Title: The Cohomology of Groups (Johnson-Freyd/Guo) - Lecture 9

Speakers: Meng Guo, Theo Johnson-Freyd

Collection: The Cohomology of Groups (Johnson-Freyd/Guo)

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URL: http://pirsa.org/19110045

1707.08388 + 1810.00463
Cijt w/ D. Treumann.

Goal: Thm. $H^3(\mathbb{M}; U(1)) \cong \mathbb{Z}_{24} \oplus \underbrace{(\leq 4)}_{\text{prob trivial}}$

$\mathbb{M}$ = Fischer-Griess monster is the largest sporadic simple gp.

$$\#\mathbb{M} = 2^{46} \cdot 3^{20} \cdot 5^{9} \cdot 7^{6} \cdot 13^{3} \cdot 11^{2} \cdot 17 \cdot 19 \cdot 23 \cdot 29 \cdot 31 \cdot 41 \cdot 47 \cdot 59 \cdot 71 \sim 10^{54}$$

$\#\mathbb{M} = 2^{46} \cdot 3^{20} \cdot 5^9 \cdot 7^6 \cdot 13^3 \cdot 11^2 \cdot 17 \cdot 19 \cdot 23 \cdot 29 \cdot 31 \cdot 41 \cdot 47 \cdot 59 \cdot 71 \sim 10^{54}$

remark: $\in \#SL_{15}(\mathbb{F}_2)$.

$H^1(\mathbb{M}, U(1)) = \hom(\mathbb{M}, U(1)) = \{*\}$ because $\mathbb{M}$ is simple

$H^2(\mathbb{M}, U(1)) = \{\text{central extensions}\} = \{*\}$ ~1980s.

$H^3(\mathbb{M}, U(1))$ $\exists$ 't Hooft anomalies of actions of $\mathbb{M}$ on (1+1)d QFTs

Method. work prime by prime. $H^\bullet(\mathbb{M}, U(1))$ is a finite ab. gp.

$\bigoplus_{p \mid \#\mathbb{M}} H^\bullet(\mathbb{M}; U(1))_{(p)}$ "p-part"; order is a power of p

s.t. $\chi(\#\mathbb{M}) = 0$ on this finite gp.

$\mathbb{Z}_p \oplus \mathbb{Z}_p$ or $\mathbb{Z}_{p^2}$ or ...

Sylow theory: $\forall p$. $\exists S \subseteq M$ s.t. $\frac{\#M}{\#S}$ is coprime to p.

$H^\cdot(M; \mathcal{U}(1))_{(p)} \underset{\text{"transfer"}}{\overset{\text{rest}}{\rightleftarrows}} H^\cdot(S, \mathcal{U}(1))_{(p)}$

a sort of integral / push forward

$\circlearrowleft$ = mult by $\frac{\#M}{\#S}$.

is an IO on $H^\cdot_{(p)}$.

rest is an injection onto a direct summand.
on (p)-parts.

rest is an injection onto a direct summand on (p)-parts.

Eg: $p \geq 17$ p-Sylow in M is a $\mathbb{Z}_p$. in fact $M \geq S = \mathbb{Z}_p \rtimes \mathbb{Z}_{\frac{p-1}{2}}$.

$$\begin{pmatrix} * & * \\ 0 & * \end{pmatrix} \in PSL_2(\mathbb{F}_p).$$

$$H^3(M; \mathcal{U}(1))_{(p)} \subseteq H^3(S; \mathcal{U}(1))_{(p)}$$

$\uparrow$ compat w/ S.S.

$$\#\mathbb{M} = 2^{46}\cdot 3^{20}\cdot 5^{9}\cdot 7^{6}\cdot 13^{3}\cdot \underbrace{11^{2}\cdot 17\cdot 19\cdot 23\cdot 29\cdot 31\cdot 41\cdot 47\cdot 59\cdot 71}\sim 10^{54}$$

remark: $\leq \# SL_{13}(\mathbb{F}_2)$.

$$E_2^{i,j} = H^i\Big(\overbrace{\mathbb{Z}_{p-\frac{1}{2}}}^{\text{no (p)s.}} ; \overbrace{H^j(\mathbb{Z}_p ; U(1))}^{\text{purely (p)}}\Big)_{(p)} \Rightarrow H^{i+j}(S ; U(1))_{(p)}$$

if $i>0$, then H^i = "purely p" = "no p's" $= 0$.

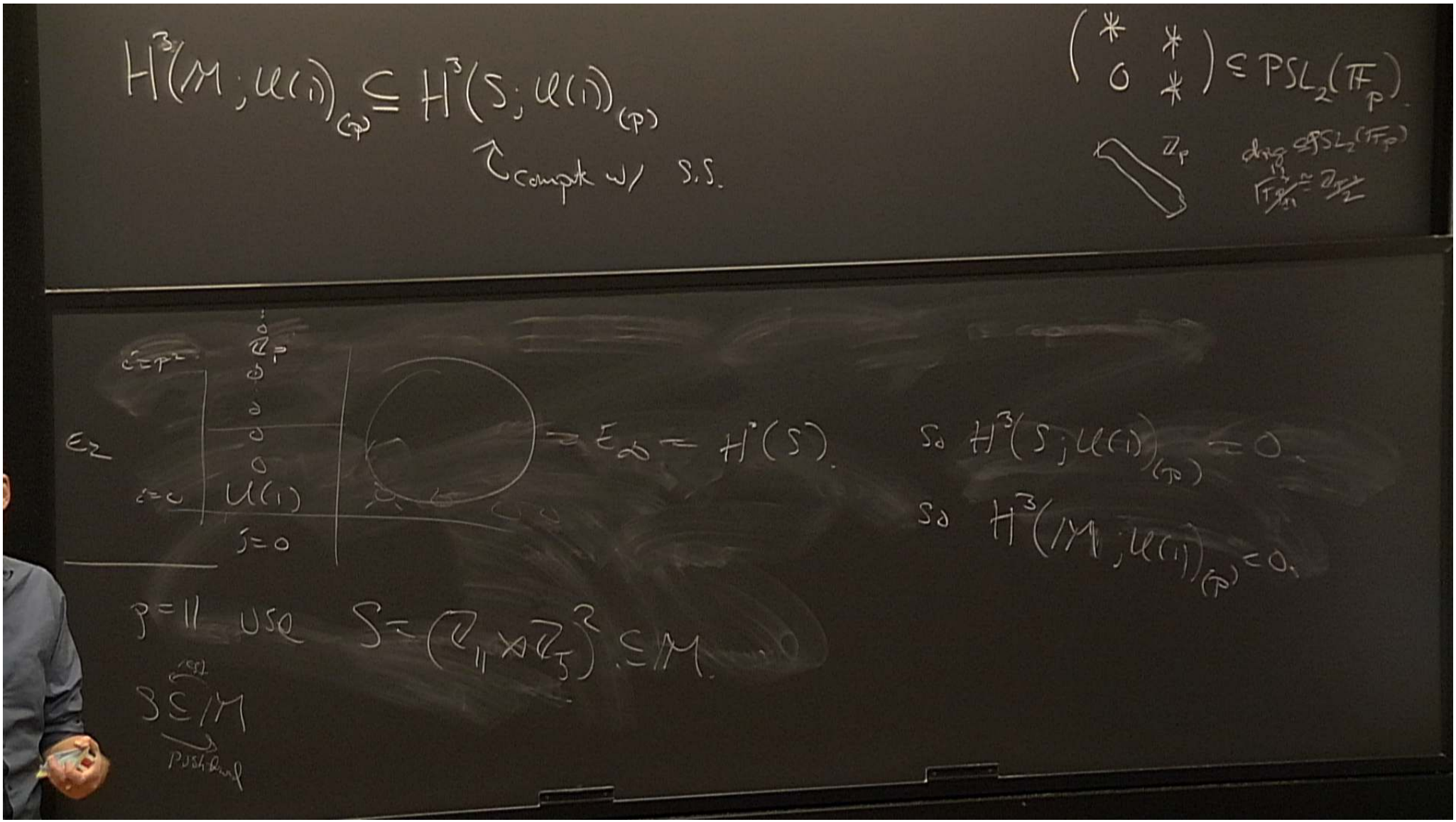
$E_2^{i,j} = \{i=0 \text{ column}\}$

$$H^j(\mathbb{Z}_p ; U(1)) = \begin{cases} \mathbb{Z}_p & \text{if } j \text{ odd} \\ 0 & \text{if } j \text{ even.} \end{cases}$$

$$H^{j+1}(\mathbb{Z}_p ; \mathbb{Z}) = \mathbb{Z}[t]/pt=0 \quad (\deg 2)$$

$\mathbb{Z}_{p-\frac{1}{2}}$ acts faithfully on $\mathbb{Z}_p \cdot t$

acts by k^{th} power of the faithful action on $\mathbb{Z}_p \cdot t^k$


$H^3(M;U(1))_{(p)} \subseteq H^3(S;U(1))_{(p)}$
compute w/ S.S.
$\begin{pmatrix} * & * \\ 0 & * \end{pmatrix} \subseteq PSL_2(\mathbb{F}_p)$
$\mathbb{Z}_p$
$\mathbb{F}_{p+1}^\times \cong \mathbb{Z}_{p+1}$
E_2
$i=p^2$
$\mathbb{Z}_p$
$i=0$
$U(1)$
$j=0$
$= E_\infty = H^\bullet(S)$
So $H^3(S;U(1))_{(p)} = 0$.
So $H^3(M;U(1))_{(p)} = 0$.
$p=11$ use $S = (\mathbb{Z}_{11} \rtimes \mathbb{Z}_5)^2 \subseteq M$.
$S \subseteq M$
push-forward

$$\#\mathbb{M} = 2^{46}\cdot 3^{20}\cdot 5^{9}\cdot 7^{6}\cdot 13^{3}\cdot 11^{2}\cdot 17\cdot 19\cdot 23\cdot 29\cdot 31\cdot 41\cdot 47\cdot 59\cdot 71 \sim 10^{54}$$

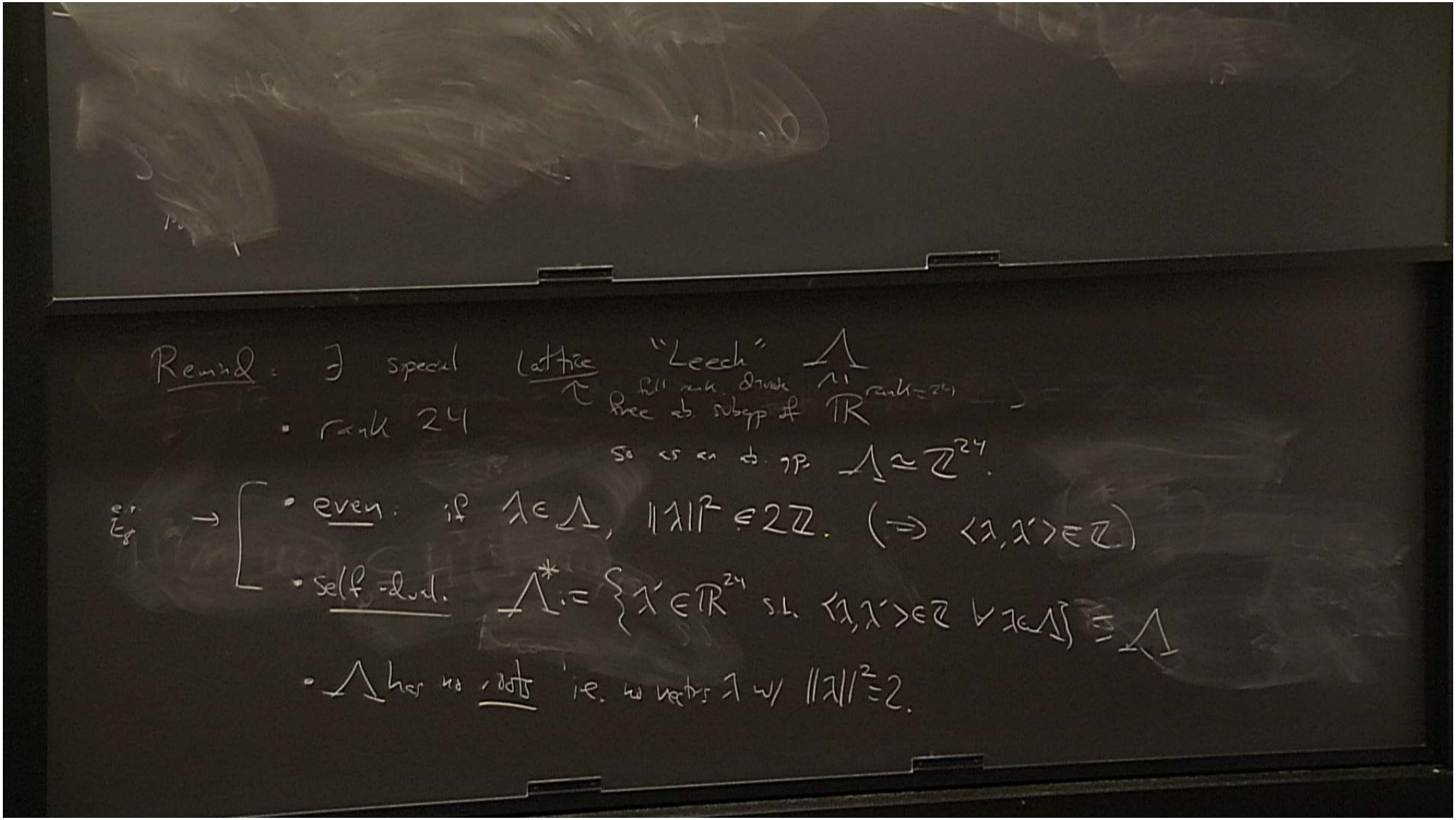
✓

remark: $\in \#SL_{13}(\mathbb{F}_2)$.

The primes $p = 2, 3, 5, 7, 13$ are special. (cf. Conway–Norton)
Monstrous moonshine conjectures

- there are non-conjugate cyclic subgps of $\mathbb{M}$ of order p
- p-Sylow in $\mathbb{M}$ is nonabelian.
- $p-1$ divides 24.

Why? I don't know. But I know the main ingredient.



Remind: $\exists$ special lattice "Leech" Λ

full rank, discrete $\wedge$ free ab. subgp of $\mathbb{R}^{\text{rank}=24}$

- rank 24

so as an ab. gp. $\Lambda \simeq \mathbb{Z}^{24}$.

e.g. E_8 $\rightarrow$

- even: if $\lambda \in \Lambda$, $\|\lambda\|^2 \in 2\mathbb{Z}$. $(\Rightarrow \langle \lambda, \lambda' \rangle \in \mathbb{Z})$
- self-dual. $\Lambda^* := \{\lambda' \in \mathbb{R}^{24} \text{ s.t. } \langle \lambda, \lambda' \rangle \in \mathbb{Z} \ \forall \lambda \in \Lambda\} = \Lambda$
- Λ has no roots i.e. no vectors λ w/ $\|\lambda\|^2 = 2$.

$\Leftrightarrow \langle \lambda, \lambda' \rangle \in \mathbb{Z}$

• self-dual. $\Lambda^* := \{ \lambda' \in \mathbb{R}^{24} \text{ s.t. } \langle \lambda, \lambda' \rangle \in \mathbb{Z} \ \forall \lambda \in \Lambda \} = \Lambda$

• Λ has no roots i.e. no vectors λ w/ $\|\lambda\|^2 = 2$.

• $O(\Lambda) \subseteq O(\mathbb{R}^{24}) = C_{00} = 2 C_{01}$, C_{01} is sporadic simple.

Given even + self dual lattice,

$\rightsquigarrow$ holomorphic bosonic (1+1)d CFT

no right movers

V_Λ

$c_L = \text{rank} = 24$

$c_R = 0$.

$\sim$ EOM: holomorphic map from (1+1)d spacetime to $\mathbb{R}^{24}/\Lambda = T$

abelian WZW model.

Some manifest automorphisms of V_Λ

for Leech, these are all the automorphisms

but for a general lattice, there are more

in particular: every root in Λ generates syms.

• $T = \mathbb{R}^{24}/\Lambda$ acts by translation;

• $O(\Lambda)$ acts by rotation.

$\exists$ anomaly to lifting these classical symmetries to quantum thy.

$\rightsquigarrow$ actually $\mathrm{Aut}(V_\Lambda) \supseteq T \cdot O(\Lambda)$. non-split.

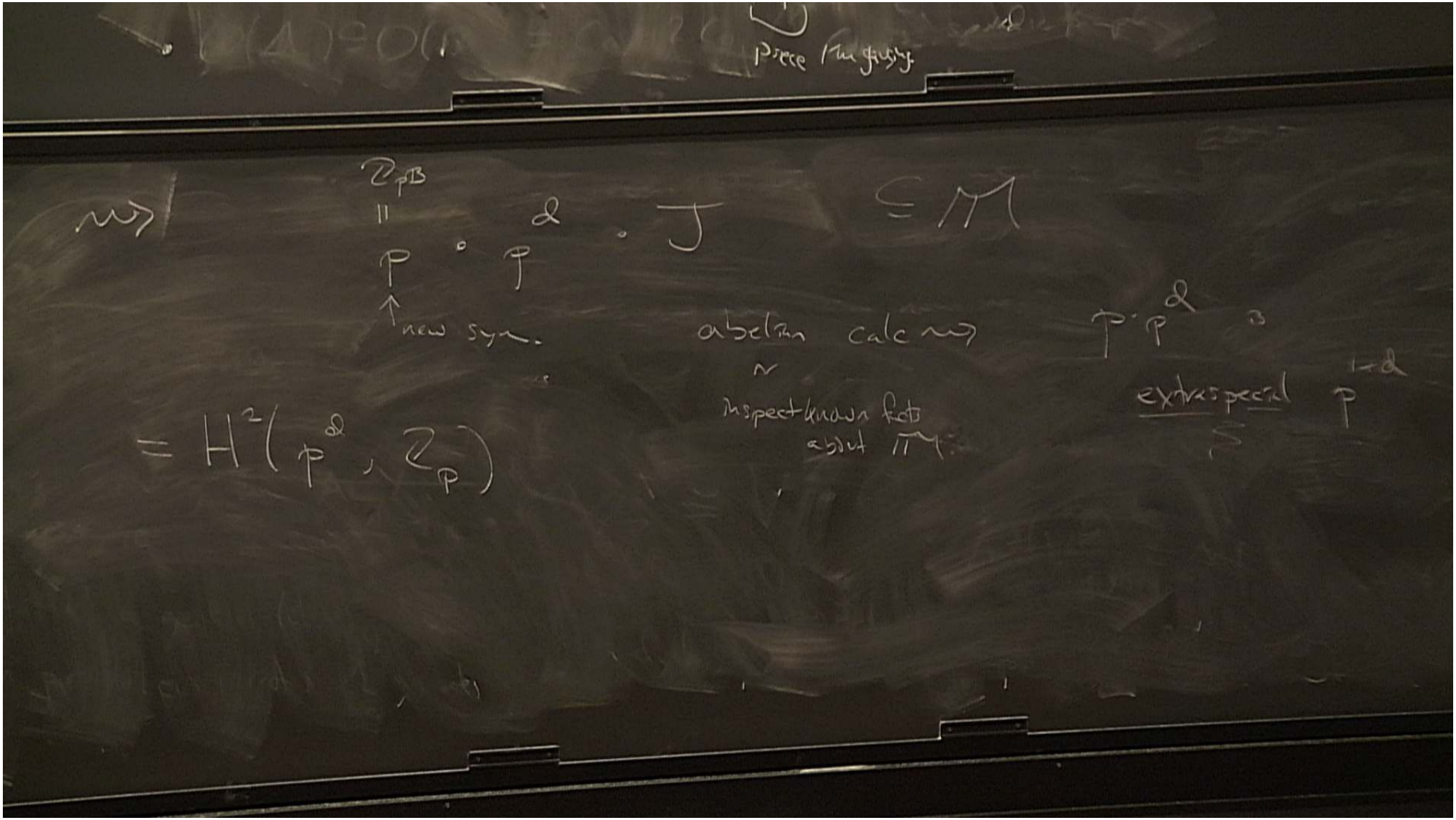
$$\#\mathbb{M} = 2^{46}\cdot 3^{20}\cdot 5^{9}\cdot 7^{6}\cdot 13^{3}\cdot \underbrace{11^{2}\cdot 17\cdot 19\cdot 23\cdot 29\cdot 31\cdot 41\cdot 47\cdot 59\cdot 71}_{\checkmark} \sim 10^{54}$$

remark: $\in \#SL_{13}(\mathbb{F}_2)$.

The primes $p \in \{2, 3, 5, 7, 13\}$ are special. (cf. Conway–Norton Monstrous moonshine conjectures)

- $\exists\, g \in O(\Lambda)$ of order p s.t. $g \subset \mathbb{R}^{24}$ has no (nonzero) fixed pts
  Unique up to conjugacy.
- $\mathbb{Z}_p \subseteq O(\Lambda)$ means this $\mathbb{Z}_p$ lifts in a unique-up-to-conjugacy way to $\mathbb{Z}_p \subseteq \mathrm{Aut}(V_\Lambda)$.

In fact, to a monstrous auto.



piece / ungauging

$\rightsquigarrow$ $\mathbb{Z}_{p^B} = p \cdot p^d \cdot J \subseteq M$

$\uparrow$ new sym.

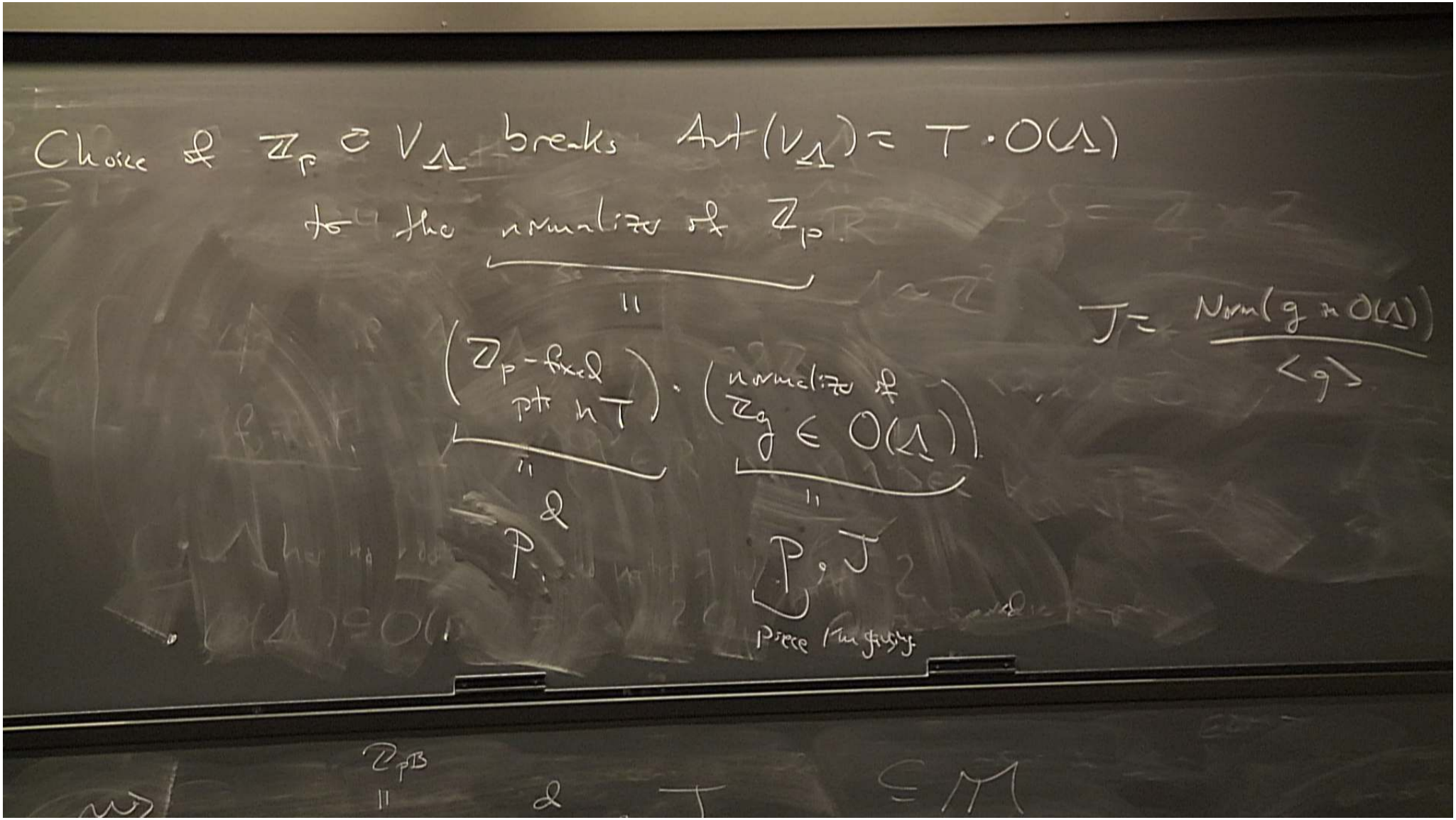
abelian calc $\rightsquigarrow$ $p \cdot p^d$

$\sim$

inspect known facts about M

extraspecial p^{1+d}

$= H^2(p^d, \mathbb{Z}_p)$



Choice of $\mathbb{Z}_p \circlearrowright V_\Lambda$ breaks $\mathrm{Aut}(V_\Lambda) = T \cdot O(\Lambda)$

to the $\underbrace{\text{normalizer of } \mathbb{Z}_p}$.

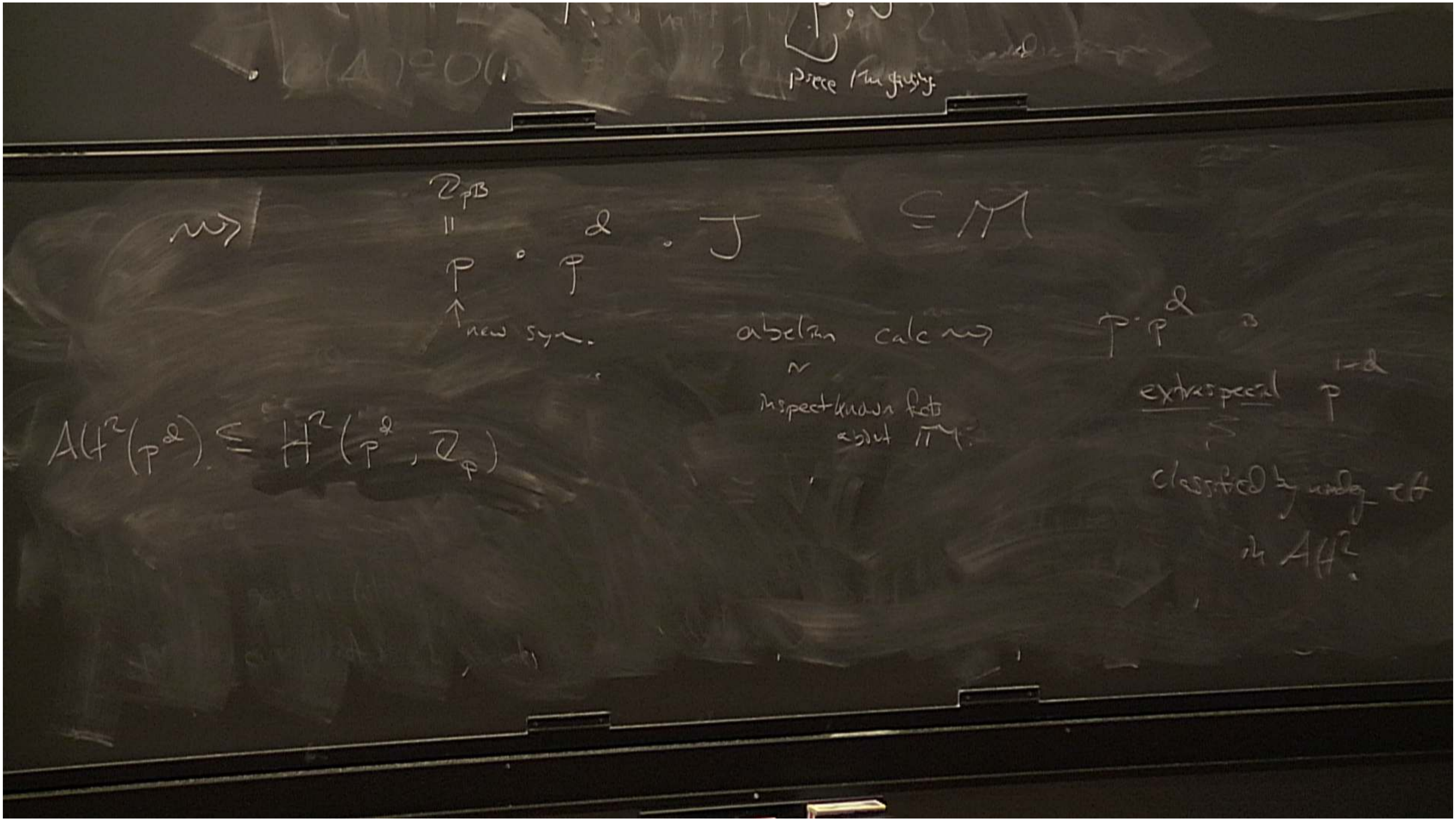
$$= \underbrace{\left(\begin{array}{c}\mathbb{Z}_p\text{-fixed} \\ \text{pts in } T\end{array}\right)}_{= P} \cdot \underbrace{\left(\begin{array}{c}\text{normalizer of} \\ \mathbb{Z}_g \in O(\Lambda)\end{array}\right)}_{= P \cdot J}$$

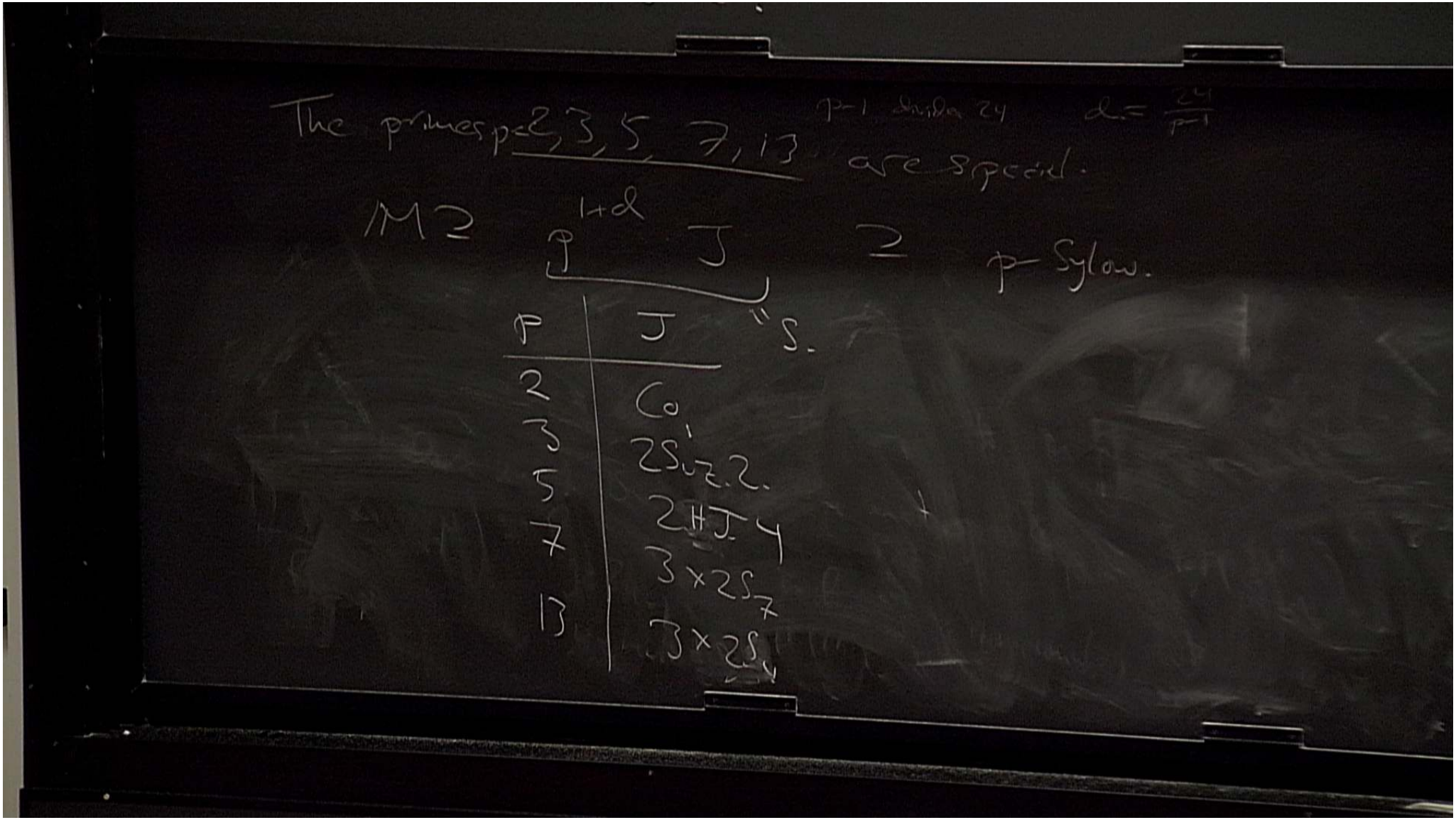
P & P piece lin gauging

$$J = \frac{\mathrm{Norm}(g \text{ in } O(\Lambda))}{\langle g \rangle}$$

$\mathbb{Z}_{pB}$

$\|$ & T $\subseteq M$


new sym.
abelian calc
inspect known facts
extraspecial
classified by
$\mathrm{AH}^2(p^d) \subseteq H^2(p^d, \mathbb{Z}_p)$

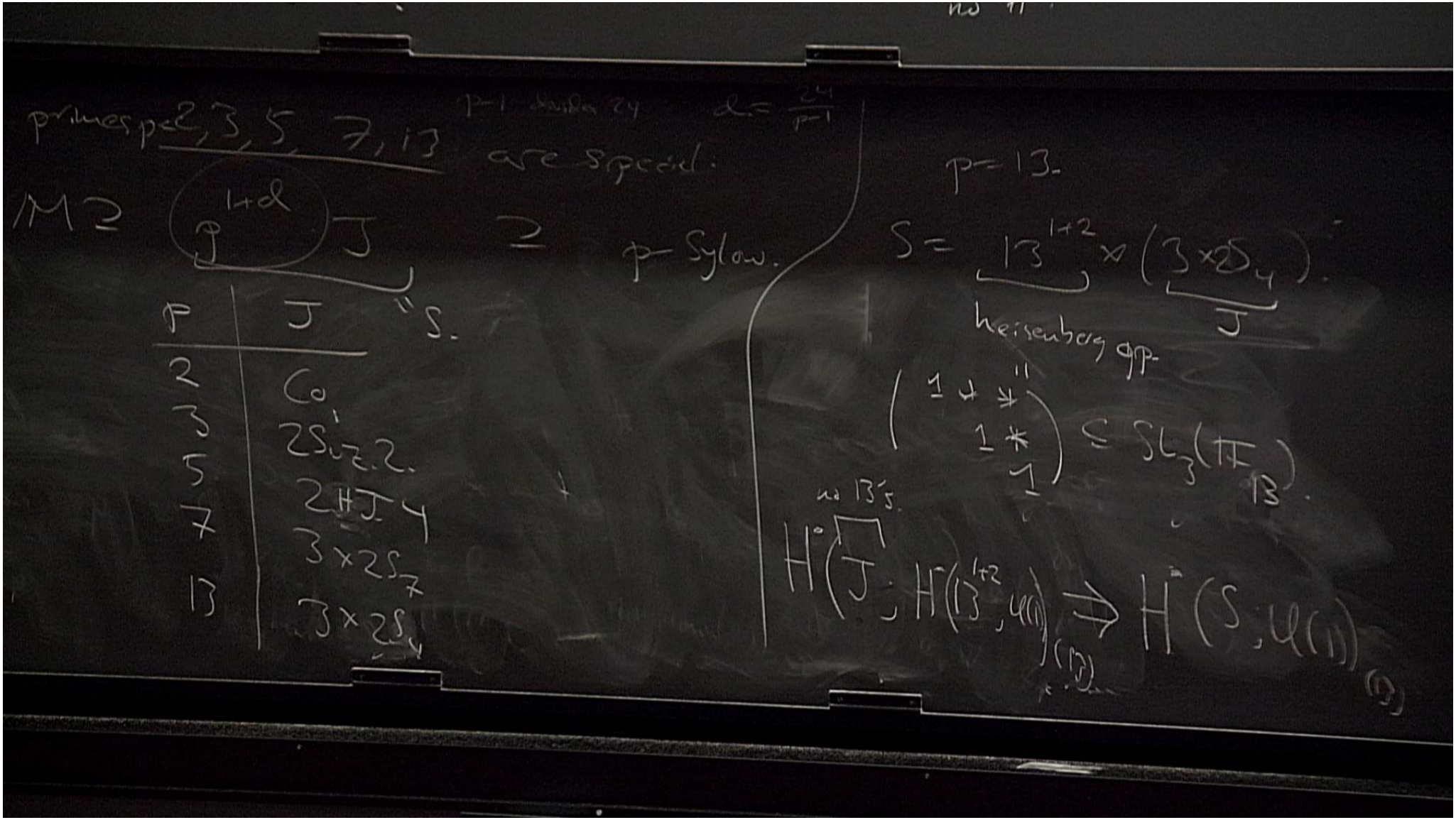


The primes $p = 2, 3, 5, 7, 13$ are special.

$p-1$ divides 24 $\quad d = \frac{24}{p-1}$

$$M \supseteq \underbrace{p^{1+d} \quad J}_{\text{"}S.} \supseteq p\text{-Sylow.}$$

p	J
2	Co_1
3	$2Suz.2.$
5	$2HJ.4$
7	$3 \times 2S_7$
13	$3 \times 2S_4$



primes $p = 2, 3, 5, 7, 13$ are special.

$p-1$ divides 24 $\quad d = \frac{24}{p-1}$

$$M \supseteq \underbrace{p^{1+d} J}_{= S} \supseteq p\text{-Sylow.}$$

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13	$3 \times 2S_4$

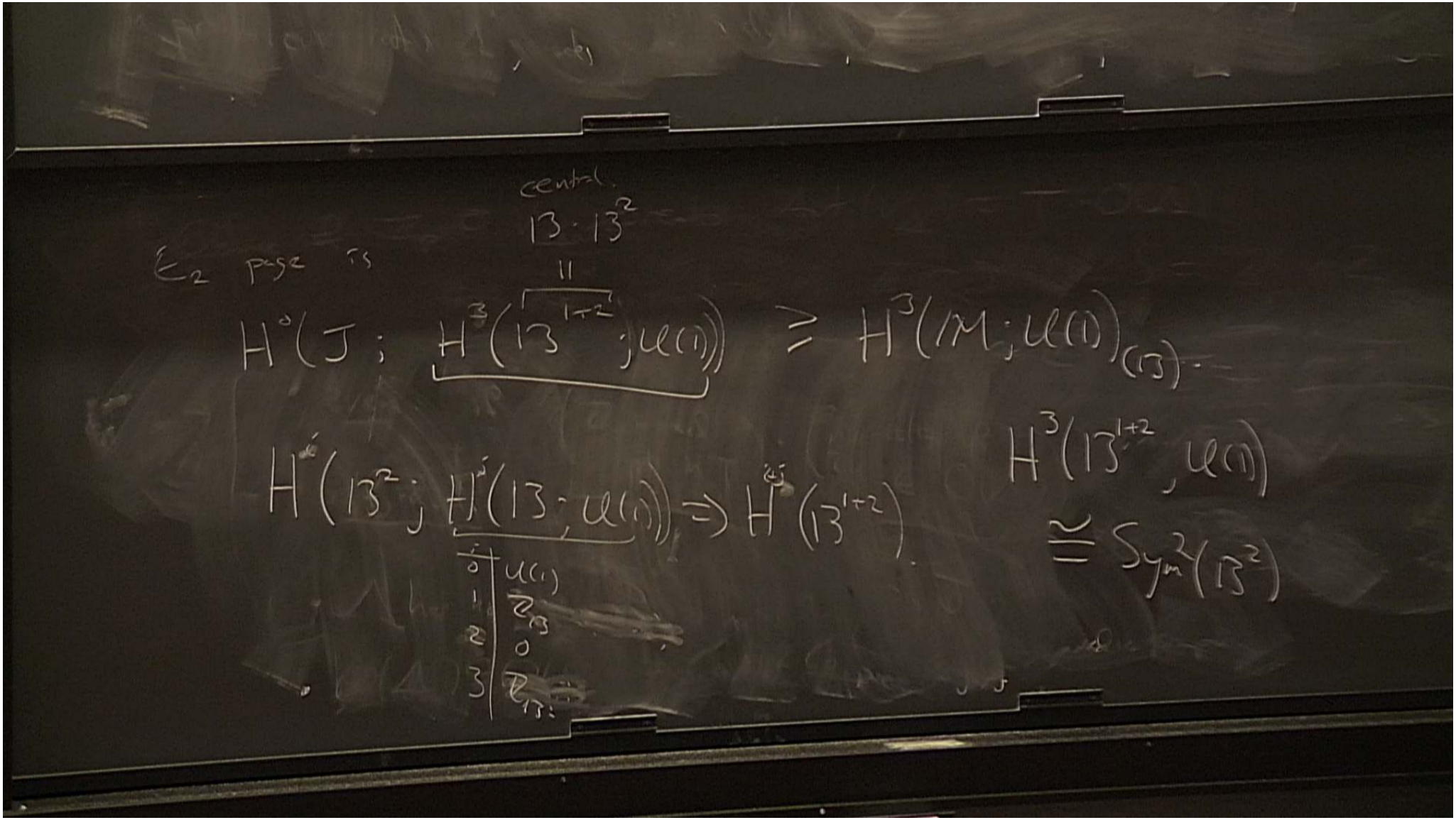
$p = 13$.

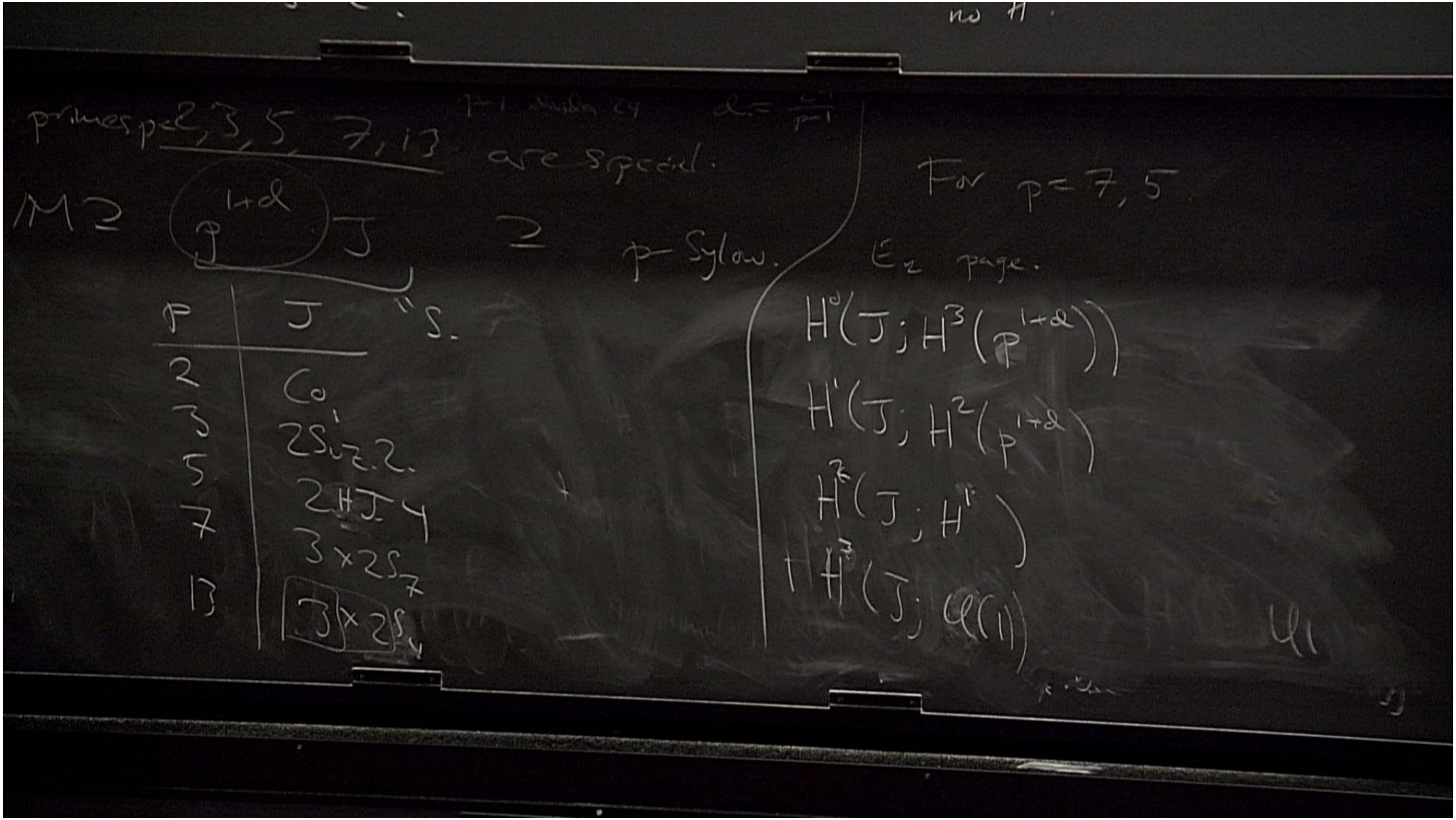
$$S = \underbrace{13^{1+2}}_{\text{heisenberg gp.}} \rtimes (\underbrace{3 \times 2S_4}_{J}).$$

$$\begin{pmatrix} 1 & * & * \\ & 1 & * \\ & & 1 \end{pmatrix} \subseteq SL_3(\mathbb{F}_{13}).$$

no 13's.

$$H^\bullet(J; H^\bullet(13^{1+2}; U(1))_{(13)}) \Rightarrow H^\bullet(S; U(1))_{(13)}$$


central
$13 \cdot 13^2$
E_2 page is
$H^0(J; H^3(13^{1+2}; \mathcal{U}(1))) \cong H^3(M; \mathcal{U}(1))_{(13)}$
$H^3(13^{1+2}, \mathcal{U}(1))$
$\cong Sym^2(13^2)$
$H^i(13^2; H^j(13; \mathcal{U}(1))) \Rightarrow H^{i+j}(13^{1+2})$
$\mathcal{U}(1)$
$\mathbb{Z}_{13}$
0
$\mathbb{Z}_{13}$



no H.

primes $p = 2, 3, 5, 7, 13$ are special.

$p-1$ divides 24 $\quad d = \frac{2^{-1}}{p-1}$

$M \supseteq \underbrace{p^{1+d} J}_{= S} \supseteq$ p-Sylow.

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2	Co_1
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13	$\boxed{3} \times 2S_4$

For $p = 7, 5$

E_2 page.

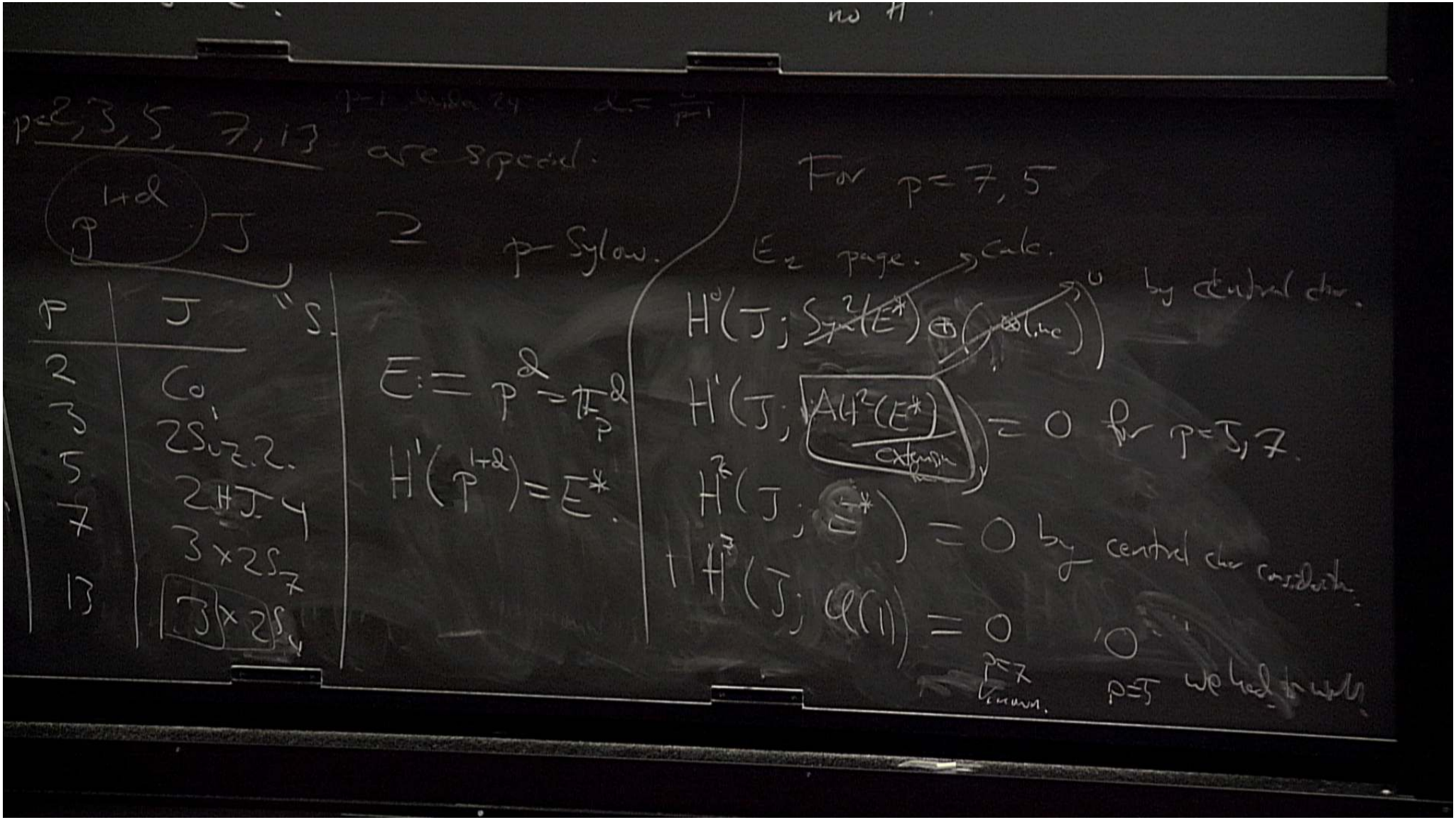
$H^0(J; H^3(p^{1+d}))$

$H^1(J; H^2(p^{1+d})$

$H^2(J; H^1)$

$H^3(J; U(1))$

$U(1$



no H.

$p=2,3,5,7,13$ are special.

p^{1+d} J $\supseteq$ p-Sylow.

p	J
2	Co_1
3	$2Suz.2$
5	$2HJ.4$
7	$3\times 2S_7$
13	$\boxed{3}\times 2S_4$

$E := p^d = \mathbb{F}_p^d$

$H^1(p^{1+d}) = E^*$.

For $p=7,5$

E_2 page. calc.

$H^0(J; \cancel{Sym^2(E^*)} \oplus (\cancel{\otimes \text{line}}))$ 0 by central char.

$H^1(J; \boxed{\frac{Alt^2(E^*)}{\text{extension}}}) = 0$ for $p=5,7$.

$H^2(J; E^*) = 0$ by central char consideration.

$H^3(J; \mathbb{Q}(1)) = 0$ $p=7$ known. 0 $p=5$ we had to work.

$\# SL_{13}(\mathbb{F}_2)$.

$\times \quad \times \quad \times \quad \underbrace{\quad}_{\text{done, no } H^3} \sim 10$

$p = 2, 3, 5, 7, 13$ are special.

$p^{1+d} J \supseteq$ p-Sylow.

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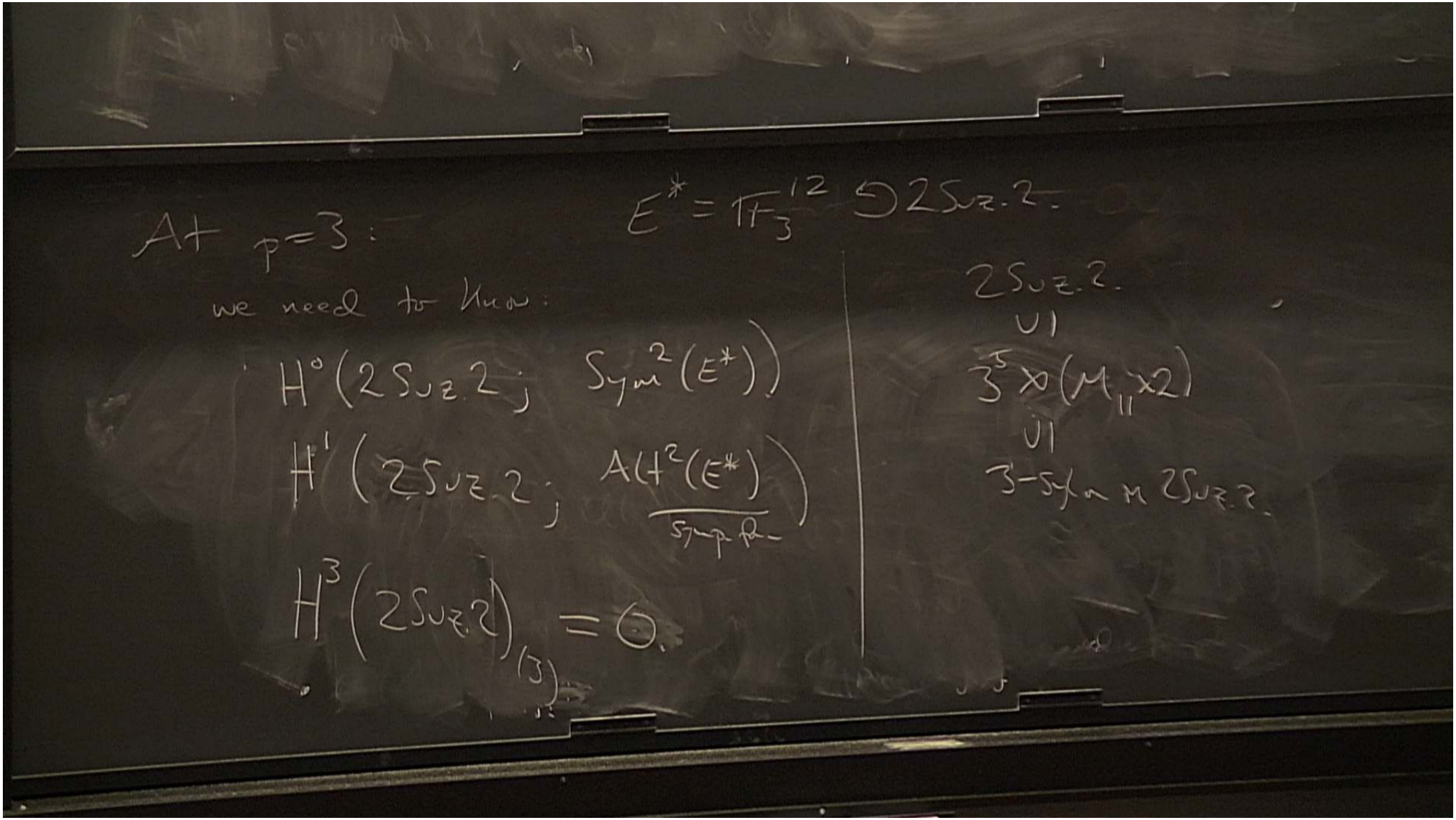
$H^2(J; E^*) = 0$ by central char consideration.

$H^3(J; \mathbb{Q}(1)) = 0$ 0

$p = 7$



At $p=3$:

$E^* = \mathbb{F}_3^{12} \circlearrowleft 2\mathrm{Suz}.2$.

we need to know:

$H^0(2\mathrm{Suz}.2;\ \mathrm{Sym}^2(E^*))$

$H^1(2\mathrm{Suz}.2;\ \underbrace{\mathrm{Alt}^2(E^*)}_{\text{symp. form}})$

$H^3(2\mathrm{Suz}.2)_{(3)} = 0.$

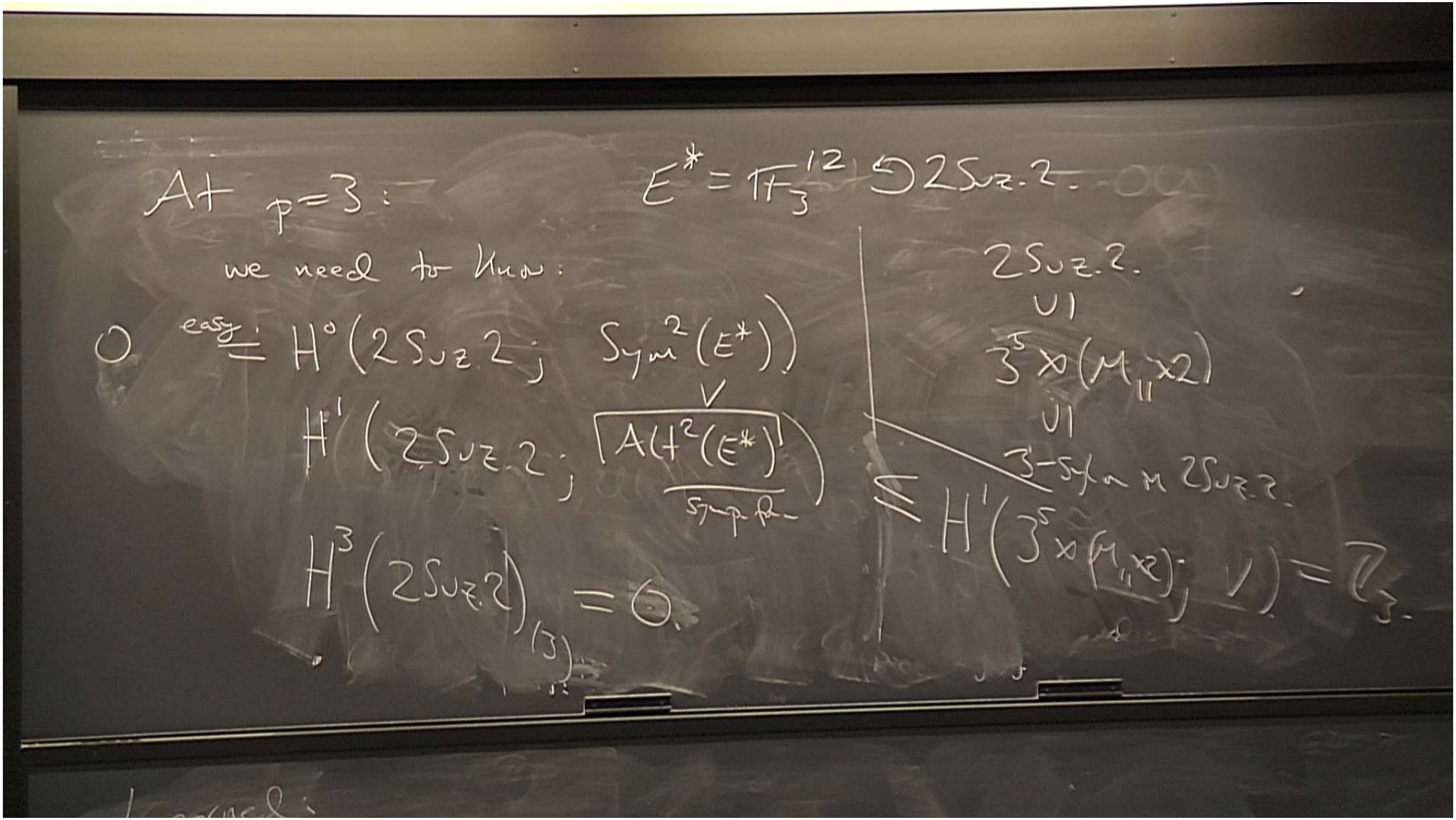
$2\mathrm{Suz}.2$

$\cup\!|$

$3^5 \rtimes (M_{11} \times 2)$

$\cup\!|$

3-Sylow in $2\mathrm{Suz}.2$.



At $p=3$: $\quad E^* = \mathbb{F}_3^{12} \circlearrowleft 2\mathrm{Suz}.2.$

we need to know:

$$0 \overset{\text{easy}}{=} H^0(2\mathrm{Suz}.2;\ \mathrm{Sym}^2(E^*))$$

$$\vee$$

$$H^1(2\mathrm{Suz}.2;\ \underbrace{\boxed{\mathrm{Alt}^2(E^*)}}_{\text{symp. form}})$$

$$H^3(2\mathrm{Suz}.2)_{(3)} = 0.$$

$$2\mathrm{Suz}.2.$$

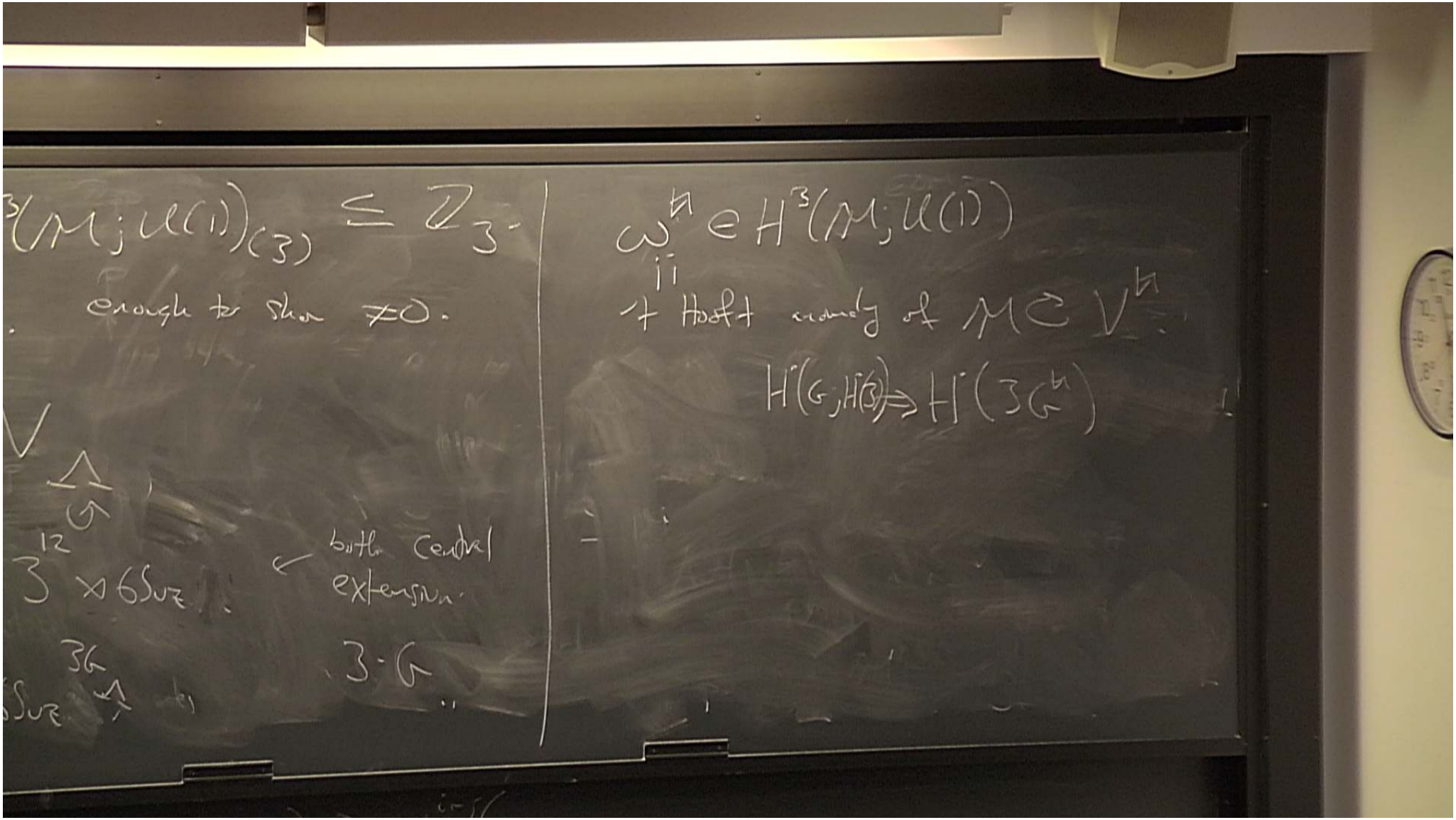
$$\cup\!|$$

$$3^5 \rtimes (M_{11} \times 2)$$

$$\cup\!|$$

$$3\text{-Sylow in } 2\mathrm{Suz}.2.$$

$$\leq H^1(3^5 \rtimes (M_{11} \times 2);\ \vee) = \mathbb{Z}_3.$$



$^3(M; U(1))_{(3)} \leq \mathbb{Z}_3$.

enough to show $\neq 0$.

V

Λ

$3^{12} \rtimes 6Suz$ ← both central extension.

$3G$

$6Suz$ Λ

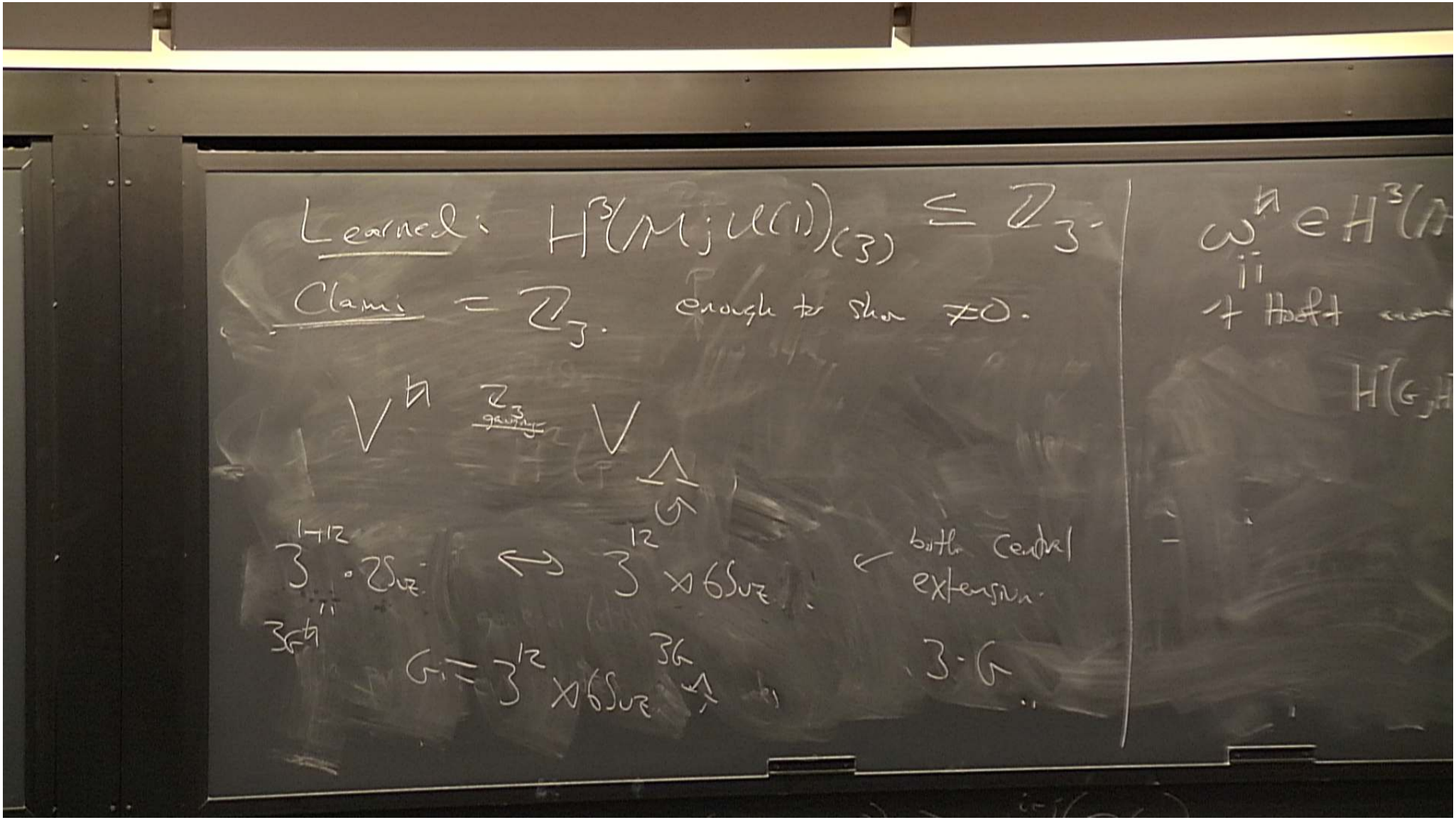
$3 \cdot G$

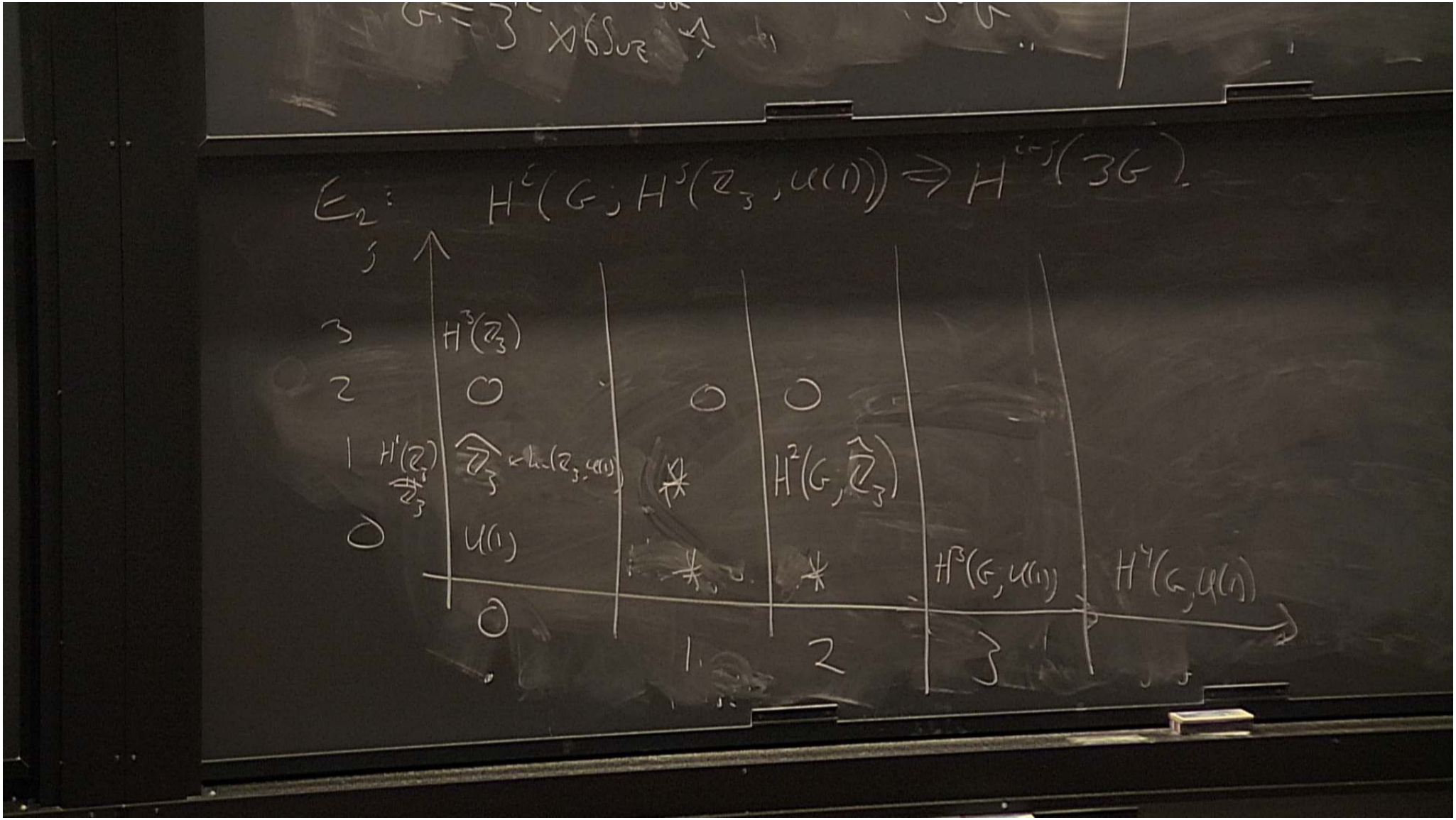
$\omega^\natural \in H^3(M; U(1))$

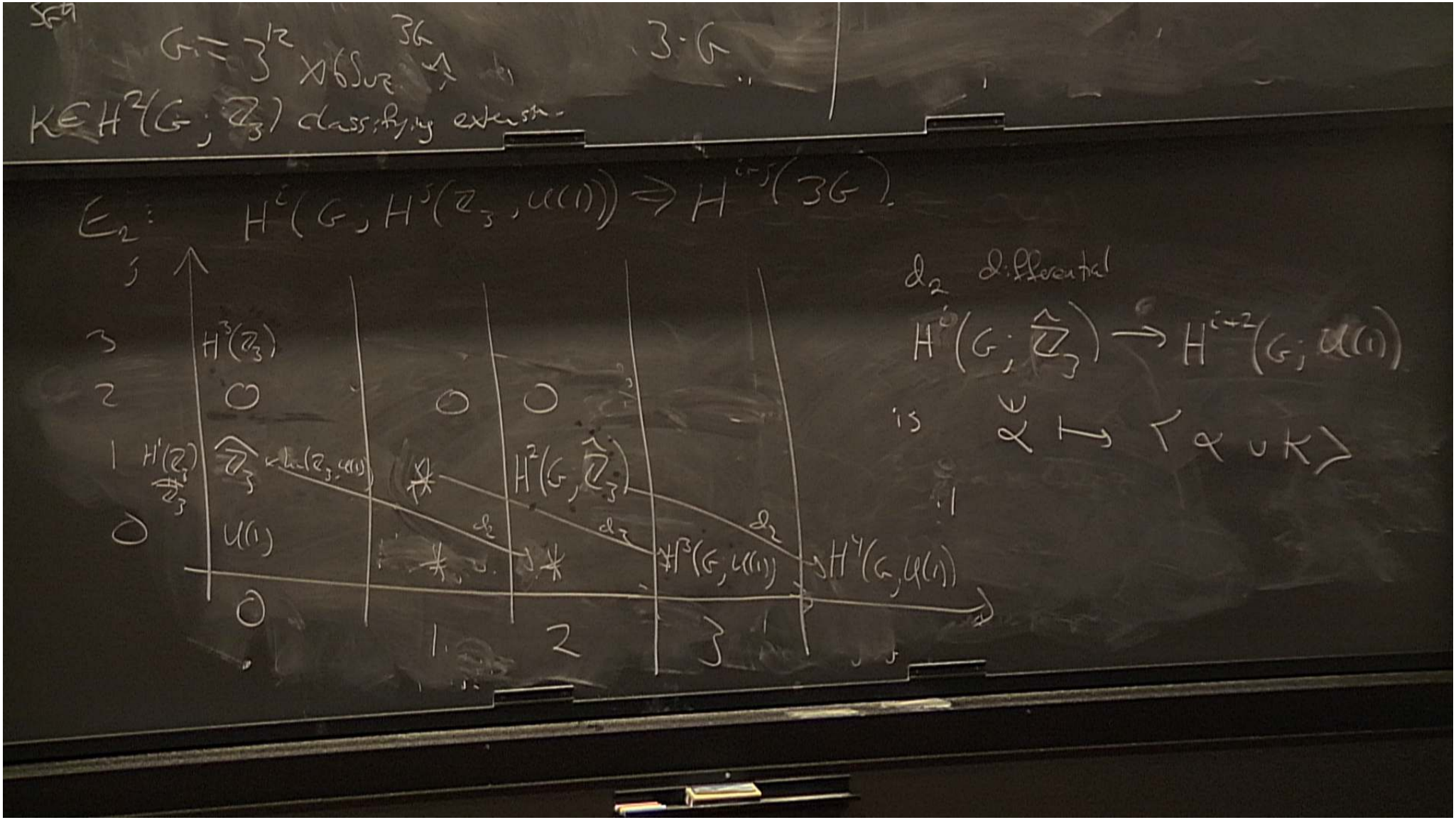
$\shortparallel$

't Hooft anomaly of $M \circlearrowright V^\natural$

$H^\cdot(G; H^\cdot(3)) \Rightarrow H^\cdot(3G^\natural)$


Learned: $H^3(M; U(1))_{(3)} \leq \mathbb{Z}_3$.
Claim: $= \mathbb{Z}_3$. enough to show $\neq 0$.
$V^{\uparrow}$ $\mathbb{Z}_3$ gauging V
$\uparrow G$
$3^{1+12} \cdot 2S_{U_2}$ $\leftrightarrow$ $3^{12} \rtimes 6S_{U_2}$ ← both central extensions
$3G$
$G = 3^{12} \rtimes 6S_{U_2}$ $3G$ $3 \cdot G$
$\omega_3^{\uparrow} \in H^3(M$
't Hooft
$H(G,$


E_2: $H^i(G; H^j(\mathbb{Z}_3, U(1))) \Rightarrow H^{i+j}(3G)$
j
3
2
1
0
$H^3(\mathbb{Z}_3)$
0
$H^1(\mathbb{Z}_3, \mathbb{Z}_3)$
$\mathbb{Z}_3$
$U(1)$
0
0
$H^2(G, \mathbb{Z}_3)$
$H^3(G, U(1))$
$H^4(G, U(1))$
0
1
2
3



$G = 3^{12} \rtimes 6Sur$ $\quad 3G \quad 3 \cdot G$

$K \in H^2(G; \mathbb{Z}_3)$ classifying extension

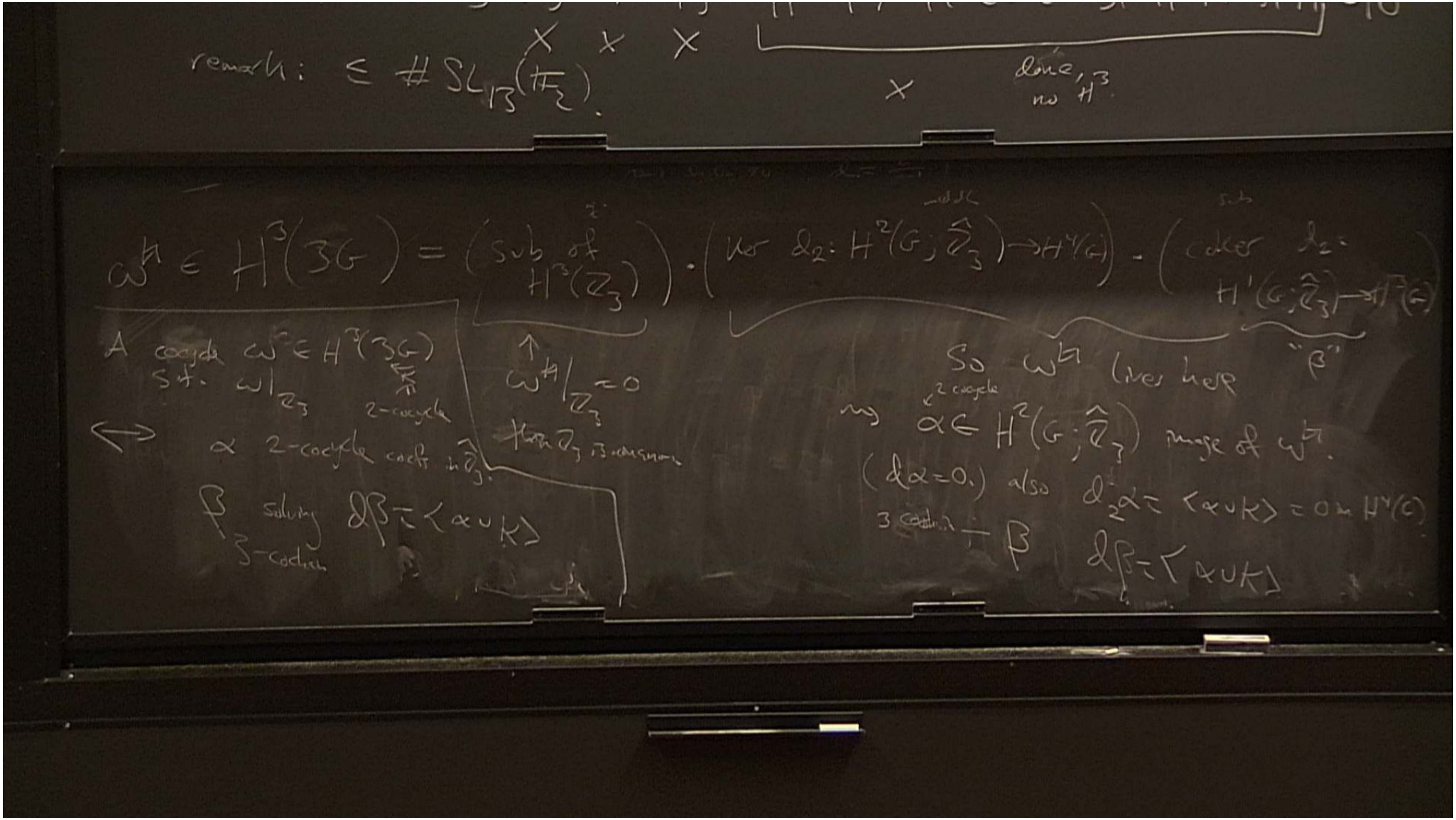
E_2: $\quad H^i(G, H^j(\mathbb{Z}_3, U(1))) \Rightarrow H^{i+j}(3G)$

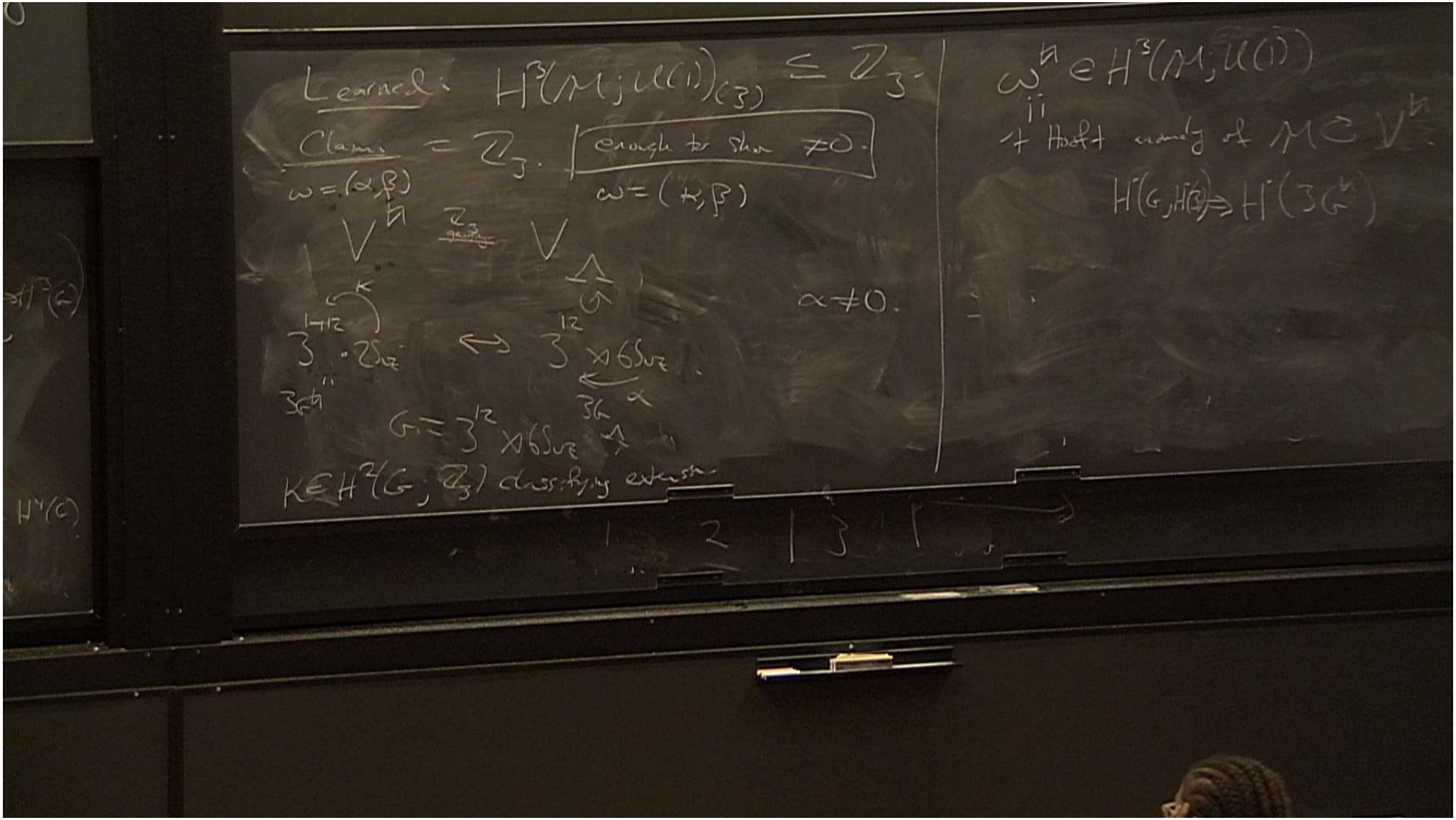
j					
3	$H^3(\mathbb{Z}_3)$				
2	0	0	0		
1	$H^1(\mathbb{Z}_3, \hat{\mathbb{Z}}_3)$ $\hat{\mathbb{Z}}_3 \leftarrow \mathrm{Hom}(\mathbb{Z}_3, U(1))$	$*$	$H^2(G, \hat{\mathbb{Z}}_3)$		
0	$U(1)$	$*$	$*$	$H^3(G, U(1))$	$H^4(G, U(1))$
	0	1	2	3	

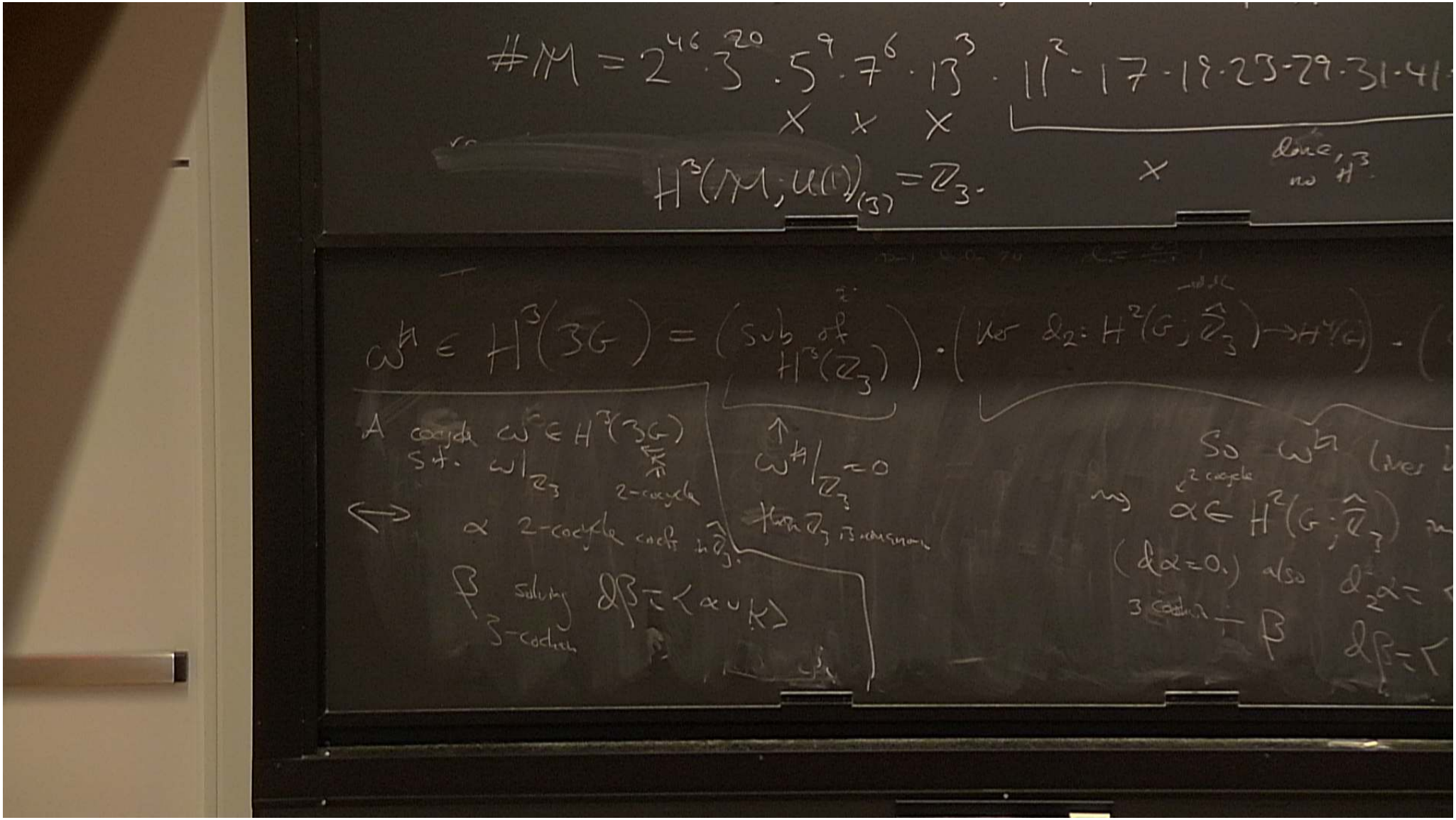
d_2 differential

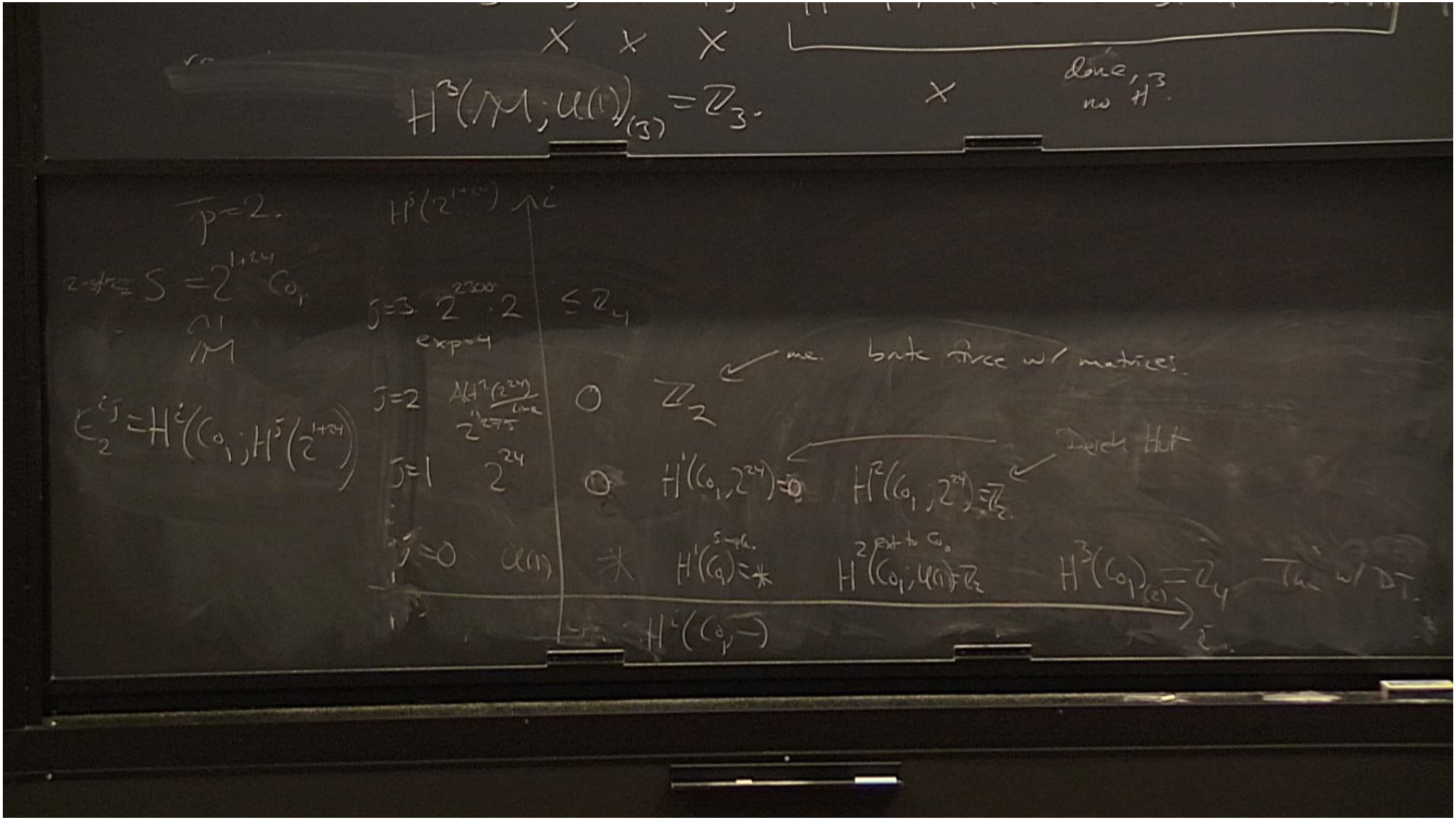
$H^i(G; \hat{\mathbb{Z}}_3) \longrightarrow H^{i+2}(G; U(1))$

is $\quad \alpha \mapsto \langle \alpha \cup K \rangle$

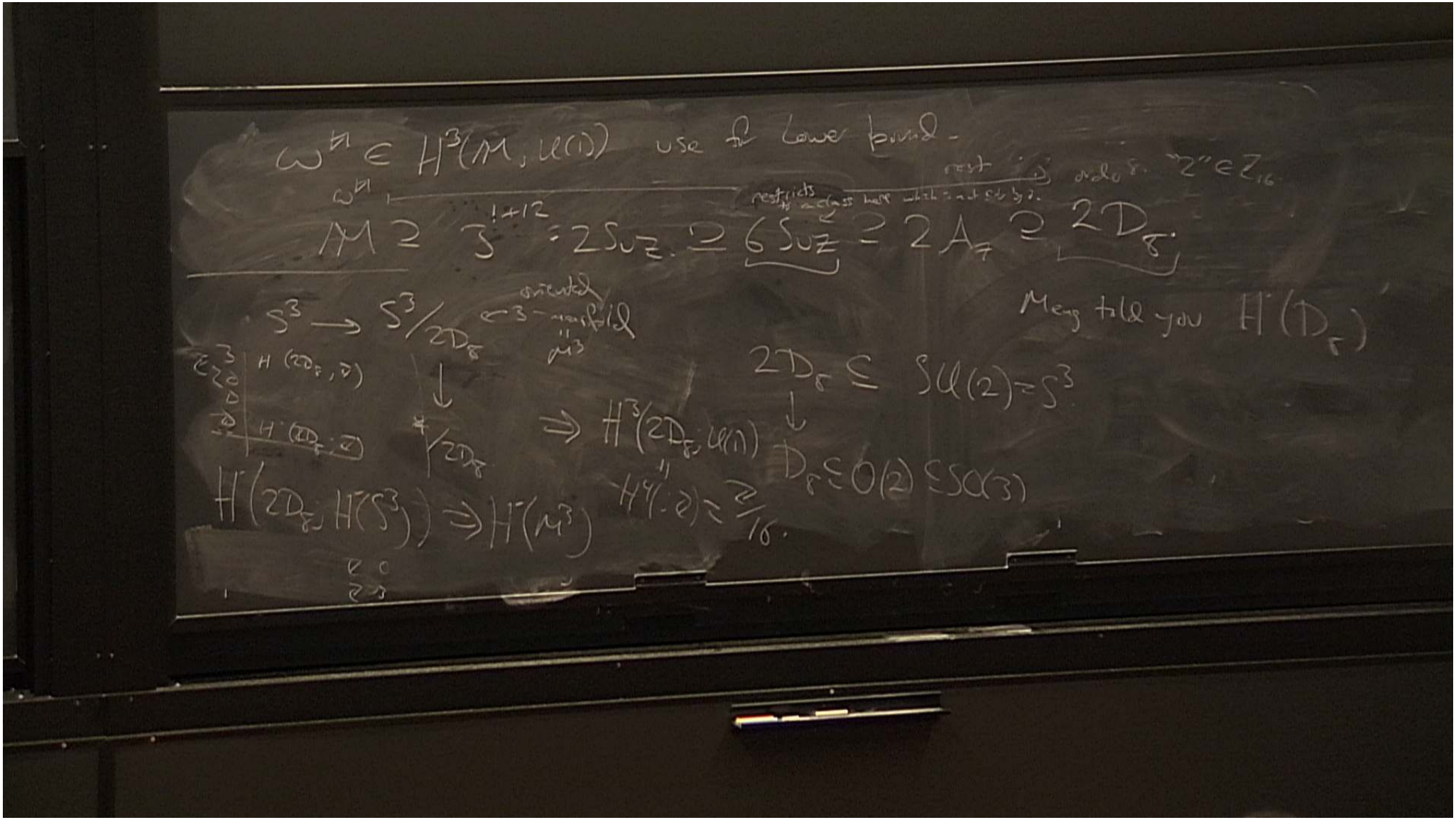

remark: $\in \#SL_{13}(\mathbb{F}_2)$.
done, no H^3.
$\omega^4 \in H^3(BG) = (\text{sub of } H^3(\mathbb{Z}_3)) \cdot (\ker d_2: H^2(G;\hat{\mathbb{Z}}_3) \to H^4(G)) \cdot (\text{coker } d_2: H^1(G;\hat{\mathbb{Z}}_3) \to H^3(G))$
"β"
A cocycle $\omega \in H^3(BG)$ s.t. $\omega|_{\mathbb{Z}_3}$ 2-cocycle
α 2-cocycle coeff in $\hat{\mathbb{Z}}_3$.
β solving $d\beta = \langle \alpha \cup k \rangle$ 3-cochain
$\omega^4|_{\mathbb{Z}_3} = 0$ from $\mathbb{Z}_3$ 3-manifold
So ω^4 lives here
2-cocycle
$\alpha \in H^2(G;\hat{\mathbb{Z}}_3)$ image of ω^4.
$(d\alpha = 0.)$ also $d_2\alpha = \langle \alpha \cup k \rangle = 0$ in $H^4(G)$
3 cochain β $d\beta = \langle \alpha \cup k \rangle$


Learned: $H^3(M; U(1))_{(3)} \leq \mathbb{Z}_3$.
Claim: $= \mathbb{Z}_3$.
enough to show $\neq 0$.
$\omega = (\alpha, \beta)$
$\omega = (\kappa, \beta)$
$V^\natural$
$\mathbb{Z}_3$ gauging
V
Λ
$\alpha \neq 0$.
$3^{1+12} \cdot 2Suz$
$\leftrightarrow$
$3^{12} \rtimes 6Suz$
$3G^\natural$
$3G$
α
$G = 3^{12} \rtimes 6Suz$
$\kappa \in H^2(G; \mathbb{Z}_3)$ classifying extension
$\omega^\natural \in H^3(M; U(1))$
't Hooft anomaly of $M \circlearrowright V^\natural$
$H^\cdot(G, H^\cdot(3)) \Rightarrow H^\cdot(3G^\natural)$


#M = 2^46 · 3^20 · 5^9 · 7^6 · 13^3 · 11^2 · 17 · 19 · 23 · 29 · 31 · 41
H^3(M; U(1))_(3) = Z_3.
done, no H^3
ω^H ∈ H^3(3G) = (sub of H^3(Z_3)) · (ker d_2 : H^2(G; Z̃_3) → H^4(G)) · (
A cocycle ω ∈ H^3(3G) s.t. ω|_Z_3
2-cocycle
ω^H|_Z_3 = 0
then Z_3 is a coboundary
⟺ α 2-cocycle coefs in Z̃_3.
β solving dβ = ⟨α ∪ K⟩
3-cochain
So ω^H lives
2 cocycle
⇝ α ∈ H^2(G; Z̃_3)
(dα = 0.) also d_2 α =
3 cochain — β
dβ = ⟨



$H^3(\mathbb{M}; U(1))_{(3)} = \mathbb{Z}_3$.
done, no H^3.
$p=2$.
$H^j(2^{1+24})$
2-str $S = 2^{1+24} Co_1$
$\cap$
$\mathbb{M}$
$E_2^{ij} = H^i(Co_1; H^j(2^{1+24}))$
$j=3$ $2^{2300} \cdot 2$ $\leq \mathbb{Z}_4$
exp=4
$j=2$ $Alt^2(2^{24})$ line 2^{275} 0 $\mathbb{Z}_2$
me. brute force w/ matrices.
$j=1$ 2^{24} 0 $H^1(Co_1, 2^{24}) = 0$ $H^2(Co_1, 2^{24}) = \mathbb{Z}_2$
Dereth Hut
$j=0$ $U(1)$ $*$ $H^1(Co_1) = *$ (simple) $H^2(Co_1; U(1)) = \mathbb{Z}_2$ (ext to Co_0) $H^3(Co_1)_{(2)} = \mathbb{Z}_4$ Tue w/ DT.
$H^i(Co_1, -)$



$\omega^{\natural} \in H^3(M, U(1))$ use for lower bound.

$\omega^{\natural}$ restricts to a class here which is not stable by ∂. rest ... order 8. "2" $\in \mathbb{Z}_{16}$

$$M \supseteq 3^{1+12} \cdot 2Suz \supseteq \underbrace{6Suz} \supseteq 2A_7 \supseteq \underbrace{2D_8}$$

$S^3 \longrightarrow S^3/2D_8 \leftarrow$ oriented 3-manifold $= M^3$

$\downarrow$

$*/2D_8$

$\Rightarrow H^3(2D_8, U(1)) = H^4(\cdot, \mathbb{Z}) = \mathbb{Z}/16.$

$H^{\cdot}(2D_8, H^{\cdot}(S^3)) \Rightarrow H^{\cdot}(M^3)$

$\mathbb{Z}$ 3 | $H^{\cdot}(2D_8, \mathbb{Z})$

$\mathbb{Z}$ 0

$\mathbb{Z}$ 0

$\mathbb{Z}$ 0 | $H^{\cdot}(2D_8, \mathbb{Z})$

$\mathbb{Z}$ 0

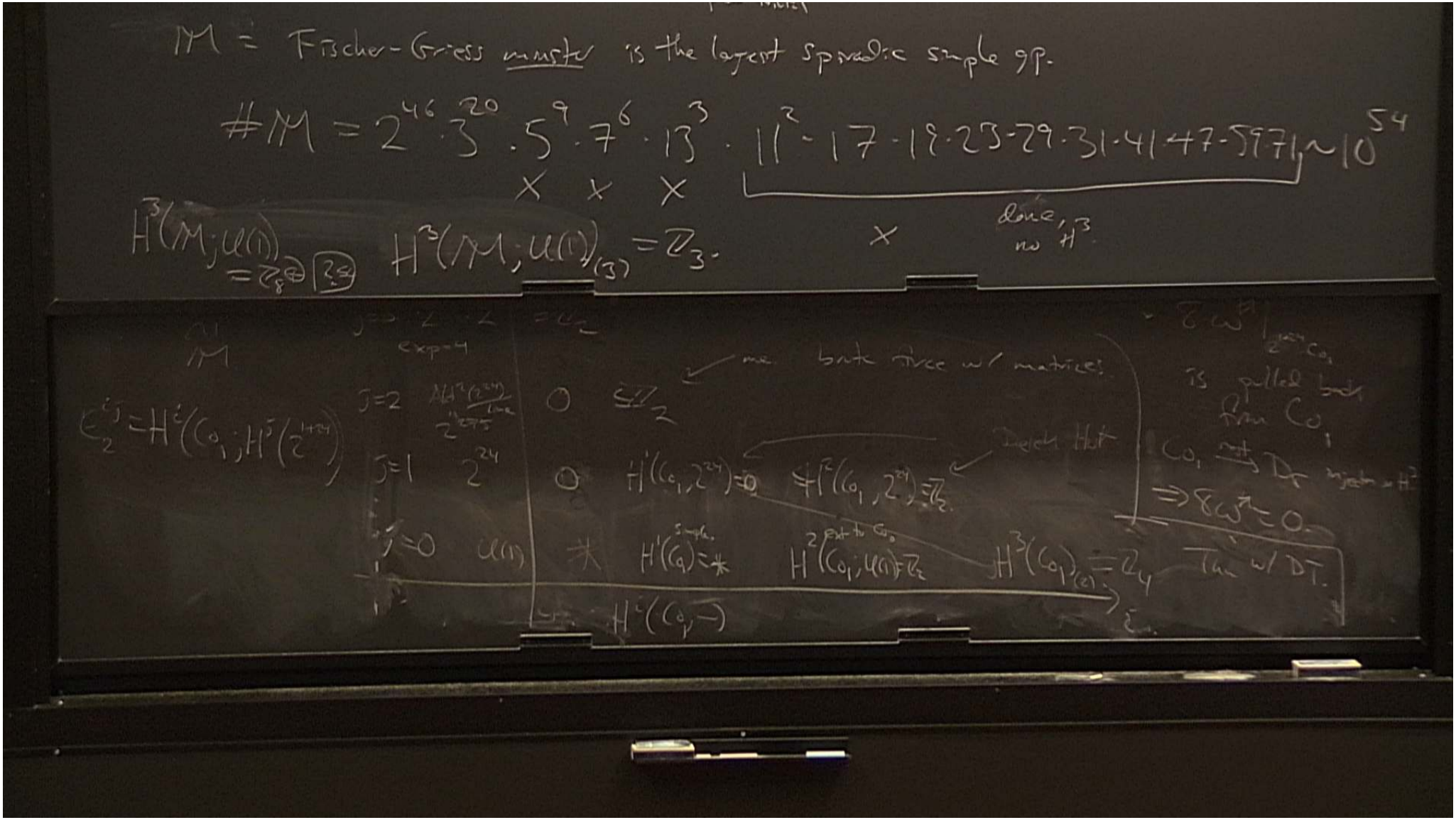
$\mathbb{Z}$ 3

Meng told you $H^{\cdot}(D_8)$

$2D_8 \subseteq SU(2) = S^3$

$\downarrow$

$D_8 \subseteq O(2) \subseteq SO(3)$



$\mathbb{M}$ = Fischer-Griess <u>monster</u> is the largest sporadic simple gp.

$$\#\mathbb{M} = 2^{46}\cdot 3^{20}\cdot 5^{9}\cdot 7^{6}\cdot 13^{3}\cdot \underbrace{11^{2}\cdot 17\cdot 19\cdot 23\cdot 29\cdot 31\cdot 41\cdot 47\cdot 59\cdot 71}_{\times\ \text{done, no } H^3} \sim 10^{54}$$

$\times \quad \times \quad \times$

$H^3(\mathbb{M};U(1)) = \mathbb{Z}_8 \oplus \boxed{?\leq}$ $\quad H^3(\mathbb{M};U(1))_{(3)} = \mathbb{Z}_3.$

$\mathbb{M}$

$E_2^{ij} = H^i(Co_1; H^j(2^{1+24}))$

$j=3$	$2\cdot 2$ exp=4	$=\mathbb{Z}_2$			
$j=2$	$AH^2(2^{24})$ line 2^{1+24}	0	$\leq \mathbb{Z}_2$		
$j=1$	2^{24}	0	$H^1(Co_1, 2^{24}) = 0$	$H^2(Co_1, 2^{24}) = \mathbb{Z}_2$	
$j=0$	$U(1)$	$*$	$H^1(Co_1) = *$ (simple)	$H^2(Co_1; U(1)) = \mathbb{Z}_2$ (ext to Co_1)	$H^3(Co_1)_{(2)} = \mathbb{Z}_4$
			$H^i(Co_1, -)$		

me. brute force w/ matrices

Deich Hut

$\cdot\ 8\omega^{\#}|_{2^{1+24}\cdot Co_1}$ is pulled back from Co_1

$Co_1 \xrightarrow{\text{rest}} D_8$ injection on H^3

$\Rightarrow 8\omega^{\#} = 0.$

Thm w/ DT.

1707.08388 + 1810.00463
Z_2 w/ D. Treumann.

Goal: Thm. $H^3(\mathbb{M}, U(1)) \cong \mathbb{Z}_{24} \oplus (\underbrace{\leq 4}_{\text{probably trivial}})$

$\mathbb{M}$ = Fischer-Griess monster is the largest sporadic simple gp.

$$\#\mathbb{M} = 2^{46}\cdot 3^{20}\cdot 5^9\cdot 7^6\cdot 13^3\cdot \underbrace{11^2\cdot 17\cdot 19\cdot 23\cdot 29\cdot 31\cdot 41\cdot 47\cdot 59\cdot 71}_{\text{done, no } H^3} \sim 10^{54}$$

$H^3(\mathbb{M}; U(1)) = \mathbb{Z}_8 \oplus$ [illegible] $H^3(\mathbb{M}; U(1))_{(3)} = \mathbb{Z}_3$.