Title: Defect Monstrous Moonshine

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Series: Quantum Fields and Strings

Date: November 26, 2019 - 2:30 PM

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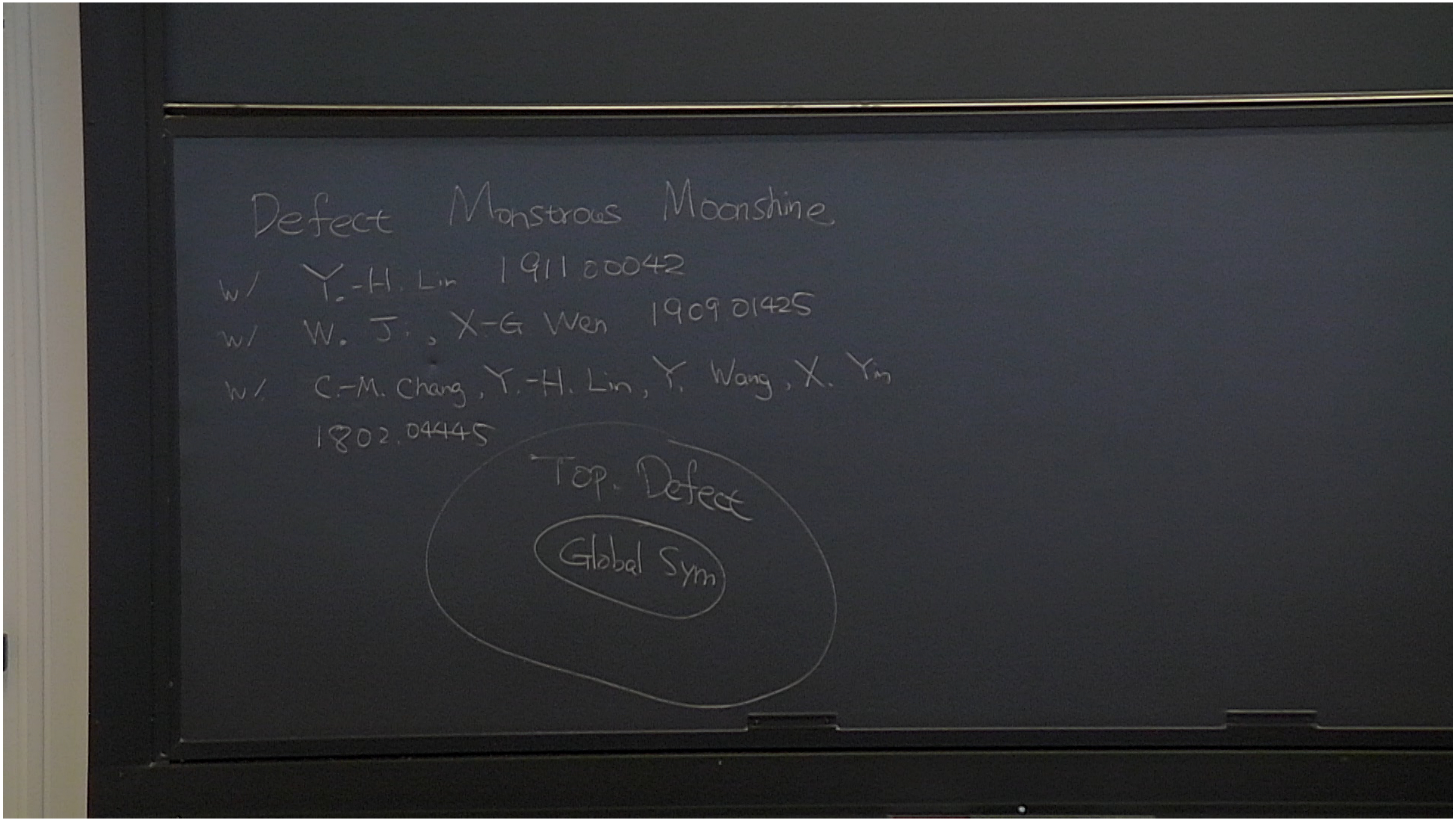
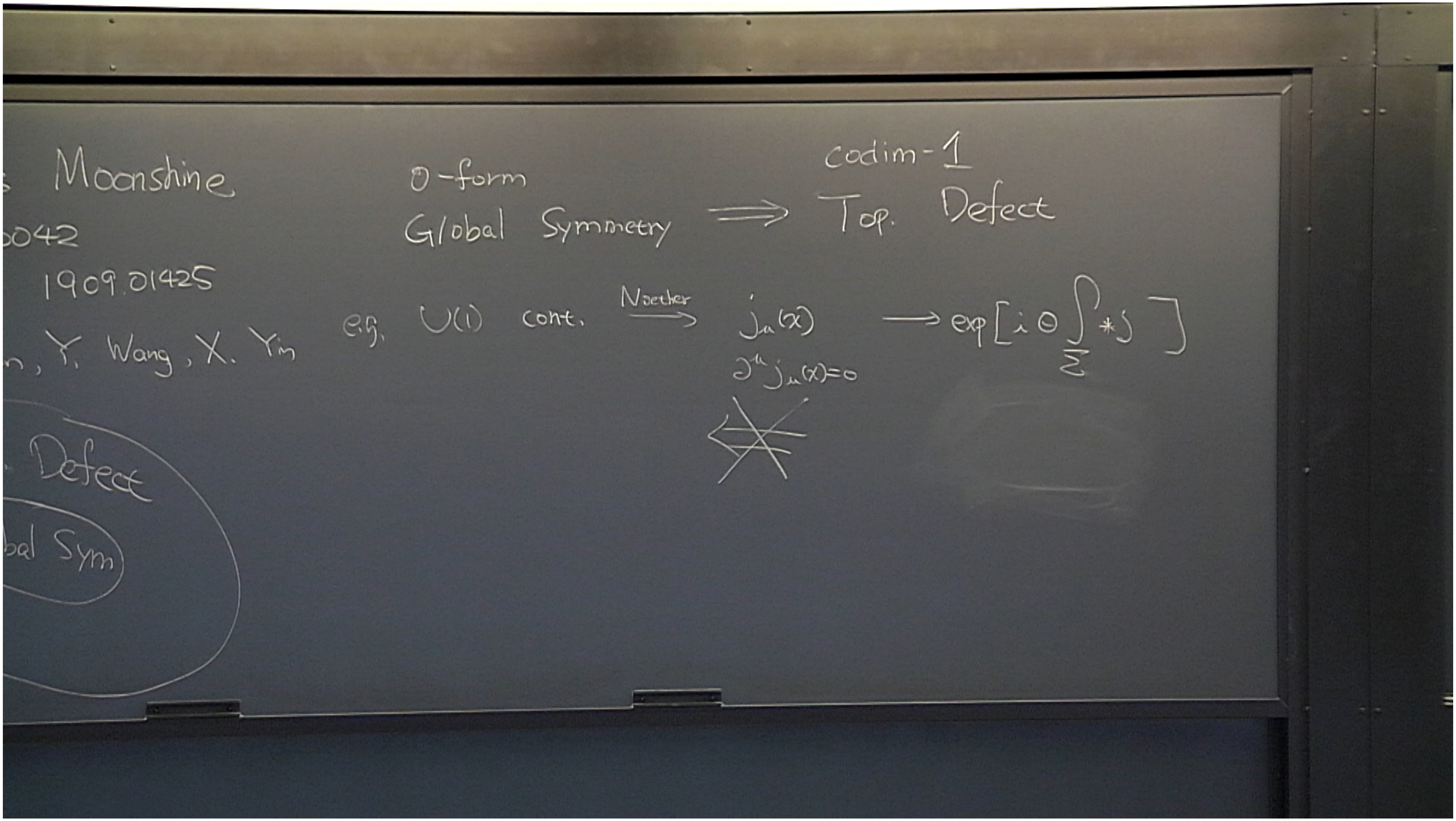
Abstract: The Monster CFT is a (1+1)d holomorphic CFT with the Monster group global symmetry. The symmetry twisted partition functions exhibit the celebrated Monstrous Moonshine Phenomenon. From a modern point of view, topological defects generalize the notion of global symmetries. We argue that the Monster CFT has a Kramers-Wannier duality defect that is not associated with any global symmetry. The duality defect extends the Monster group to a larger category of topological defects that contains an Ising subcategory. We introduce the defect McKay-Thompson series defined as the Monster partition function twisted by the duality defect, and find that it is invariant under the genus-zero congruence subgroup 16D0 of PSL(2,Z).


Defect Monstrous Moonshine
w/ Y.-H. Lin 1911 00042
w/ W. J. , X-G Wen 1909 01425
w/ C.-M. Chang, Y.-H. Lin, Y. Wang, X. Yin
1802.04445
Top. Defect
Global Sym


Moonshine
1909.01425
Y. Wang, X. Yin
0-form
Global Symmetry ⟹ Top. Defect
codim-1
e.g. U(1) cont. Noether
$j_\mu(x)$
$\partial^\mu j_\mu(x)=0$
$\exp[i\theta\int_\Sigma *j]$
Defect
bal Sym

Bosonization / Fermionization in 1+1 d

$c_L = c_R = \frac{1}{2}$

$m = 0$

Majorana fermion $\xrightarrow[\text{the spin str.}]{\text{sum over}}$ Ising CFT

(3,4) minimal model

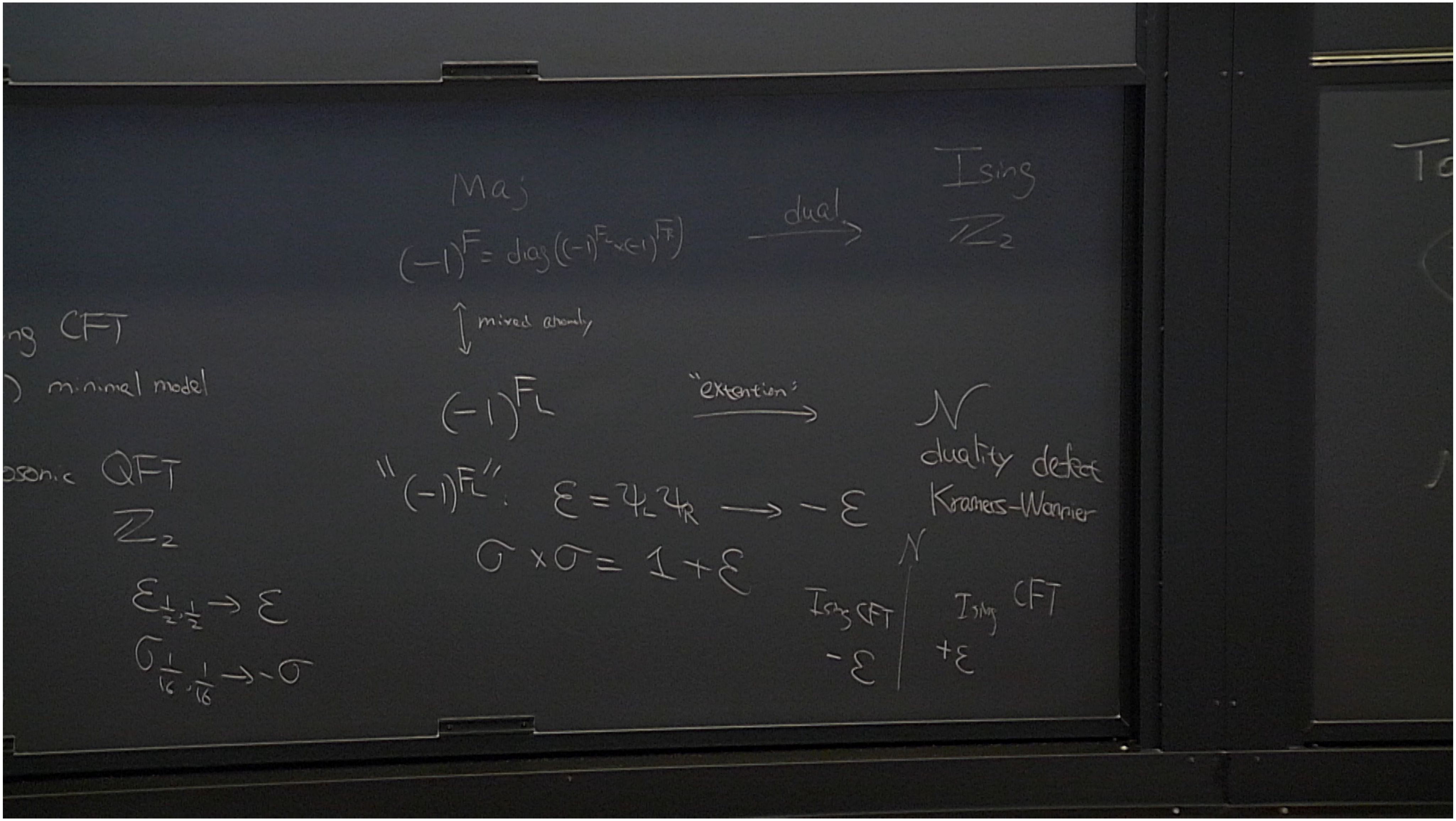
	fermionic QFT	bosonic QFT
Global Sym:	$\mathbb{Z}_2 \times \mathbb{Z}_2$	$\mathbb{Z}_2$

$(-1)^{F_L}$: $\psi_L(z) \to -\psi_L(z)$, $\psi_R(\bar{z}) \to \psi_R(\bar{z})$

$(-1)^{F_R}$: $\psi_L(z) \to \psi_L(z)$, $\psi_R(\bar{z}) \to -\psi_R(\bar{z})$

$\varepsilon_{\frac{1}{2},\frac{1}{2}} \to \varepsilon$

$\sigma_{\frac{1}{16},\frac{1}{16}} \to -\sigma$


Maj
(−1)^F = diag((−1)^{F_L} × (−1)^{F_R})
dual
Ising
Z_2
mixed anomaly
ng CFT
) minimal model
(−1)^{F_L}
"extention"
N
duality defect
Kramers-Wannier
osonic QFT
Z_2
"(−1)^{F_L}": ε = ψ_L ψ_R → −ε
σ × σ = 1 + ε
ε_{1/2, 1/2} → ε
σ_{1/16, 1/16} → −σ
N
Ising CFT
−ε
Ising CFT
+ε
To

Top. Defects of Ising

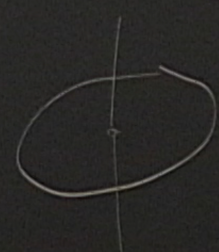
I $\quad \eta$ $\quad \mathbb{Z}_2$

$\mathcal{N}$

Kramers-Wannier duality defect

$\eta \times \eta = I, \quad \eta \times \mathcal{N} = \mathcal{N} \times \eta = \mathcal{N}$

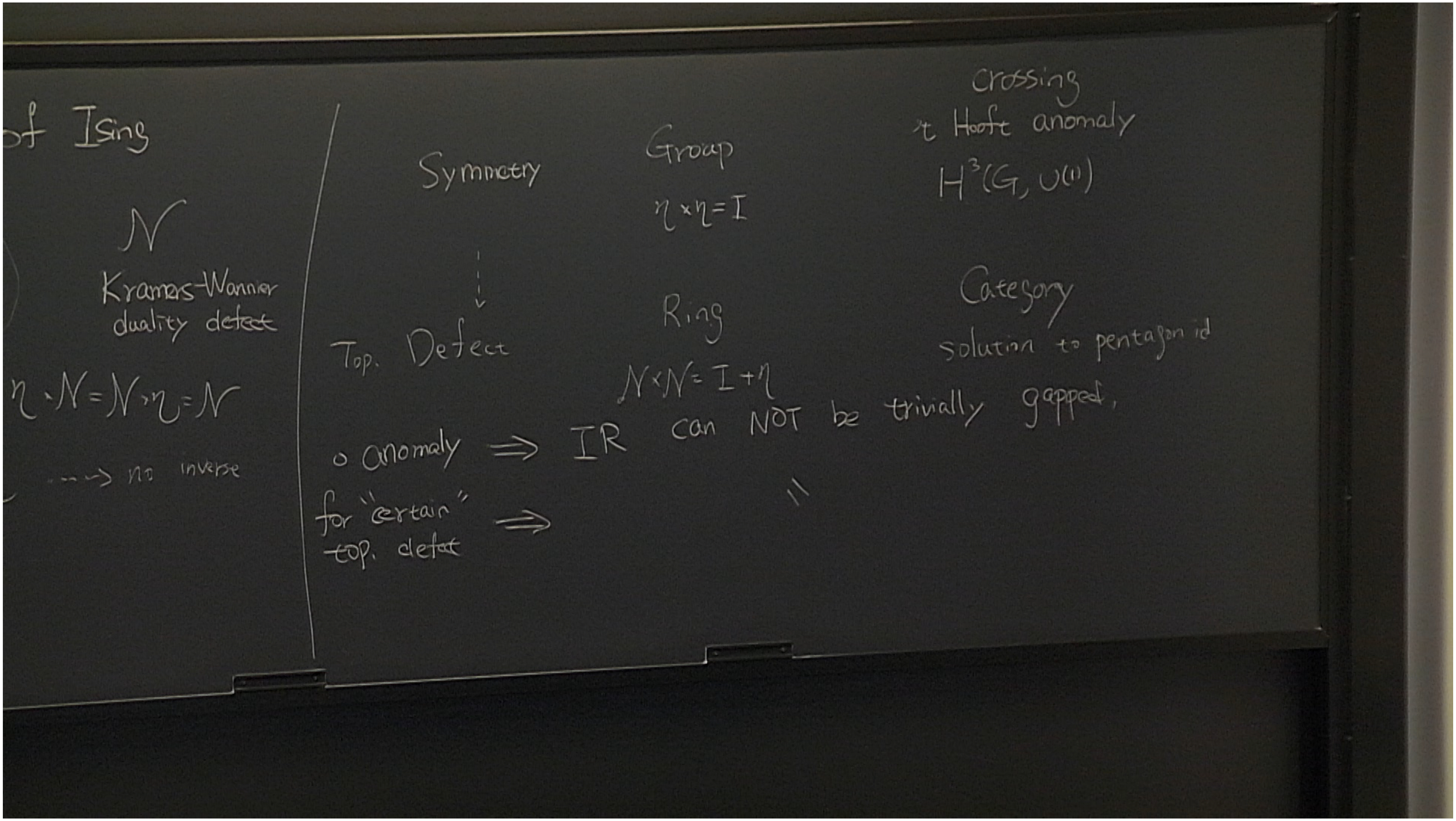
$\mathcal{N} \times \mathcal{N} = I + \eta$ ----> no inverse

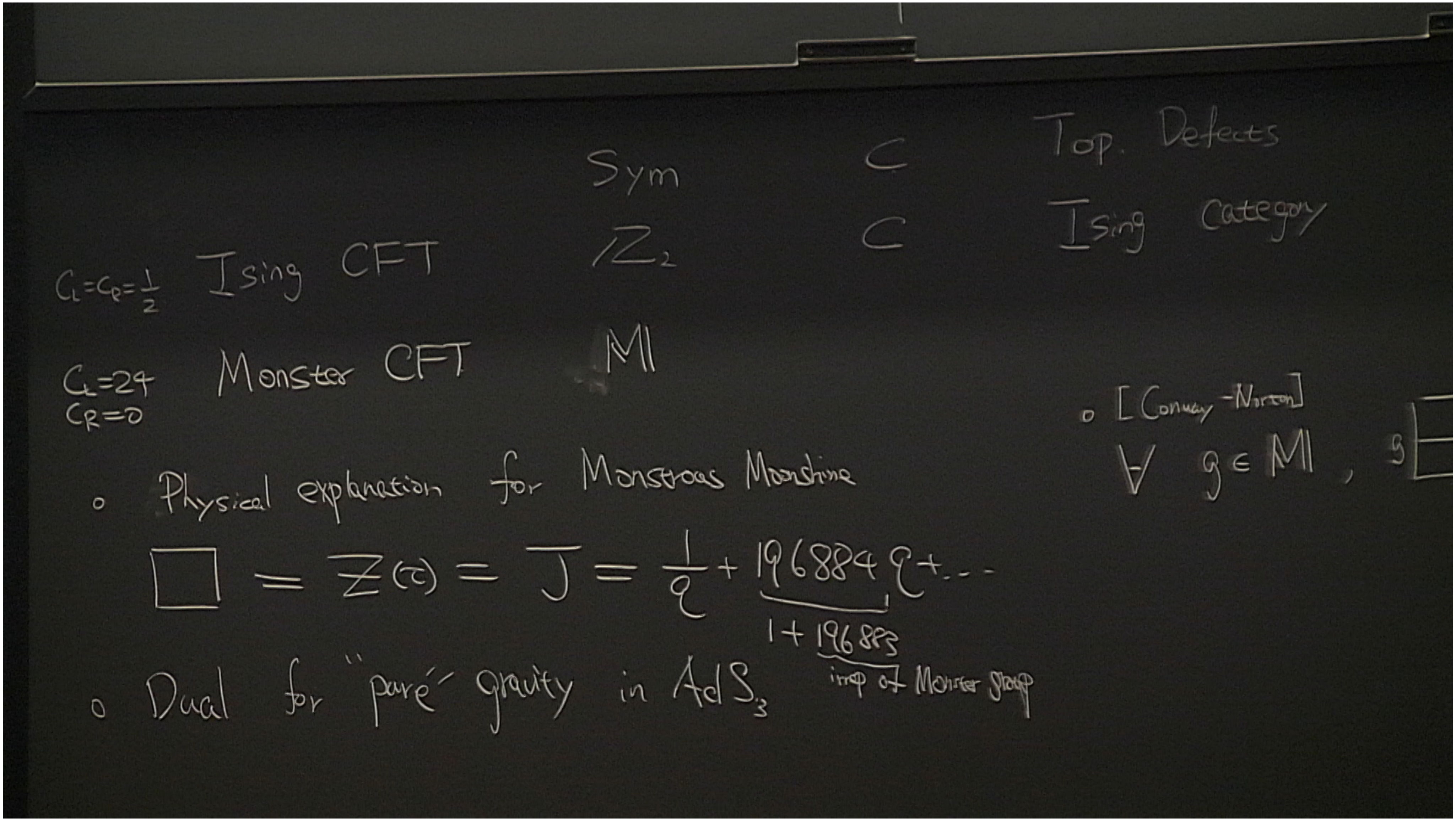


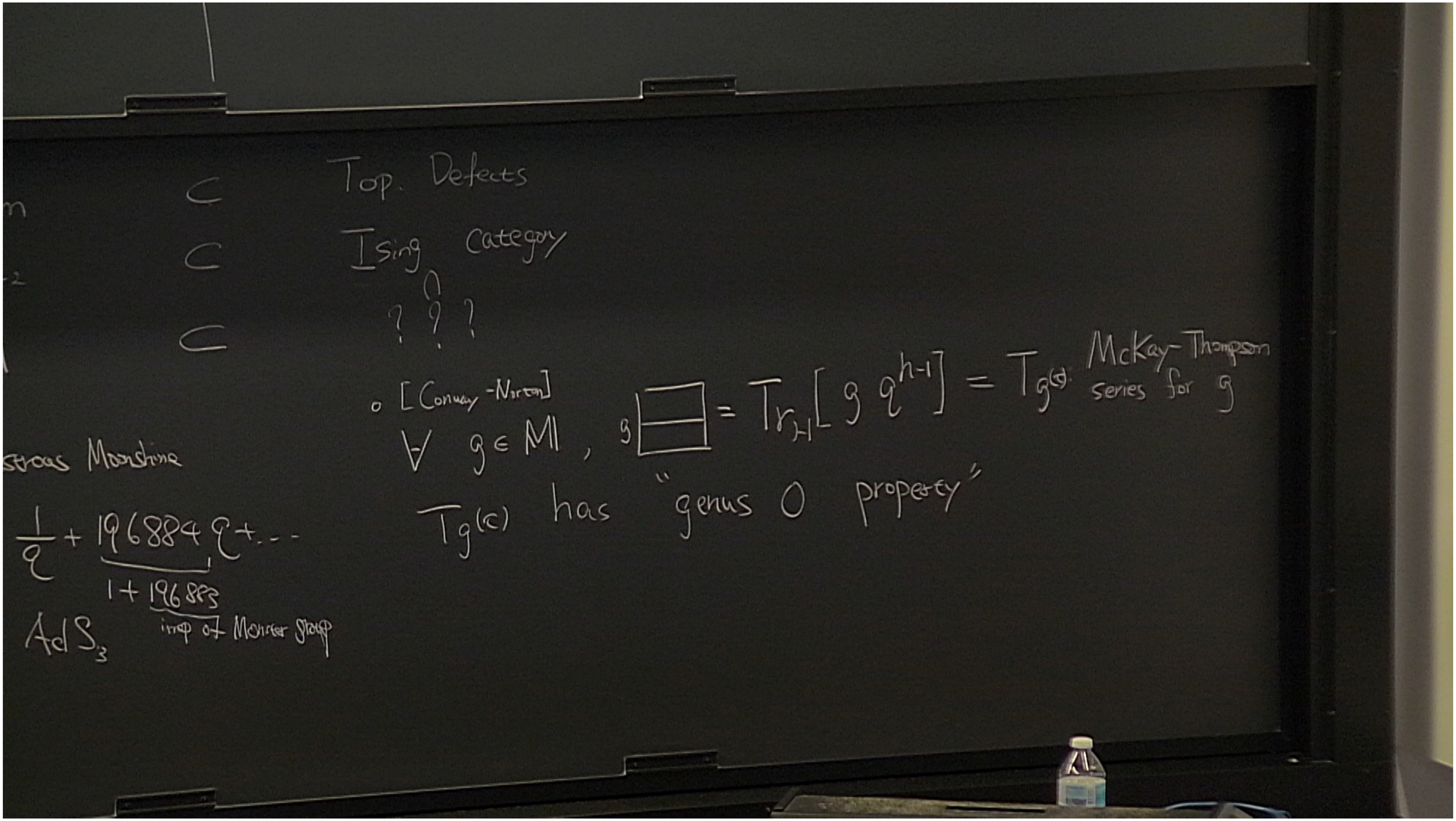
Symmetry

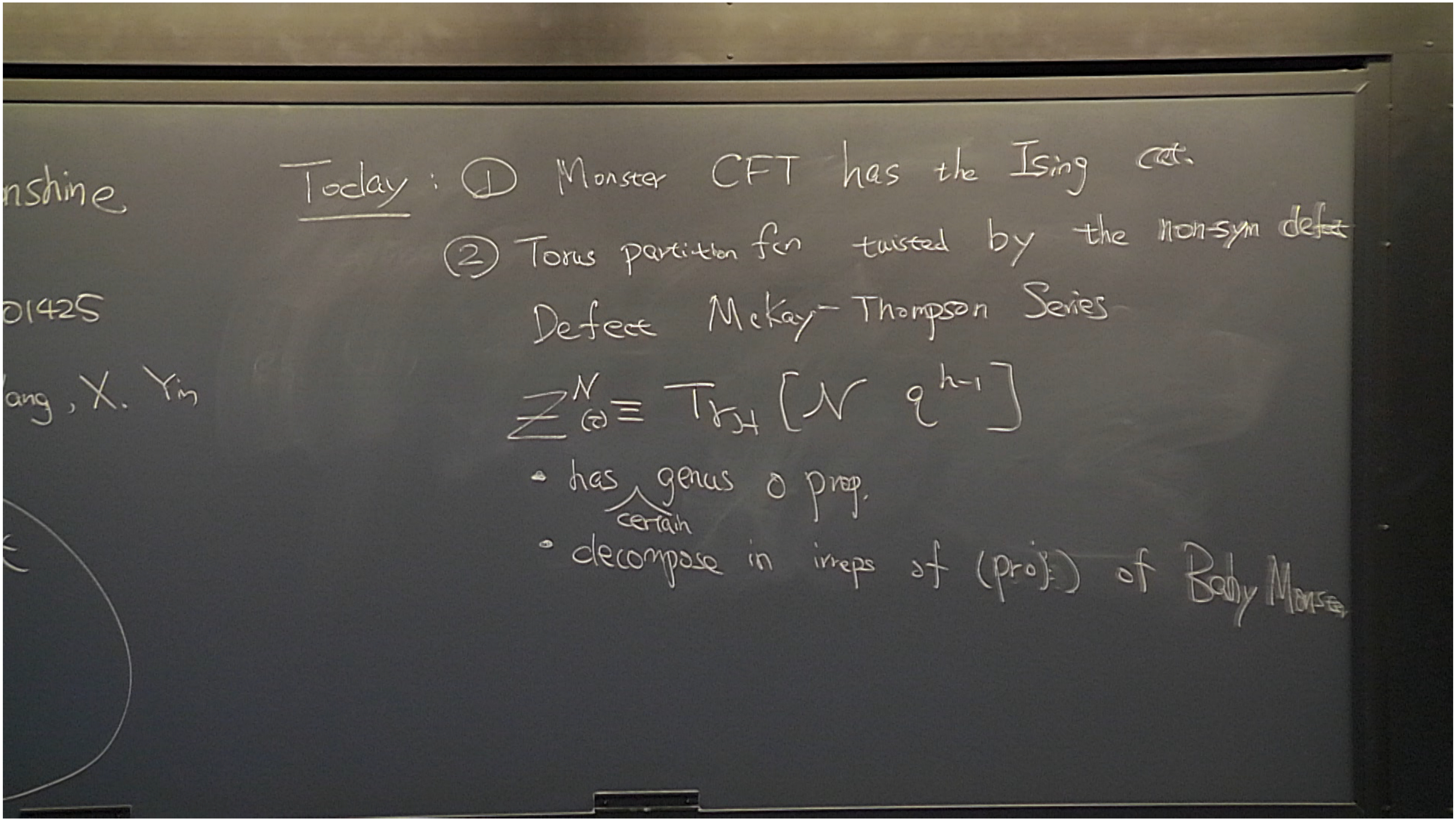
↓

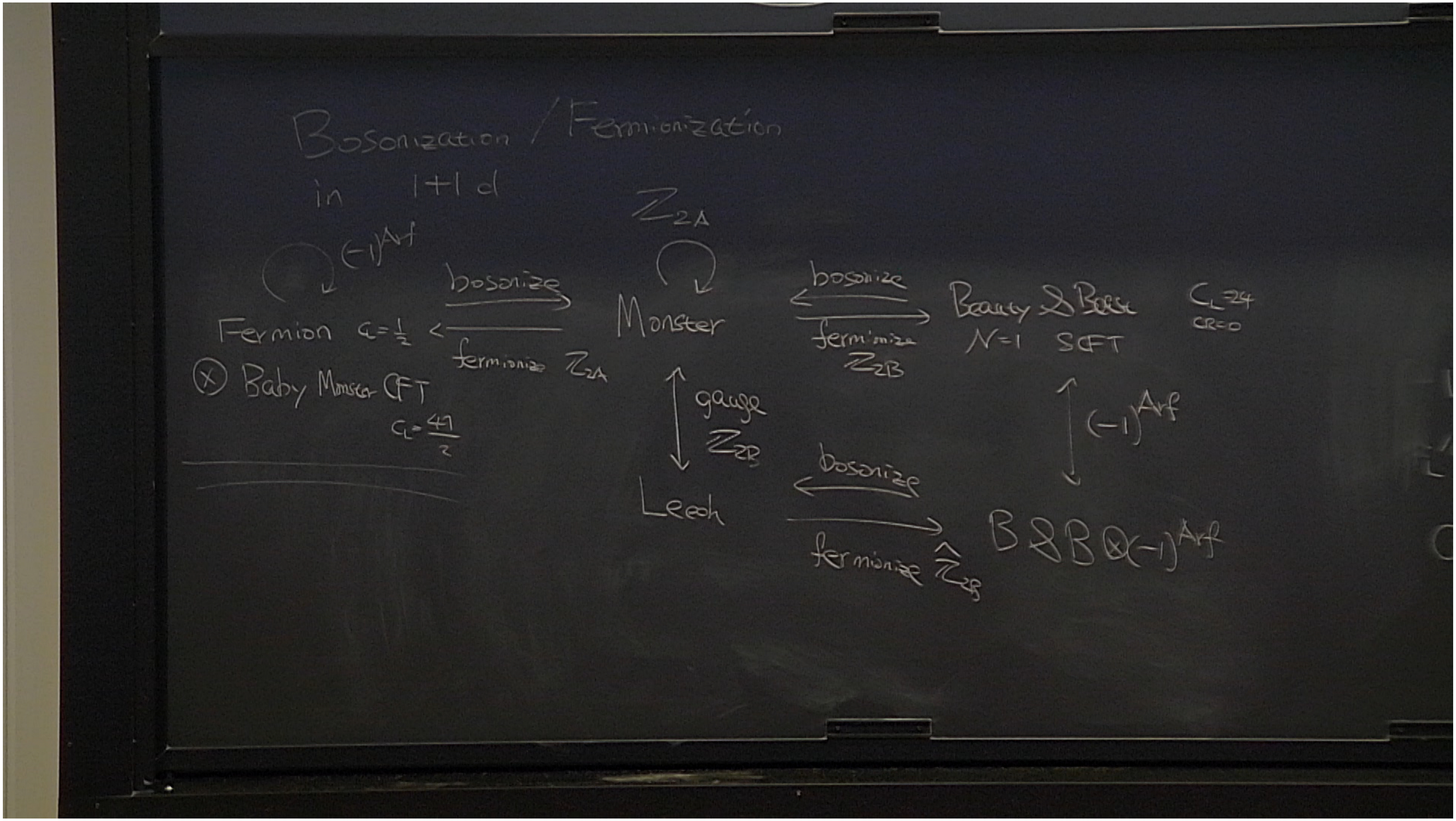
Top. Defect


of Ising
Kramers-Wannier
duality defect
no inverse
Symmetry
Top. Defect
Group
Ring
crossing
't Hooft anomaly
Category
solution to pentagon id
anomaly ⇒ IR can NOT be trivially gapped.
for "certain" top. defect ⇒


Sym ⊂ Top. Defects
$c_L = c_R = \frac{1}{2}$ Ising CFT $\mathbb{Z}_2$ ⊂ Ising Category
$c_L = 24$, $c_R = 0$ Monster CFT $\mathbb{M}$
∘ Physical explanation for Monstrous Moonshine
$\square = Z(\tau) = J = \frac{1}{q} + 196884\, q + \dots$
$1 + 196883$
irrep of Monster group
∘ Dual for "pure" gravity in AdS_3
∘ [Conway-Norton]
$\forall\, g \in \mathbb{M}$, g


Top. Defects
Ising category
???
[Conway-Norton]
$\forall\, g \in \mathbb{M}$, $= \mathrm{Tr}_{\mathcal{H}}[\, g\, q^{h-1}] = T_g(\tau)$
McKay-Thompson series for g
$T_g(\tau)$ has "genus 0 property"
Moonshine
$\frac{1}{q} + 196884\, q + \cdots$
$1 + 196883$
irrep of Monster group
AdS_3


nshine
01425
ang, X. Yin
Today: ① Monster CFT has the Ising cat.
② Torus partition fcn twisted by the non-sym defect
Defect McKay-Thompson Series
$Z^{\mathcal{N}}(\tau) \equiv \mathrm{Tr}_{\mathcal{H}} [\mathcal{N} q^{h-1}]$
• has certain genus 0 prop.
• decompose in irreps of (proj.) of Baby Monster


Bosonization / Fermionization
in 1+1 d
$\mathbb{Z}_{2A}$
$(-1)^{Arf}$
bosonize
Fermion $c=\frac{1}{2}$
fermionize $\mathbb{Z}_{2A}$
⊗ Baby Monster CFT
$c_L = \frac{47}{2}$
Monster
gauge $\mathbb{Z}_{2B}$
Leech
bosonize
fermionize $\mathbb{Z}_{2B}$
Beauty & Beast
$\mathcal{N}=1$ SCFT
$c_L=24$
$c_R=0$
$(-1)^{Arf}$
bosonize
fermionize $\hat{\mathbb{Z}}_{2B}$
$B\&B \otimes (-1)^{Arf}$

ality & Bose $\quad c_L = 24,\ c_R = 0$

$\mathcal{N}=1$ SCFT

$\updownarrow \ (-1)^{\mathrm{Arf}}$

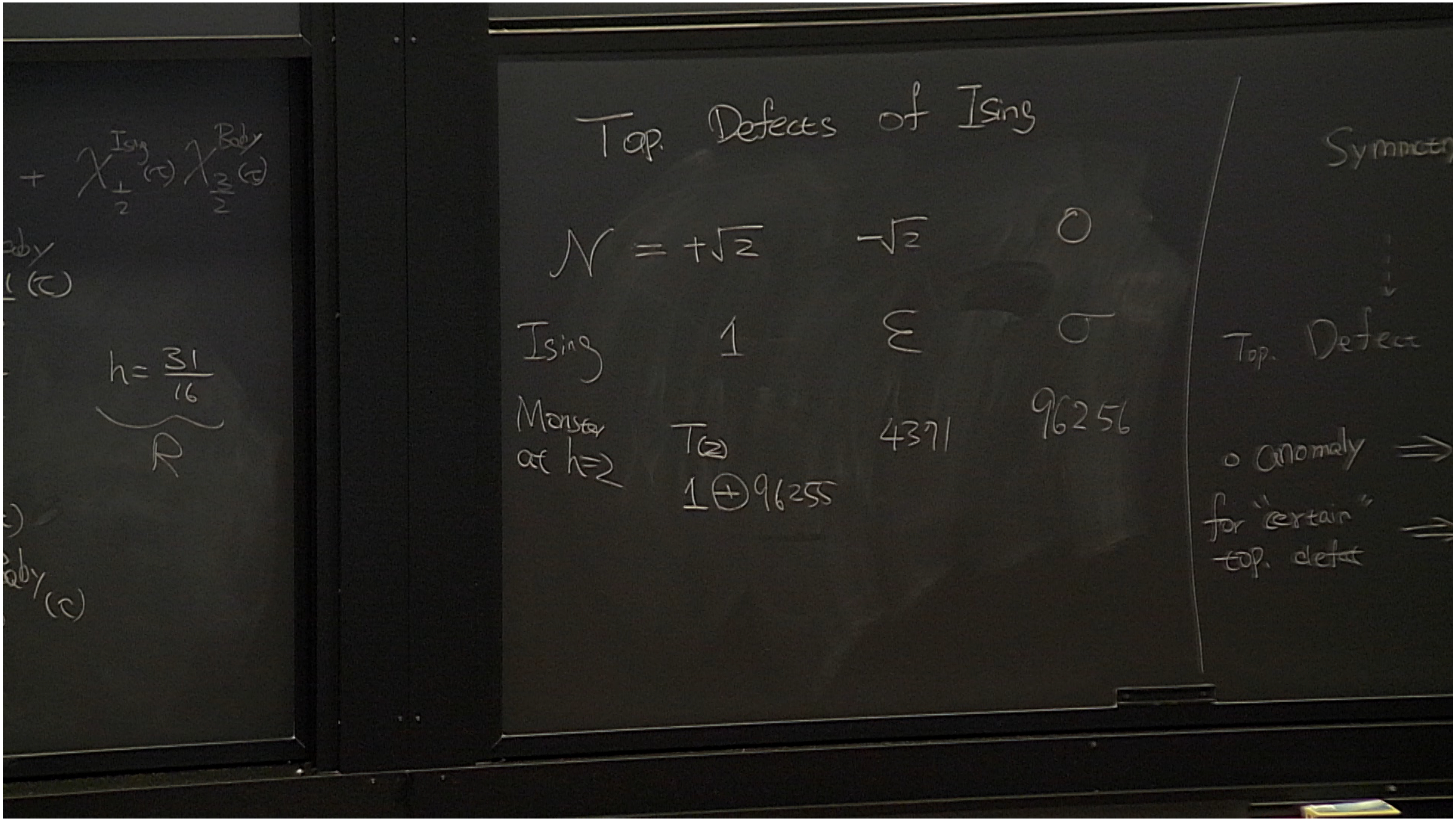
B & B $\otimes (-1)^{\mathrm{Arf}}$

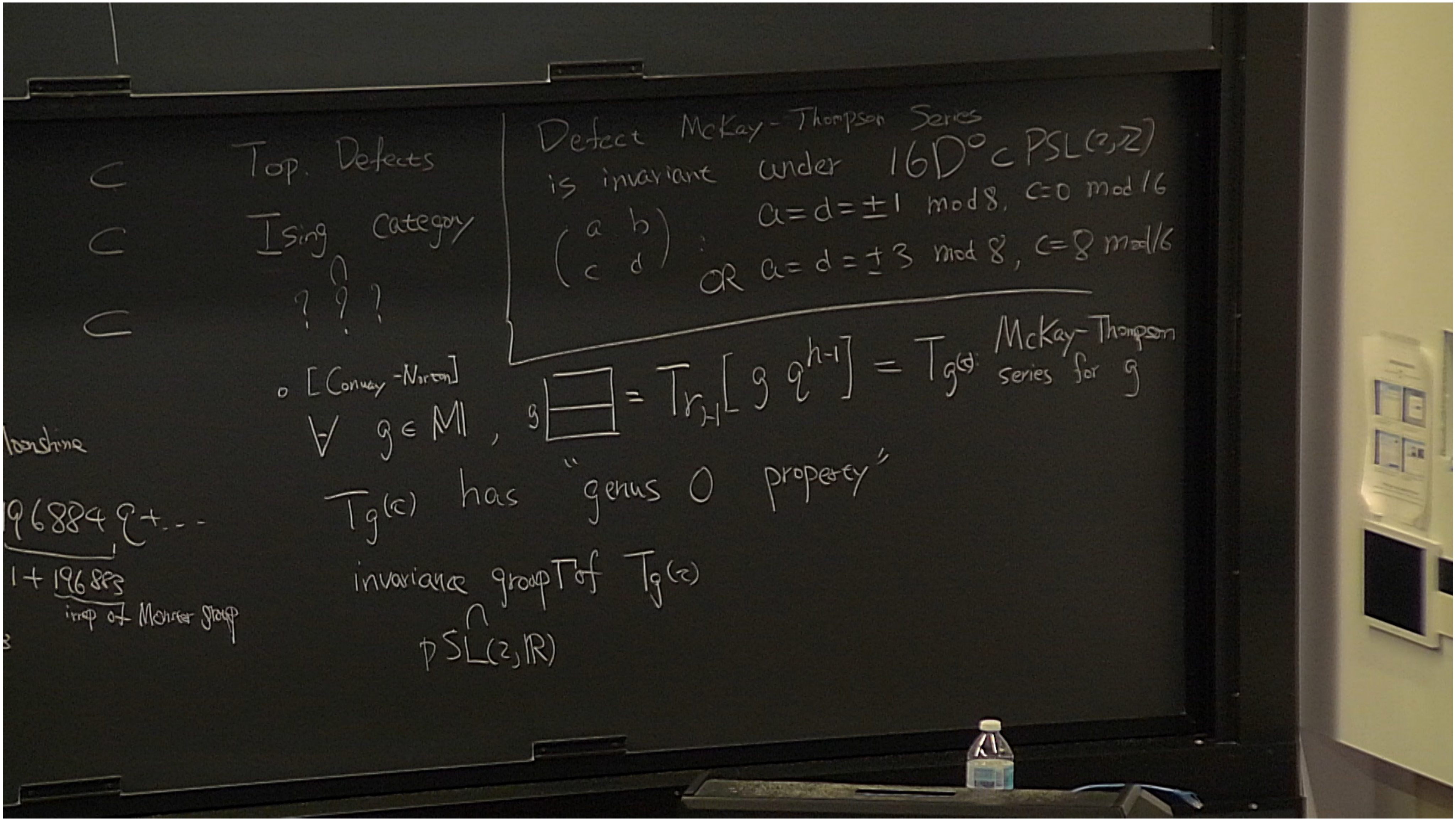
Monster

$$J(\tau) = \chi_0^{\mathrm{Ising}}(\tau)\,\chi_0^{\mathrm{Baby}}(\tau) + \chi_{\frac{1}{2}}^{\mathrm{Ising}}(\tau)\,\chi_{\frac{3}{2}}^{\mathrm{Baby}}(\tau) + \chi_{\frac{1}{16}}^{\mathrm{Ising}}(\tau)\,\chi_{\frac{31}{16}}^{\mathrm{Baby}}(\tau)$$

Baby Monster: $\underbrace{h = 0,\ \tfrac{3}{2}}_{NS} \qquad \underbrace{h = \tfrac{31}{16}}_{R}$

$$\mathcal{N}\ \boxminus \quad Z^{\mathcal{N}}(\tau) = \sqrt{2}\,\chi_0^{\mathrm{Ising}}(\tau)\,\chi_0^{\mathrm{Baby}}(\tau) - \sqrt{2}\,\chi_{\frac{1}{2}}^{\mathrm{Ising}}(\tau)\,\chi_0^{\mathrm{Baby}}(\tau)$$


Top. Defects of Ising
N = +√2   -√2   0
Ising   1   ε   σ
Monster at h=2   T(z)   4371   96256
1 ⊕ 96255
h = 31/16
R
Symmetry
Top. Defect
0 anomaly ⇒
for "certain" top. defect ⇒


Top. Defects
Ising category
? ? ?
Defect McKay-Thompson Series
is invariant under 16D° ⊂ PSL(2,Z)
(a b / c d) : a=d=±1 mod 8, c=0 mod 16
OR a=d=±3 mod 8, c=8 mod 16
[Conway-Norton]
∀ g ∈ M,
= Tr_{H}[g q^{h-1}] = T_g(τ): McKay-Thompson series for g
T_g(τ) has "genus 0 property"
invariance group Γ of T_g(τ)
PSL(2,R)
Moonshine
196884 q + ...
1 + 196883
irrep of Monster group