Title: Numerical approach to infrared conformality and associated dualities in 2+1 dimensions

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Series: Condensed Matter

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Abstract: QFTs in 2+1 dimensions are powerful systems to understand the emergence of mass-gap and particle spectrum in QCD-like theories that describe our 3+1 dimensional world. Recently, these 2+1 dimensional systems have attracted even more attention due to conjectured dualities between seemingly very different theories and due to their applications to condensed matter systems. In this talk, I will describe our numerical investigations of the infrared behaviors of 2+1 dimensional U(1) and SU(N) gauge theories coupled to many favors of massless fermions using lattice regularization. I will also explain how lattice formulation is a potential tool to study and check particle-vortex dualities.

Numerical approach to IR conformality and associated dualities in 2+1 dimensions

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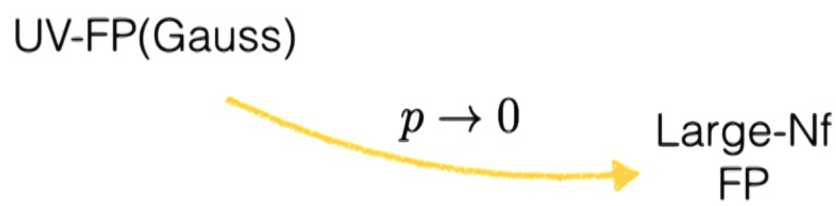
Perimeter Institute Condensed Matter Seminar

November 5, 2019

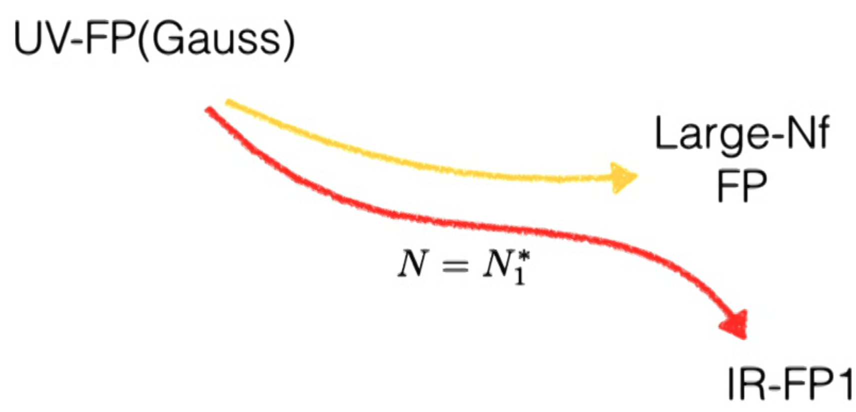
In collaboration with Rajamani Narayanan (FIU)

1. Motivation
2. Lattice regulated 3d fermions and parity-anomaly
3. Infrared fate of gauge theories coupled to Dirac fermions
4. Approaching dualities by computing monopole scaling dimensions
5. Studies on charge-2 $N = 1$ QED_3

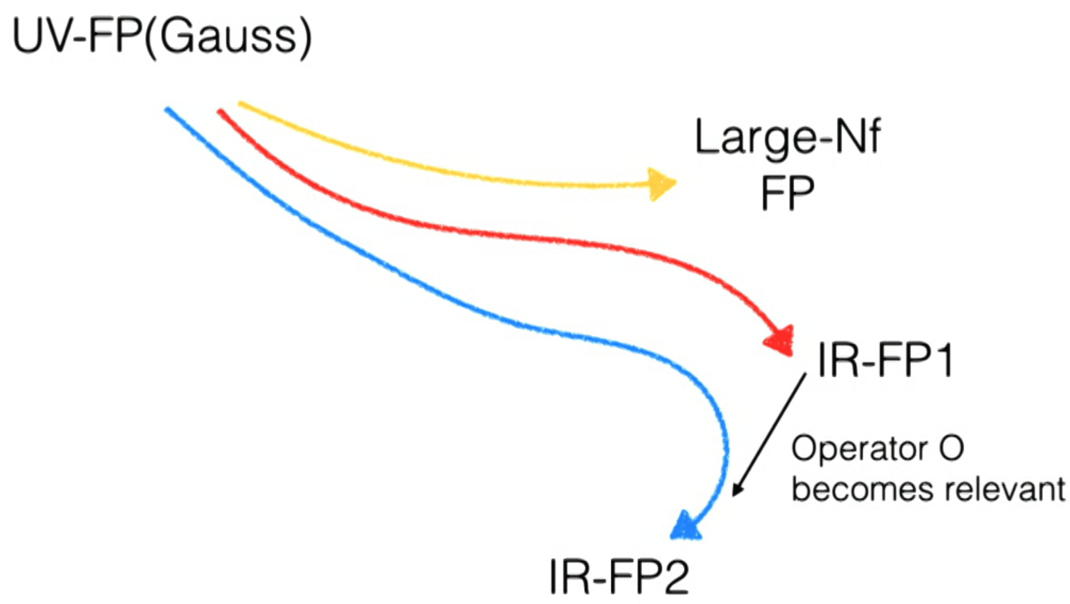
QFTs: flow from UV-FP to an IR-FP or mass-gapped



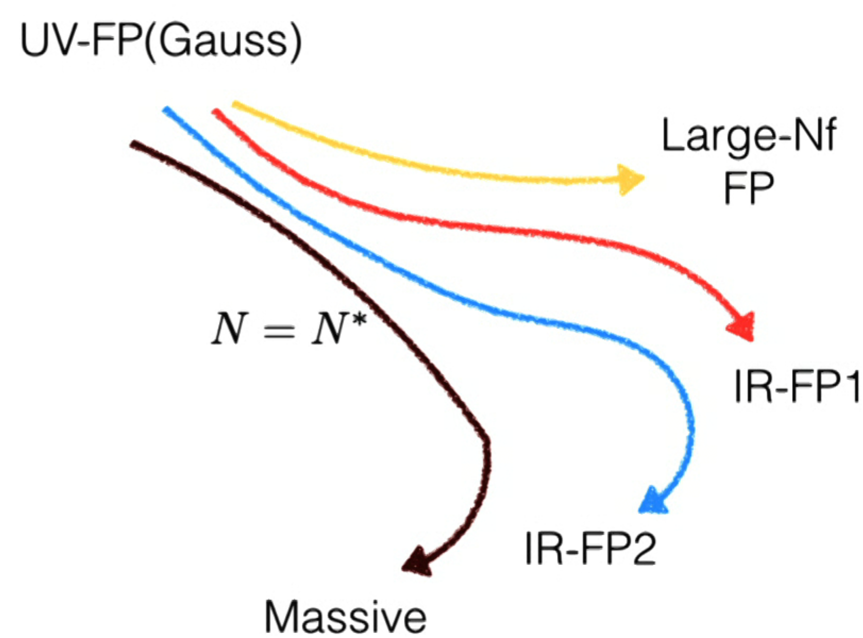
QFTs: flow from UV-FP to an IR-FP or mass-gapped



QFTs: flow from UV-FP to an IR-FP or mass-gapped



QFTs: flow from UV-FP to an IR-FP or mass-gapped



N

$$\langle \mathcal{O}(x)\mathcal{O}(0)\rangle \sim \frac{1}{|x|^{2\Delta}} \quad \text{as} \quad |x| \to \infty$$

$$\langle \mathcal{O}(x)\mathcal{O}(0)\rangle \sim e^{-M|x|} \quad \text{as} \quad |x| \to \infty$$

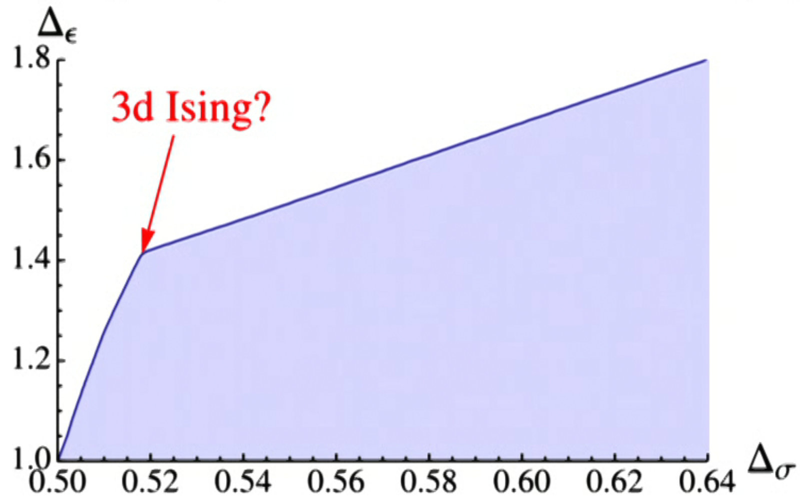
$\mathcal{O}(x)$

→ composites like $\overline{\psi}(x)\sigma_\mu\psi(x)$

→ topological defects like monopole $M_Q(x)$

Classifying the CFTs in 2+1d

- One way is the ongoing non-perturbative conformal bootstrap approach



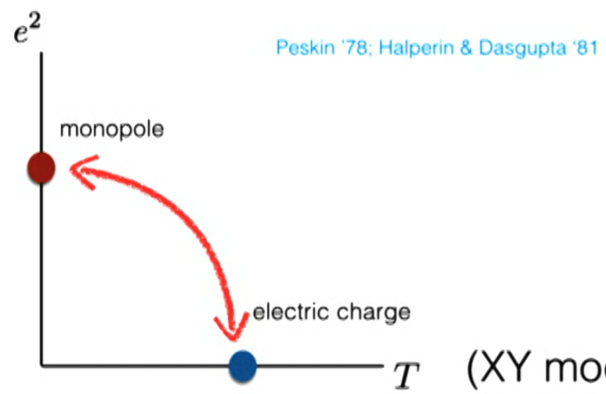
El-Showk et. al '12 (bootstrap collaboration)

- This talk: use non-perturbative lattice simulations.

Recent developments in particle-vortex dualities (but an old example)

Lattice model: $S = \frac{1}{T}\sum_x \cos\left(\partial_\mu\theta(x) - ea_\mu\right) + \frac{1}{4}F_{\mu\nu}(a)^2$

(FZS model)



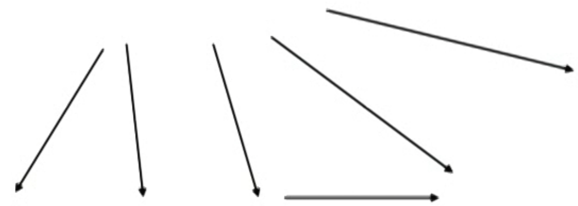
(XY model)

$$\langle e^{-iQ\theta_x} e^{iQ\theta_y}\rangle_{\mathrm{XY}} \propto \langle M_{-Q}(x)M_Q(y)\rangle_{\mathrm{FZS}} \propto \frac{1}{|x-y|^{2\Delta}}$$

Recent developments in particle-vortex dualities

Seiberg et al '16, Karch and Tong '16

$$\text{Free fermion} \xleftrightarrow{\text{conjectured duality}} \text{Wilson} - \text{Fisher boson} + U(1)\ \text{CS term}$$



fermion-fermion dualities

fermion-boson dualities

Use lattice to "prove" or "disprove" conjectures?

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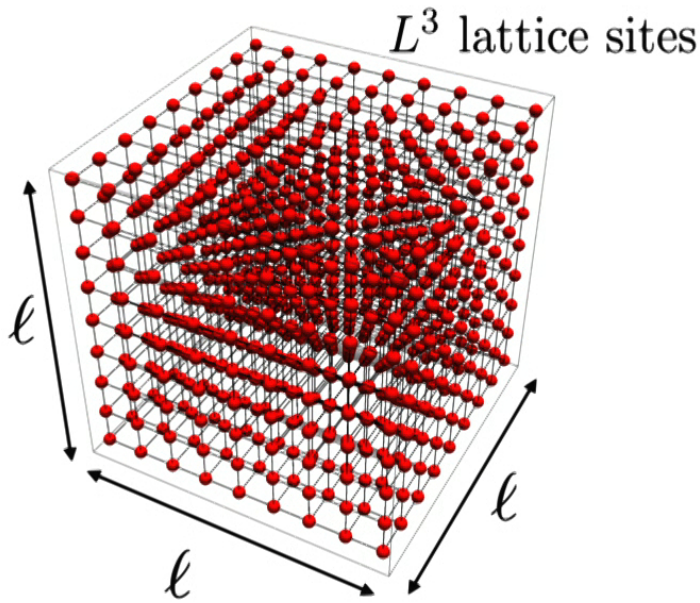
Euclidean 3d Gauge theories in the continuum

Lagrangian

$$L = \sum_{i=1}^{N} \{\overline{\psi}_i \sigma_\mu (\partial_\mu + iq_i A_\mu) \psi_i + m_i \overline{\psi}_i \psi_i\} + \frac{1}{g^2} S_g(A)$$

- $\psi \rightarrow$ 2-component complex fermion
- $g^2 \rightarrow$ coupling constant of dimension $[\text{mass}]^1$
  Scale setting $\Rightarrow g^2 = 1$ and measure everything in units of g^2
- massless Dirac operator: $\not{D} = \sigma_\mu (\partial_\mu + iA_\mu)$
  A special property in 3d: $\not{D}^\dagger = -\not{D}$
- $S_g(A)$ is for can be "compact" or "non-compact" for U(1) theory i.e., Monopoles are allowed or disallowed in continuum limit.

Euclidean 3d Gauge theories in the continuum



- Study the theory on a periodic Euclidean torus with dimensionless volume $\ell^3 = (g^2 l_{\rm ph})^3$. Discretize ℓ^3 torus into L^3 lattice.
- Super-renormalizability: take $L \to \infty$ keeping ℓ fixed.

Compact or non-compact theory $\Rightarrow$ U(1) theory with or without monopoles

Parition function for lattice QED$_3$ in terms of fermion and gauge parts:

$$Z = \left(\prod_{x,\mu} \int_{-\infty}^{\infty} d\theta_\mu(x)\right) \det^{N/2}\left[\not{D}^\dagger(e^{i\theta})\not{D}(e^{i\theta})\right] \times \mathcal{W}_g,$$

The gauge part satisfying the compactness $\theta \to \theta + 2\pi n$ of U(1) gauge group:

$$\mathcal{W}_g \equiv \sum_{\{N_{\mu\nu}\}} e^{-\frac{L}{\ell}\sum_{x,\mu>\nu}(F_{\mu\nu}(x) - 2\pi N_{\mu\nu}(x))^2}$$

C QED$_3$: $\epsilon_{\mu\nu\rho}\nabla_\mu N_{\nu\rho}(x) \neq 0$ generically

NC QED$_3$: $\epsilon_{\mu\nu\rho}\nabla_\mu N_{\nu\rho} = 0$ at all x

NC QED$_3$ with monopole insertion: $\epsilon_{\mu\nu\rho}\nabla_\mu N_{\nu\rho} = 2Q\delta_{x,0} - 2Q\delta_{x,x'}$

Regulating the fermion determinant: Parity anomaly

Parity in Euclidean: $x \to -x$; $A(x) \to -A(-x)$; $\psi \to \psi$; $\overline{\psi} \to -\overline{\psi}$

On a gauge field background A:

$$\det \not{D} = (i)^{n_+ - n_-} |\det \not{D}| \xrightarrow{\text{UV regularization}} e^{i\Gamma(A;m)} |\det \not{D}|$$

- $m = \infty \Rightarrow \Gamma(A;m) = q^2 \eta(A) \sim \frac{q^2}{4\pi} \int d^3x \epsilon_{\mu\nu\rho} A_\mu \partial_\nu A_\rho$
- $m = 0 \Rightarrow \Gamma(A;m) = \frac{q^2 \eta(A)}{2}$ (Parity anomaly)
- $m = -\infty \Rightarrow \Gamma(A;m) = 0$

Making the massless fermion theory parity-invariant

- **Theory I:**

  $\{$ 2 massless $q = 1$ fermions with anomaly $\frac{\eta(A)}{2}$ each $\}$+ $\{$ 1 Pauli-Villars regulator $q = 1$ fermion with anomaly $-\eta(A)\}$

- **Theory II:**

  $\{$ 1 massless $q = 2$ fermion with anomaly $2\eta(A)\}$ + $\{$2 Pauli-Villars regulator $q = 1$ fermions each with anomaly $-\eta(A)\}$

  $\rightarrow$ relevant to FQHE at $\nu = 1/2$ (D.T.Son '15)

Lattice regularization: Overlap fermions

Generating function for lattice 'overlap' fermions (with electric charge q for U(1) theory) with lattice mass $\hat{m}$:

$$Z(\overline{\psi}, \psi, A) = \det \left[\frac{(1-\hat{m})}{2} + \frac{(1+\hat{m})}{2} V_{qA} \right] e^{-\overline{\psi} G \psi}$$

with a $2L^3 \times 2L^3$ unitary matrix V_{qA} (constructed out of Wilson-Dirac operator... but not relevant to talk)

Pattern of anomaly is apparent here:

$$\hat{m} = 1 \Rightarrow \det V_A = e^{i\eta(A)} \qquad \text{and} \qquad \hat{m} = 0 \Rightarrow \det(1 + V_A) \sim e^{i\frac{\eta(A)}{2}}$$

Constructing parity-invariant theories on the lattice

- Theory I:

$\{$ 2 massless $q = 1$ fermions with anomaly $\frac{\eta(A)}{2}$ each $\}$+ $\{$ 1 Pauli-Villars regulator $q = 1$ fermion with anomaly $-\eta(A)\}$ $\Rightarrow$

$$Z = \left\{ \det(1 + V_A) \det(1 + V_A) \right\} \left\{ \det V_A^\dagger \right\} = \det \left[(1 + V_A)(1 + V_A^\dagger) \right]$$

- Theory II:

$\{$ 1 massless $q = 2$ fermion with anomaly $2\eta(A)\}$ + $\{$2 Pauli-Villars regulator $q = 1$ fermions each with anomaly $-\eta(A)\}$

$$Z = \left\{ \det(1 + V_{2A}) \right\} \left\{ \left(\det V_A^\dagger \right)^2 \right\}$$

How scale-breaking would come about in parity-invariant theories...

- Parity invariant theory with even N flavors of massless two-component fermion has $SU(N)$ flavor symmetry

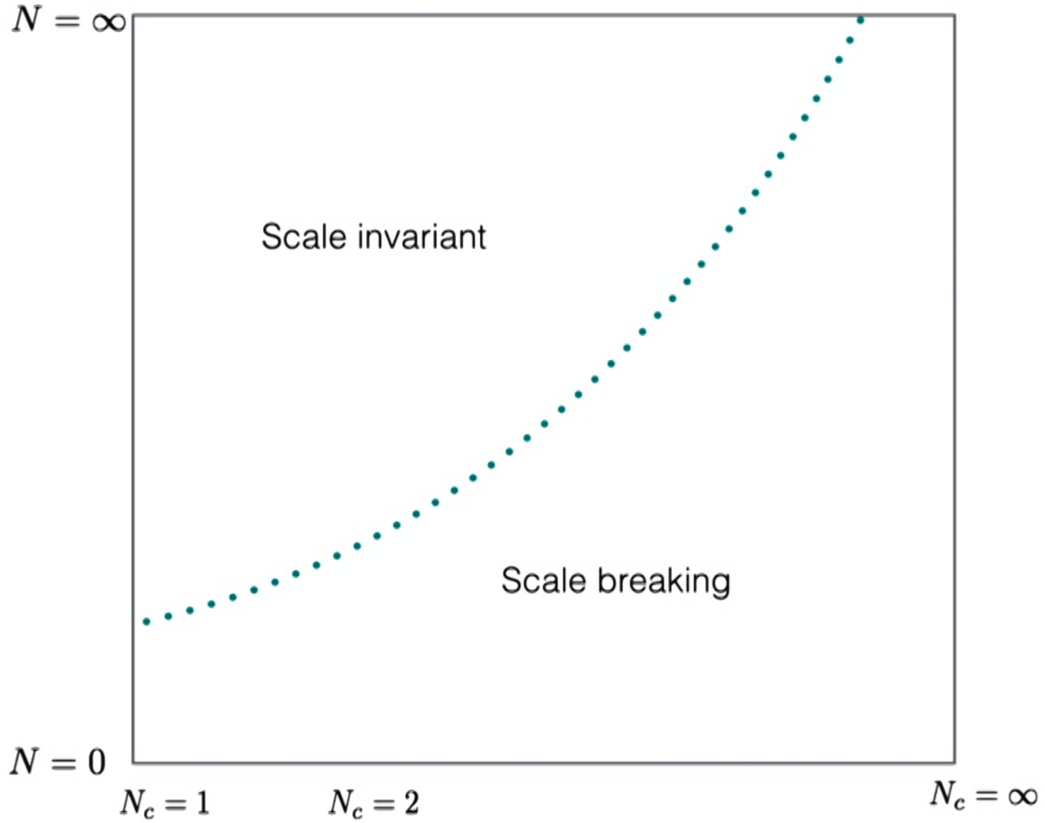
$$\begin{pmatrix} \psi_1 \\ \psi_2 \\ \vdots \\ \psi_N \end{pmatrix} \to U \begin{pmatrix} \psi_1 \\ \psi_2 \\ \vdots \\ \psi_N \end{pmatrix}; \quad U \in SU(N)$$

- Spontaneous symmetry breaking of $SU(N) \to SU(N/2) \times SU(N/2) \times U(1)$ producing a non-zero condensate bilinear condensate

$$\Sigma = \frac{1}{N} \sum_{i}^{N/2} \left\langle \overline{\psi}_i \psi_i - \overline{\psi}_{N/2+i} \psi_{N/2+i} \right\rangle \neq 0,$$

which sets a scale even when $\ell \to \infty$ (Pisarski '84, Vafa and Witten '84)

A generic phase diagram in $N - N_c$ plane



Telling apart theories with and without IR scale-invariance

- Study the theory at finite ℓ
- The low-lying eigenvalues of Dirac operator $\not{D}$ are discrete. When ordered by magnitude:
$$0 < \lambda_1(\ell) < \lambda_2(\ell) < \dots$$
- Take continuum limit of $\lambda_i(\ell)$ at fixed ℓ by taking $L \to \infty$.

Telling apart theories with and without IR scale-invariance

Look at the low-lying eigenvalues of dirac operator $\not{D}$ ordered by their magnitudes: $0 < \lambda_1 < \lambda_2 < \ldots$ **Three different scenarios:**

- Free theory$\Rightarrow$ $\lambda_i \propto \frac{1}{\ell^{1}}$
- Condensate $\Sigma \Rightarrow$ $\lambda_i = \frac{z_i}{\Sigma \ell^{3}}$; z_i from nonχ random matrix model

  Banks,Casher+Leutwyler, Smilga, Verbaarschot, Shuryak,Zahed...
- Scale-invariance$\Rightarrow$ $\lambda_i \propto \frac{1}{\ell^{1+\gamma_m}}$

  c.f., Debbio and Zwicky '10

Telling apart theories with and without IR scale-invariance

For $\ell \ll$ Goldstone boson compton wavelength
and $\ell \gg$ (typical "hadron" scale)$^{-1}$

$$\left\{ \lambda_i \sim \int [dA] \, \det{}^{N/2} \left[\not{D} \not{D}^\dagger \right] e^{-S_g} \right\} \to \left\{ z_i \sim \int [dH] \, \det{}^{N/2} (H.H) e^{-H^2/2} \right\}$$

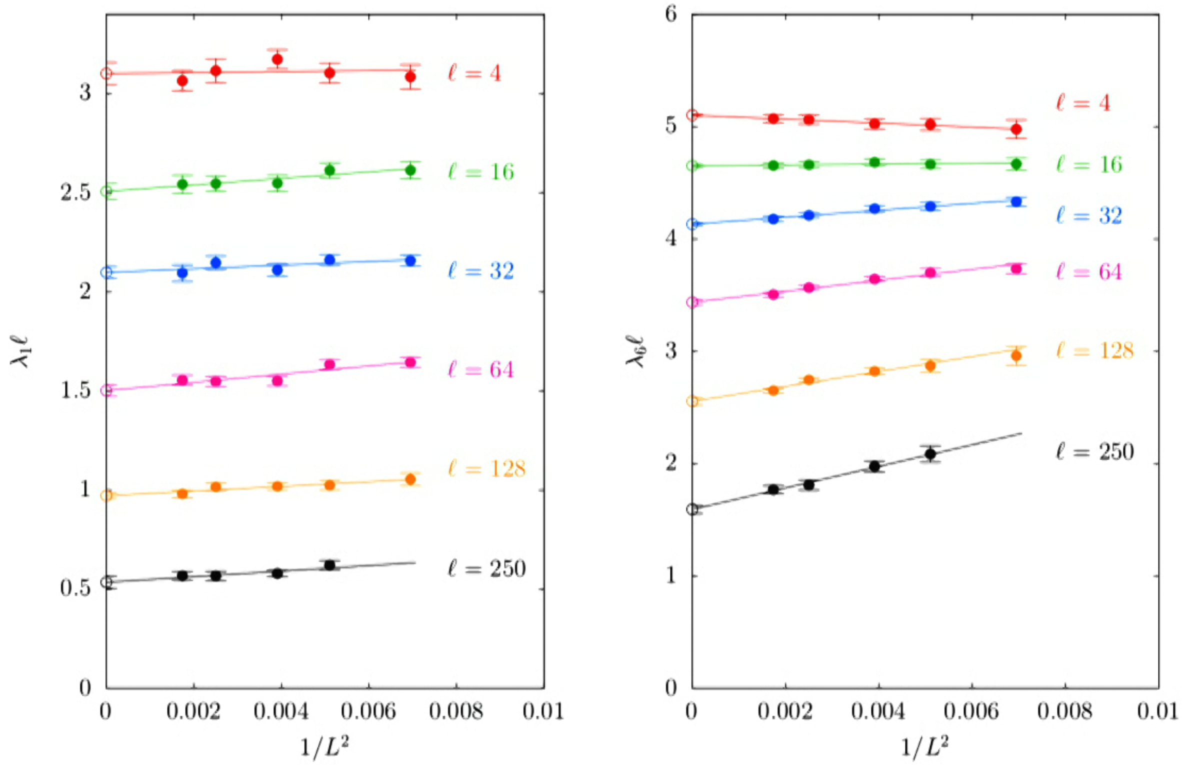
$H = H^\dagger$ (or $H = H^T$ for SU(2) theory)

Verbaarschot,Zahed '94

Define: $\Sigma_i(\ell) = \dfrac{z_i}{\ell^3 \lambda_i(\ell)}$

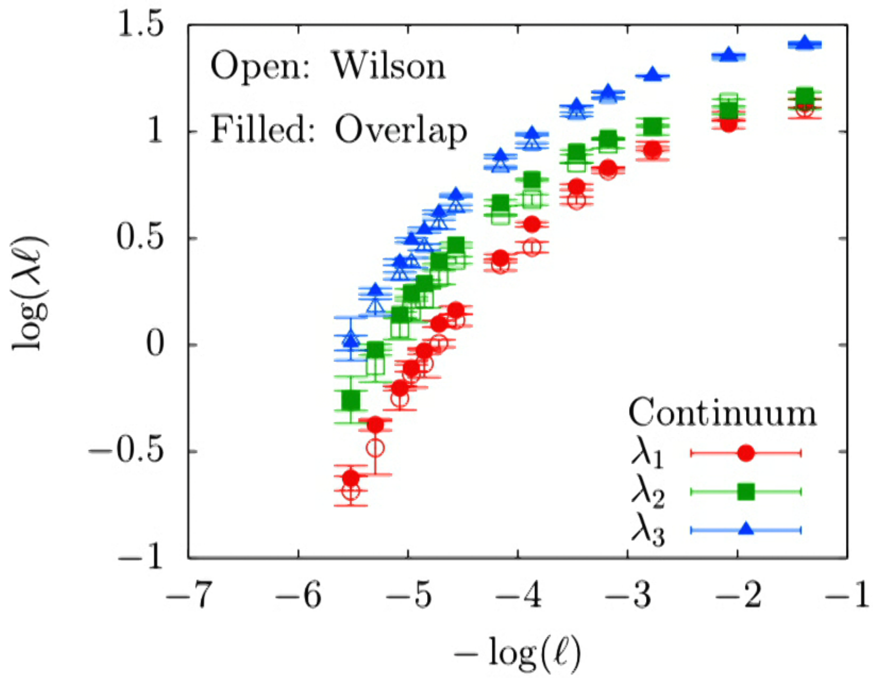
Check: $\lim_{\ell \to \infty} \Sigma_i(\ell) = \Sigma > 0$ (or vanishes like $\lim_{\ell \to \infty} \Sigma_i(\ell) \sim \ell^{-2+\gamma_m}$?)

Taking continuum limits at fixed ℓ



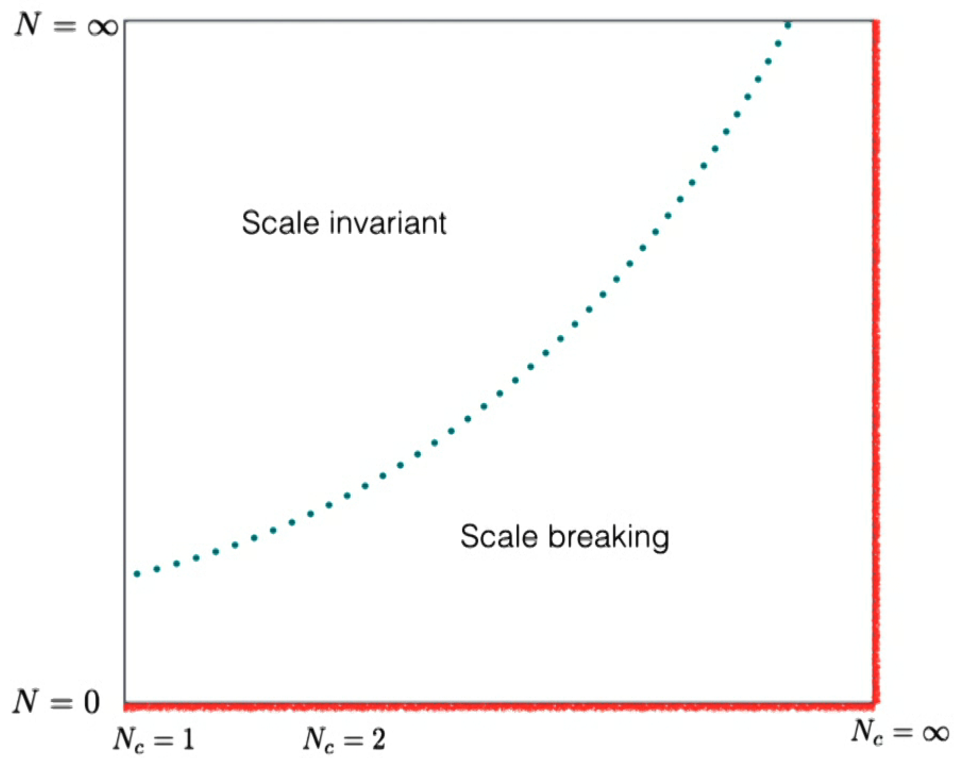
$N = 2$ QED$_3$ with overlap

Taking continuum limits at fixed ℓ

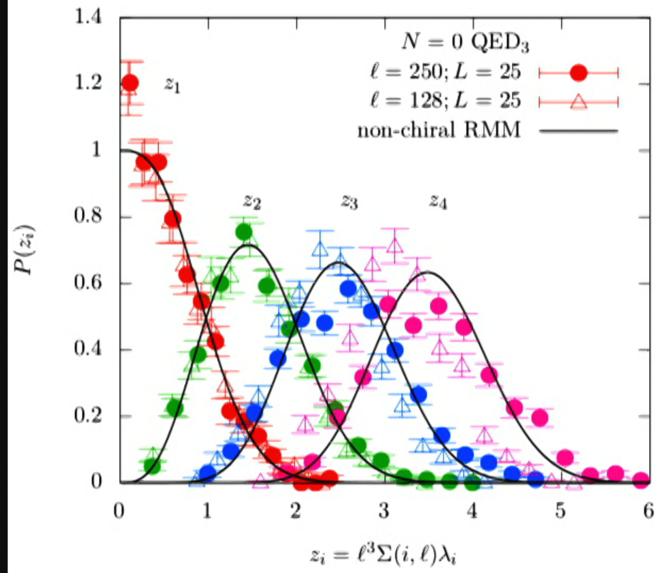


A sanity check... lattice regulator shouldn't matter (if done correctly!)

Where you definitely expect scale-breaking...

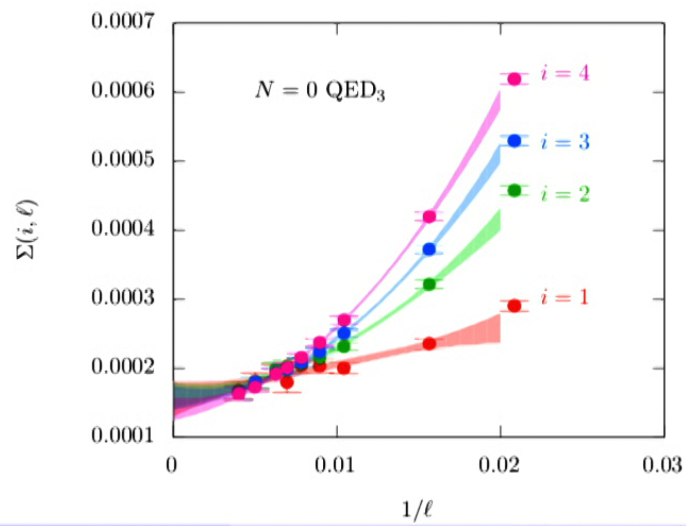


Quenched $U(1)$

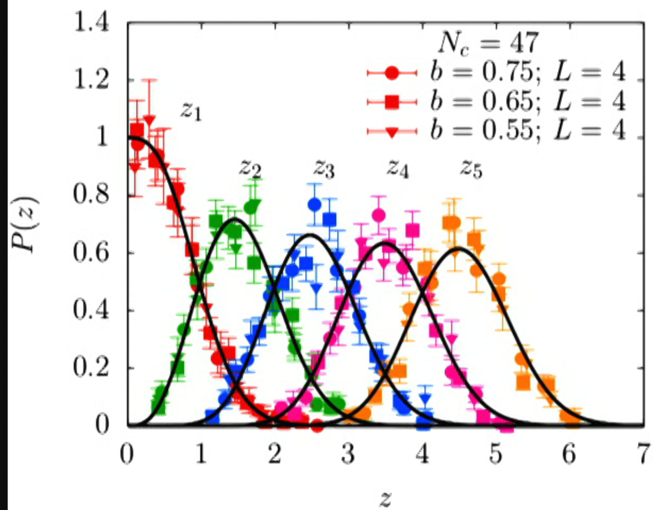


Comparison to nonχ-GUE

$$\Sigma(i, \ell) \equiv \frac{z_i}{\lambda_i \ell^3}$$

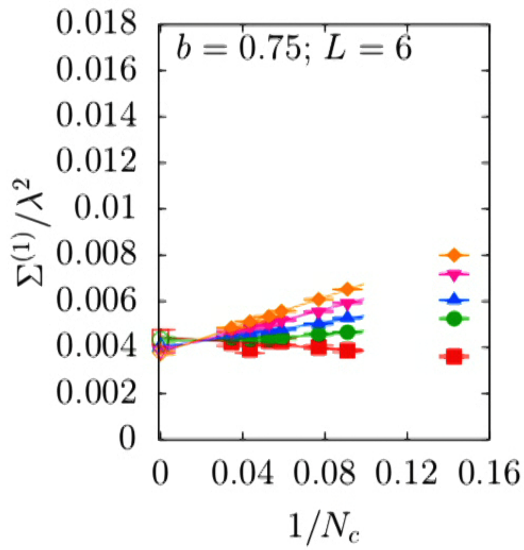


't Hooft Limit

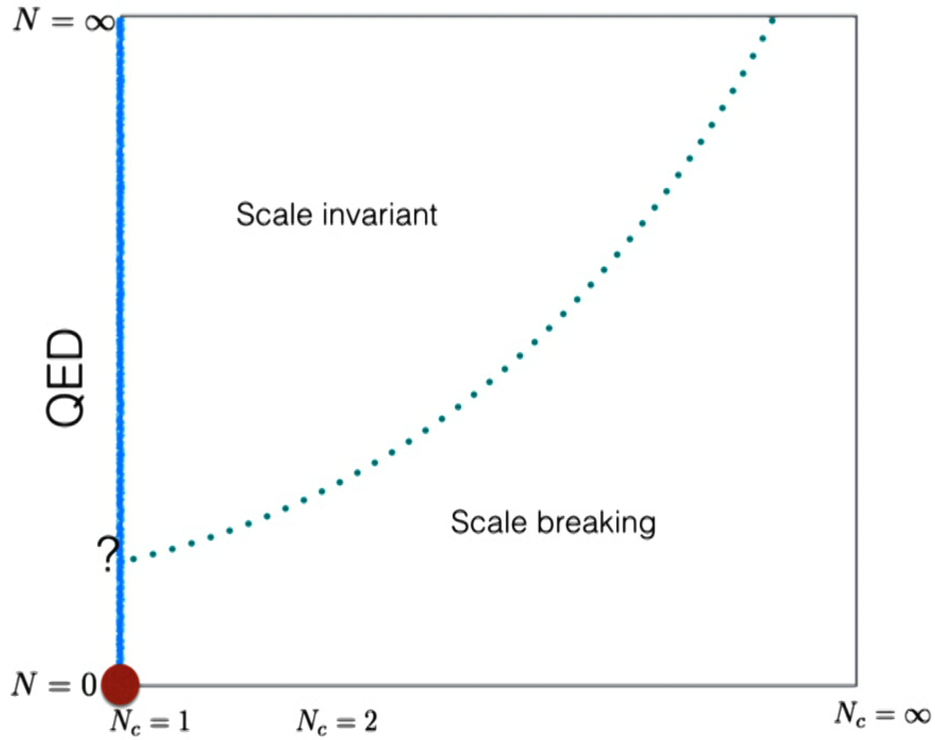


Comparison to nonχ-GUE

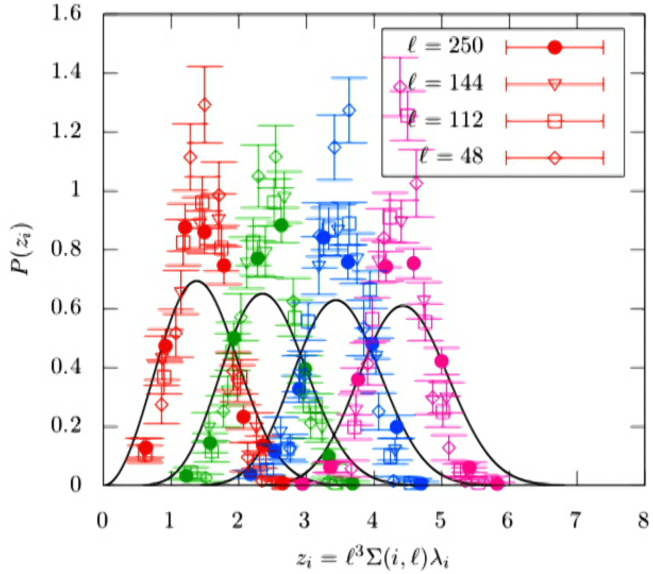
't Hooft limit $\lambda = N_c g^2$



QED_3 with backreaction from massless fermions...



QED$_3$ with $N = 2$ flavors



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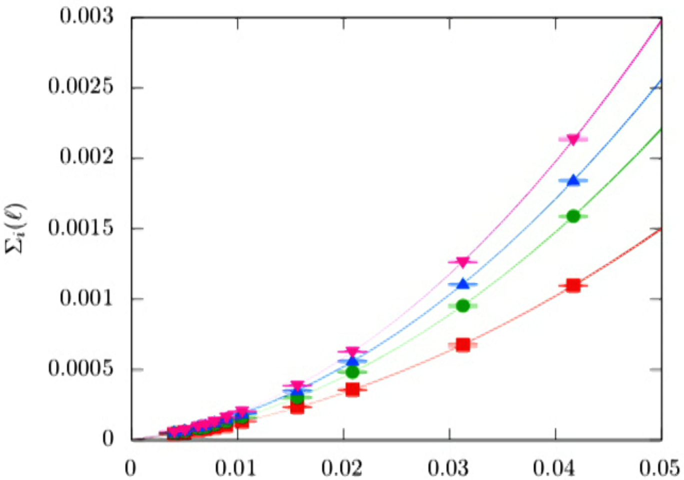
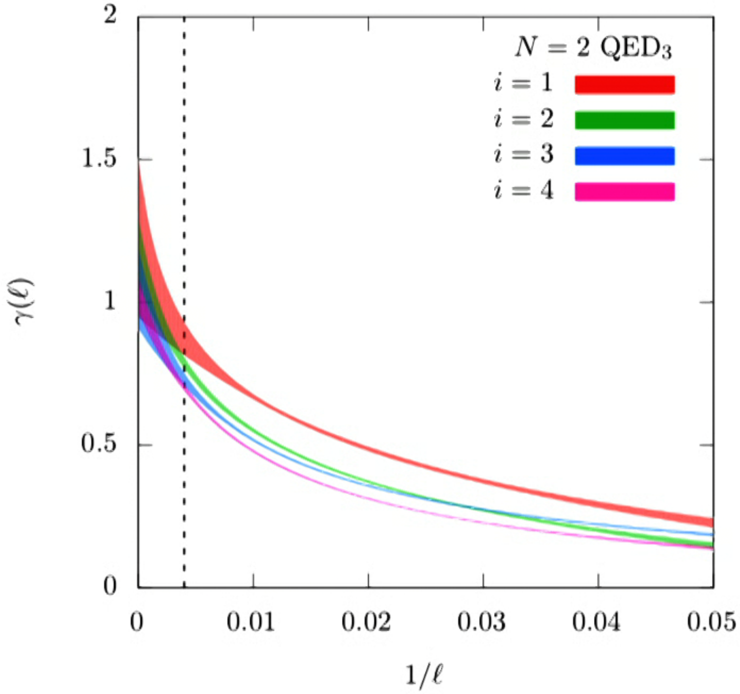
$$\Sigma(i, \ell) \equiv \frac{z_i}{\lambda_i \ell^3}$$



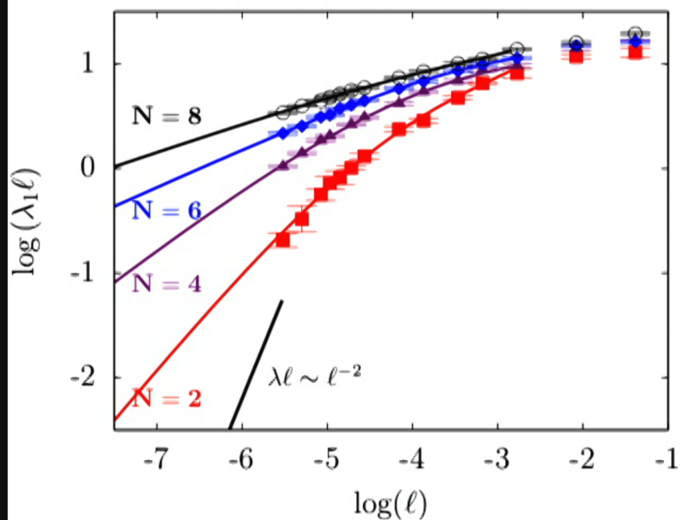
QED_3 with $N = 2$ flavors

Instead, lets assume a conformal behavior with $1/\ell$ corrections

$$\Sigma(\ell) \sim \ell^{-2+\gamma_m}\left(1 + a_1/\ell + a_2/\ell^2 + \dots\right); \quad \gamma(\ell) = 2 + \frac{\partial \log \Sigma(\ell)}{\partial \log \ell}$$

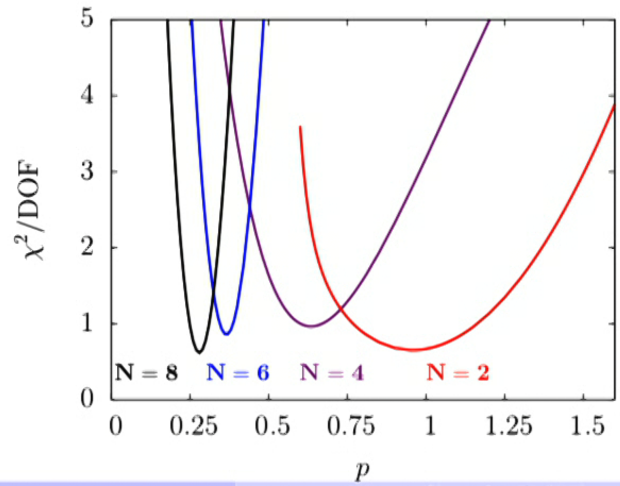


QED$_3$ with $N \geq 2$ flavors



- γ_m decreases with N:
  trend$\Rightarrow$ $\gamma_m \approx \dfrac{2}{N}$

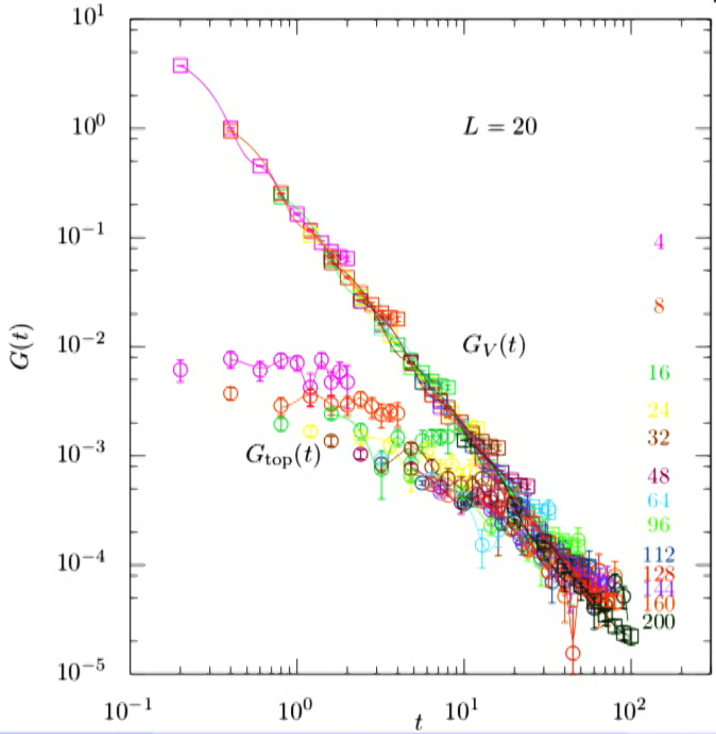
χ^2/DOF for $\lambda_1(\ell) \sim \ell^{-(1+\gamma_m)}$



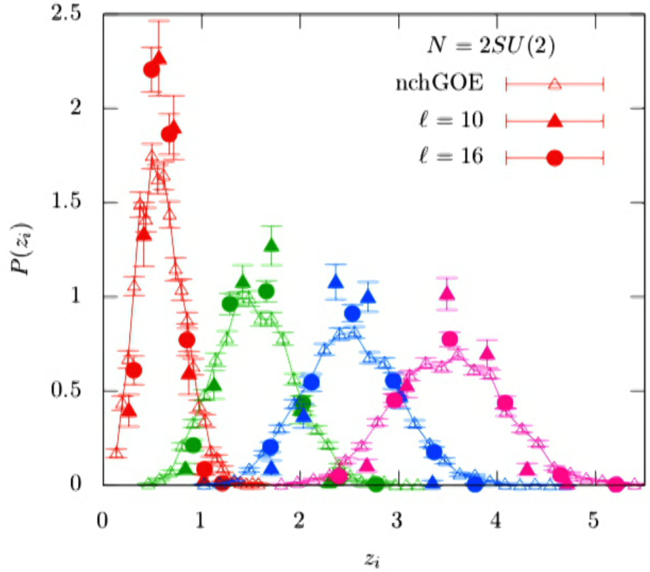
Self-duality of $N = 2$ QED$_3$?

Conjectured in Wang et al'17

$$J_\mu = \overline{\psi}_1(x)\sigma_\mu\psi_1(x) - \overline{\psi}_2(x)\sigma_\mu\psi_2(x) \quad ; \quad J_\mu^{\mathrm{top}} = \frac{1}{2\pi}\epsilon_{\mu\nu\rho}F^{\mu\nu}$$



$SU(2)$ QCD with $N = 2$ flavors



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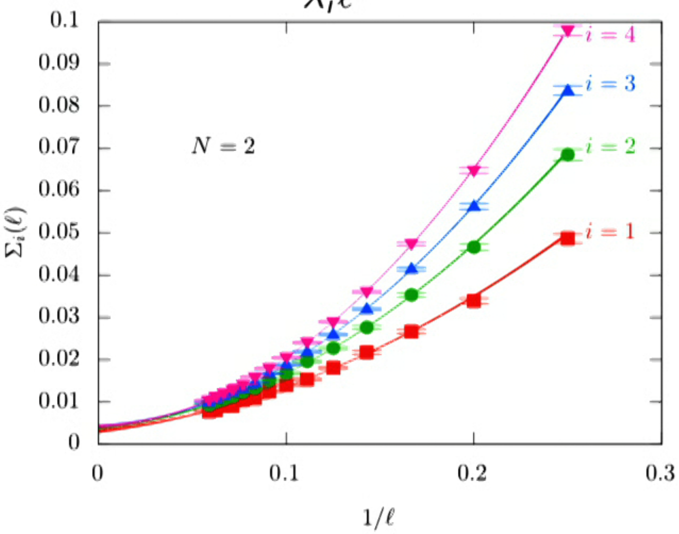
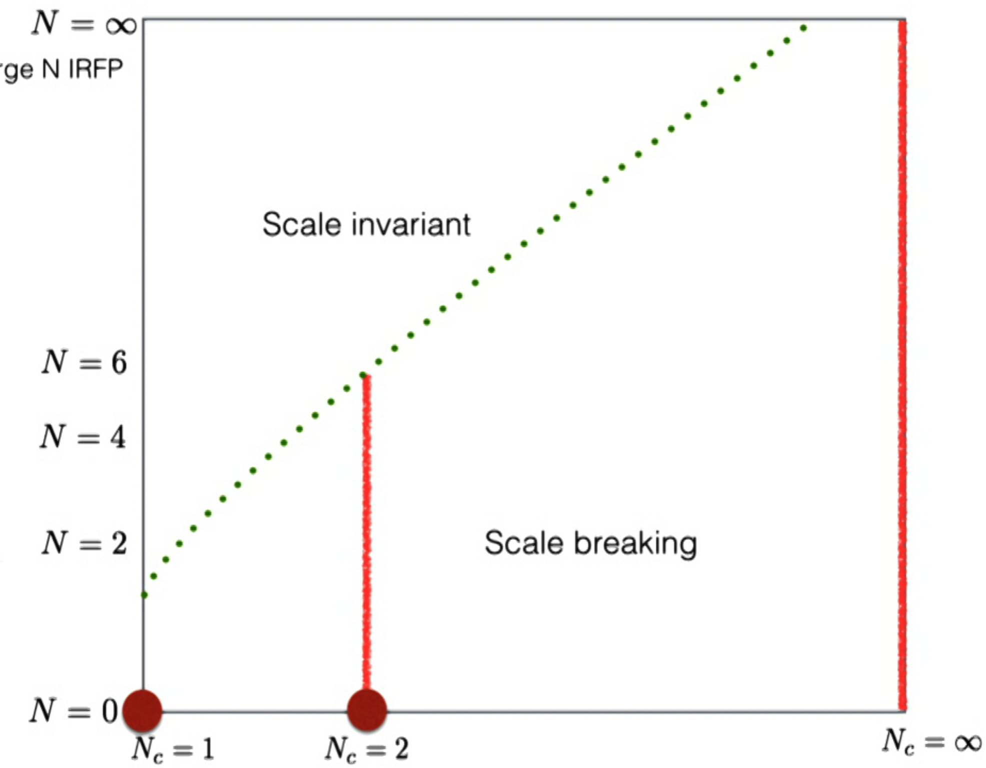


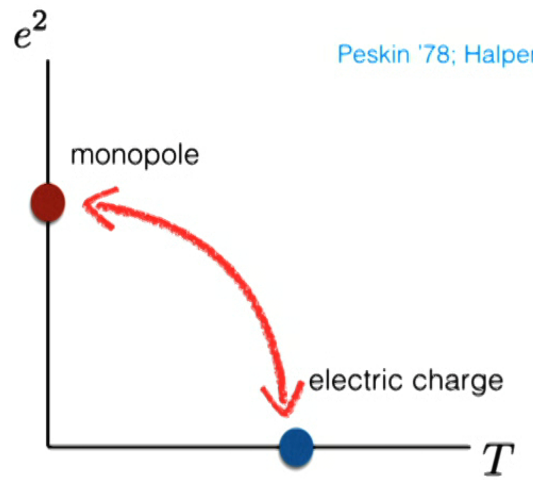
N-N_c phase diagram as we understand it



Consider the simplest and mathematically rigorous particle-vortex duality

(FZS model)

Peskin '78; Halperin & Dasgupta '81

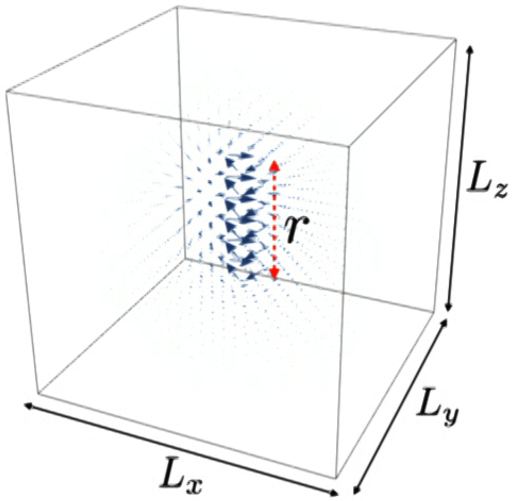


(XY model)

$$\langle e^{-iQ\theta_x} e^{iQ\theta_y} \rangle_{\mathrm{XY}} \propto \langle M_{-Q}(x) M_Q(y) \rangle_{\mathrm{FZS}} \propto \frac{1}{|x-y|^{2\Delta}}$$

Computing monopole scaling dimensions

Couple the critical system with $U(1)$ symmetry (gauge/ungauged) to background field of classical Q-monopole-antimonopole pair separated by distance $r = \rho L$.



Look for the logarithmic finite-size dependence of free energy to introduce monopole-antimonopole pair:

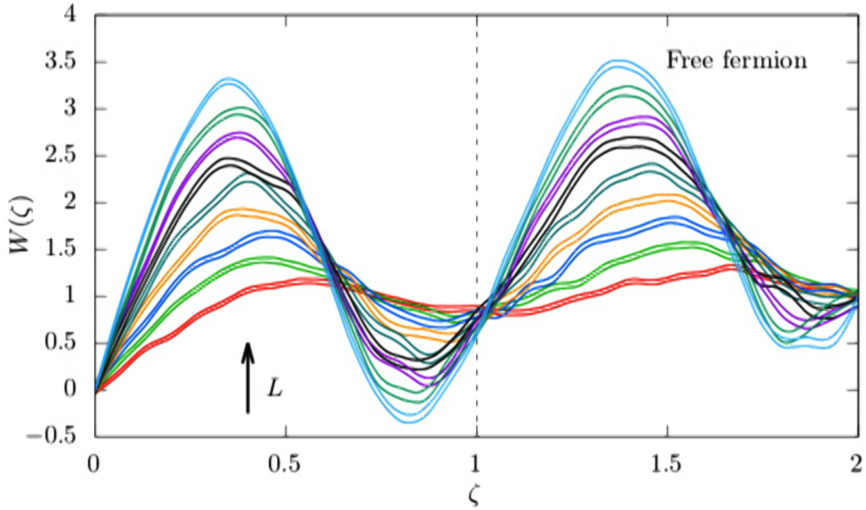
$$F_Q(r = \rho L) \sim 2\Delta_Q \log(L)$$

Slowly increase monopole charge from 0 to Q

overcoming the overlap problem $\Rightarrow$ Differentiate and then integrate:

$$F_Q(\rho L) = \int_0^Q W(\zeta) d\zeta \quad ; \quad W(\zeta) = \frac{-1}{Z(\zeta A_Q)} \frac{\partial}{\partial \zeta} Z(\zeta A_Q).$$

Free fermion case: Matches with computation of Casimir energy of free fermions on S^2.



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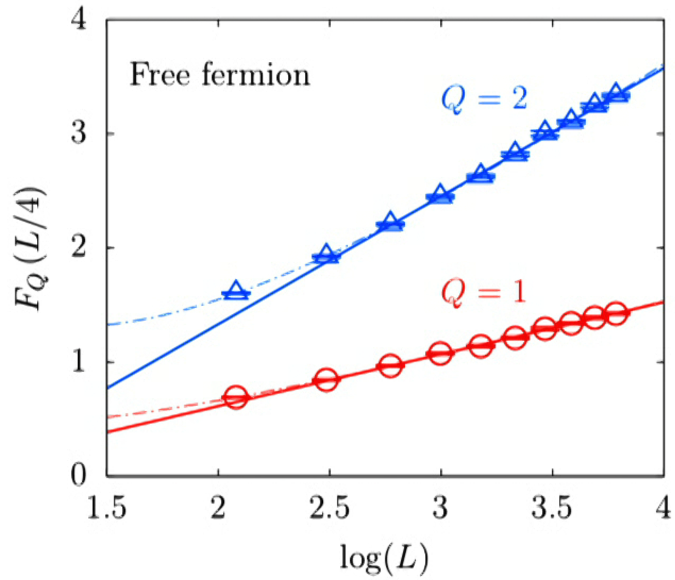
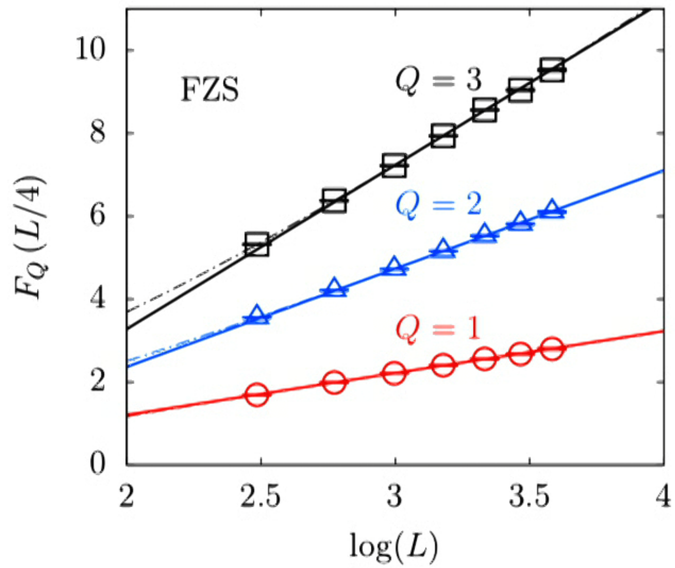


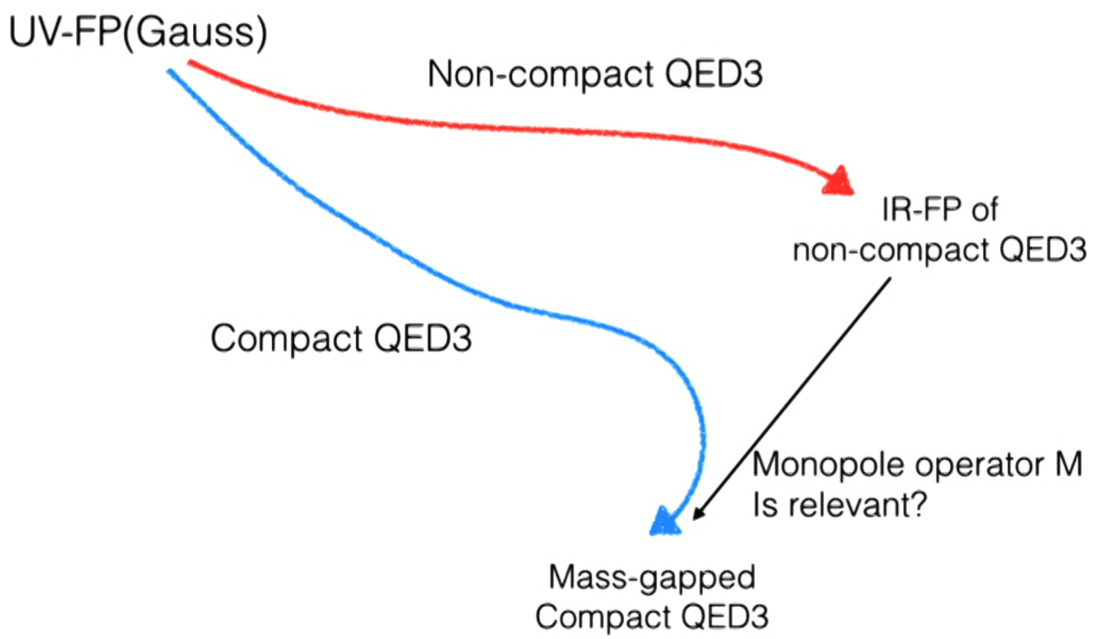
Numerically demonstrating the basic particle-vortex duality



Model	Q	FZS Monopole Δ	XY charge Δ
FZS	1	0.48(2)	0.516(3)
	2	1.23(2)	1.238(5)
	3	2.15(4)	2.116(6)

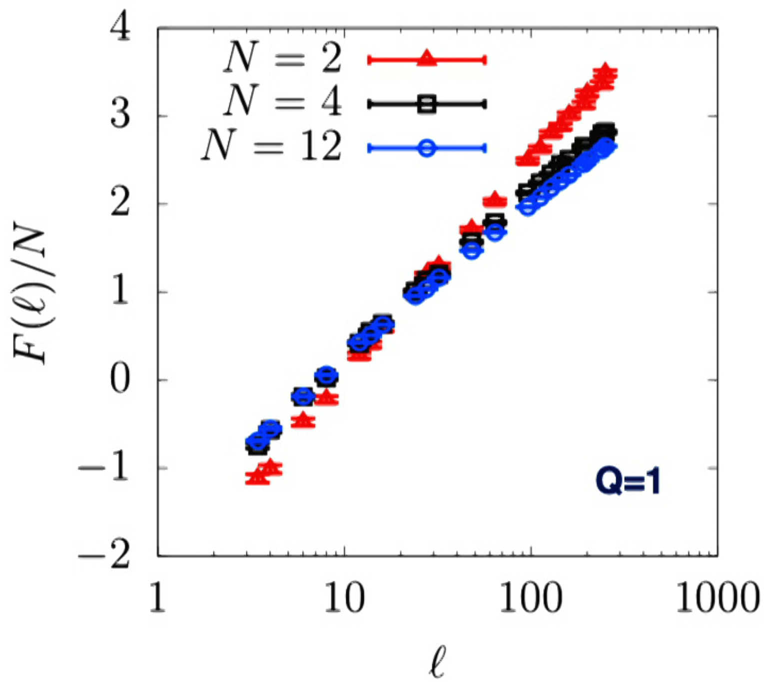
Can we repeat this for some chosen conjectured particle-vortex dualities?

Monopole operator as a relevant perturbation



Monopole correlator in NC-QED$_3$

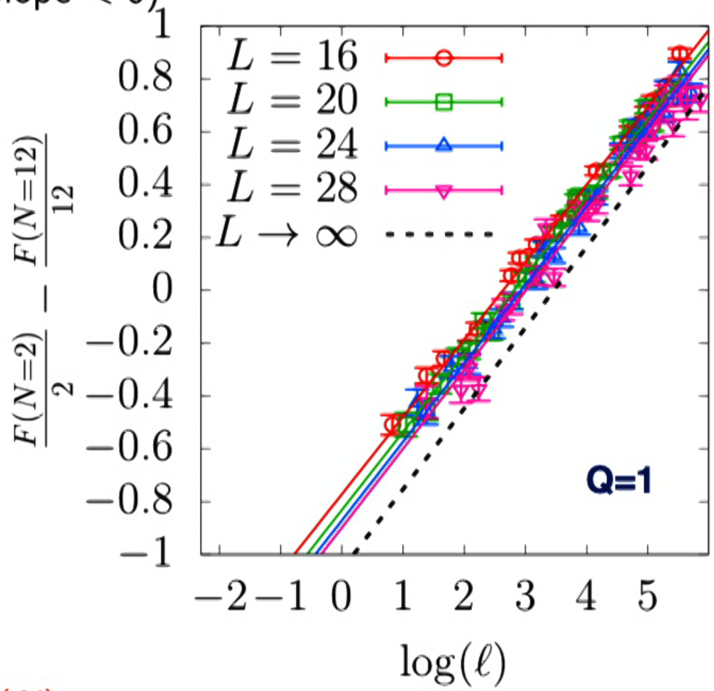
Results for $N = 2, 4, 12$ QED$_3$: $Q\overline{Q}$ pair separated by $\ell/4$ as box-size ℓ is increased.



$\Delta(N = 12) = 3.2(2) \Rightarrow$ marginal

Monopole correlator in NC-QED$_3$

Difference in free-energy per flavor between $N = 2$ and $N = 12$ (Prediction from $1/N$-expansion: Slope < 0)



$$\frac{\Delta(N)}{N} > \lim_{N\to\infty} \frac{\Delta(N)}{N} \text{ for } N \sim \mathcal{O}(1)$$

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