Title: Entanglement and extended conformal field theory

Speakers: Gabriel Wong

Series: Quantum Fields and Strings

Date: November 05, 2019 - 2:30 PM

URL: http://pirsa.org/19110041

Abstract: Defining entanglement in a continuum field theory is a subtle challenge, because the Hilbert space does not naively factorize into local products.   For gauge theories, the problem arises from the gauss law constraint, and it can be resolved by an extension of the Hilbert space which introduces edge modes at the entangling surface.  Recently we showed how this extension  fits inside the frame work of 2D extended topological field theory.  In this talk we attempt a generalization to 2D CFTs in which factorization is determined by the fusion rules.  Time permitting we will speculate on applications, e.g to continuum constructions of tensor networks and to entanglement in string theory 

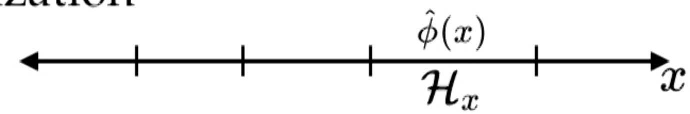
# What is a QFT?

$$Z[J] = \int D[\phi] e^{-S(\phi,J)}$$

Correlators /Scattering Amplitudes
Topological invariants
Replica trick computation of EE

~~Local Hilbert Space factorization~~

$$\mathcal{H} = \otimes_x \mathcal{H}_x$$



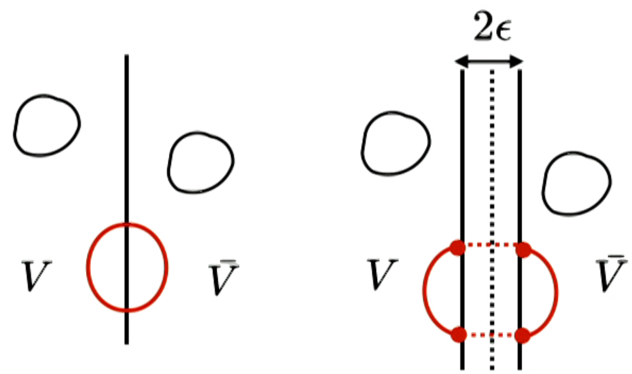

1 A subregion has a a boundary and therefore edge modes, even for a scalar field! ( Agon, HeadrickJefferis,Kasko ) (Campaglia, Freidal,et al)

2 Degrees of freedom in subregions are not independent

- continuity in a quantum field theory
- Gauss Law constraint in gauge theory. Even on a lattice !

## How do we define entanglement in a QFT?

# Extended Hilbert space for gauge theories




$$\mathcal{H}_{\text{physical}} \subset \mathcal{H}_V \otimes \mathcal{H}_{\bar{V}}$$

Contains edge modes transforming under boundary symmetry group $G \otimes \bar{G}$

**Gauss law** $(J_n \otimes 1 + 1 \otimes \bar{J}_{-n})|\psi\rangle = 0 \qquad |\psi\rangle \in \mathcal{H}_V \otimes \mathcal{H}_{\bar{V}}$

(Can be viewed as an equivalence relation )

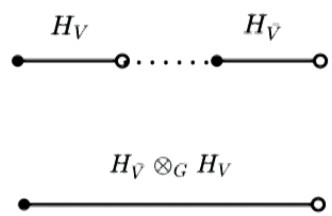
**Entangling product** $\mathcal{H}_{\text{physical}} = \mathcal{H}_V \otimes_G \mathcal{H}_{\bar{V}}$

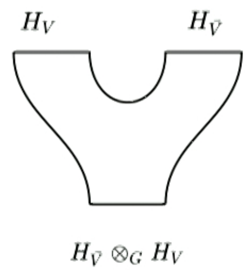
**Reduced density matrix** $\rho_V = \text{tr}_{\bar{V}} |\psi\rangle\langle\psi|$

**Entanglement Entropy** $S_V = -\text{tr}\ \rho_V \log\ \rho_V$

# Extended Hilbert space and extended QFT

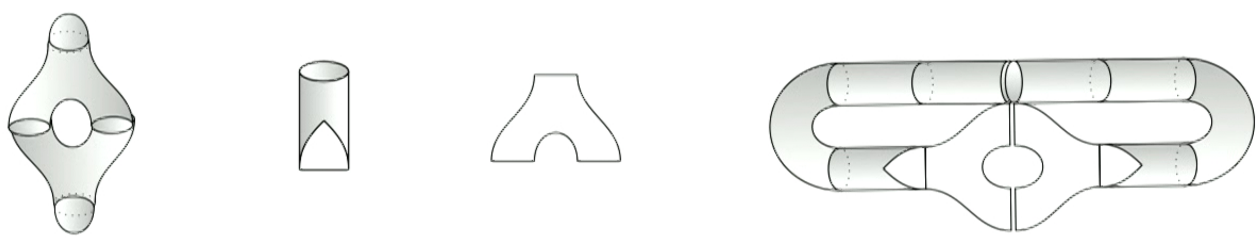
- What are the rules for determining the "correct" edge modes and their gluings?
- In 1+1 D, we showed the extension is part of the data of the extended topological quantum field theory (Donnelly-Wong 2018)
- Key insight: View the entangling product as a spacetime process=cobordism

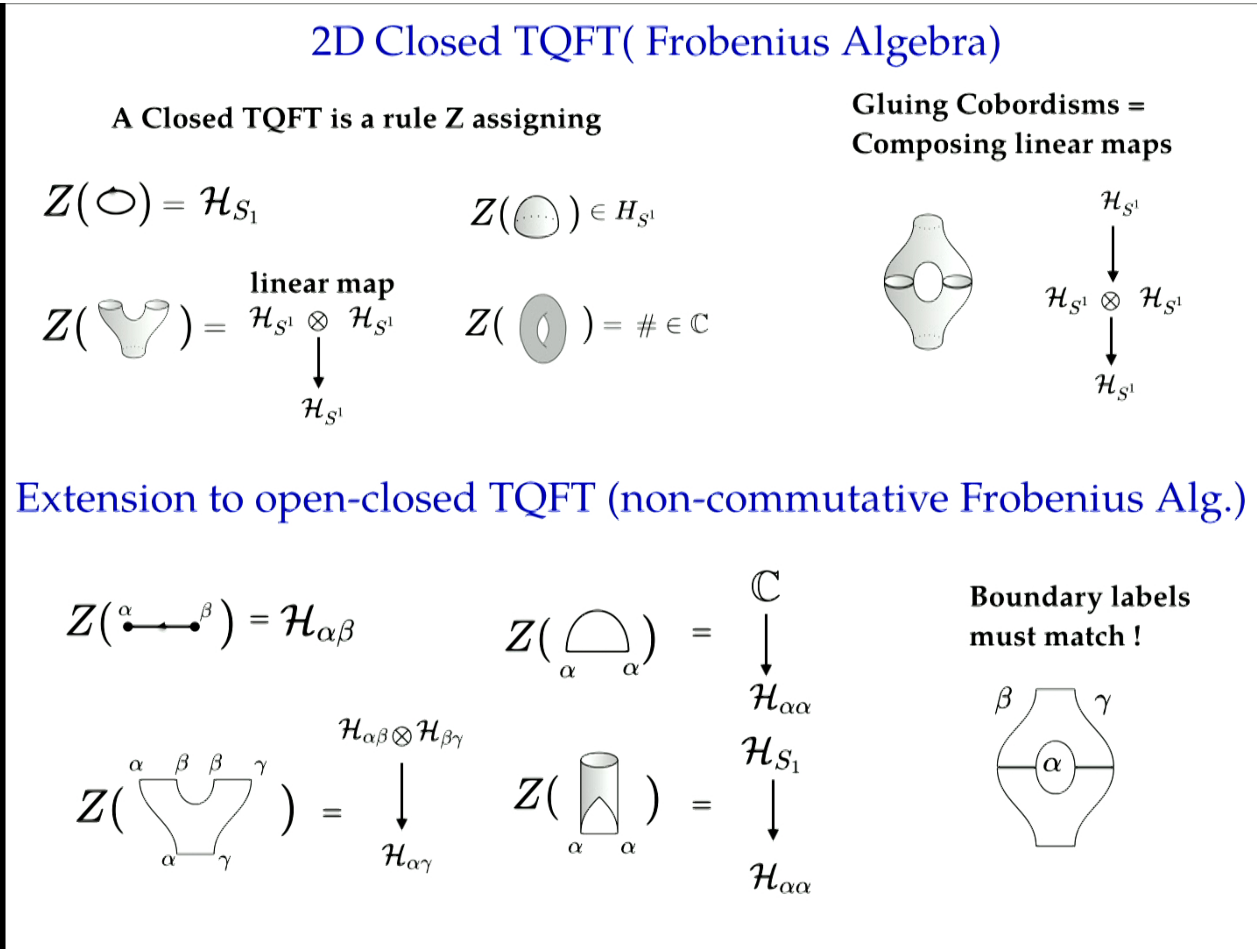


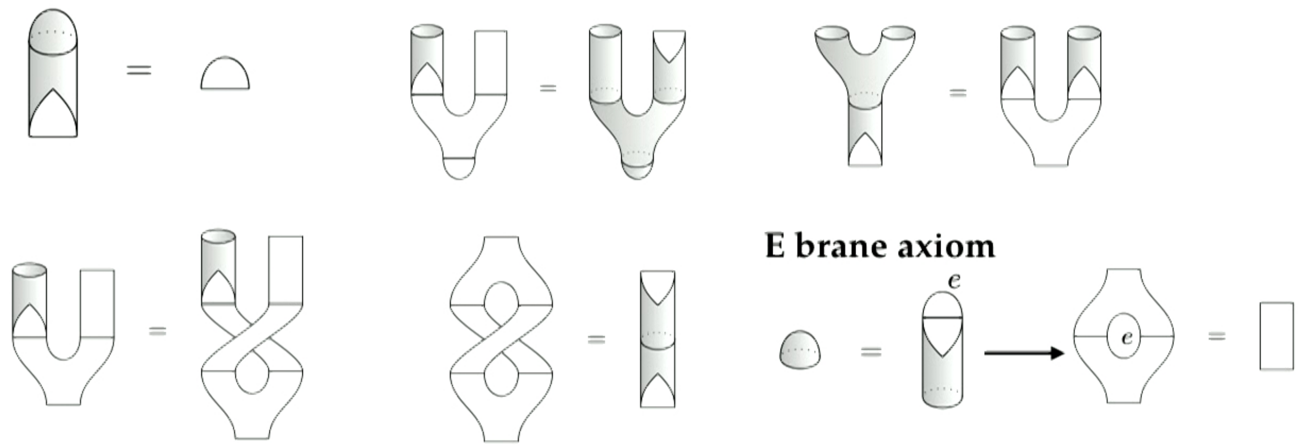

## Extended QFT (Atiyah, Segal, Freed, Baez,…)

Cut path integral along surfaces of increasing codimension




2D Closed TQFT( Frobenius Algebra)
A Closed TQFT is a rule Z assigning
$Z(\ ) = \mathcal{H}_{S_1}$
$Z(\ ) \in H_{S^1}$
linear map
$Z(\ ) = \mathcal{H}_{S^1} \otimes \mathcal{H}_{S^1} \to \mathcal{H}_{S^1}$
$Z(\ ) = \# \in \mathbb{C}$
Gluing Cobordisms =
Composing linear maps
$\mathcal{H}_{S^1} \to \mathcal{H}_{S^1} \otimes \mathcal{H}_{S^1} \to \mathcal{H}_{S^1}$
Extension to open-closed TQFT (non-commutative Frobenius Alg.)
$Z(\ ) = \mathcal{H}_{\alpha\beta}$
$Z(\ ) = \mathbb{C} \to \mathcal{H}_{\alpha\alpha}$
Boundary labels
must match !
$Z(\ ) = \mathcal{H}_{\alpha\beta} \otimes \mathcal{H}_{\beta\gamma} \to \mathcal{H}_{\alpha\gamma}$
$Z(\ ) = \mathcal{H}_{S_1} \to \mathcal{H}_{\alpha\alpha}$
α
β
γ

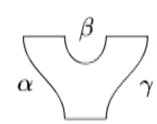
# Extended TQFT  Sewing relations



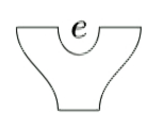

Moore-Segal:

Q: Given a closed string theory,  what are the possible boundaries, i.e. D Branes?

A: D branes correspond to extensions to an open string algebra satisfying these constraints.



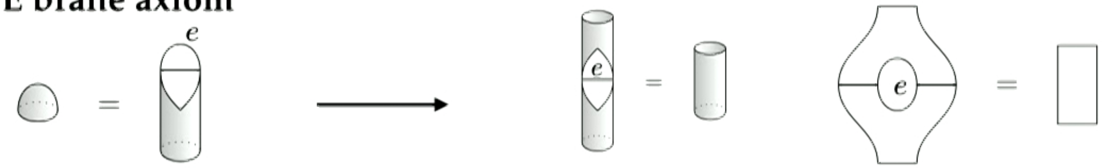

For us: **Open string algebra ~ choice of Hilbert space extension**



# The Entanglement Brane boundary condition

Holes originating from splitting the Hilbert space can be sewed up

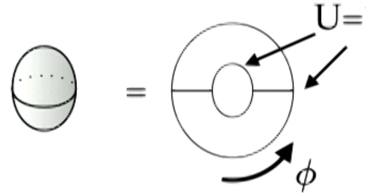
**E brane axiom**




**e**= choice of (possibly nonlocal ) boundary conditions

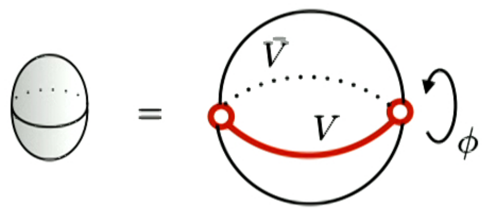
In 2D Yang Mills: e = trivial holonomy along boundary circles
~sum over electric boundary conditions.

$$U = \mathrm{P}\exp(i\int A_x\, dx)$$



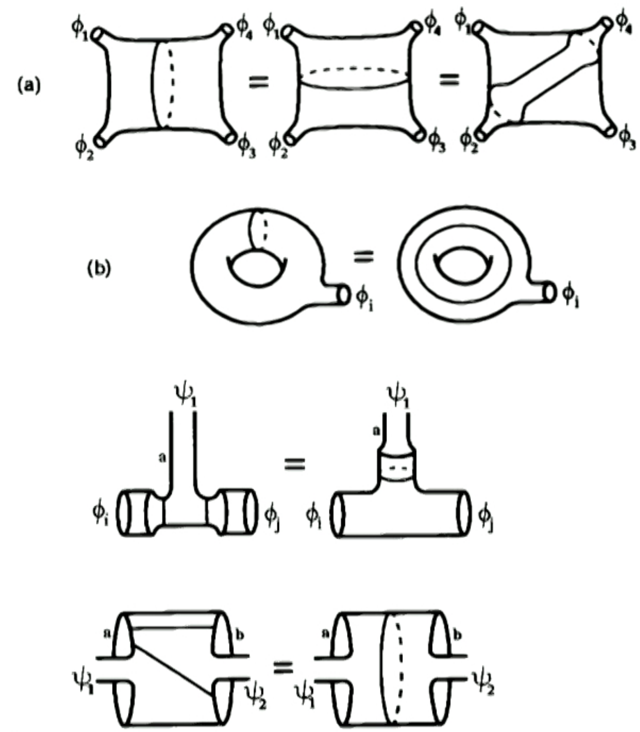

**Implies correlations are preserved under reduction to V:**

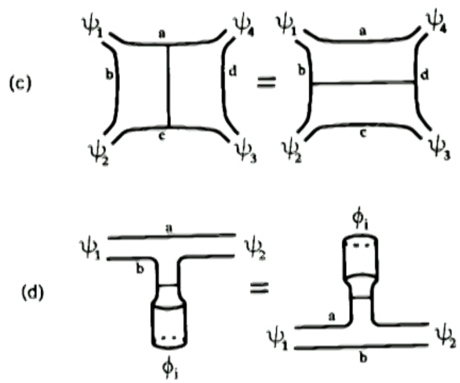
$$\langle O_V \rangle = \mathrm{tr}(\rho_V O_V)$$



# Extended Conformal field theory

Lewellen (1992) has formulated open-closed CFT in terms of sewing axioms



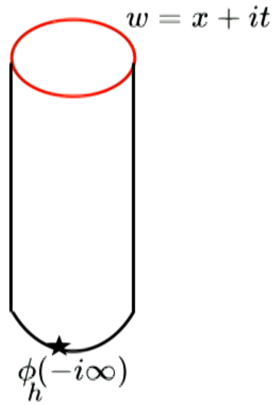



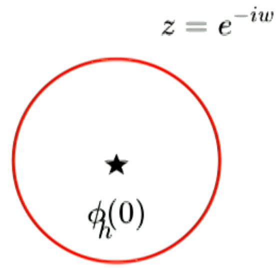

Extended Hilbert space? Edge modes?

Entanglement Brane boundary condition?

Cobordism description of Hilbert space factorization?

# Local Hilbert space for closed CFT




Primaries

$|h>= \phi_h(0)|0>_{\text{disk}}$

Representation of Kacs-Moody

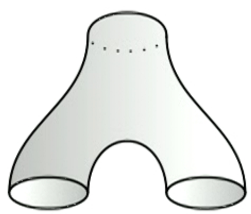
$$J_n = \int dz\, z^n J(z)$$

$$[J_n, J_m] = m\delta_{m+n}$$

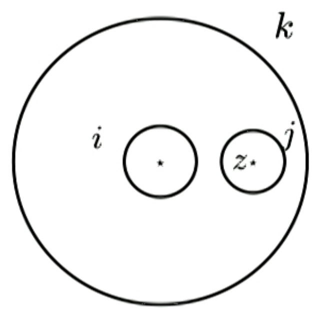
n>0 $\quad J_n|h>= 0$ Highest weight

$J_{-n_1}\cdots J_{-n_k}|h>$ Excited states

Pair of pants ~ OPE Coefficients



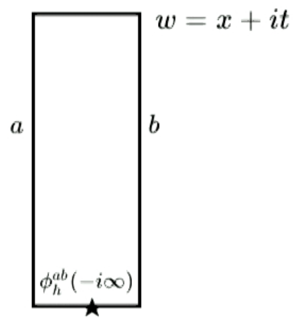
Chiral Vertex operator (Moore-Seiberg)

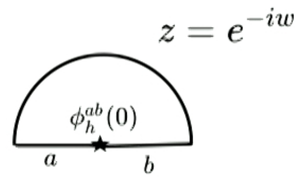



$$V_{ij}^k(z) : H_i \otimes H_j \to H_k$$

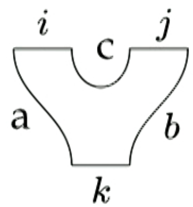
$$V_{ij}^k(z) = \frac{C_{ijk}}{z^{\Delta_i+\Delta_j-\Delta_k}}$$

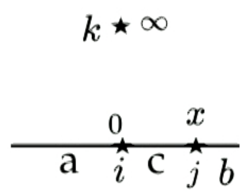
# Local Hilbert space for the Open CFT



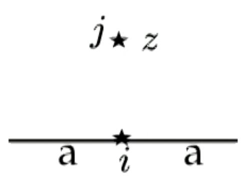




$$|h >_{ab} = \phi_h^{ab}(0)|0 >_{\text{half disk}}$$



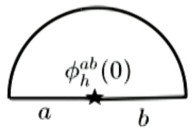




$$= \frac{C_{ijk}^{abc}}{x^{\Delta_i + \Delta_j - \Delta_k}}$$


$$(2\mathrm{Im} z)^{\Delta_i - \Delta_j - \bar{\Delta}_j} C_{ji}^a$$

# Conformal BC and unfolding trick




$$|h>_{ab} = \phi_h^{ab}(0)|0>_{\text{half disk}}$$

a,b = conformally invariant boundary conditions for the open string

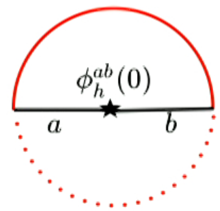
$$T(z) = \bar{T}(\bar{z}) \text{ for real } z$$

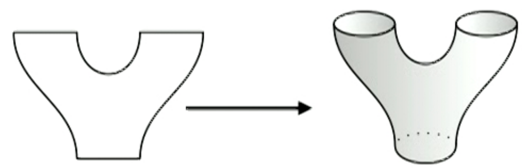
$$J(z) = \pm \bar{J}(\bar{z}) \text{ for real } z$$

Extend to z in LHP

$$T(z) = \bar{T}(\bar{z}^*)$$

$$J(z) = \pm \bar{J}(\bar{z}^*)$$



Preserves half the symmetry generators

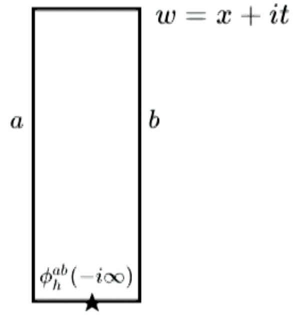
$$L_n = \int dz\, z^{n+1} T(z) \mp \int d\bar{z}\, \bar{z}^{n+1} \bar{T}(\bar{z}) = \oint dz z^{n+1} T(z)$$

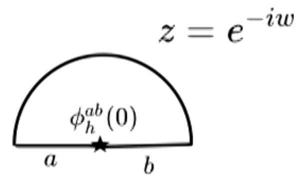
$$J_n = \int dz\, z^n J(z) \mp \int d\bar{z}\, \bar{z}^n \bar{J}(\bar{z}) = \oint dz\, z^n J(z)$$

n>0 $\quad J_n |h_{ab}> = 0$ Highest weight

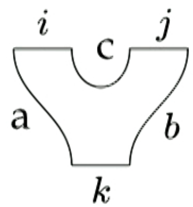
$J_{-n_1} \cdots J_{-n_k} |h_{ab}>$ Excited states

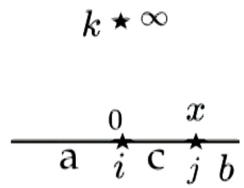
# Local Hilbert space for the Open CFT



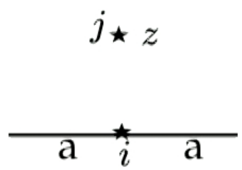




$$|h>_{ab}= \phi_h^{ab}(0)|0>_{\text{half disk}}$$



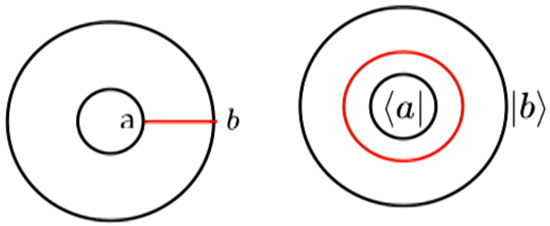

$$= \frac{C_{ijk}^{abc}}{x^{\Delta_i+\Delta_j-\Delta_k}}$$


$$(2\text{Im}z)^{\Delta_i-\Delta_j-\bar{\Delta}_j} C_{ji}^{a}$$

# Entanglement Brane boundary condition for a CFT

"Closed string" description of conformally invariant boundary condition.



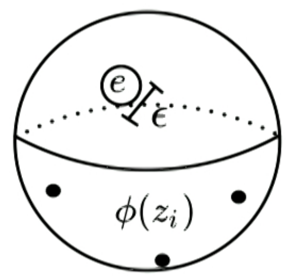

$$(L_n^C - \bar{L}_{-n}^C)|a\rangle = 0$$

$$(J_n^C \pm \bar{J}_{-n}^C)|a\rangle = 0 \qquad \text{Neumann/Dirichlet}$$

Solutions: Ishibashi states $|h\rangle\rangle = \sum_N |h, N\rangle \otimes |\bar{h}, \bar{N}\rangle$ $\quad h = \text{Bulk Primary}$

Proposal: Entanglement brane |e> = Vacuum Ishibashi state |0>>

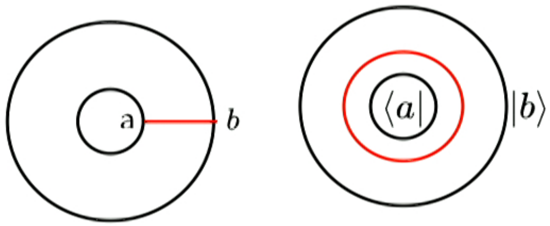
$$\lim_{\epsilon \to 0} \langle \phi(z_1) \dots \phi(z_n) \rangle_e = \langle \phi(z_1) \dots \phi(z_n) \rangle_{\text{bulk}}$$




**Does not satisfy cardy condition**, i.e. not a local open string BC !
As in gauge theory, this corresponds to a sum over local open string boundary conditions

# Entanglement Brane boundary condition for a CFT

"Closed string" description of conformally invariant boundary condition.



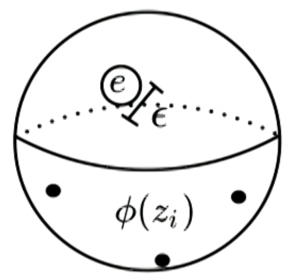

$$(L_n^C - \bar{L}_{-n}^C)|a\rangle = 0$$

$$(J_n^C \pm \bar{J}_{-n}^C)|a\rangle = 0 \qquad \text{Neumann/Dirichlet}$$

Solutions: Ishibashi states $|h\rangle\rangle = \sum_N |h, N\rangle \otimes |\bar{h}, \bar{N}\rangle$ $\qquad h = \text{Bulk Primary}$

Proposal: Entanglement brane |e> = Vacuum Ishibashi state |0>>

$$\lim_{\epsilon \to 0} \langle \phi(z_1) \dots \phi(z_n) \rangle_e = \langle \phi(z_1) \dots \phi(z_n) \rangle_{\text{bulk}}$$




**Does not satisfy cardy condition**, i.e. not a local open string BC !
As in gauge theory, this corresponds to a sum over local open string boundary conditions

# E brane for a Free, Compact Boson

Zero modes $$|p_L, p_R\rangle = |\frac{m}{2R} + nR, \frac{m}{2R} - nR\rangle \qquad \phi \sim \phi + 2\pi R$$

Ishibashi states: $$|nR, -nR\rangle\rangle = \exp \sum_{l=1} \frac{-\alpha_{-l}\tilde{\alpha}_{-l}}{l} |nR, -nR\rangle$$

$$|(\frac{m}{2R}, \frac{m}{2R})\rangle\rangle = \exp \sum_{l=1} \frac{-\alpha_{-l}\tilde{\alpha}_{-l}}{l} |\frac{m}{2R}, \frac{m}{2R}\rangle$$

Local open boundary conditions are given by the **Cardy** boundary states:

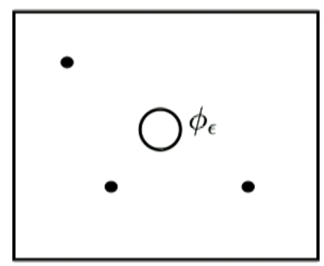
Neumann $$||w\rangle\rangle = R^{1/2} \sum_{n\in\mathbb{Z}} e^{iwnR} |nR, -nR\rangle\rangle$$

Dirichlet $$||\phi_\epsilon\rangle\rangle = (2R)^{-1/2} \sum_{m\in\mathbb{Z}} e^{i\frac{m\phi_\epsilon}{R}} |\frac{m}{2R}, \frac{m}{2R}\rangle\rangle$$

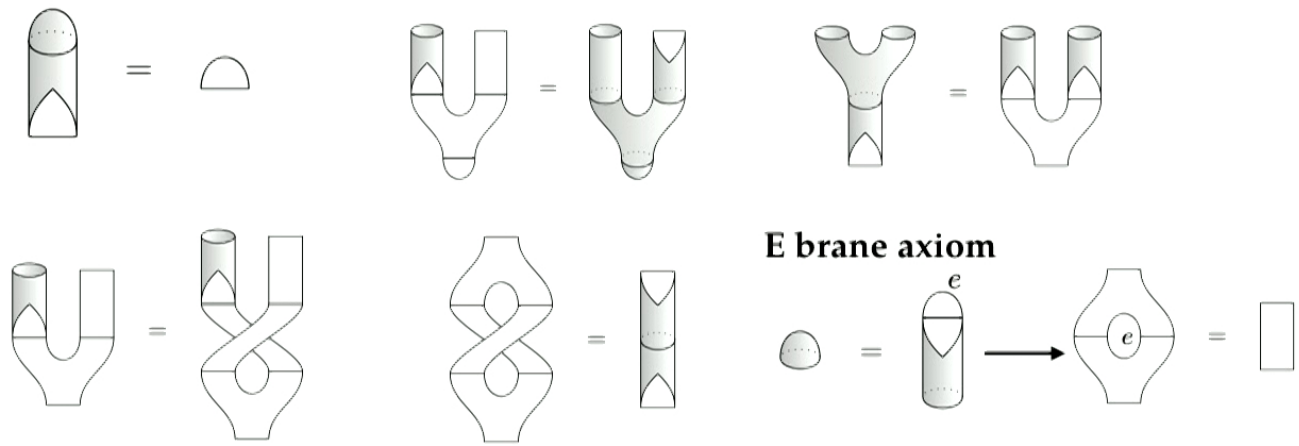
Vacuum Ishibashi/E brane state

$$|e\rangle = \begin{cases} |0,0\rangle\rangle \sim \int_0^{2\pi R} d\phi_\epsilon ||\phi_\epsilon\rangle\rangle \\ |0,0\rangle\rangle \sim \int_0^{\frac{2\pi}{R}} dw ||w\rangle\rangle \end{cases}$$

Restore shift invariance



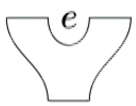
# Extended TQFT Sewing relations




Moore-Segal:

Q: Given a closed string theory, what are the possible boundaries, i.e. D Branes?

A: D branes correspond to extensions to an open string algebra satisfying these constraints.


For us: **Open string algebra ~ choice of Hilbert space extension**

# E brane for a Free, Compact Boson

Zero modes $|p_L, p_R\rangle = |\frac{m}{2R} + nR, \frac{m}{2R} - nR\rangle$ $\qquad \phi \sim \phi + 2\pi R$

Ishibashi states:

$$|nR, -nR\rangle\rangle = \exp \sum_{l=1} \frac{-\alpha_{-l}\tilde{\alpha}_{-l}}{l} |nR, -nR\rangle$$

$$|(\frac{m}{2R}, \frac{m}{2R})\rangle\rangle = \exp \sum_{l=1} \frac{-\alpha_{-l}\tilde{\alpha}_{-l}}{l} |\frac{m}{2R}, \frac{m}{2R}\rangle$$

Local open boundary conditions are given by the **Cardy** boundary states:

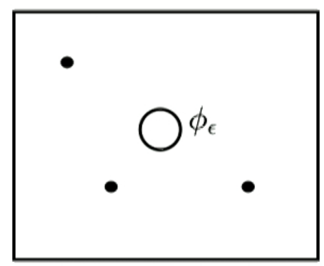
Neumann $$||w\rangle\rangle = R^{1/2} \sum_{n\in\mathbb{Z}} e^{iwnR} |nR, -nR\rangle\rangle$$

Dirichlet $$||\phi_\epsilon\rangle\rangle = (2R)^{-1/2} \sum_{m\in\mathbb{Z}} e^{i\frac{m\phi_\epsilon}{R}} |\frac{m}{2R}, \frac{m}{2R}\rangle\rangle$$

Vacuum Ishibashi/E brane state

$$|e\rangle = \begin{cases} |0,0\rangle\rangle \sim \int_0^{2\pi R} d\phi_\epsilon ||\phi_\epsilon\rangle\rangle \\ |0,0\rangle\rangle \sim \int_0^{\frac{2\pi}{R}} dw ||w\rangle\rangle \end{cases}$$

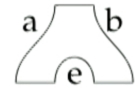
Restore shift invariance



# Dirichlet E brane and edge mode entanglement entropy

Dirichlet extension to open strings :

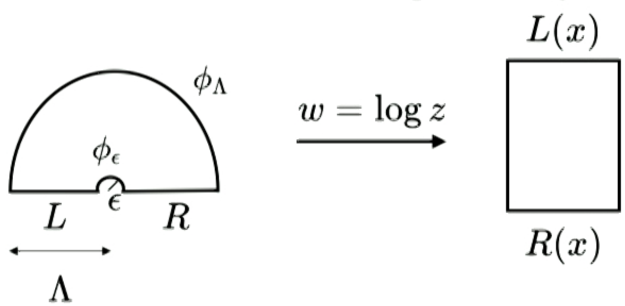

$\mathcal{H}_{S^1} \to \bigoplus_{\phi_0} \mathcal{H}_{\phi_0 \phi_0}$




$\mathcal{H}_{ac} \to \bigoplus_{\phi_0} \mathcal{H}_{a\phi_0} \otimes \mathcal{H}_{\phi_0 c}$

Factorizing the close string vacuum


Wavefunctional from the path integral




$$\phi(x,t) = \phi_0 + \phi_1 x + \cdots$$

$$\phi_0 \in [0, 2\pi R]$$

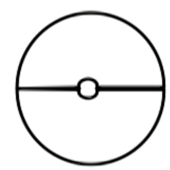
$$\phi_1 \in \mathbb{R}$$

$$\phi_\epsilon = \phi_0 + \phi_1 \log \epsilon$$

$$\phi_\Lambda = \phi_0 + \phi_1 \log \Lambda$$

$$|\Psi\rangle_D = \int d\phi_\epsilon d\phi_\Lambda \int D[R(x)]D[L(x)] e^{-S_{\text{on-shell}}} |\phi_\Lambda, \phi_\epsilon, L(x)\rangle \otimes |\phi_\epsilon, \phi_\Lambda, R(x)\rangle$$

$$Z(\beta) = \mathrm{tr}_V \rho_V^{\frac{\beta}{2\pi}} = l \int_0^{2\pi R} d\phi_0 \int_{-\infty}^{\infty} d\phi_1 e^{-\beta l \phi_1^2} Z_{\text{osc}} \qquad l = \log \frac{\Lambda}{\epsilon}$$



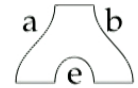
$$S = -\mathrm{tr}\rho_V \log \rho_V = \frac{1}{3} \log \frac{\Lambda}{\epsilon} + \frac{1}{2} \log \pi R^2 + \cdots \longleftarrow$$

Michel-Srednicki
"Edge mode EE"

# Dirichlet E brane and edge mode entanglement entropy

Dirichlet extension to open strings :

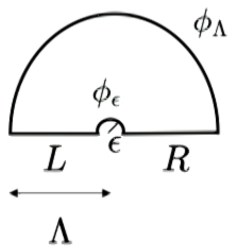

$\mathcal{H}_{S^1} \to \bigoplus_{\phi_0} \mathcal{H}_{\phi_0 \phi_0}$

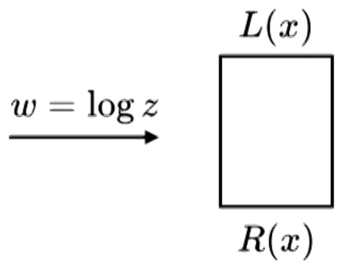



$\mathcal{H}_{ac} \to \bigoplus_{\phi_0} \mathcal{H}_{a\phi_0} \otimes \mathcal{H}_{\phi_0 c}$

Factorizing the close string vacuum


Wavefunctional from the path integral




$$\phi(x,t) = \phi_0 + \phi_1 x + \cdots$$

$$\phi_\epsilon = \phi_0 + \phi_1 \log \epsilon$$

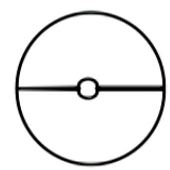
$$\phi_\Lambda = \phi_0 + \phi_1 \log \Lambda$$

$$\phi_0 \in [0, 2\pi R]$$

$$\phi_1 \in \mathbb{R}$$

$$|\Psi\rangle_D = \int d\phi_\epsilon d\phi_\Lambda \int D[R(x)]D[L(x)] e^{-S_{\text{on-shell}}} |\phi_\Lambda, \phi_\epsilon, L(x)\rangle \otimes |\phi_\epsilon, \phi_\Lambda, R(x)\rangle$$

$$Z(\beta) = \mathrm{tr}_V \rho_V^{\frac{\beta}{2\pi}} = l \int_0^{2\pi R} d\phi_0 \int_{-\infty}^{\infty} d\phi_1 e^{-\beta l \phi_1^2} Z_{\text{osc}} \qquad l = \log \frac{\Lambda}{\epsilon}$$

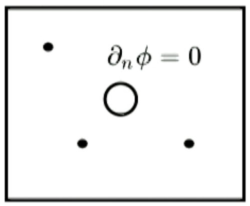


$$S = -\mathrm{tr}\rho_V \log \rho_V = \frac{1}{3}\log\frac{\Lambda}{\epsilon} + \frac{1}{2}\log \pi R^2 + \cdots \longleftarrow$$

Michel-Srednicki
"Edge mode EE"

# The Neumann E brane and T duality

$\phi \sim \phi + 2\pi R$



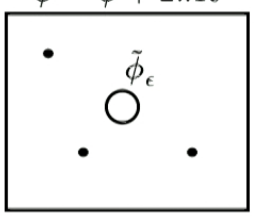

Shift invariance/Fusion rules preserved

$$\langle V_{p_1} V_{p_2} \cdots V_{p_n} \rangle \sim \delta(\sum_i p_i) \qquad V_p = \exp \frac{ip\phi}{R}, p \in \mathbb{Z}$$

Bulk correlation of local operators preserved up to $O(\epsilon)$

But there are non-local vortex operators which we have to check: Use T- Duality

$\tilde{\phi} \sim \tilde{\phi} + 2\pi \tilde{R}$



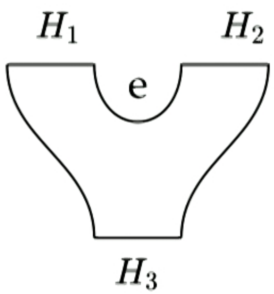

Integrating over $\tilde{\phi}_\epsilon$ preserves shift invariance of $\tilde{\phi}$

$$\langle \tilde{V}_{q_1} \tilde{V}_{q_2} \cdots \tilde{V}_{q_n} \rangle \sim \delta(\sum_i q_i) \qquad \tilde{V}_p = \exp \frac{iq\tilde{\phi}}{\tilde{R}}, q \in \mathbb{Z}$$

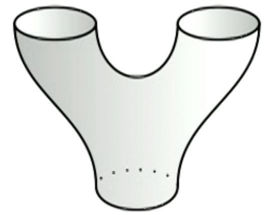
This correspond to summing over Wilson lines of $\phi$

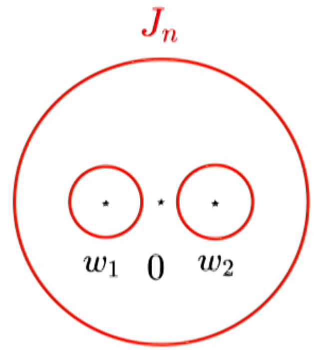
# Tensor product factorization from CFT fusion

Gaberdiel, Moore-Seiberg




$H_1 \otimes H_2 / \sim \quad \to H_3$






$$\int_C \frac{dz}{2\pi i} \langle \chi | z^n J(z) \psi(w_1) \phi(w_2) | 0 \rangle = \left( \int_0 + \int_{w_1} + \int_{w_2} \right) \frac{dz}{2\pi i} z^n \langle \chi | J(z) \psi(w_1) \phi(w_2) | 0 \rangle$$

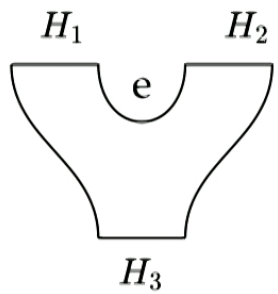
$$= \sum \langle \chi | \Delta^1(J_n) \psi(w_1) \Delta^2(J_n) \phi(w_2) | 0 \rangle$$

Co-product $\quad n > 0$

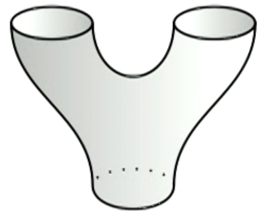
$$\Delta_{w_1, w_2}(J_n) = \sum_{m=0}^{n} \binom{n}{m} \left( w_1^{n-m} J_m \otimes 1 + w_2^{n-m} 1 \otimes J_m \right)$$

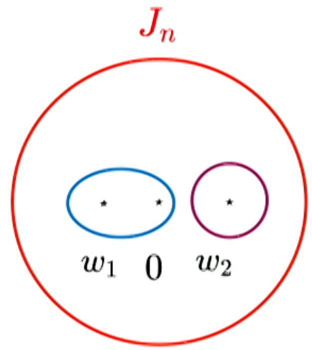
# Tensor product for CFT fusion

Gaberdiel, Moore-Seiberg




$H_1 \otimes H_2 / \sim \quad \to H_3$






$$\int_C \frac{dz}{2\pi i} \langle \chi | z^n J(z) \psi(w_1) \phi(w_2) | 0 \rangle = \Big( \int_0 + \int_{w_1} + \int_{w_2} \Big) \frac{dz}{2\pi i} \, z^n \langle \chi | J(z) \psi(w_1) \phi(w_2) | 0 \rangle$$

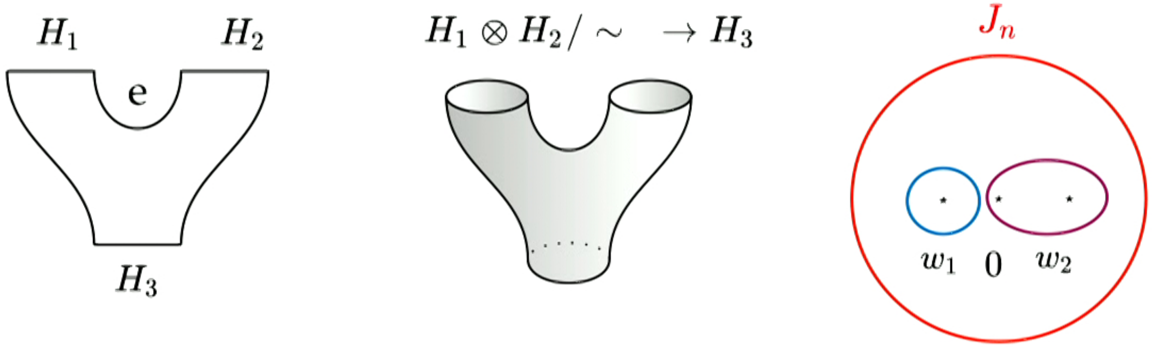
$$= \sum \langle \chi | \Delta^1(J_n) \psi(w_1) \Delta^2(J_n) \phi(w_2) | 0 \rangle$$

Co-product for negative modes

$$\Delta_{w_1,w_2}(J_{-n}) = \sum_{m=0}^{n} \binom{n+m-1}{m} (-1)^m w_1^{-(n+m)} J_m \otimes 1 + \sum_{l=n}^{\infty} \binom{l-1}{n-1} (-w_2)^{l-n} 1 \otimes J_{-l}$$

# Tensor product for CFT fusion

Gaberdiel, Moore-Seiberg




$$\int_C \frac{dz}{2\pi i}\,\langle\chi|z^n J(z)\psi(w_1)\phi(w_2)|0\rangle = \left(\int_0 + \int_{w_1} + \int_{w_2}\right)\frac{dz}{2\pi i}\, z^n\langle\chi|J(z)\psi(w_1)\phi(w_2)|0\rangle$$

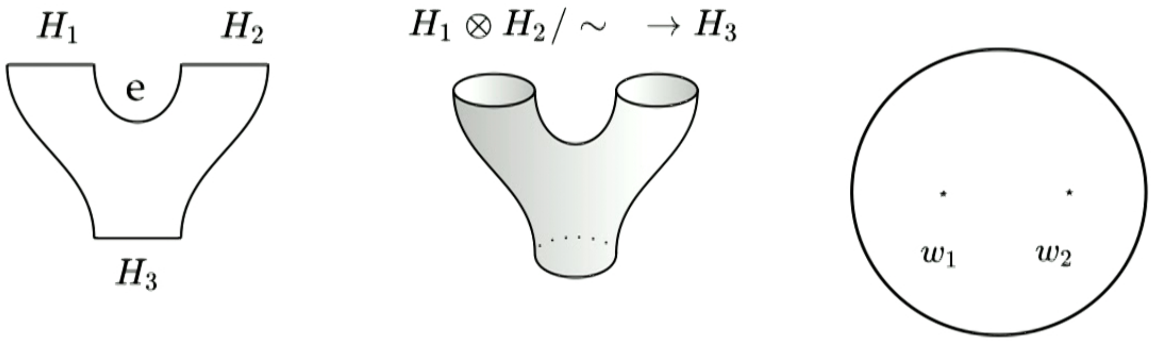
$$= \sum \langle\chi|\Delta^1(J_n)\psi(w_1)\Delta^2(J_n)\phi(w_2)|0\rangle$$

Co-product for negative modes

$$\tilde{\Delta}_{w_1,w_2}(J_{-n}) = \sum_{m=0}^{n} \binom{n+m-1}{m}(-1)^m w_2^{-(n+m)}\, 1\otimes J_m + \sum_{l=n}^{\infty}\binom{l-1}{n-1}(-w_1)^{l-n}\otimes J_{-l}\otimes 1$$

# Tensor product for CFT fusion

Gaberdiel, Moore-Seiberg




$$\Delta_{w_1,w_2}(J_{-n}) = \sum_{m=0}^{n} \binom{n+m-1}{m} (-1)^m w_1^{-(n+m)} J_m \otimes 1 + \sum_{l=n}^{\infty} \binom{l-1}{n-1} (-w_2)^{l-n} 1 \otimes J_{-l}$$

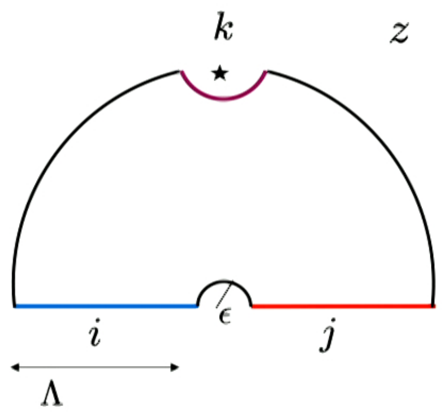
$$\tilde{\Delta}_{w_1,w_2}(J_{-n}) = \sum_{m=0}^{n} \binom{n+m-1}{m} (-1)^m w_2^{-(n+m)} 1 \otimes J_m + \sum_{l=n}^{\infty} \binom{l-1}{n-1} (-w_1)^{l-n} \otimes J_{-l} \otimes 1$$

$H_1 \otimes H_2 / \sim$ is the largest quotient of $H_1 \otimes H_2$ where $\tilde{\Delta}(J_{-n}) = \Delta(J_{-n})$

Restores symmetry of the co-product

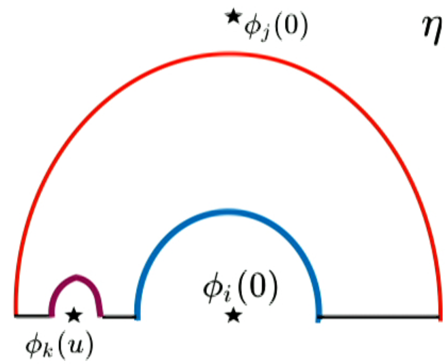
Preserves the level/central charge of the Kacs-Moody/Virasoro Algebra

# Entangling product and CFT fusion




$$\eta = \left(\frac{z}{\epsilon}\right)^{i\frac{\pi}{l}}$$

$$l = \log\frac{\Lambda}{\epsilon}$$

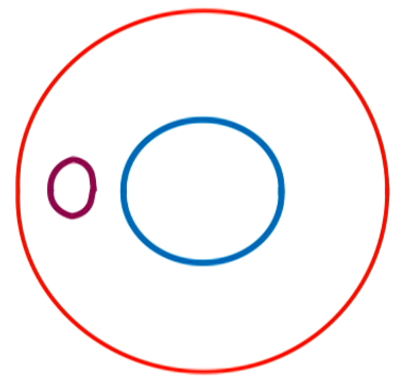



Kacs-Moody Charges:

$$J_n(u) = \int d\eta (\eta - u)^n J(\eta)$$

$$J_m(\eta = 0) = \int d\eta\, \eta^m J(\eta) = \int dz \left(\frac{z}{\epsilon}\right)^{\frac{im\pi}{l}} J(z)$$

$$J_l(\rho = 0) = \int d\rho\, \rho^l J(\rho)$$

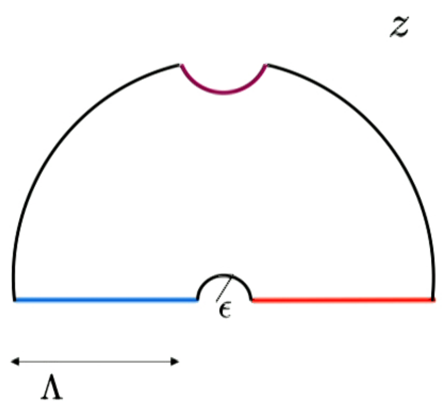


$$\Delta\Big(J_n(u)\Big) = \sum_{m=0}^{n} (-u)^m \binom{n}{m} J_m \otimes 1 + \sum_{l=0}^{n} (-u)^{n-l} \binom{n}{l} 1 \otimes J_{-l}$$

annihilation

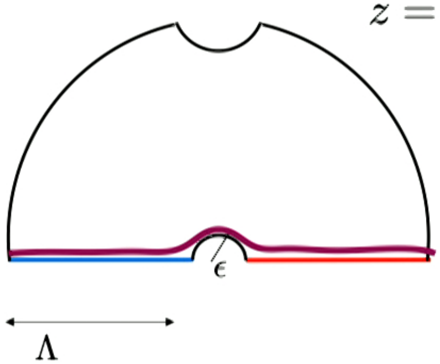
creation

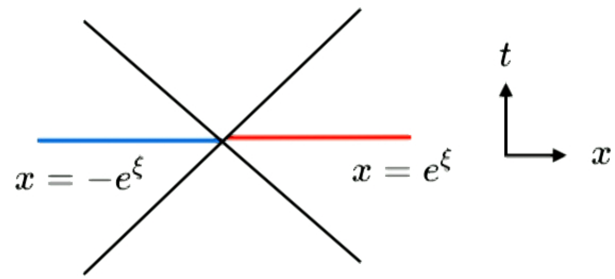
# Comparison with Bogoliubov Transformation



# Comparison with Bogoliubov Transformation

$z = x + i\tau$




Rindler modes

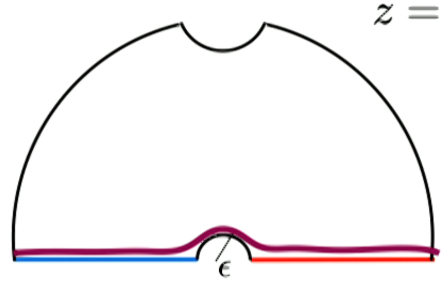
$$\phi(x+t)|_{t=0} = \int_{-\infty}^{\infty} dp\ a_p e^{ipx} = \int_{-\infty}^{\infty} dk\, \alpha_k^L x^{ik} \theta(-x) + \alpha_k^R x^{ik} \theta(x)$$

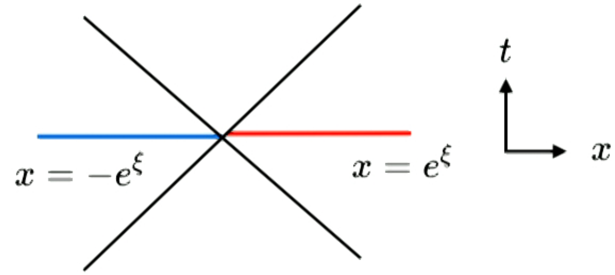
$$a_{-p} = a_p^\dagger$$
$$\alpha_{-k} = \alpha_k^\dagger$$

Bogoliubov transformation

$$p a_p = \int_0^{\infty} dk\ \frac{p^{-ik} e^{-\frac{\pi k}{2}}}{\Gamma(-ik)} \alpha_k^R + \int_{-\infty}^{0} dk \frac{p^{-ik} e^{-\frac{\pi k}{2}}}{\Gamma(-ik)} \alpha_k^L$$

# Comparison with Bogoliubov Transformation




$\lim_{\epsilon \to 0} -u = 1$

$$\Delta(J_n(u)) = \sum_{m=0}^{n} \binom{n}{m} J_m \otimes 1 + \sum_{m=0}^{n} \binom{n}{q} 1 \otimes J_{-q}$$

Large n $\quad n >> m, q \quad \eta \sim 0$

$$J_n(u) = \int d\eta (\eta - u)^n J(\eta) \sim \int d\eta \exp(n\eta) J(\eta) \qquad \binom{n}{q} \to \frac{n^q}{\Gamma(q+1)}$$

Wick rotation $\quad n \to -ip \quad m \to ik \quad q \to -ik$

$$J_m(\eta = 0) = \int d\eta\, \eta^m J(\eta) = \int dz \left(\frac{z}{\epsilon}\right)^{im\pi/l} J(z)$$

$$J_q(\rho = 0) = \int d\rho\, \rho^q J(\rho) = \int dz \left(\frac{z}{\epsilon}\right)^{-iq\pi/l} J(z)$$

$$\binom{n}{q} \to \frac{p^{-ik} e^{-\frac{\pi k}{2}}}{-ik\,\Gamma(-ik)}$$

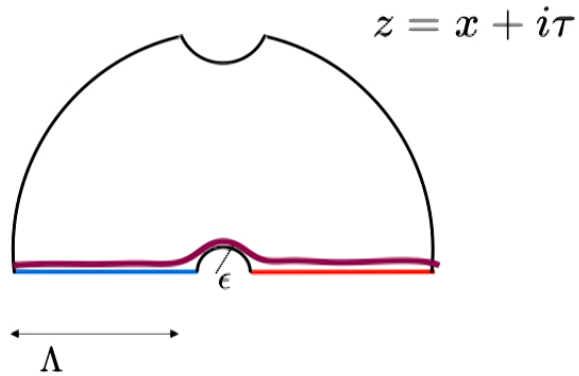
Co-Product

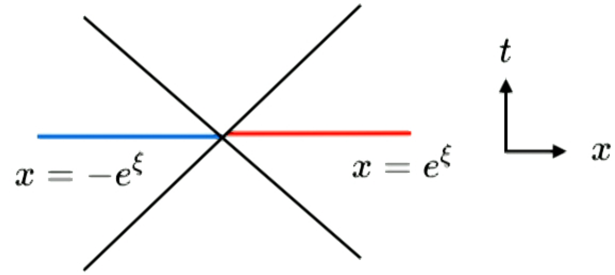
$$\Delta(J_{-ip}(u)) = \int_0^\infty dk\, \frac{p^{ik} e^{\frac{\pi k}{2}}}{\Gamma(ik)} \frac{J_{ik}}{ik} + \int_0^\infty dk\, \frac{p^{-ik} e^{\frac{-\pi k}{2}}}{\Gamma(-ik)} \frac{J_{ik}}{-ik}$$

Bogoliubov

$$p\, a_p = \int_0^\infty dk \frac{p^{ik} e^{\frac{\pi k}{2}}}{\Gamma(ik)} \alpha_k^L + \int_0^\infty dk\, \frac{p^{-ik} e^{-\frac{\pi k}{2}}}{\Gamma(-ik)} \alpha_k^R$$

# Comparison with Bogoliubov Transformation




$$\lim_{\epsilon \to 0} -u = 1 \qquad \Delta(J_n(u)) = \sum_{m=0}^{n} \binom{n}{m} J_m \otimes 1 + \sum_{m=0}^{n} \binom{n}{q} 1 \otimes J_{-q}$$

Large n $\quad n >> m, q \quad \eta \sim 0$

$$J_n(u) = \int d\eta (\eta - u)^n J(\eta) \sim \int d\eta \, \exp(n\eta) J(\eta) \qquad \binom{n}{q} \to \frac{n^q}{\Gamma(q+1)}$$

Wick rotation $\quad n \to -ip \quad m \to ik \quad q \to -ik$

$$J_m(\eta = 0) = \int d\eta \, \eta^m J(\eta) = \int dz \left(\frac{z}{\epsilon}\right)^{im\pi/l} J(z)$$

$$J_q(\rho = 0) = \int d\rho \, \rho^q \, J(\rho) \quad = \int dz \left(\frac{z}{\epsilon}\right)^{-iq\pi/l} J(z)$$

$$\binom{n}{q} \to \frac{p^{-ik} e^{-\frac{\pi k}{2}}}{-ik \, \Gamma(-ik)}$$

Co-Product

$$\Delta(J_{-ip}(u)) = \int_0^{\infty} dk \, \frac{p^{ik} e^{\frac{\pi k}{2}}}{\Gamma(ik)} \frac{J_{ik}}{ik} + \int_0^{\infty} dk \, \frac{p^{-ik} e^{\frac{-\pi k}{2}}}{\Gamma(-ik)} \frac{J_{ik}}{-ik}$$

Bogoliubov

$$p \, a_p = \int_0^{\infty} dk \frac{p^{ik} e^{\frac{\pi k}{2}}}{\Gamma(ik)} \alpha_k^L + \int_0^{\infty} dk \, \frac{p^{-ik} e^{-\frac{\pi k}{2}}}{\Gamma(-ik)} \alpha_k^R$$

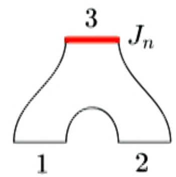
## Summary

We propose an extension of 2D CFT incorporating the E brane BC ~ vacuum Ishibashi state.

Factorization of CFT Hilbert space via a co- product formula (Gaberdiel/Moore Seiberg)

$$J_n|0\rangle_3 = \Delta(J_n)|\Omega\rangle_{12} = 0$$

$$J_{-n}|0\rangle_3 = \Delta(J_{-n})|\Omega\rangle_{12}$$




$|\Omega\rangle_{12} =$ 

## Puzzles and generalizations

Explicit understanding of the quotient on the tensor product

Relation to Conne fusion of Bi modules over Von Neumann algebras? (Wasserman, Henrique,Conne..)

Generalization to QFT( Segal unpublished) — Alternative to the Algebraic QFT approach?

Higher co-dimension= richer structure: e.g. co-dimension 3 in Chern Simons theory

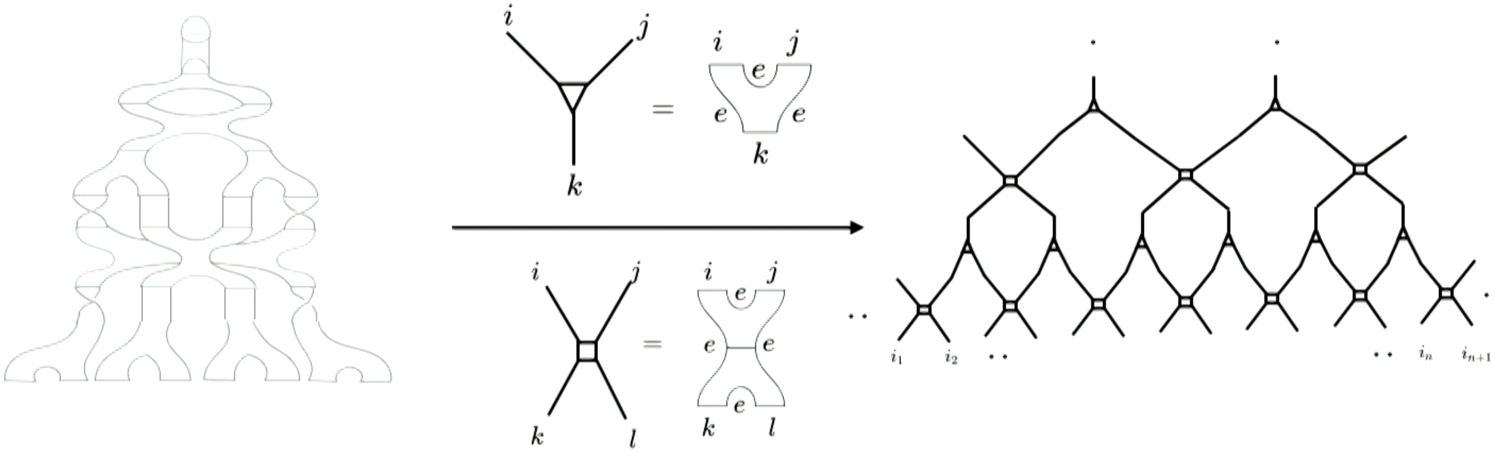
## Applications

Tensor Network

AdS/BCFT

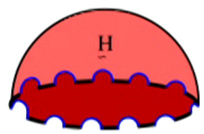
Entanglement in string theory

# Tensor Networks from OPE's

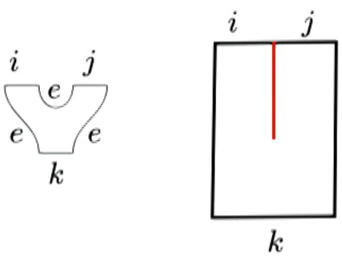



## AdS/BCFT - bulk dual to the boundary extended CFT?

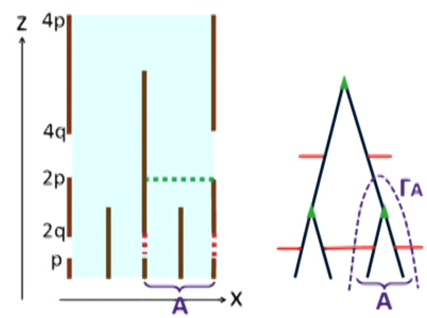
Relation to Raamsdonk's BC bits?




"String theory representation"

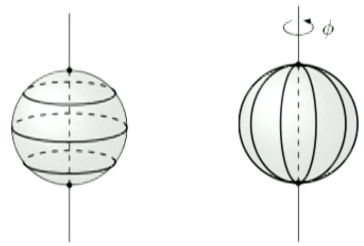



Takayanagi et. al :Bulk dual to local quench



## Entanglement in string theory

$$\phi \sim \phi + \beta$$

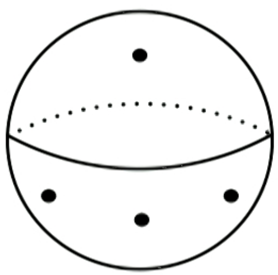



BH Entropy = Entanglement entropy

$$S = (1 - \beta\partial_\beta)|_{\beta=2\pi} Z_{string}(\beta) = \frac{A}{4G}$$

**c.f. Susskind-Uglum, Tseytlin**

## State counting interpretation from open-closed string duality



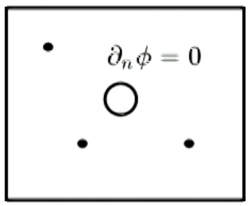
Punctures by the entangling surface ~ holes on the worldsheet

E brane boundary condition on the Worldsheet

Factorization of the string theory Hilbert space?

# The Neumann E brane and T duality

$\phi \sim \phi + 2\pi R$



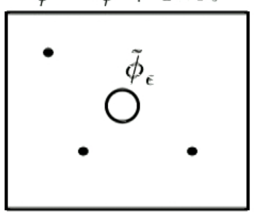

Shift invariance/Fusion rules preserved

$$\langle V_{p_1} V_{p_2} \cdots V_{p_n} \rangle \sim \delta(\sum_i p_i) \qquad V_p = \exp \frac{ip\phi}{R}, p \in \mathbb{Z}$$

Bulk correlation of local operators preserved up to $O(\epsilon)$

But there are non-local vortex operators which we have to check: Use T- Duality

$\tilde{\phi} \sim \tilde{\phi} + 2\pi \tilde{R}$




Integrating over $\tilde{\phi}_\epsilon$ preserves shift invariance of $\tilde{\phi}$

$$\langle \tilde{V}_{q_1} \tilde{V}_{q_2} \cdots \tilde{V}_{q_n} \rangle \sim \delta(\sum_i q_i) \qquad \tilde{V}_p = \exp \frac{iq\tilde{\phi}}{\tilde{R}}, q \in \mathbb{Z}$$

This correspond to summing over Wilson lines of $\phi$