Title: Relativity, Particle localizability, and Entanglement

Speakers: Jason Pye

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Abstract: Can a relativistic quantum field theory be consistently described as a theory of localizable particles? There are many well-known obstructions to such a description. Here, we trace exactly how such obstructions arise in the regime between nonrelativistic quantum mechanics and relativistic quantum field theory. Perhaps unexpectedly, we find that in the nonrelativistic limit of QFT, there are persisting issues with the localizability of particle states. Related via the Reeh-Schlieder theorem, we also show that the fate of ground state entanglement and the Unruh effect is nontrivial in the nonrelativistic limit.

Relativity, Particle localizability and Entanglement

Jason Pye

PI Quantum Foundations Seminar
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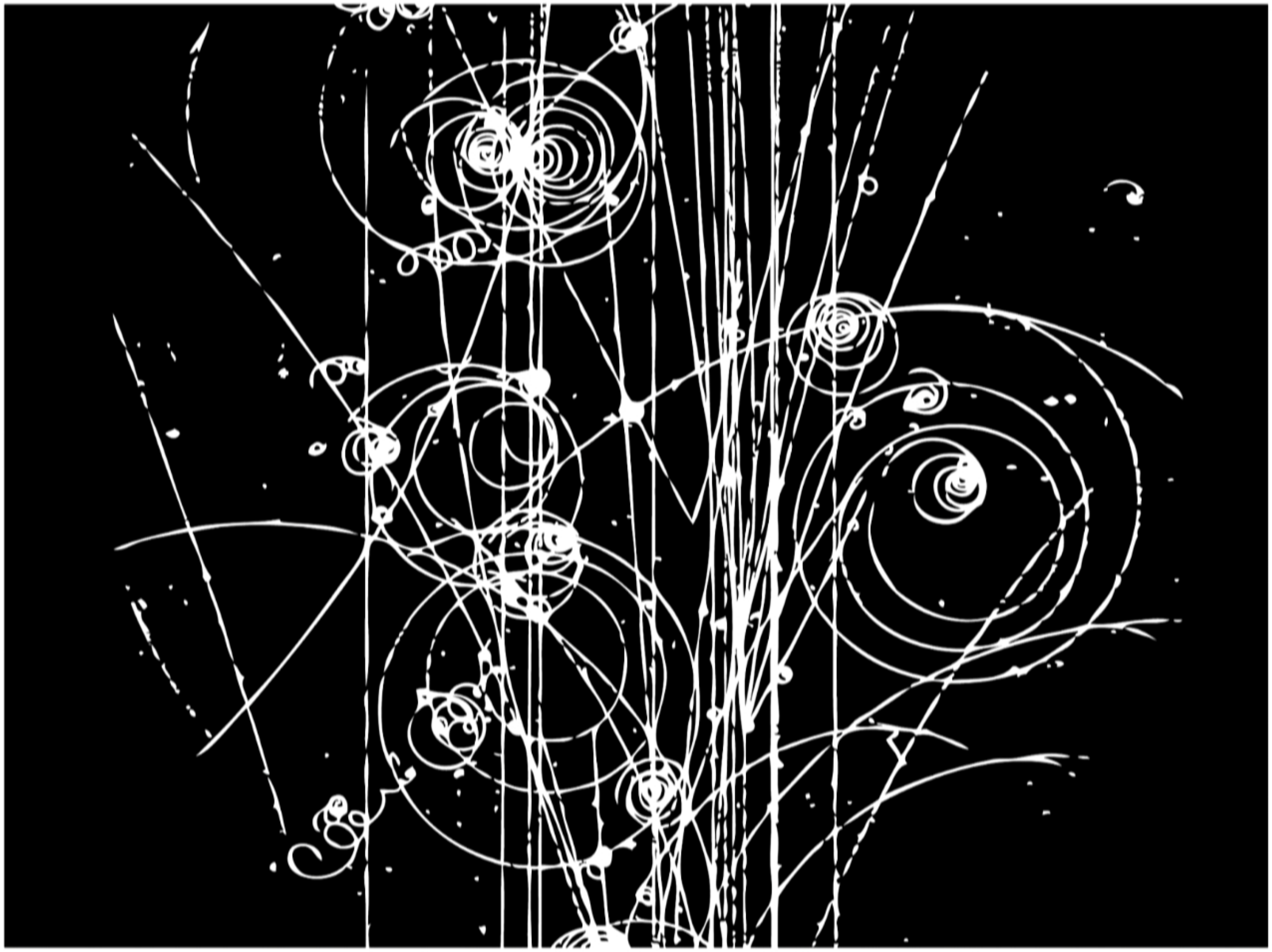
M. Papageorgiou, JP (2019) *J. Phys. A: Math. Theor.* **52** 375304

UNIVERSITY OF WATERLOO



What is a particle?

→ Entity which is **localizable**

Other considerations:

- Can be counted $\mathcal{F}[\mathcal{H}] = \mathbb{C} \oplus \mathcal{H} \oplus (\mathcal{H}^{\otimes 2})_{S,A} \oplus \cdots$
  - observable number operator, Fock space
- Relativistic dispersion relation $E = \sqrt{\mathbf{p}^2 + m^2}$
- …

Where is a particle?

- Operators are local

$$d(x,y) \text{ spacelike} \implies [\phi(x), \phi(y)] = 0$$

- But particles are represented by states!

$$\mathcal{F}[\mathcal{H}] = \mathbb{C} \oplus \mathcal{H} \oplus (\mathcal{H}^{\otimes 2})_{S,A} \oplus \cdots$$

→ Can we find states that describe localizable particles?

How can we characterize particle localizability in QFT?

Position operator? Local number operator?

Position operator in QFT?

- Analogue of $|\psi(x)|^2$, $X = \int dx\, x|x\rangle\langle x|$?
- (No-go) Malament theorem

Hilbert space $\mathcal{H}$, Projs. $\Delta \to P_\Delta$, Transl. rep. $x \to U(x)$

(1) Translation covariance: $P_{\Delta+x} = U(x)P_\Delta U(-x)$

(2) Energy condition: time transl $U(x) = e^{-itH(x)}$ s.t. $H(x) > 0$

(3) Localizability: Δ_1, Δ_2 disjoint $\implies P_{\Delta_1}P_{\Delta_2} = P_{\Delta_2}P_{\Delta_1} = 0$

(4) Locality: Δ_1, Δ_2 spacelike $\implies P_{\Delta_1}P_{\Delta_2} = P_{\Delta_2}P_{\Delta_1}$

$\implies P_\Delta = 0$

[1] D. Malament, (1996) in *Perspectives on Quantum Reality* (ed. R. Clifton)

Local number operator?

- Count particles in finite volume? $N_L = \int_V d\mathbf{x} a^\dagger_{\mathbf{x}} a_{\mathbf{x}}$

- (No-go) Reeh-Schlieder (1961)

  In AQFT, $\mathcal{O} \to \mathcal{A}(\mathcal{O})$

  e.g., generated by $\phi(f) = \int dx f(x)\phi(x)$

  <u>cor.</u> $\text{If } A \in \mathcal{A}(\mathcal{O}) \text{ and } A|0\rangle_G = 0, \text{ then } A = 0.$

  $\therefore a_{\mathbf{x}}|0\rangle_G = 0 \implies a_{\mathbf{x}} = 0, a^\dagger_{\mathbf{x}} = 0$

Concrete attempts at localization

- How do obstructions appear in a concrete scenario?
- Schemes for attempting localization for free KG field

  1) Fourier transform of standard Fock states

$$H = \int \frac{d\mathbf{k}}{(2\pi)^n} \hbar\omega_{\mathbf{k}} a_{\mathbf{k}}^{\dagger} a_{\mathbf{k}}$$

$$a_{\mathbf{x}} = \int \frac{d\mathbf{k}}{(2\pi)^n} e^{i\mathbf{k}\cdot\mathbf{x}} a_{\mathbf{k}}$$

$$|\psi\rangle = \int d\mathbf{x}\, \psi(\mathbf{x}) a_{\mathbf{x}}^{\dagger} |0\rangle_G$$

Alternative schemes?

2) Local harmonic oscillators

$$c^{-2}\partial_t^2\phi(t,\mathbf{x}) - \nabla^2\phi(t,\mathbf{x}) + (\tfrac{mc}{\hbar})^2\phi(t,\mathbf{x}) = 0$$

Generate Fock space using:

$$b_{\mathbf{x}} := \sqrt{\frac{m}{2\hbar^2}}\phi(\mathbf{x}) + \frac{i}{\sqrt{2m}}\pi(\mathbf{x})$$

$$H = \int d\mathbf{x}\left[mc^2 b^\dagger_{\mathbf{x}} b_{\mathbf{x}} - \frac{\hbar^2}{4m}(b_{\mathbf{x}} + b^\dagger_{\mathbf{x}})\nabla^2(b_{\mathbf{x}} + b^\dagger_{\mathbf{x}})\right]$$

What does "local" particle number in these schemes represent?

$$N_1 = \int d\mathbf{x}\ a_{\mathbf{x}}^{\dagger} a_{\mathbf{x}} \qquad\qquad N_2 = \int d\mathbf{x}\ b_{\mathbf{x}}^{\dagger} b_{\mathbf{x}}$$

Attempts at local number operators

- How do localizability obstructions appear for number operators in these two schemes?

  1) Fourier transformed: $a_{\mathbf{x}} = \int \frac{d\mathbf{k}}{(2\pi)^n} e^{i\mathbf{k}\cdot\mathbf{x}} a_{\mathbf{k}}$

  - Non-locally related to field operators

$$a_{\mathbf{y}} = \int d\mathbf{x}\, [F_+(\mathbf{y}-\mathbf{x}) b_{\mathbf{x}} + F_-(\mathbf{y}-\mathbf{x}) b^\dagger_{\mathbf{x}}]$$

$F_\pm \sim e^{-k_c|\mathbf{y}-\mathbf{x}|}$, recall: $b_{\mathbf{x}} := \sqrt{\frac{m}{2\hbar^2}}\phi(\mathbf{x}) + \frac{i}{\sqrt{2m}}\pi(\mathbf{x})$

$\Longrightarrow |\psi\rangle = \int d\mathbf{x}\, \psi(\mathbf{x}) a^\dagger_{\mathbf{x}} |0\rangle_G$ is non-local

Attempts at local number operators

- How do localizability obstructions appear for number operators in these two schemes?

  2) Local oscillators: $b_{\mathbf{x}} := \sqrt{\frac{m}{2\hbar^2}}\phi(\mathbf{x}) + \frac{i}{\sqrt{2m}}\pi(\mathbf{x})$

$$a_{\mathbf{y}} = \int d\mathbf{x}\,[F_+(\mathbf{y}-\mathbf{x})b_{\mathbf{x}} + F_-(\mathbf{y}-\mathbf{x})b^\dagger_{\mathbf{x}}]$$

  - $a_{\mathbf{x}}, b_{\mathbf{x}}$ generate different Fock spaces: $b_{\mathbf{x}}|0\rangle_G \neq 0$
  - Representations are unitarily inequivalent

$$\mathrm{tr}(\beta\beta^\dagger) = \int d\mathbf{x}d\mathbf{y}|F_-(\mathbf{x}-\mathbf{y})|^2 = \infty$$

1) $a_{\mathbf{x}} = \int \frac{d\mathbf{k}}{(2\pi)^n} e^{i\mathbf{k}\cdot\mathbf{x}} a_{\mathbf{k}}$	2) $b_{\mathbf{x}} := \sqrt{\frac{m}{2\hbar^2}}\phi(\mathbf{x}) + \frac{i}{\sqrt{2m}}\pi(\mathbf{x})$
✗ Non-local in space $a_{\mathbf{y}} = \int d\mathbf{x}\, [F_+(\mathbf{y}-\mathbf{x}) b_{\mathbf{x}} + F_-(\mathbf{y}-\mathbf{x}) b^\dagger_{\mathbf{x}}]$	✓ Label dofs in space
✓ $[H, N] = 0$	✗ Not preserved in time
✓ $E = \sqrt{\mathbf{p}^2 + m^2}$	✗ Dispersion relation

Impact of special relativity?

Relativistic QFT

- ✗ Position operator
- ✗ Local number op

Non-rel. approx $v \ll c$ — Fix particle number, N

$\longleftrightarrow$

NR-QFT $\approx$ NRQM

Non-relativistic QM

- ✓ Position operator
- ✓ Local number op

<u>Concretely</u>: How does the tension between the two schemes for attempted localization subside?

How to take non-relativistic approx?

- "$c \to \infty$" : $|\mathbf{v}| \ll c$ gives $|\mathbf{k}| \ll k_c := mc/\hbar$

$$\frac{|\mathbf{v}|}{c} = \frac{|\mathbf{p}|}{mc} = \frac{\hbar|\mathbf{k}|}{mc} \ll 1$$

- hard cutoff $|\mathbf{k}| \leq \Lambda \ll k_c$, expand up to $(\Lambda/k_c)^2$

  since

$$E = \sqrt{\mathbf{p}^2 c^2 + m^2 c^4} \approx mc^2 + \frac{\mathbf{p}^2}{2m}$$

How are schemes related after NR approx?

- Does the Bogoliubov transformation become the identity?
- No! Remaining discrepancy between schemes

$$a_{\mathbf{y}} = b_{\mathbf{y}} + \int d\mathbf{x}\; F_{-}^{(2)}(\mathbf{y}-\mathbf{x})b_{\mathbf{x}}^{\dagger}$$

  Also remain unitarily inequivalent

- Fully recovering particle localizability in NRQM involves more than just removing (special) relativistic features.

How to recover non-relativistic QM?

- Use $a_{\mathbf{y}}, a^\dagger_{\mathbf{y}}$ to recover NRQM

$$|\Psi(t)\rangle := \frac{1}{\sqrt{N!}} \int d\mathbf{y}_1 \cdots d\mathbf{y}_N \Psi(\mathbf{y}_1, \ldots, \mathbf{y}_N; t) a^\dagger_{\mathbf{y}_1} \cdots a^\dagger_{\mathbf{y}_N} |0\rangle_G$$

$$i\hbar \partial_t \Psi(\mathbf{y}_1, \ldots, \mathbf{y}_N; t) = \left(E_0 + mc^2 N - \frac{\hbar^2}{2m} \sum_{i=1}^{N} \nabla_i^2 \right) \Psi(\mathbf{y}_1, \ldots, \mathbf{y}_N; t)$$

$$\mathbf{X} = \frac{1}{N} \int d\mathbf{y} \mathbf{y} a^\dagger_{\mathbf{y}} a_{\mathbf{y}}, \qquad [X_i, P_j] = i\hbar \delta_{ij}$$

How to bridge the gap?

- Local dofs in QFT (even after NR approx) vs. localization in recovered NRQM
- Foldy-Wouthuysen trsf implicit in Bjorken & Drell (1964)

$$a_{\mathbf{y}} = U^\dagger b_{\mathbf{y}} U = b_{\mathbf{y}} + \int d\mathbf{x}\, F_{-}^{(2)}(\mathbf{y}-\mathbf{x}) b_{\mathbf{x}}^\dagger$$

(note: unitary inequivalence)

- Extra transformation step involves a non-local reshuffling of the degrees of freedom.

Recap

- Obstructions to particle localizability (Malament, Reeh-Schlieder) do not subside in non-relativistic approximation of QFT

- Recovery requires non-local reshuffling of degrees of freedom

Entanglement and particle localizability

- Entanglement gives vacuum fundamentally non-local character. Is this obstructing particle localizability?
- What happens to entanglement in non-relativistic limit?

- Entanglement in QFT of independent interest for: Unruh/Hawking effects, holography, condensed matter, quantum information, …
- Here, examining role in foundational aspects of QFT

Revisiting Reeh-Schlieder

- How is entanglement related to localizability?
- Recall corollary

$$\mathcal{O} \to \mathcal{A}(\mathcal{O})$$

$$\text{If } A \in \mathcal{A}(\mathcal{O}) \text{ and } A|0\rangle_G = 0, \text{ then } A = 0.$$

- Reeh-Schlieder theorem (1961)

$$\text{For any } \mathcal{O}, |0\rangle_G \text{ is cyclic for } \mathcal{A}(\mathcal{O}).$$

i.e., can approximate any state by acting $\mathcal{A}(\mathcal{O})$ on $|0\rangle_G$

→ Related to vacuum entanglement!

Redhead's intuition[1]

- How are state cyclicity and entanglement related?
- Can reproduce with two qubits!

  $$\mathcal{H}_1 \otimes \mathcal{H}_2 = \mathbb{C}^2 \otimes \mathbb{C}^2$$

- "Baby" Reeh-Schlieder theorem

  $$|\Psi\rangle = |01\rangle - |10\rangle \text{ is cyclic for } \mathcal{B}(\mathcal{H}_1)$$

  idea: any state in $\mathcal{H}_1 \otimes \mathcal{H}_2$ can be written

  $$(\alpha 1 \otimes 1 + \beta X \otimes 1 + \gamma Z \otimes 1 + \delta XZ \otimes 1)|\Psi\rangle$$

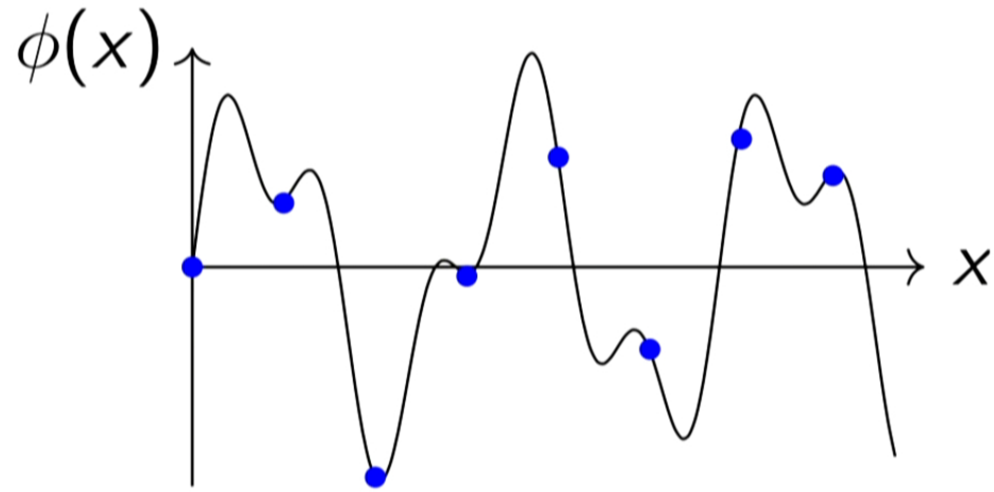
  $$\alpha, \beta, \gamma, \delta \in \mathbb{C}$$

[1]M. Redhead, (1995) *Found. Phys.* 25(1)

Persisting entanglement?

- Entanglement between local dofs $b_{\mathbf{x}}, b_{\mathbf{x}}^{\dagger}$
  - Cutoff $|\mathbf{k}| \leq \Lambda \ll k_c$ (sampling theory[1])

1. Can represent fields on a lattice
2. Local dofs are at lattice points



[1]JP, W. Donnelly, A. Kempf, (2015) *Phys. Rev. D* 92 105022

Persisting entanglement?

- Formally remains entangled

$$|0\rangle_G^\Lambda = \mathcal{N}\exp\left[-\frac{1}{2}\sum\nolimits_{\mathbf{m},\mathbf{m}'\in\mathbb{Z}^n}(\alpha^{-1}\beta)_{\mathbf{m}\mathbf{m}'}b^\dagger_{\mathbf{m}}b^\dagger_{\mathbf{m}'}\right]|0\rangle_L^\Lambda$$

$$= |00\cdots\rangle - \frac{1}{2}\sum\nolimits_{\mathbf{m}\neq\mathbf{m}'}(\alpha^{-1}\beta)_{\mathbf{m}\mathbf{m}'}|0\cdots 01_{\mathbf{m}}0\cdots 01_{\mathbf{m}'}0\cdots\rangle$$

$$-\frac{1}{\sqrt{2}}\sum\nolimits_{\mathbf{m}}(\alpha^{-1}\beta)_{\mathbf{m}\mathbf{m}}|0\cdots 02_{\mathbf{m}}0\cdots\rangle$$

- But unitary inequivalence!

Persisting entanglement?

- Can we quantify entanglement more carefully?
- Temperature of single oscillator

$$k_B T \sim \frac{mc^2}{\log[(\Lambda/k_c)^{-4}]}$$

- Logarithmic Negativity between two local oscillators
  - e.g., $n = 1$ dimensions, distance M lattice spacings

$$E_N \sim \tfrac{1}{M^2 \pi^2} (\Lambda/k_c)^2$$

Summary

- Obstructions in the extent to which QFT can describe localizable particles
- These obstructions persist after non-relativistic approximation
- Related to vacuum entanglement, which also persists in this regime