementing diffeomorphism invariance beyond perturbation theory is a major source of technical and conceptual problems.
- In CDT, we have got rid of this problem by eliminating coordinates and any coordinate redundancy. However, what *are* quantum-gravitational observables in the absence of diffeomorphisms?

[health warning: "observable" does not imply a direct link to phenomenology]

The story of observables: quantum

- Observables in CDT QG are formulated in terms of geometric notions, like distances and volumes, but do not rely on smoothness.
- Like in the continuum, one is not allowed to 'mark' a point (curve, surface, …) in terms of some extrinsic labels. A quantity "at a point" is still not a meaningful concept, and must be averaged over spacetime, in addition to summing over geometries, to obtain an observable.
- Computer simulations work with labelled triangulations, but are set up to be invariant under relabellings.
- discrete relabellings are merely 'moral equivalents' of smooth coordinate transformations; the two groups are not related, but we contend that their corresponding quotient spaces are:

Metrics(M)/Diff(M) ↭ (unlabelled) DTs

Given that quantum observables are nonlocal quantities, can we get a handle on any short-scale properties of quantum spacetime?

Yes, in an average sense

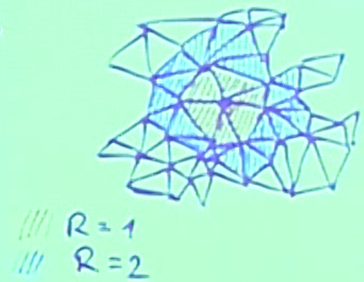
- We can define quantum observables $\hat{O}_\delta$ which are spacetime averages, but nevertheless depend on a length scale δ that is being probed,

$$\langle \hat{\mathcal{O}}_\delta \rangle = \frac{1}{Z} \int \mathcal{D}g \, \mathcal{O}_\delta[g] \, e^{-S[g]}, \quad \mathcal{O}_\delta[g] = \frac{1}{V(M)} \int_M d^4x \, \sqrt{g} \, \mathcal{O}_{\delta,x}[g]$$

- This is important, because we want to identify short-scale quantum effects and verify the presence of long-distance classical behaviour.
- We have several observables of this type in CDT quantum gravity. An example is the volume of a geodesic ball of radius R,

$$\langle \mathrm{Vol}(B_R) \rangle \propto R^{d_H},$$

from which we extract the Hausdorff dimension d_H.



geodesic balls B_R of radius R in 2D

- For the spectral dimension D_S, the relevant observable is the average return probability

$$\mathcal{R}_V(\sigma) := \frac{1}{V(M)} \int_M d^dx \, P(x, x; \sigma) \propto \frac{1}{\sigma^{D_s/2}}$$

diffusion time

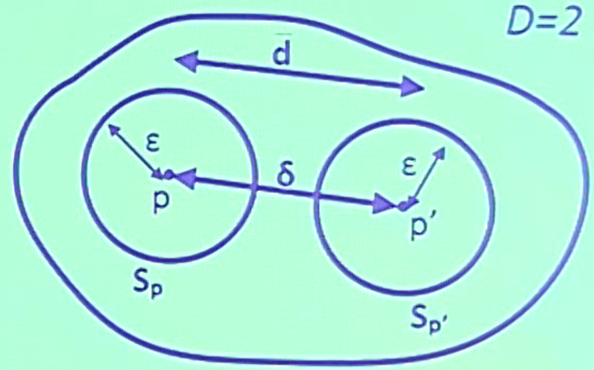
sol.n to heat equation

A new example: quantum Ricci curvature

In D dimensions, the key idea is to compare the distance $\bar{d}$ between two *(D-1)*-spheres with the distance δ between their centres.

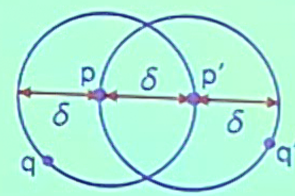
The sphere-distance criterion:
"On a metric space with positive (negative) Ricci curvature, the distance $\bar{d}$ of two nearby spheres S_p and $S_{p'}$ is smaller (bigger) than the distance δ of their centres."

(c.f. Y. Ollivier, J. Funct. Anal. 256 (2009) 810)



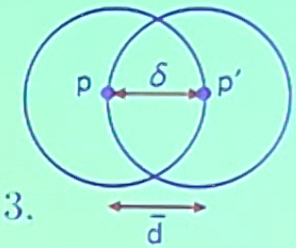
Our variant uses the ***average sphere distance*** $\bar{d}$ of two spheres of radius δ whose centres are a distance δ apart,

- involves only distance and volume measurements
- the directional/tensorial character is captured by the "double sphere"
- again, coarse-graining is captured by a ***variable length scale*** δ



Implementing quantum Ricci curvature

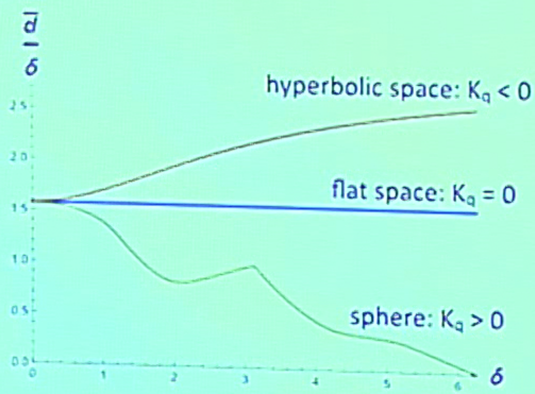
From the quotient of sphere distance and centre distance we extract the **"quantum Ricci curvature K_q at scale δ"**,



$$\frac{\bar{d}(S_p^\delta, S_{p'}^\delta)}{\delta} = c_q(1 - K_q(p,p')), \quad \delta = d(p,p'), \quad 0 < c_q < 3,$$

where c_q is a non-universal constant. The simplest observable (Ricci scalar) is obtained by first averaging over p′, and then p.

D=2



To interpret quantum results, we are currently building a reference library, computing $K_q(\delta)$ on various classical spaces (constantly curved, ellipsoids, orbifolds, ...), which is very nontrivial and gives us a new way of thinking of their invariant properties.

(N. Klitgaard & RL, Phys. Rev. D 97 (2018) 046008 & 106017, and w.i.p.)

Summary

Finding quantum spacetime is a dynamical issue that requires a sufficiently complete candidate theory of nonperturbative QG.

I argued for "fundamental continuity" as a guiding principle.

The full power of Regge's idea of describing geometry without coordinates can unfold in nonperturbative QG, yielding a manifestly **diffeomorphism-invariant** formulation.

I used CDT quantum gravity to illustrate all of these points. They lead to a new perspective on the all-important issue of **observables**. Despite the absence of smoothness, one can define a new class of global, but scale-dependent observables, including curvature.

They will hopefully enable us to uncover ***universal properties of quantum spacetime and quantum gravity in a Planckian regime***, and bring us closer to finding true quantum signatures.


CAUTION