Title: Dirty Quantum Criticality

Speakers: Hart Goldman

Series: Condensed Matter

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Abstract:

Many of the most mysterious phenomena in condensed matter physics involve systems near quantum phase transitions where the interplay of quenched disorder and strong interactions likely plays an essential role. Examples include the appearance of "anomalous" 2d metallic phases and the sharing of critical exponents between seemingly different quantum Hall plateau transitions. However, few organizing principles have been developed for understanding quantum critical systems with interactions and disorder, and analytically tractable models have proven rare. In this talk, I will describe some of the first examples of quantum critical theories in 2d characterized by finite disorder and interaction strengths, focusing on particular examples of quenched disorder in systems of (i) Dirac fermions coupled to an emergent gauge field (QED3) and (ii) scalar bosons at their Wilson-Fisher fixed point. Both of these examples exhibit universal features not found together in systems with interactions or disorder alone, such as vanishing density of states, finite DC conductivities, and novel critical exponents.

Dirty Quantum Criticality

Hart Goldman

Department of Physics and Institute for Condensed Matter Theory,
University of Illinois at Urbana-Champaign

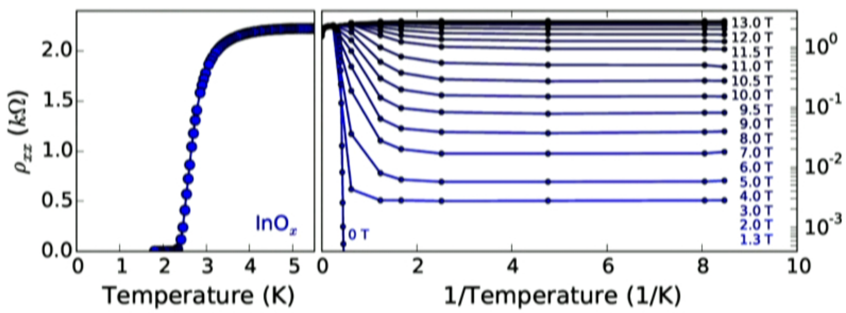
Perimeter Institute, November 12, 2019

This talk is primarily based on

- HG, A. Thomson, L. Nie, and Z. Bi, arXiv:1909.09167
- HG, M. Mulligan, S. Raghu, G. Torroba, and M. Zimet, PRB 96, 245140 (2017)
- P. Goswami, HG, and S. Raghu, PRB 95, 235145 (2017)
  [erratum: PRB 99, 079903]

Absence of quantum diffusion in 2d?

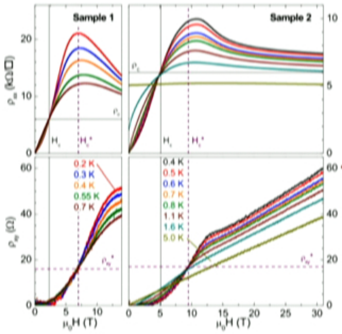
- **Conventional wisdom**: no metals in 2d at $T = 0$
- **Scaling theory of localization**: *non-interacting* electrons with weak disorder cannot exhibit a metallic ground state in 2d. [Abrahams, Anderson, Licciardello, and Ramakrishnan, PRL (1979)].
- **Experiments appear to defy this conclusion:** "anomalous metals" observed in a variety of materials
- **Classic examples:** half-filled LL, SC thin films, Si MOSFETs, …

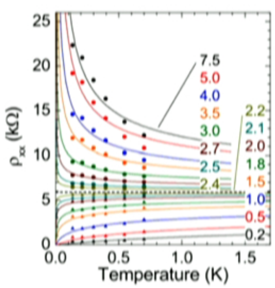


[Breznay and Kapitulnik, Science Advances (2017)]

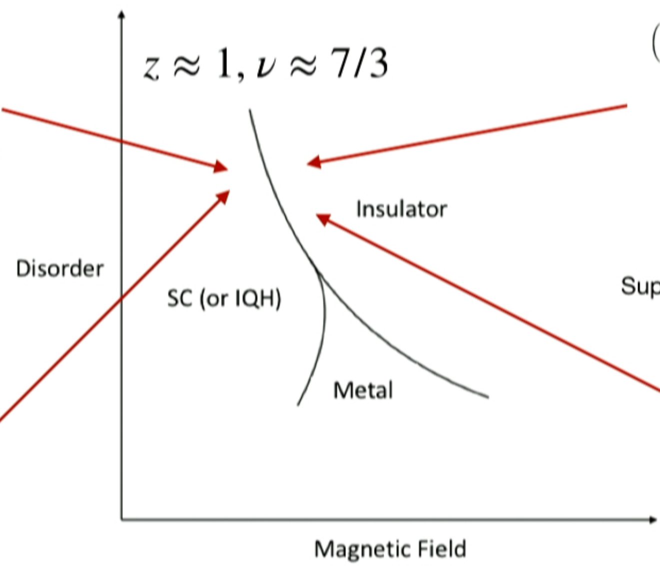
Anomalous Metals, Mysterious Quantum Critical Points

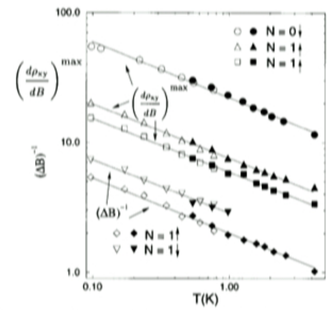
- **Anomalous metals are frequently near quantum critical points** with mysterious behavior



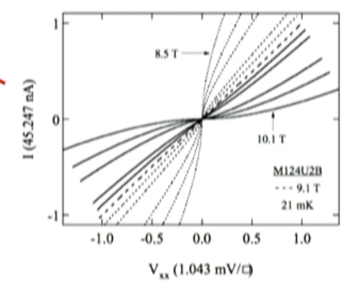


self-duality, universal transport at the field-tuned SIT
[Breznay *et al.*, PNAS (2016)]





Superuniversality of QH plateau transitions
[Wei *et al.*, PRL (1988)]



Reflection symmetry at $\nu = 1/2n$
[Shahar *et al.*, Science (1996)]

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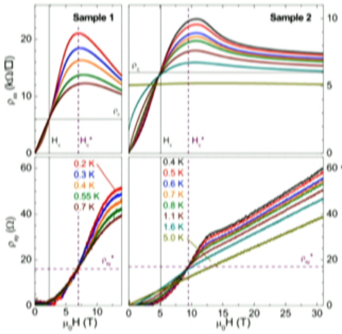

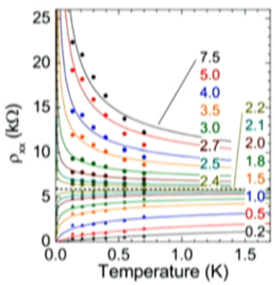

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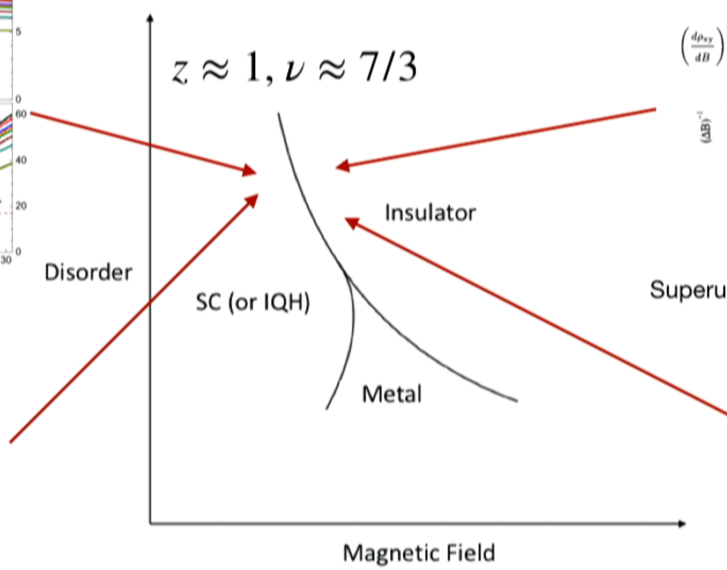

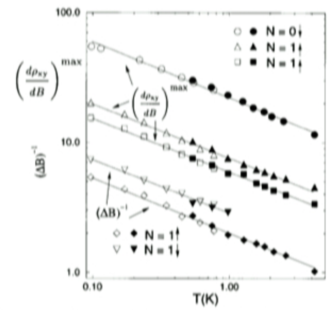

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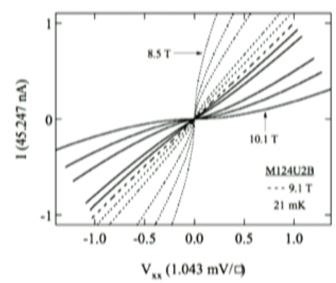

Reflection symmetry at $\nu = 1/2n$
[Shahar *et al.*, Science (1996)]

Disorder + Interactions

- None of these phenomena explainable with disorder or interactions alone, **reflect a deep interplay of disorder, interactions, and quantum criticality.**

- Two paths forward:

  1. **Target the anomalous metal (finite DoS)**: study Fermi surfaces with disorder and interactions — extremely hard! Disorder always introduces a finite (diffusive) scattering rate $\Gamma = 1/\tau \sim k_F$. Diffusive FLs always unstable in 2d, flows to strong interactions and disorder.

  2. **Target the quantum critical regime:** more tractable, scale invariance means a diverging localization length, subverts localization [Fisher, Grinstein, and Girvin, PRL (1990)].

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No collusion?

- Focus on incompressible, **critical** theories.
- Non-interacting theories (with **T** invariance) tend to flow to strong disorder and often *diffusion*, $z = 2$.

$$\omega(\mathbf{k}) \sim D|\mathbf{k}|^2$$

$$n_S(T \to 0) \to \text{finite}$$

- **What other behavior is possible?**

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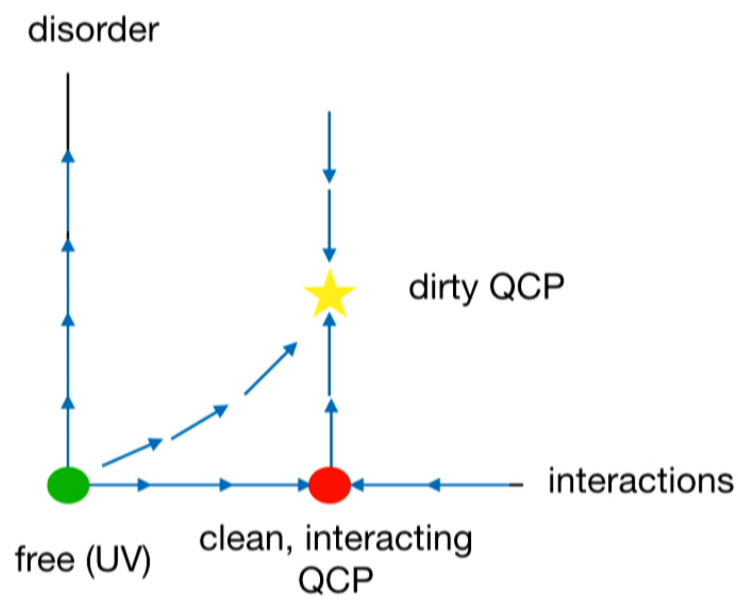
- **What other behavior is possible?**
- **Goal:** Search for 2d theories where collusion of disorder and interactions can be studied tractably.
- Develop new organizing principles for dirty quantum criticality.



Plan of this talk

We develop two major examples of theories hosting quantum critical states with *finite disorder and interactions*

I. **Bosonic quantum critical points:** disorder at Wilson-Fisher
[HG, A. Thomson, L. Nie, Z. Bi, arXiv:1909.09167]

II. **Metallic phases from gauge field fluctuations**: disorder in QED_3
[P. Goswami, HG, S. Raghu, PRB (2017)],
[A. Thomson and S. Sachdev, PRB (2017)]

To access these fixed points, we employ both **perturbative RG** methods and glean non-perturbative lessons from particle-vortex **duality**
[HG, M. Mulligan, S. Raghu, G. Torroba, M. Zimet, PRB (2017)].

The dirty boson problem

- **What is the simplest interacting theory?** Consider a system of bosons undergoing **a** $T = 0$ **superfluid-insulator transition.**

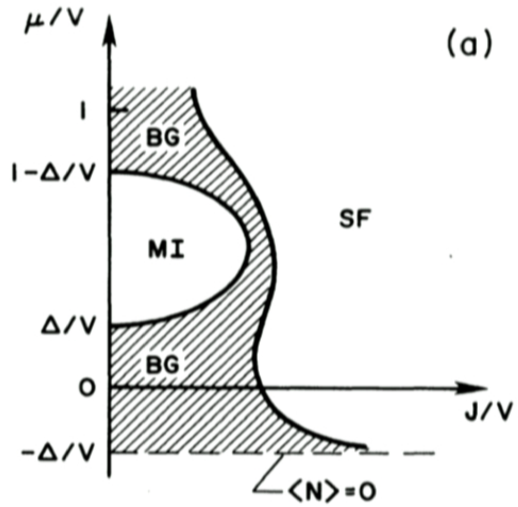
- **Quenched disorder:** a random mass $R(\mathbf{x})$ *depending only on space.* This is Always consistent with symmetry,

$$\mathcal{L}_{\text{dis}} = R(\mathbf{x})|\phi|^2$$

$$\overline{R(\mathbf{x})} = 0 \qquad \overline{R(\mathbf{0})R(\mathbf{x})} \sim \Delta\delta(\mathbf{x})$$

← Gaussian white noise

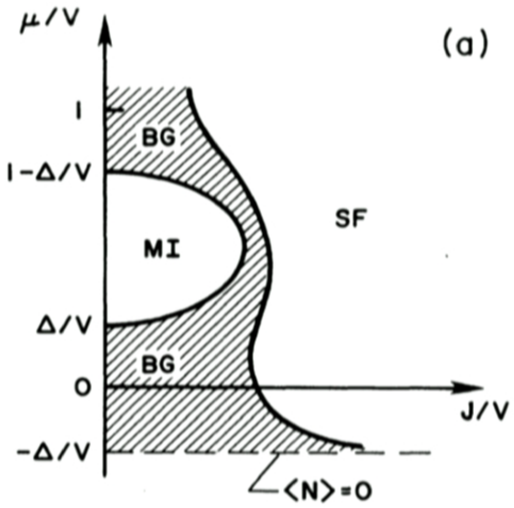
The dirty boson problem



- Very **physically relevant**: 4He in porous Vycor, Josephson junction arrays, cold atoms, …

[Fisher, Weichman, Grinstein, and Fisher, PRB (1989)]

The dirty boson problem



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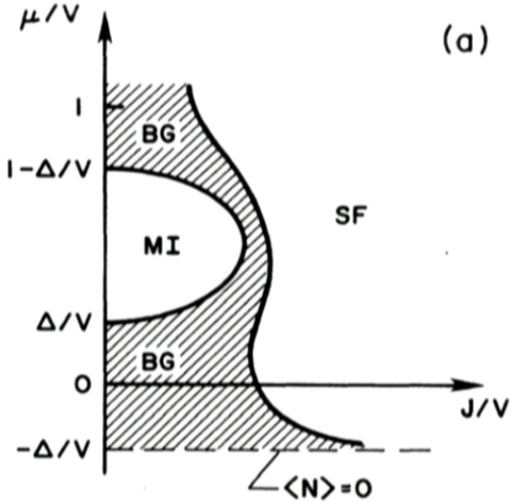

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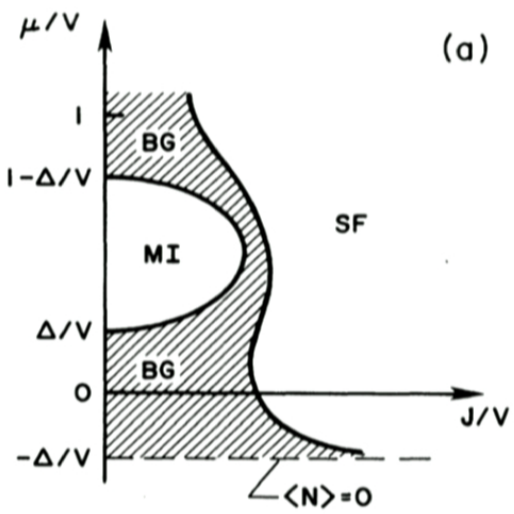

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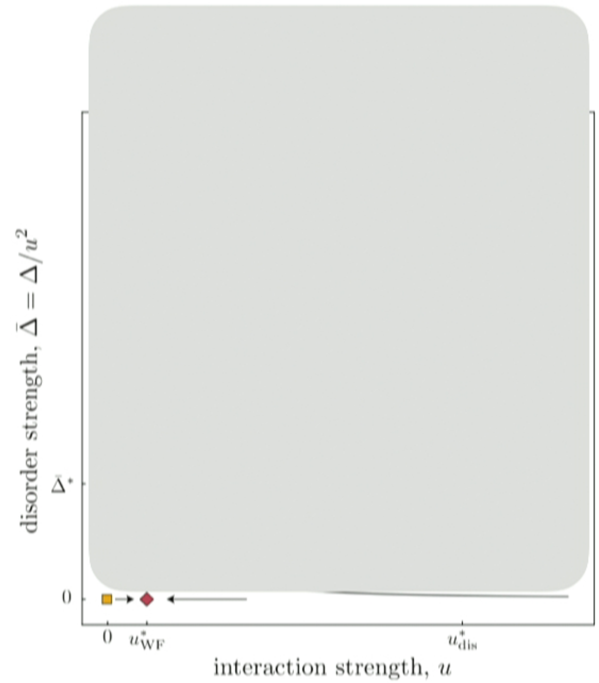
The double-ϵ expansion

- **Clean limit:** Wilson-Fisher fixed point. What happens when we introduce disorder?



The double-ϵ expansion

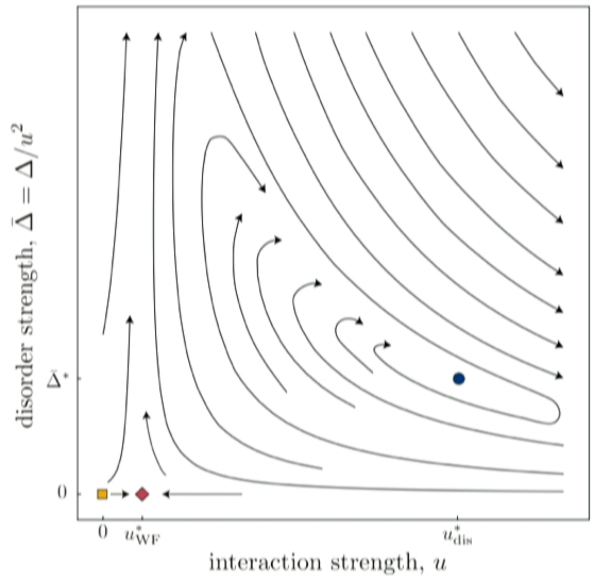
- **There is a fixed point, but the RG flows are spirals!** Takes a very long time to reach the fixed point, contrasts with universal behavior observed in numerics:

$$\nu \approx 1, z \approx 1.5$$

[Vojta *et al.*, PRB (2016)], …

- non-unitary behavior, imaginary eigenvalues

$(\epsilon = 2, \epsilon_t = 1)$



This is alarming

- Appears to be a consequence of perturbing a free, classical fixed point.

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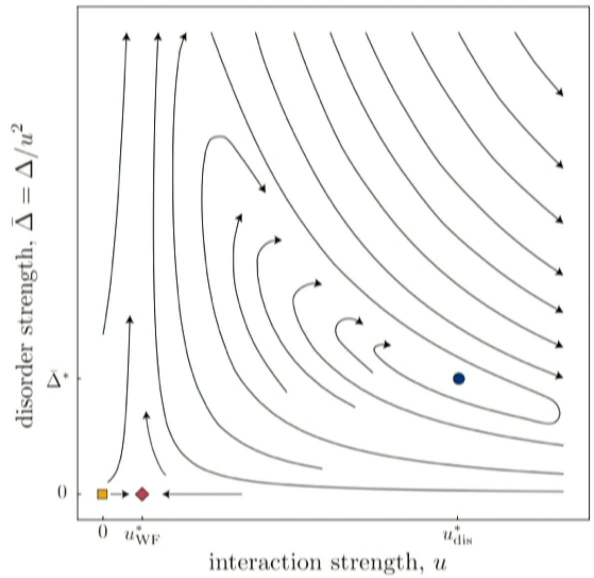

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  - More technical explanation: operator degeneracy. Similar to $\mathbf{CP}^N$ model (SIT in smectic crystals), why 182 is a famous number. [Halperin, Lubensky, Ma, PRL (1974)]

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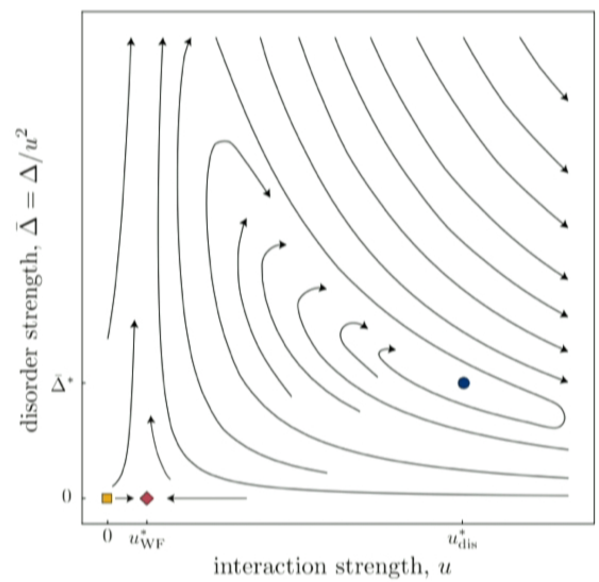

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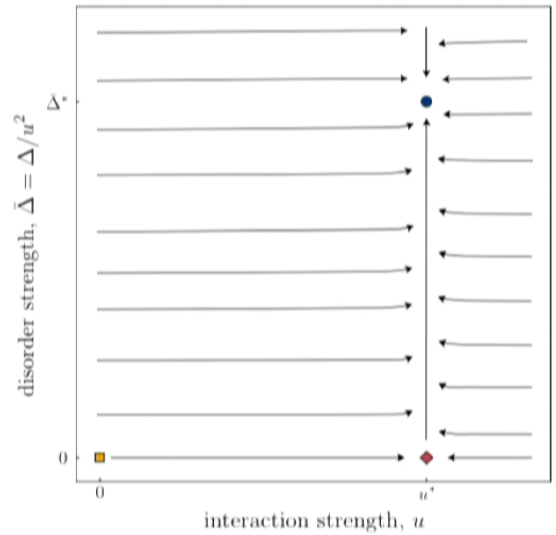
  - More technical explanation: operator degeneracy. Similar to $\mathbf{CP}^N$ model (SIT in smectic crystals), why 182 is a famous number. [Halperin, Lubensky, Ma, PRL (1974)]
- **What happens if we perturb Wilson-Fisher instead?**
- **Preview:** Will find far more germane RG flows. Both screening and anomalous dim effects play a role.



Organizing principles: interacting bosons

- $d < 3$: (relativistic) bosons are always interacting at $T = 0$, flow to Wilson-Fisher fixed point

$$\mathcal{L}_E = |\partial\phi|^2 + r_c|\phi|^2 + \frac{u}{2}|\phi|^4$$

- Here the operator $|\phi|^2$ has a *positive* anomalous dimension, $\eta_{|\phi|^2} > 0$.

- **Disorder:** $\mathcal{L}_{dis} = R(\mathbf{x})|\phi|^2, \quad \overline{R(\mathbf{0})R(\mathbf{x})} \sim \Delta\delta(\mathbf{x})$

$[\Delta] = 4 - d - 2\eta_{|\phi|^2} \Rightarrow$ **Stable if** $d > 4 - 2\eta_{|\phi|^2}$

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Organizing principles: interacting bosons

- It is useful to **think of interactions as modifying the disorder correlations.** Imagine we started with

$$\overline{R(\mathbf{x})} = 0\,,\ \overline{R(\mathbf{0})R(\mathbf{x})} \sim \frac{\Delta}{|\mathbf{x}|^{\chi_0}}$$

- Gaussian white noise: $\chi_0 \to d$,

$$\overline{R(\mathbf{0})R(\mathbf{x})} \to \Delta\delta(\mathbf{x})$$

Large-N limit of the dirty O(2N) model

- **Strategy:** (i) flow to the Wilson-Fisher fixed point in the large-N limit, then (ii) introduce disorder.

- **Model:** N complex scalars ϕ_I in 3 Euclidean spacetime dimensions. Hubbard-Stratonovich the $|\phi|^4$ interaction with a field $\sigma \sim |\phi|^2$ (disorder will couple to σ)

$$\mathcal{L}_E = |\partial\phi_I|^2 + \frac{i}{\sqrt{N}}\sigma|\phi_I|^2 + \frac{i}{u}R(\mathbf{x})\sigma + \frac{1}{2u}\sigma^2$$

- **Global symmetries**: $U(1)_{EM}$ charge conservation, time reversal **T**, particle-hole **PH**, parity **P**, $U(N)$ flavor ($N=1$: 2d quantum XY model).

⟶ flavor + **T**, **PH**, **P**: $O(2N)$ symmetry

Large-N limit of the dirty O(2N) model

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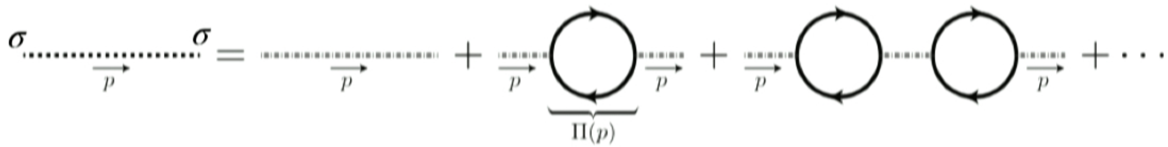
- Take $N \to \infty$, holding u fixed, sums **“screening” bubbles**,

$G_\sigma(p) =$ 

- IR limit ($u \to \infty$): $G^\sigma(p) \to 8|p|$, $[\sigma] = 1 + \eta_{|\phi|^2} = 2$

- Running coupling: $\bar{u} = u/8\,|\,p\,|$,

$$\beta_{\bar{u}} = \frac{d\bar{u}}{d\ell} = \bar{u}(1-\bar{u})$$ (Wilson-Fisher: $\bar{u}_* = 1$)

Enter disorder

- To study the effects of disorder, our interest is in the disorder-averaged free energy

$$\overline{F} = -\overline{\log Z[R]} = -\int \mathcal{D}R\, e^{-\int_{\mathbf{x}} R^2/2\Delta} \log Z[R]$$

- Studying $\overline{F}$ directly is normally intractable. Instead use the "replica trick," introduce n_r copies of the theory and take the limit $n_r \to 0$,

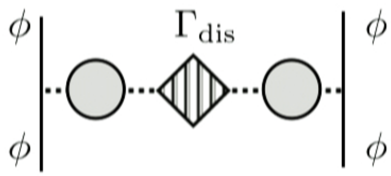
$$\log Z = \lim_{n_r \to 0} \frac{Z^{n_r} - 1}{n_r}$$

- **Integrate out R:** disorder looks like "mass" for σ that is non-local in imaginary time,

$$S_{\text{dis}} = \frac{\bar{\Delta}}{2} \int d\tau d\tau' d^2\mathbf{x} \sum_{n,m}^{n_r} \sigma_n(\mathbf{x},\tau)\sigma_m(\mathbf{x},\tau') \qquad \bar{\Delta} = \Delta/u^2$$

Screening

- **How is disorder affected by interactions in the large-N limit?** Look at the disorder vertex



$$\sim \frac{\bar{\Delta}}{|\mathbf{x}|^4}$$

$\chi_{\rm int} = 4$, short-ranged in $d = 2$!

- Summing bubbles means that **the disorder potential is screened non-perturbatively** as we flow to Wilson-Fisher. Manifestation of $\eta_{|\phi|^2} = 1$.

Quantum corrections at Wilson-Fisher

- In the limit $N \to \infty$, disorder is exactly marginal. **What about $1/N$ corrections?**

- Leading correction to $\eta_{|\phi|^2}$ at clean Wilson-Fisher is $\delta\eta_{|\phi|^2} < 0$, **makes disorder more relevant.** Makes sense: $N = 1$ (XY model) has $\eta_{|\phi|^2} \approx 0.5$.

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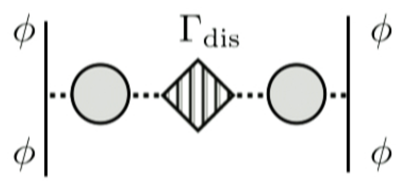

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Universal features

- **The RG flows remain nice even for small N.** We obtain exponents

$$\nu = 1 + \mathcal{O}(1/N^2),\ z = 1 + \frac{16}{3\pi^2 N} + \mathcal{O}(1/N^2)$$

From balancing disorder and interactions

$1 < z < 2$

compressibility:
$\kappa \sim |r - r_c|^{\nu(d-z)} \to 0$

- $N = 1 : O(2)$ (XY) model:

$$\nu \approx 1,\ z \approx 1.5$$

Agreement with numerics for the superfluid-“Mott glass” transition!

[Vojta *et al.*, PRB (2016)], …

- **Universal transport**: Expect finite $\sigma_{xx}(\omega/T \to 0)$ at the transition. Future work with P. Nosov and S. Raghu.

Breaking PH symmetry: Disorder coupling to global currents

- Imagine disorder coupling to the global charge current

$$\mathcal{L}_{\rm dis} = V(\mathbf{x})J^\tau + \mathcal{A}_i(\mathbf{x})J^i$$

$$J_\tau = \phi^\dagger\partial_\tau\phi - \partial_\tau\phi^\dagger\,\phi \qquad J_i = i(\phi^\dagger\partial_i\phi - \partial_i\phi^\dagger\,\phi)$$

- Disorder generates $z > 1$, current conservation $\partial_\mu J^\mu = 0$ implies

$[\Delta_V] = 2z - 2$ → $[J^\tau] = 2,\ [J^i] = 1 + z$ ← $[\Delta_{\mathcal{A}}] = 0$

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$$[\Delta_V] = 2z - 2 \rightarrow [J^\tau] = 2,\ [J^i] = 1 + z \leftarrow [\Delta_\mathcal{A}] = 0$$

- Means **a random chemical potential for the bosons** (breaks **PH**) **is relevant** (non-perturbative), sends the theory into a strong disorder phase.

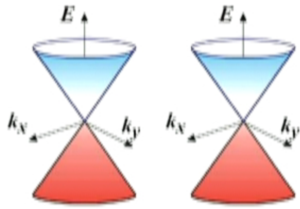
- **Possibly an interesting glassy state**, but more work has to be done to understand this state. Exponents happen to be close to what is seen in numerically for disorder-tuned superfluid-“Bose glass” transitions.

Part II:
2d metallic phases from dirty QED_3

[Pallab Goswami, HG, and Sri Raghu, PRB (2017)],
[HG, Mike Mulligan, Sri Raghu, Gonzalo Torroba, and Max Zimet, PRB (2017)]

Weakly interacting example

- **This part of the talk:** dirty metallic *phases* with vanishing DoS
- **Graphene + Coulomb interactions**



$+ \ \frac{e}{r}$ + random potential $V(\mathbf{x})$

[Fradkin, PRB (1986)], [Ludwig, Fisher, Shankar, and Grinstein, PRB (1994)], [Foster and Aleiner, PRB (2008)]

Organizing principles:
Diracs + disorder

- **Want**: general understanding of effects of disorder on Diracs in d spatial dimensions *at neutrality* (vanishing DoS).

- Again consider a random *spatial* potential $V(\mathbf{x})$ with variance Δ,

$$\overline{V(\mathbf{x})} = 0\,, \qquad \overline{V(\mathbf{x})V(\mathbf{0})} \sim \frac{\Delta}{|\mathbf{x}|^{\chi_0}}$$

- Gaussian white noise: $\chi_0 \to d$

$$\overline{V(\mathbf{x})V(\mathbf{0})} \to \Delta\delta(\mathbf{x})$$

- $[V(\mathbf{x})] = 1$: random mass, scalar, vector potential

$$\Rightarrow [\Delta] = 2 - \chi_0 \Rightarrow \text{Stable if } \chi_0 = d > 2 \quad \textbf{(Harris criterion)}$$

- **Effects of interactions:** screening and quantum corrections again modify χ_0 so that the interacting stability criterion is

$$\chi_0 \to \chi_{\rm int} = \chi_0 + 2\eta > 2$$

Clean QED_3

- **Natural suspects:** emergent gauge fields.
    - Play important roles e.g. half-filled LL, field-tuned SIT, spin liquids, …

Input: DVI - 1920x1080p@60Hz
Output: SDI - 1920x1080i@60Hz

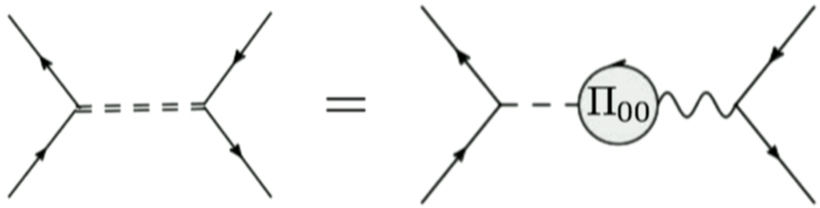
Disorder at the QED_3 fixed point

- The large-N_f QED_3 fixed point is a **quantum critical phase**. We now wish to introduce disorder, look at interplay with gauge field fluctuations.
- Our main interest is in disorder respecting **T** symmetry:
  - scalar potentials (density, chemical potential — stable metallic behavior!)
  - **T**-preserving mass (leads to a dirty-interacting fixed point!)
- To study disorder, we will **again make use of the replica trick.**

Screening again

$$S_{\text{dis}} = \int d\tau d\tau' d^2\mathbf{x}\, \psi^\dagger_{I,n}\psi_{I,n}(\mathbf{x},\tau)\psi^\dagger_{J,m}\psi_{J,m}(\mathbf{x},\tau')$$

- **The random chemical potential is screened** by the same bubble diagrams that screened the photon propagator!



$$\Gamma_{\text{dis}} \sim \frac{\Delta}{\alpha^2}|\mathbf{p}|^2$$

- Disorder correlations are now short-ranged:

$$\overline{V(\mathbf{0})V(\mathbf{x})} \sim \frac{\Delta}{\alpha^2}\frac{1}{|\mathbf{x}|^4} \qquad \chi_{\text{int}} = \chi_0 + 2$$

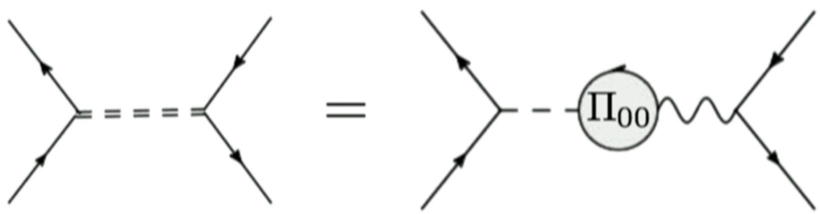
(note erratum)

- **QED_3 is stable to a random chemical potential!** Stark contrast with non-interacting case.

Screening again

$$S_{\text{dis}} = \int d\tau d\tau' d^2\mathbf{x}\, \psi^\dagger_{I,n}\psi_{I,n}(\mathbf{x},\tau)\psi^\dagger_{J,m}\psi_{J,m}(\mathbf{x},\tau')$$

- **The random chemical potential is screened** by the same bubble diagrams that screened the photon propagator!



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- **QED_3 is stable to a random chemical potential!** Stark contrast with non-interacting case.

What about the vortices?

- **Not so fast:** a chemical potential couples to the *gauged* density. But there is a conserved global current corresponding to the *vortices* of a_μ:

$$J^\mu = \frac{1}{2\pi}\varepsilon^{\mu\nu\lambda}\partial_\nu a_\lambda$$

- Disorder coupling to global currents *cannot be screened.* Consider random charge density (also breaks **PH**),

$$\frac{1}{2\pi}\varepsilon^{i\nu\lambda}\mathcal{A}_i(\mathbf{x})\partial_\nu a_\lambda \sim a_0\rho(\mathbf{x})$$

[HG, Mulligan, Raghu, Torroba, and Zimet, PRB (2017)], [Thomson and Sachdev, PRB (2017)]

Random mass: a dirty-interacting metallic phase

- Now consider disorder coupling to the mass operator

$$M(\mathbf{x})\bar{\psi}\sigma^z\psi, \quad \overline{M(\mathbf{0})M(\mathbf{x})} \sim \Delta\delta(\mathbf{x}) \quad (\chi_0 \to 2)$$

$$\bar{\psi}\sigma^z\psi = \sum_{I=1}^{N_f}(\bar{\psi}_I\psi_I - \bar{\psi}_{I+N_f}\psi_{I+N_f})$$

- Preserves **PH** and **T**, breaks the flavor/valley symmetry, $SU(2N_f) \to SU(N_f) \times SU(N_f) \times U(1)$

Random mass:
a dirty-interacting metallic phase

- **Can these effects compete?** Lead to a dirty metallic phase?
- For various reasons, we used a special type of ϵ-expansion (large-N_f also works), still use the replica trick

$$d = 3 - \epsilon \quad \longleftarrow \quad \text{spatial dimensions}$$

$$\overline{M(\mathbf{0})M(\mathbf{x})} \sim \frac{\Delta}{|\mathbf{x}|^2} \quad \longleftarrow \quad \text{disorder correlations fixed}$$

- broadly applicable type of expansion, may capture small-N_f behavior well.
- Running couplings are $\bar{\alpha}$, $\bar{\Delta}$, and the velocity, v.

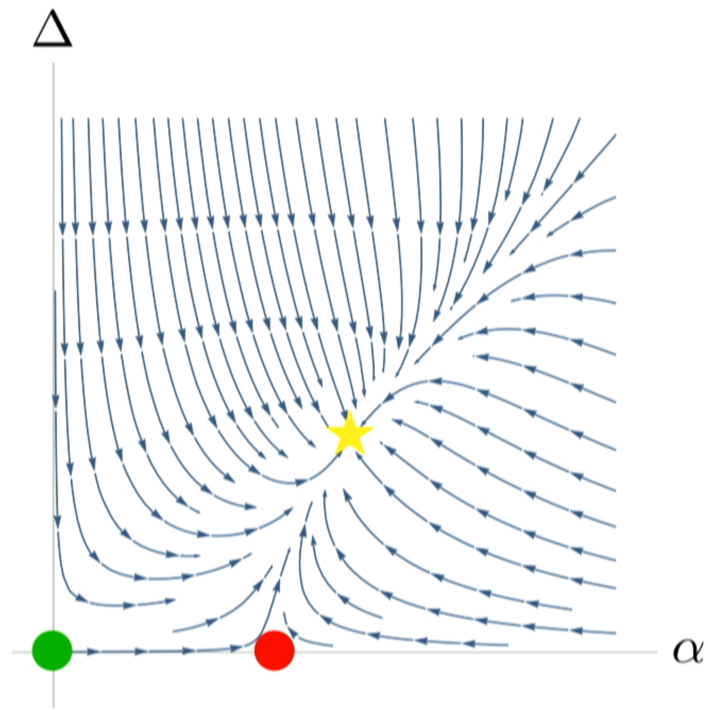
Random mass: a dirty-interacting metallic phase

- RG equations are

$$z = 1 + \frac{1}{3}\bar{\alpha}\left(1 - v^2\right)$$
$$\frac{dv}{d\ell} = v\left[-\frac{2}{3}\bar{\Delta} - \frac{\bar{\alpha}}{N}g_1(v) + \frac{1}{3}\bar{\alpha}\left(1 - v^2\right)\right]$$
$$\frac{d\bar{\alpha}}{d\ell} = \bar{\alpha}\left[\epsilon + \frac{2}{3}\bar{\Delta} - \frac{2}{3}\bar{\alpha} + \frac{\bar{\alpha}}{N}g_1(v)\right]$$
$$\frac{d\bar{\Delta}}{d\ell} = \frac{2\bar{\alpha}}{N}g_2(v)\Delta - \frac{8}{3}\bar{\Delta}^2$$

- **Non-trivial stable fixed point** (linearized in ϵ, N_f)

$$z_* = 1 + \frac{9\epsilon}{8N} \qquad \bar{\Delta}_* = \frac{27\epsilon}{16N}$$
$$\bar{\alpha}_* = \frac{3\epsilon}{2} \qquad v_* = 1 - \frac{9}{8N}$$



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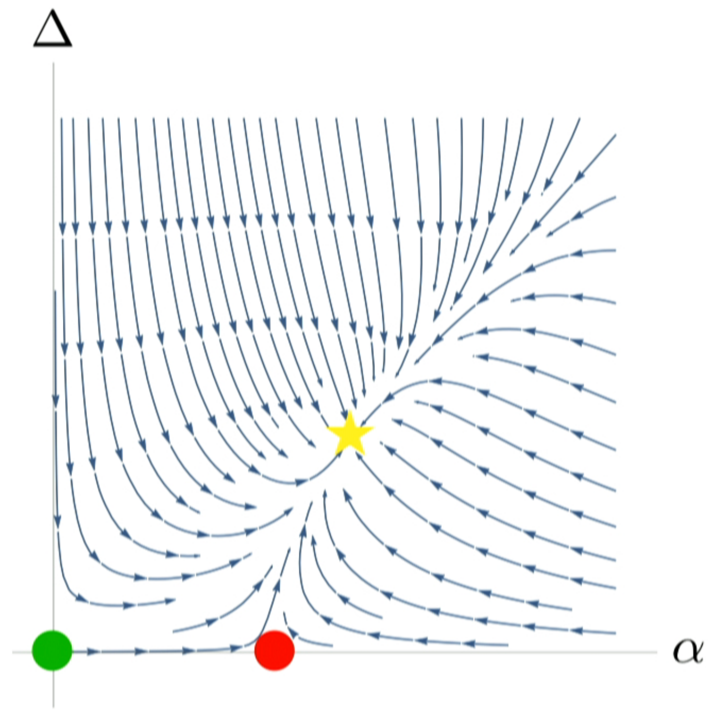

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Universal features

- Dynamical exponent $1 < z < 2$ (for large enough N_f), neither (quantum) diffusive nor conventionally ballistic!
- **Means DoS and compressibility continue to vanish** as $T \to 0$, $\rho(T) \sim T^{(d-z)/z}$
- **Expect a finite, universal DC conductivity,** crude Drude estimate:

inelastic scattering rate: $\frac{1}{\tau} \sim (a_1 \bar{\Delta}_* + a_2 \bar{\alpha}_*) T$

$$\sigma_{xx}\left(\frac{\omega}{T} \to 0\right) \sim \frac{ne^2}{m}\tau \sim \frac{1}{a_1 \bar{\Delta}_* + a_2 \bar{\alpha}_*} \qquad \begin{array}{c} m \sim T^{(2-z)/z} \\ n \sim T^{d/z} \end{array}$$

- Precise calculation requires studying the polarization tensor at finite temperature — quite hard!
- **First example of a dirty, interacting 2d quantum critical phase!**

What have we learned?

- Introducing a fluctuating gauge field **dramatically** changes the behavior of Dirac fermions in the presence of disorder.
- While before **T**-invariant disorder generically led to a diffusive regime with finite DoS, **this fate is arrested in QED_3**, leading to new types of quantum critical phases with vanishing DoS.
- **Intuition from particle-vortex duality**: introduction of the gauge field has exchanged roles of electric and magnetic impurities [HG, M. Mulligan, S. Raghu, G. Torroba, M. Zimet, PRB (2017)].

Lots of future directions

- **Holy grail:** Fermi surfaces + gauge fields + dirt, possibility of using ϵ-expansions like the one used for QED_3 + random mass.
- **DC transport** at the dirty boson fixed point, going to finite temperature. Ongoing work with P. Nosov and S. Raghu.
- Non-perturbative effects, multifractality, **rare events**.
- **New experiments:** anomalous metals near the quantum spin Hall -> SC transition. [Sajadi *et al.*, Science (2018)]
- Fermions + gauge fields + relevant disorder (random flux)? Generalization of the NLσM framework in large-N? Ongoing work with A. Thomson and P. Kumar.

Thank you!

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Thank you!