Title: Minimum length scenarios that maintain continuous symmetries

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Series: Quantum Gravity

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Abstract: It has long been argued that combining the uncertainty principle with gravity will lead to an effective minimum length at the Planck scale. A particular challenge is to model the presence of a smallest length scale in a manner which respects continuous spacetime symmetries. One path for deriving low-energy descriptions of an invariant minimum length in quantum field theory is based on generalized uncertainty principles. Here I will consider the question how this approach enables one to retain Euclidean or even Lorentzian symmetries. The Euclidean case yields a ultraviolet cutoff in the form of a bandlimit, and this then allows one to apply the powerful Shannon sampling theorem of classical information theory which establishes the equivalence between continuous and discrete representations of information. As a consequence, one obtains discrete representations of fields which are more subtle than a simple discretization of space, and are in fact equivalent to a continuum representation. Quantum fields in this model exhibit a finite density of information and a corresponding regularization of the entanglement of the vacuum, as I will demonstrate in detail. We then examine the Lorentzian symmetry generalization. This case leads to a Lorentz-invariant analogue of bandlimitation, and we discuss the nature of the corresponding sampling theory.

Minimum length scenarios that maintain continuous symmetries

PI Quantum Gravity Seminar
07 Nov 2019

Jason Pye

JP (2019) *J. Phys.: Conf. Ser.* 1275 012025
JP, W. Donnelly, A. Kempf (2015) *Phys. Rev. D* 92 105022

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Motivation
Conceptual problem:

Uncertainty principle + Gravitational field equations → ?

$$\Delta x \underset{\text{GR}}{\geq} \frac{2G\Delta p}{c^3} \underset{\text{UP}}{\geq} \frac{\hbar G}{c^3}\frac{1}{\Delta x} \quad \Longrightarrow \quad \Delta x \geq \sqrt{\frac{\hbar G}{c^3}} =: \ell_P \sim 10^{-35}$$

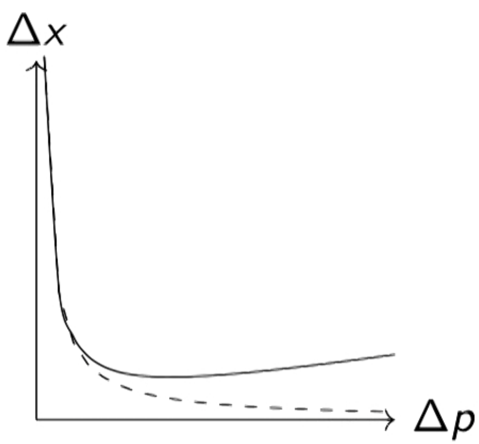
> *"Without a deep revision of classical notions it seems hardly possible to extend the quantum theory of gravity also to this domain."* *(Bronstein, 1936)*

- Does this force us into replacing spacetime with a lattice? Must we give up spacetime symmetries?

Generalised Uncertainty Principles (GUPs)

GUPs (1D)

- want: $\Delta x \geq \ell_P$



- replace

$$\Delta x \Delta p \geq \frac{1}{2}$$

with

$$\Delta x \Delta p \geq \frac{1}{2}(1 + \ell_P^2 \Delta p^2)$$

$$\implies \Delta x \geq \ell_P$$

- model using[1]:

$$[x, p] = i(1 + \ell_P^2 p^2) \quad \text{on} \quad \langle \phi | \psi \rangle = \int \frac{dp}{1 + \ell_P^2 p^2} \phi^*(p) \psi(p)$$

[1] A. Kempf, G. Mangano, R. Mann (1995) *Phys. Rev. D* 52(2)

Injecting GUPs into field theory

How do we describe (quantum) fields?

- **Repn space of Poincaré group:** $L_{\mu\nu}$, p_λ
  Build momentum space reps

$$(\tilde{\phi}|\tilde{\psi}) = \int dp\, \tilde{\phi}^*(p)\tilde{\psi}(p) \quad \text{with} \quad \tilde{\psi}(p) = (p|\psi)$$

Position space repns from: $[x^\mu, p_\nu] = i\delta^\mu{}_\nu$

$$\psi(x) = (x|\psi) = \int dp\, (x|p)(p|\psi) = \int \frac{dp}{\sqrt{2\pi}} e^{ipx}\tilde{\psi}(p)$$

fields: $\psi(x) \rightarrow$ live on joint spectra of $\{x^\mu\}_\mu$

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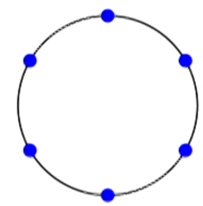
Prelude to Sampling theory

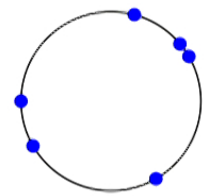
Space of functions/fields on S^1 (length L)

$$\phi(x) = \frac{1}{\sqrt{L}} \sum_{m=-M}^{M} \phi_m e^{\frac{2\pi i}{L} mx}$$

Sample points $x_1, \ldots, x_{2M+1}$



$$\phi(x_n) = \frac{1}{\sqrt{L}} \sum_{m=-M}^{M} \phi_m e^{\frac{2\pi i}{L} mx_n} =: \sum_{m=-M}^{M} A_{nm}\phi_m$$



Generically, $\det A \neq 0$, so can write

$$\phi(x) = \sum_{n=1}^{2M+1} K(x, x_n)\phi(x_n)$$

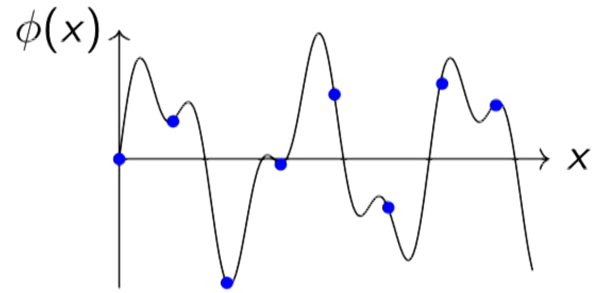
Message: Can keep the smooth manifold **and** get finite min uncertainty in position by restricting the space of functions.

Sampling theory

Interpolation formulas for bandlimited fields ($\Omega = \pi/2\ell_P$)

- Equidistant Nyquist spacing ($x_n = \pi n/\Omega$)

$$\phi(x) = \sum_n \mathrm{sinc}\,[\Omega(x - x_n)]\,\phi(x_n)$$



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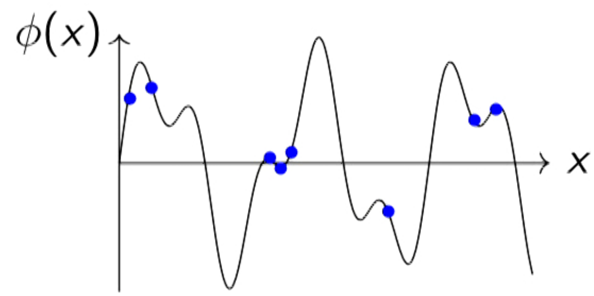


$\rightarrow$ Allows discrete representations **AND** Euclidean symmetries!

Quantization of bandlimited fields

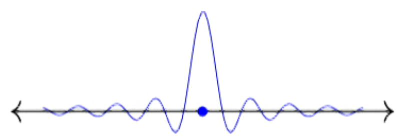
What are the independent degrees of freedom?

- Nyquist-spaced position vectors form ONB on space of (classical) fields. One-param. family of bases cover $\mathbb{R}$:

$$|x_n^{(\alpha)}) \text{ with } x_n^{(\alpha)} = \frac{\pi(n+\alpha)}{\Omega}, n \in \mathbb{Z}, \text{ fixed } \alpha \in [0,1)$$

- Sampling formula

$$\phi(x) = (x|\phi) = \sum_n (x|x_n)(x_n|\phi)$$
$$= \sum_n \text{sinc}\,[\Omega(x - x_n)]\,\phi(x_n)$$

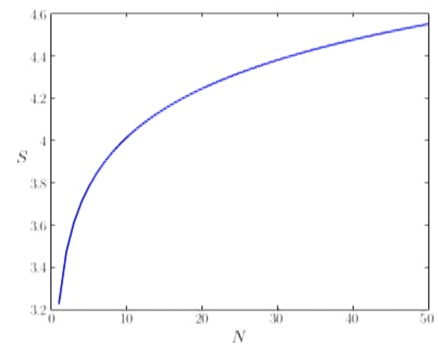


- Hilbert space of the quantum field

$$\mathcal{H} = \otimes_n \mathcal{H}_{n,\alpha} \cong \otimes_n \mathcal{H}_{n,\alpha'}$$

- Non-equidistant lattice: $\mathcal{H} \ncong \otimes_n \mathcal{H}_n$

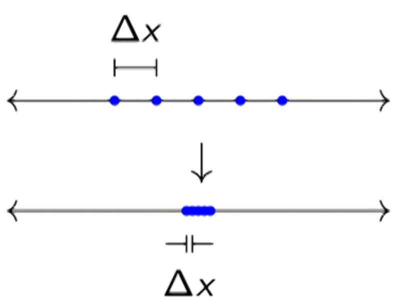
Entanglement Entropy

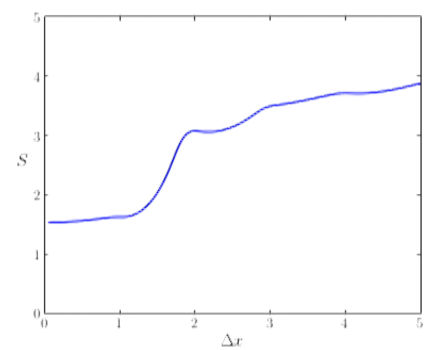


- Entropy scaling with N

$$S(N) \sim \frac{1}{3}\log(N),$$

Dependence on Δx



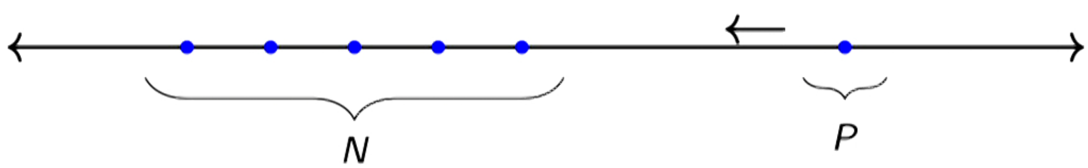


Plateau behaviour

- $S(N)$ does not decrease for $\Delta x < \pi/\Omega$!

Locality of Degrees of Freedom

- Can we use scaling behaviour to infer locality?
- Subsystem of N sample points, and probe point P



- Mutual information

$$I(N:P) = S(N) + S(P) - S(N,P)$$

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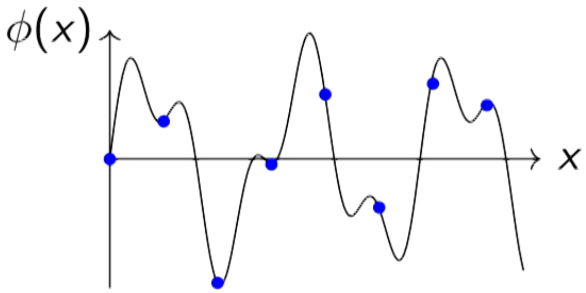

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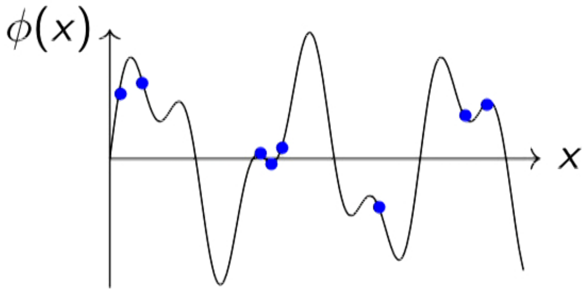
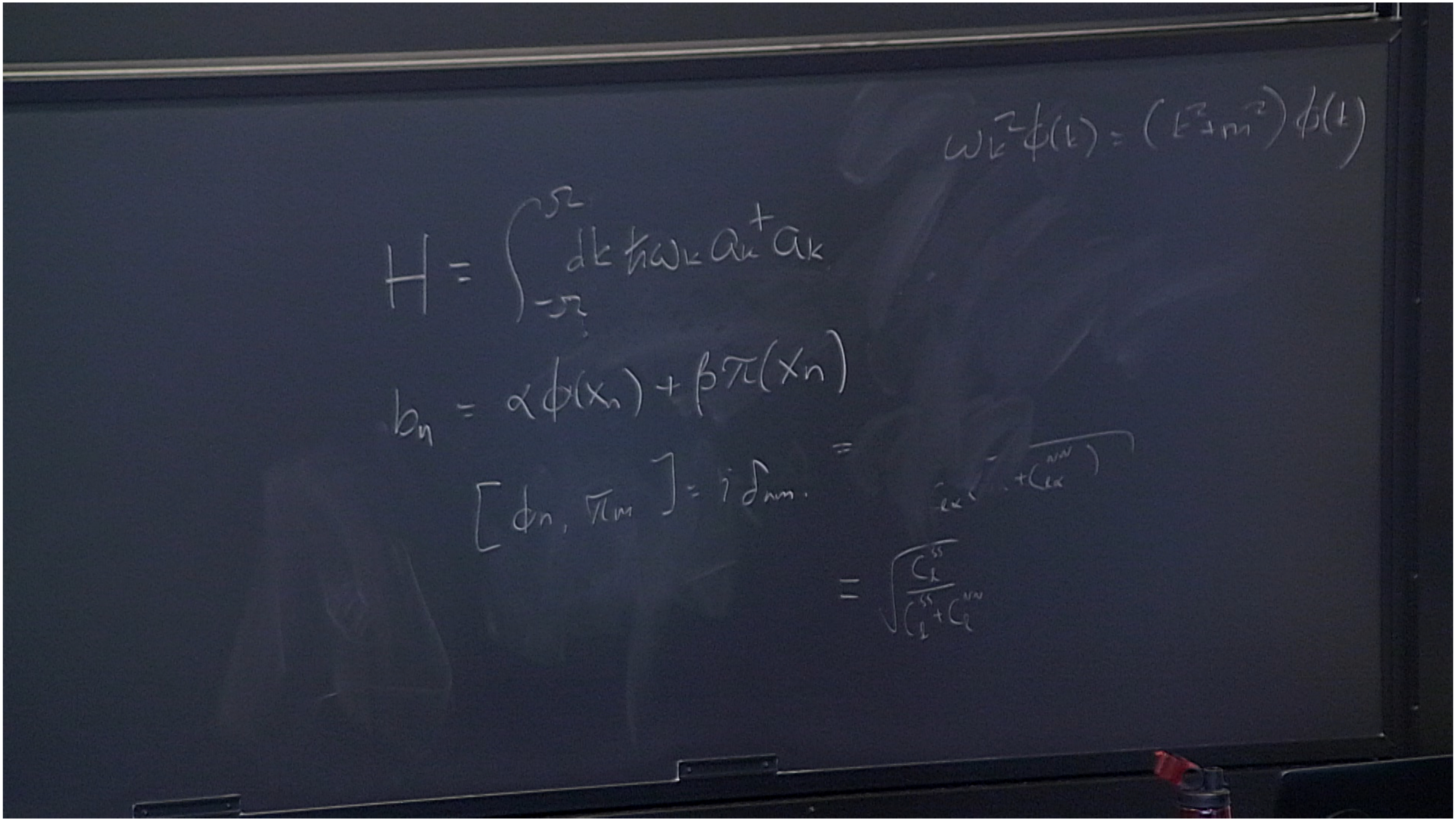


$\rightarrow$ Allows discrete representations **AND** Euclidean symmetries!



$\omega_k^2 \phi(k) = (k^2+m^2)\phi(k)$
$H = \int_{-\Omega}^{\Omega} dk\, \hbar\omega_k a_k^\dagger a_k$
$b_n = \alpha\phi(x_n) + \beta\pi(x_n)$
$[\phi_n, \pi_m] = i\delta_{nm}.$

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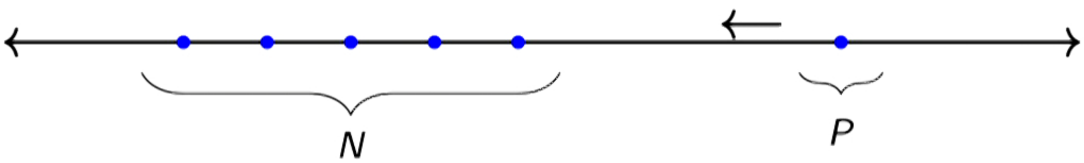

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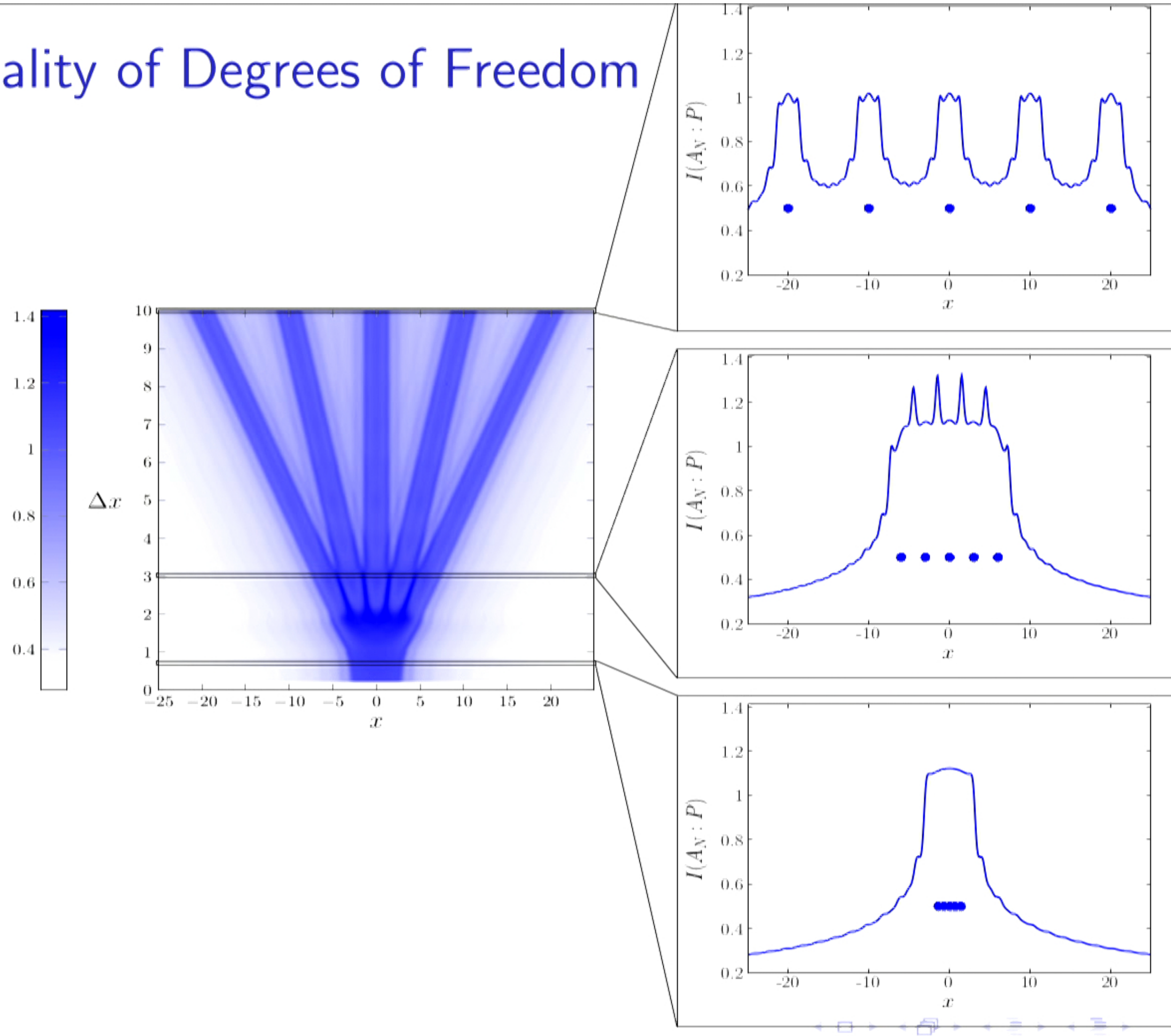

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Covariant GUPs

- How about Lorentz covariance?

$$[\hat{x}_I, \hat{p}^\mu] = i\theta_I{}^\mu(\hat{p}), \quad \text{(assume: } \det\theta \neq 0 \text{ and Jacobi ids)}$$
$$[\hat{x}_I, \hat{x}_J] = 0$$
$$[\hat{p}^\mu, \hat{p}^\nu] = 0$$

- Representation

$$(\hat{x}_I \phi)(p) = i\theta_I{}^\mu \frac{\partial}{\partial p^\mu}\phi(p)$$
$$(\hat{p}^\mu \phi)(p) = p^\mu \phi(p)$$

- Lorentz generators

$$L_{IJ} = x_I \int^p dp'^\mu (\theta^{-1})_{\mu J}(p') - x_J \int^p dp'^\mu (\theta^{-1})_{\mu I}(p')$$

Covariant GUPs → Covariant Bandlimitation

Does this induce bandlimitation as in 1D case?

- Momentum space geometry: $g_{\mu\nu} = (\theta^{-1})_\mu{}^I (\theta^{-1})_\nu{}^J \eta_{IJ}$
  $\therefore x_I = i\theta_I{}^\mu \frac{\partial}{\partial p^\mu}$ are (co-)tetrads
- Geometric constraints?

$$[x_I, x_J] = \mathcal{L}_{x_I}(x_J) \stackrel{!}{=} 0$$

$\implies \exists$ coords k^I s.t. $x_I = i\frac{\partial}{\partial k^I}$

Explicitly,

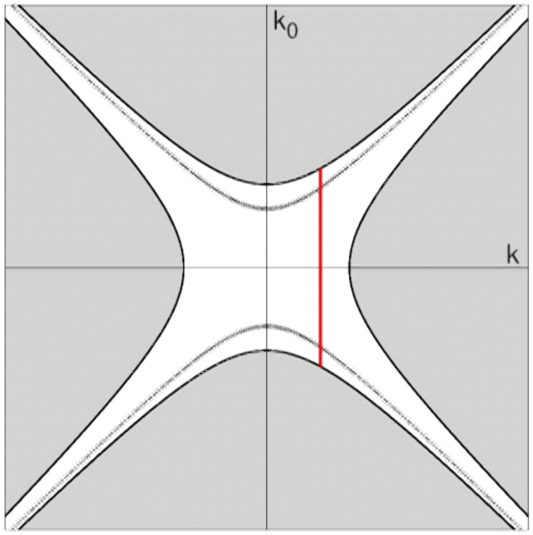
$$k^I = \int_{p_0}^{p} dp'^\mu \theta^I{}_\mu(p')$$

- These coords trivialize GUP algebra

$$[x_I, k^J] = i\delta_I{}^J$$

Covariant Bandlimitation from GUPs

Trivial geometry, but can have different topologies (as in 1D case)
i.e., choose suitable ${\theta_l}^{\mu}(p)$ so that: $|k_0^2 - \mathbf{k}^2| < \Omega^2$



$$\phi_{\mathbf{k}}(t) = \sum_n K(\mathbf{k}; t, t_n)\phi_{\mathbf{k}}(t_n)$$

$$\phi(x) = \int \frac{d\mathbf{k}}{(2\pi)^3} e^{i\mathbf{k}\cdot\mathbf{x}} \phi_{\mathbf{k}}(t)$$

(Kempf, Chatwin-Davies, Martin (2013))

$\rightarrow$ still have infinite density of dofs (Landau-Beurling)

Effects of Covariant Bandlimitation

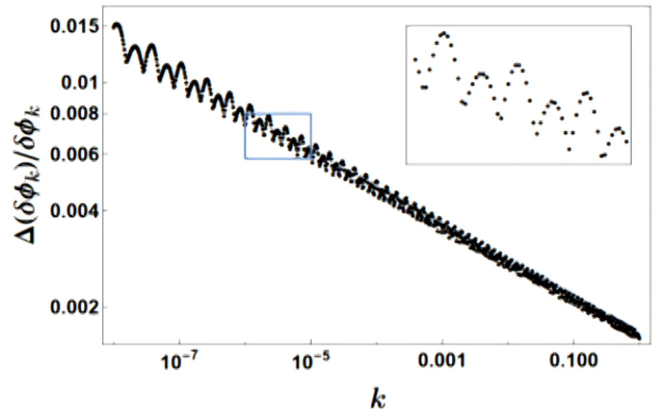
- On-shell quantities unchanged:

$$\langle\phi(x)\phi(y)\rangle, \qquad [\phi(x),\phi(y)]\,, \qquad \langle\{\phi(x),\phi(y)\}\rangle, \qquad \ldots$$

$\rightarrow$ e.g., entanglement entropy divergent, Unruh effect unchanged

- Off-shell quantities modified *(Chatwin-Davies, Kempf, Martin (2017))*: $\Box_g\phi = m^2\phi$

$$\langle T\phi(x)\phi(y)\rangle, \qquad G_R(x,y), \qquad G_A(x,y), \qquad \ldots$$



Conclusions

Euclidean:

- Bandlimitation gives lattice representations while retaining Euclidean symmetries
- Regulation of information density

Lorentzian:

- Lorentzian GUPs give covariant analogue of bandlimitation
- Restriction on off-shell fluctuations
- Effect in interacting theories?
- Further effects on curved spacetimes?