Title: Introduction Causal Sets

Speakers: Sumati Surya

Collection: Everpresent Lambda: Theory Meets Observations

Date: November 11, 2019 - 11:00 AM

URL: http://pirsa.org/19110064

# An Introduction to Causal Sets

Sumati Surya

Raman Research Institute

Everpresent Λ Meeting

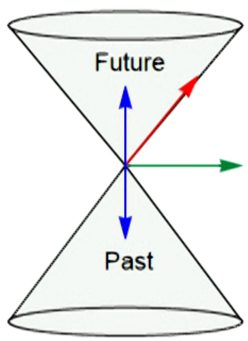
Perimeter Institute, November 2019

# Outline

- The Causal Structure Poset and the HKMM theorem
- The Causal Set Hypothesis : Spacetime $\rightarrow$ locally finite poset
- The Continuum Approximation or Causal Set Kinematics
- The Quantum Dynamics of Causal Sets
- Causal Set Phenomenology: $\Lambda$, Swerves, Non-local field theory, etc

# The causal structure poset $(M, \prec)$

- $(M, g)$ has local lightcones $\Rightarrow$ Local Causality:
  - $\prec$: causality relation (causal, $J^{\pm}(x)$)
  - $\prec\!\!\prec$: chronology relation (timelike, $I^{\pm}(x)$)
  - $\rightarrow$: horismos relation (null, $J^{\pm}(x) \backslash I^{\pm}(x)$)

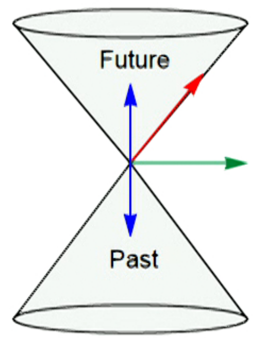


# The causal structure poset $(M, \prec)$

- $(M, g)$ has local lightcones $\Rightarrow$ Local Causality:
  - $\prec$: causality relation (causal, $J^{\pm}(x)$)
  - $\prec\!\!\prec$: chronology relation (timelike, $I^{\pm}(x)$)
  - $\rightarrow$: horismos relation (null, $J^{\pm}(x) \backslash I^{\pm}(x)$)
- In any causal spacetime $(M, \prec)$ is a poset.
  - $M$: the **set** of events.
  - $\prec$ : causal relation
    - Acyclic: $x \prec y$ and $y \prec x \Rightarrow x = y$
    - Transitive: $x \prec y$ and $y \prec z \;\Rightarrow x \prec z$

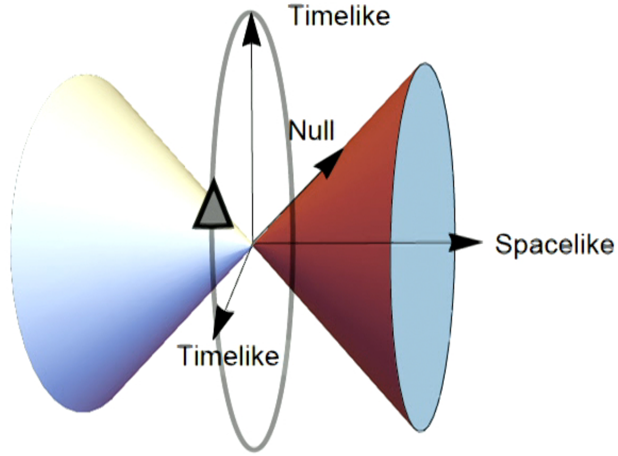
# How primitive is $(M, \prec)$ ? – Zeeman, Finkelstein, Penrose, Kronheimer, Hawking, Geroch, Ellis, Malament, Myrheim etc..

- Causal Structure remains invariant under conformal rescaling: $\tilde{g}_{ab} = \Omega^2 g_{ab}$



## How primitive is $(M, \prec)$ ? – Zeeman, Finkelstein, Penrose, Kronheimer, Hawking, Geroch, Ellis, Malament, Myrheim etc..

- Causal Structure remains invariant under conformal rescaling: $\tilde{g}_{ab} = \Omega^2 g_{ab}$
- Only spaces with signature $(-, +, +, \ldots, +)$ can have a causal structure poset



# Spacetime geometry from $(M, \prec)$

- For Minkowski spacetime, group of chronological automorphisms is isomorphic to the group of inhomogeneous Lorentz transformations and dilations.

  **Zeeman, 1964**

- The Hawking-King-McCarthy-Malament Theorem:

  *Let $f : (M_1, g_1) \to (M_2, g_2)$ be a causal bijection between two future and past distinguishing spacetimes, i.e., $x_1 \prec_1 y_1 \Leftrightarrow f(x_1) \prec_2 f(x_2)$. Then $f$ is a smooth conformal isometry: $f$ and $f^{-1}$ are smooth and $f_* g_1 = \Omega^2 g_2$.*

  **S. W. Hawking, A.R. King, P.J. McCarthy (1976); D. Malament (1977)**

## Order is most of geometry

- $(M, \prec)$ contains all but one of the $n(n+1)/2$ independent components of $(M, g)$
- "Causal structure is $9/10^{\rm th}$ of the spacetime geometry." Finkelstein (1969)
- Remaining $1/10^{\rm th}$ is the volume element

$$\epsilon = \Omega^n \times \sqrt{g} dx^1 \wedge \ldots \wedge dx^n$$

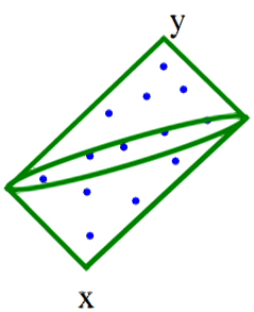
**Spacetime geometry = Causal Structure + Volume element**

# The Causal Set Hypothesis

Myrheim (1978), L.Bombelli, J.Lee, D. Meyer and R. Sorkin, (1987)

- The Causal Structure Poset: $(M, \prec) \subset (M, g)$
- Spacetime Discreteness: $N \sim V/V_p$

  *Finite spacetime volume contains finite number of spacetime "atoms"*



## Order is most of geometry

- $(M, \prec)$ contains all but one of the $n(n+1)/2$ independent components of $(M, g)$
- "Causal structure is $9/10^{\text{th}}$ of the spacetime geometry." Finkelstein (1969)
- Remaining $1/10^{\text{th}}$ is the volume element

$$\epsilon = \Omega^n \times \sqrt{g} dx^1 \wedge \ldots \wedge dx^n$$

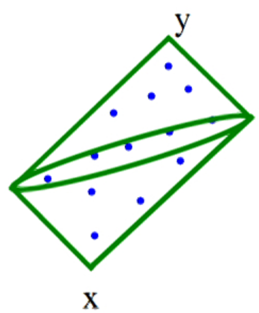
**Spacetime geometry = Causal Structure + Volume element**

# The Causal Set Hypothesis

Myrheim (1978), L.Bombelli, J.Lee, D. Meyer and R. Sorkin, (1987)

- The Causal Structure Poset: $(M, \prec) \subset (M, g)$ **Order**
- Spacetime Discreteness: $N \sim V/V_p$ **Number**

  *Finite spacetime volume contains finite number of spacetime "atoms"*



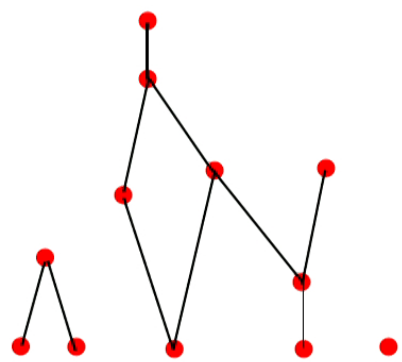

**Order + Number $\sim$ Spacetime Geometry**

# The Causal Set Hypothesis

Myrheim (1978), L.Bombelli, J.Lee, D. Meyer and R. Sorkin, (1987)

The underlying structure of spacetime is a *causal set* or locally finite poset $(C, \prec)$

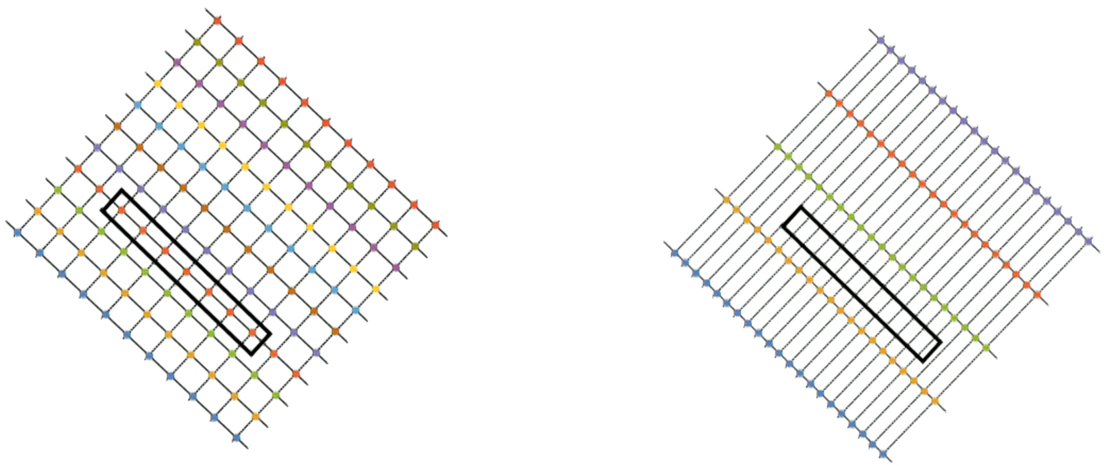
- Acyclic: $x \prec y$ and $y \prec x \Rightarrow x = y$
- Transitive: $x \prec y$ and $y \prec z \;\Rightarrow x \prec z$
- Locally finite: $|Fut(x) \cap Past(y)| < \infty$



## The Continuum Approximation

**Order + Number ∼ Spacetime Geometry**

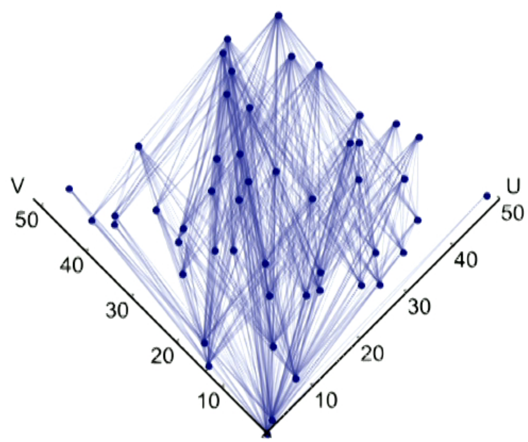
- Causal Order ⇔ Order
- Number ⇔ Volume



# The Continuum Approximation

**Order + Number ∼ Spacetime Geometry**

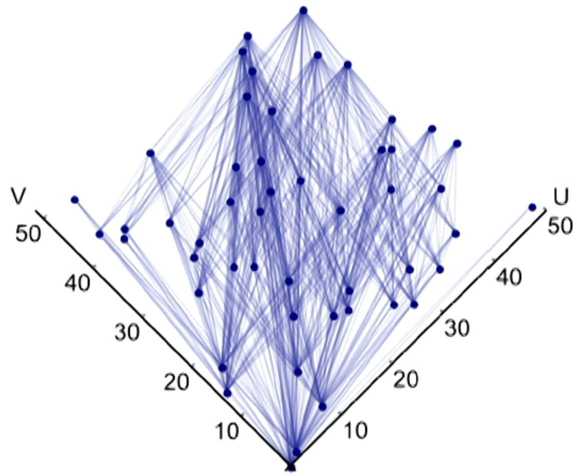
- Causal Order ⇔ Order
- Number ⇔ Volume
- Random Lattice via a Poisson process: $\mathbf{P_V(n)} \equiv \frac{1}{n!} \exp^{-\rho \mathbf{V}} (\rho \mathbf{V})^{\mathbf{n}}$, $\langle \mathbf{n} \rangle = \rho \mathbf{V}$



# Riemann's dilemma

- A discrete manifold has finite properties, whereas a continuous manifold does not. Natural quantities are to be finite. The world must be discrete.
- A discrete manifold possesses natural internal metrical structure, whereas a continuous manifold must have its metrical structure imposed from without. Natural law is to be unified. The world must be discrete.
- A continuous manifold has continuous symmetries, whereas a discrete manifold does not. Nature possesses continuous symmetries. The world must be continuous.

–from Finkelstein(1969)

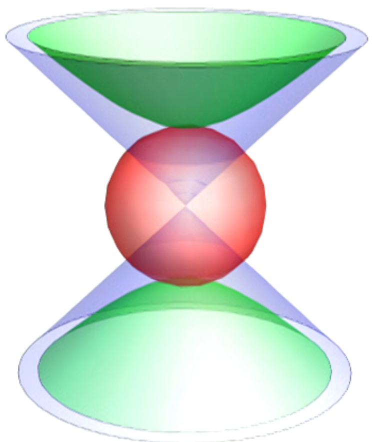



Does $C \sim \mathbb{M}^n$ violate Lorentz invariance?

# Discreteness without Lorentz violation

– L.Bombelli, J.Henson, R. Sorkin 2009, Dowker and Sorkin, 2019

- Space of all sprinklings into $\mathbb{M}^n$ : $\Omega$
- Set of all timelike directions: unit hyperboloid $H \subset \mathbb{M}^n$



# Discreteness without Lorentz violation

– L.Bombelli, J.Henson, R. Sorkin 2009, Dowker and Sorkin, 2019

- Space of all sprinklings into $\mathbb{M}^n$ : $\Omega$
- Set of all timelike directions: unit hyperboloid $H \subset \mathbb{M}^n$
- Is there a way to assign a direction $D : \Omega \to H$ consistently?
- Consistency $\Rightarrow$ under a boost $\Lambda$, $D \circ \Lambda = \Lambda \circ D$ (equivariance)
- Poisson process gives a measure $\mu$ on $\Omega$ which is volume preserving and hence Lorentz invariant. : $\mu = \mu \circ \Lambda$

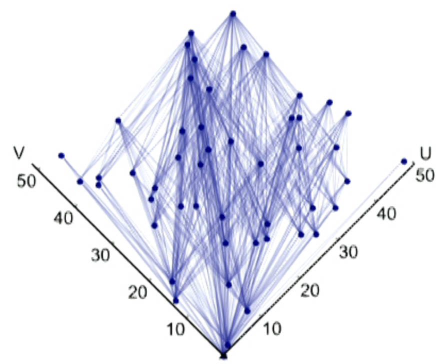
# Discreteness without Lorentz violation

– L.Bombelli, J.Henson, R. Sorkin 2009, Dowker and Sorkin, 2019

**Theorem:** There is no measurable map $D : \Omega \to H$ which is equivariant, i.e., $D \circ \Lambda = \Lambda \circ D$.

**Proof:** If such a map existed, then $\mu_D \equiv \mu \circ D^{-1}$ is a Lorentz invariant probability measure on $H$ which is not possible since $H$ is non-compact.

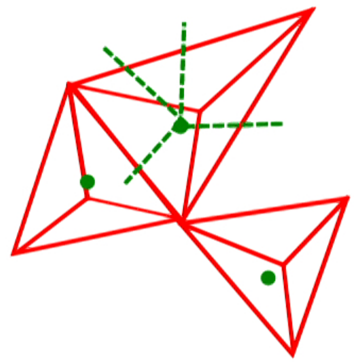
# Discreteness + Lorentz invariance ⇒ Non-locality




- A causal set need not be a fixed valency graph.

# Discreteness + Lorentz invariance ⇒ Non-locality

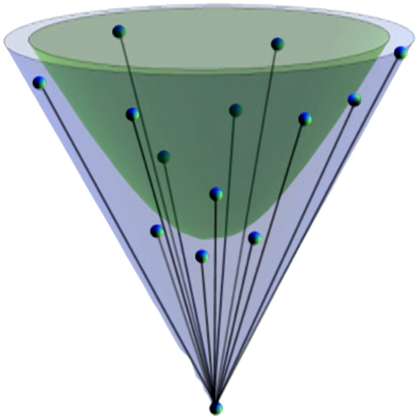
- A causal set need not be a fixed valency graph.
  - Other discretisations lead to finite/fixed valency graphs



  - In $C \sim (M, g)$, number of nearest neighbours is not fixed ($\to \infty$ in $\mathbb{M}^n$)

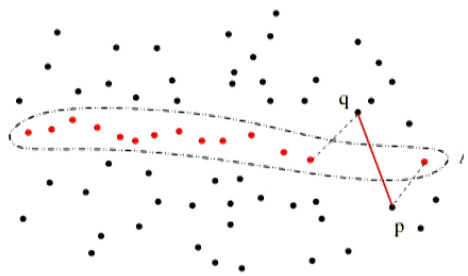
# Discreteness + Lorentz invariance ⇒ Non-locality

- A causal set need not be a fixed valency graph.
  - Other discretisations lead to finite/fixed valency graphs
  - In $C \sim (M, g)$, number of nearest neighbours is not fixed ($\to \infty$ in $\mathbb{M}^n$)



# Discreteness + Lorentz invariance ⇒ Non-locality

- A causal set need not be a fixed valency graph.
- No Cauchy evolution

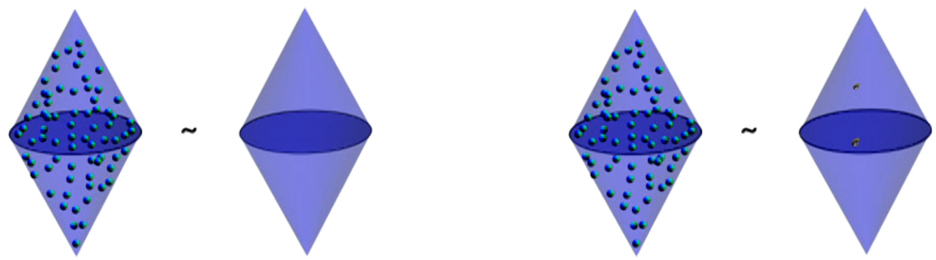



- $C$ does not admit a natural $(d-1)+1$ split into space and time.
- Initial value formulation can only be emergent: no fundamental "local" dynamics.
- Sum over Histories formulation more suitable than Hamiltonian formulation.

# The Fundamental Conjecture of CST

**Order + Number ∼ Spacetime Geometry**

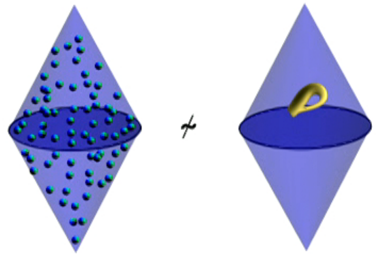
- $C \sim (M_1, g_1),\ C \sim (M_2, g_2) \Rightarrow (M_1, g_1) \sim (M_2, g_2).$



# The Fundamental Conjecture of CST

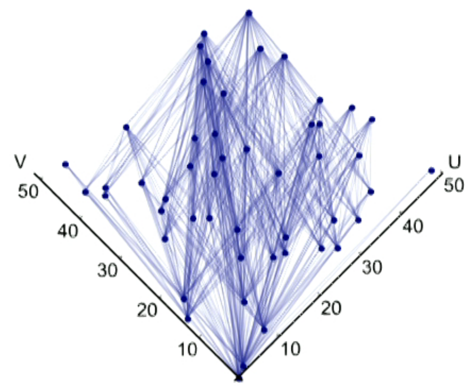
Order + Number ∼ Spacetime Geometry

- $C \sim (M_1, g_1)$, $C \sim (M_2, g_2) \Rightarrow (M_1, g_1) \sim (M_2, g_2)$.



# Geometric Reconstruction/Covariant Observables

When does a causal set look like a spacetime?



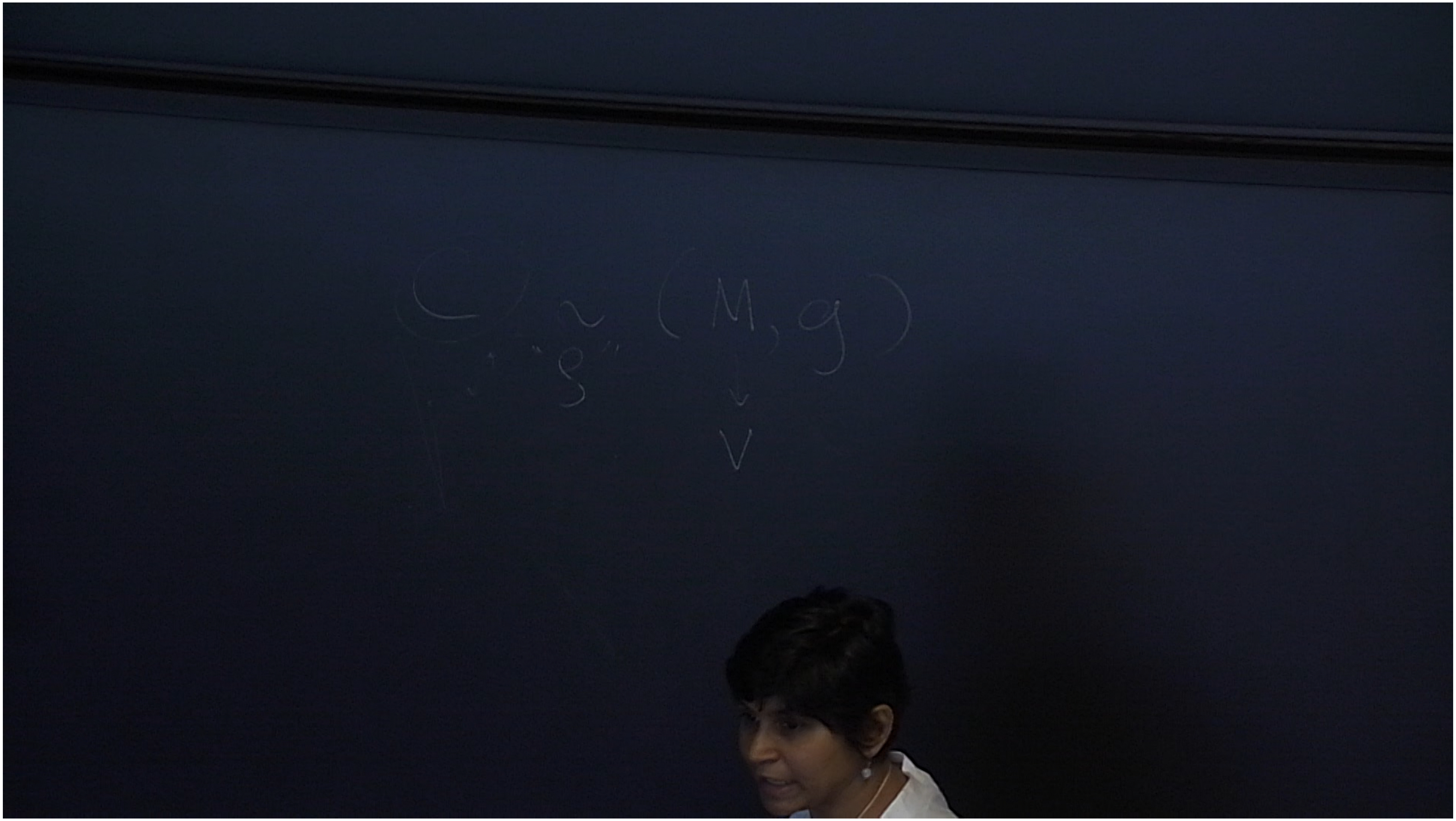

**Discrete Order $\sim$ Geometry**

# Geometric Reconstruction/Covariant Observables

- Dimension Estimators — Myrheim, Myer ..
- Timelike Distance –Brightwell & Gregory
- Spatial Homology –Major, Rideout & Surya
- Spatial and Spacelike Distance –Rideout & Wallden

  –Eichhorn, Mizera & Surya, Eichhorn, Surya & Versteegen
- D'Alembertian –Sorkin, Henson, Benincasa & Dowker, Dowker & Glaser
- Benincasa-Dowker Action –Benincasa & Dowker, Dowker & Glaser
- GHY terms in the Action – Buck, Dowker, Jubb & Surya
- Recovering Locality –Glaser & Surya
- Scalar Field Greens functions –Johnston, Dowker, Surya & Nomaan X

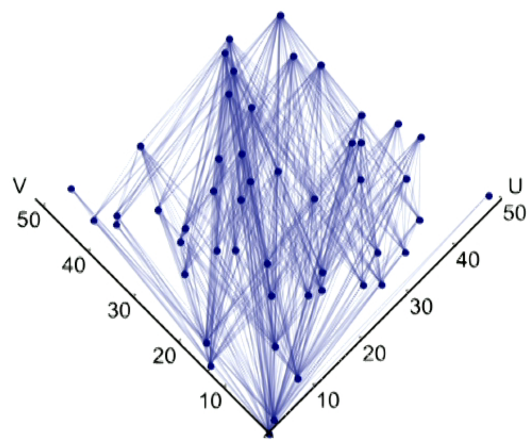
SS, Living Reviews in Relativity (2019)


(M, g)
V

# The Continuum Approximation

**Order + Number ∼ Spacetime Geometry**

- Causal Order ⇔ Order
- Number ⇔ Volume
- Random Lattice via a Poisson process: $\mathbf{P_V(n) \equiv \frac{1}{n!}\exp^{-\rho V}(\rho V)^n}$, $\langle \mathbf{n} \rangle = \rho \mathbf{V}$

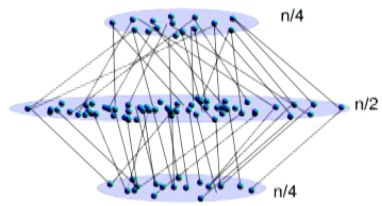


# The CST Postulate

- Spacetime is replaced by locally finite posets or causal sets

$$Z = \int Dg \exp[iS[g]/\hbar] \quad \rightarrow \quad Z = \sum_{C \in \Omega} \mu(C) \tag{1}$$

- What does a typical causal set in $\Omega_n$ look like? $|\Omega_n| \sim 2^{n^2/4}$

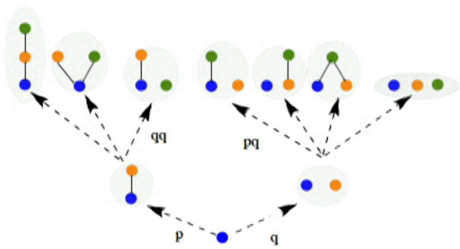



–Kleitman and Rothschild, Trans AMS, 1975

- $\mu(C)$ is a quantum measure. – R. Sorkin 2004

# Two Approaches to Dynamics

- Sequential Growth

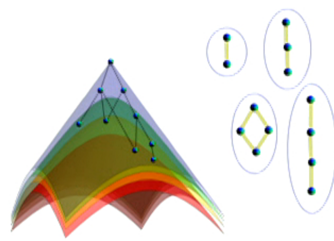



- Causality
- Covariance or Label Invariance
- “Bell causality” or spectator independence
- Markovian evolution

# Two Approaches to Dynamics

- Sequential Growth
- Continuum inspired dynamics: $Z = \sum_{C \in \Omega} \exp(iS[c]/\hbar)$
  - Benincasa Dowker Action: $\frac{1}{\hbar} S_\epsilon(c) = 4\epsilon \left( N - 2\epsilon \sum_{n=0}^{N-2} N_n \, f(n,\epsilon) \right)$

    –Benincasa and Dowker, Dowker and Glaser
    - Weighted sum over number of neighbour pairs, next to neighbour pairs, etc.



- Mesoscale $l_k >> l_p$ , $\epsilon = \left( \frac{l_p}{l_k} \right)^2 \in (0,1]$,
  $f(n,\epsilon) = (1-\epsilon)^n - 2\epsilon n (1-\epsilon)^{n-1} + \frac{1}{2}\epsilon^2 n(n-1)(1-\epsilon)^{n-2}$

# Where are we?

- Quantum Sequential Growth models: Does the quantum measure exist?

  –Dowker, Johnston and Surya 2010, Sorkin 2011, Surya and Zalel, in preparation

- Stationary phase approximations for $Z = \sum_C \exp(iS(C)/\hbar)$ –Carlip and Loomis, 2017

- Analytic continuation $Z = \sum_C \exp(i\beta S(C)/\hbar) \rightarrow Z = \sum_C \exp(-\beta S(C)/\hbar)$

  –Surya 2011, Glaser and Surya 2015, Glaser, O'Connor and Surya, 2017, Cunningham and Surya, 2019

## Where are we?

- Observables in the theory are order invariants (includes all the geometric invariants like dimension, action, etc.)
- But..
  - If the path is clear, the technical hurdles are fairly great (calculability in sequential growth models)
  - If there is calculability, there is conceptual difficulty (analytic continuation)

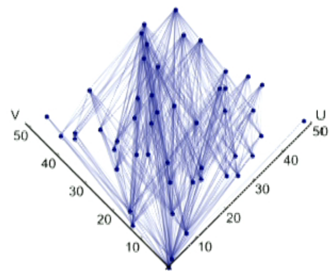
Ultimately quantum gravity needs a quantum theory of closed systems

–Sorkin, 2007 and others..

# What is Causal Set Phenomenology?

- Does the continuum emerge from causal set quantum gravity ??

- The continuum approximation itself is distinct and "quantum"




- Characteristic features: Discreteness, Lorentz invariance, non-locality..

# Examples of Causal Set Phenomenology

- Sorkin's 1987 prediction for $\Delta\Lambda$

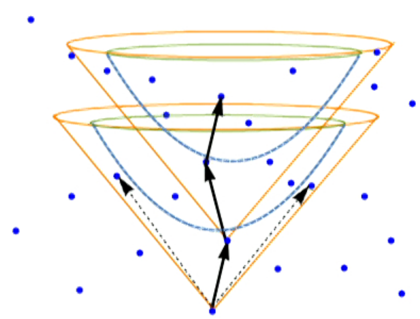
  Sorkin (1987), Sorkin (1997), Ahmed, Dodelson, Greene, Sorkin (2004), ..

- Swerves

  Dowker, Henson and Sorkin (2003), Dowker, Philpott and Sorkin (2009), Mattingly and Kaloper(2006)

  - Particles hopping on the causal set
  - No straight lines: particles must "swerve just a little"
  - Momentum Diffusion in $\mathbb{M}^4 \times \mathbb{H}^3$:

$$\frac{\partial}{\partial \tau}\rho(\mathbf{x}^\mu, \mathbf{p}^\nu) = \mathbf{k}\nabla^2_{\mathbf{p}}\rho(\mathbf{x}^\mu, \mathbf{p}^\nu) - \frac{1}{\mathbf{mc}^2}\mathbf{p}^\mu \partial_\mu \rho(\mathbf{x}^\mu, \mathbf{p}^\nu)$$



Causal Set Phenomenology is rich – we are just seeing the tip of the iceberg..