Title: PSI 2019/2020 - Condensed Matter (Wang) - Lecture 12

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Collection: PSI 2019/2020 - Condensed Matter (Wang)

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Ginzburg-Landau Meissner effect (Anderson-Higgs mechanism)

Landau + minimal coupling to E&M. $\vec{A}(\vec{x})$, $A_0 = 0$ $(T < T_c)$

$$f[\Phi, A] = \frac{1}{2}\left|(\vec{\partial} - i2\vec{A})\Phi\right|^2 - \frac{1}{\xi^2(T)}|\Phi|^2 + b|\Phi|^4 \qquad \frac{1}{\xi^2(T)} \propto T_c - T$$

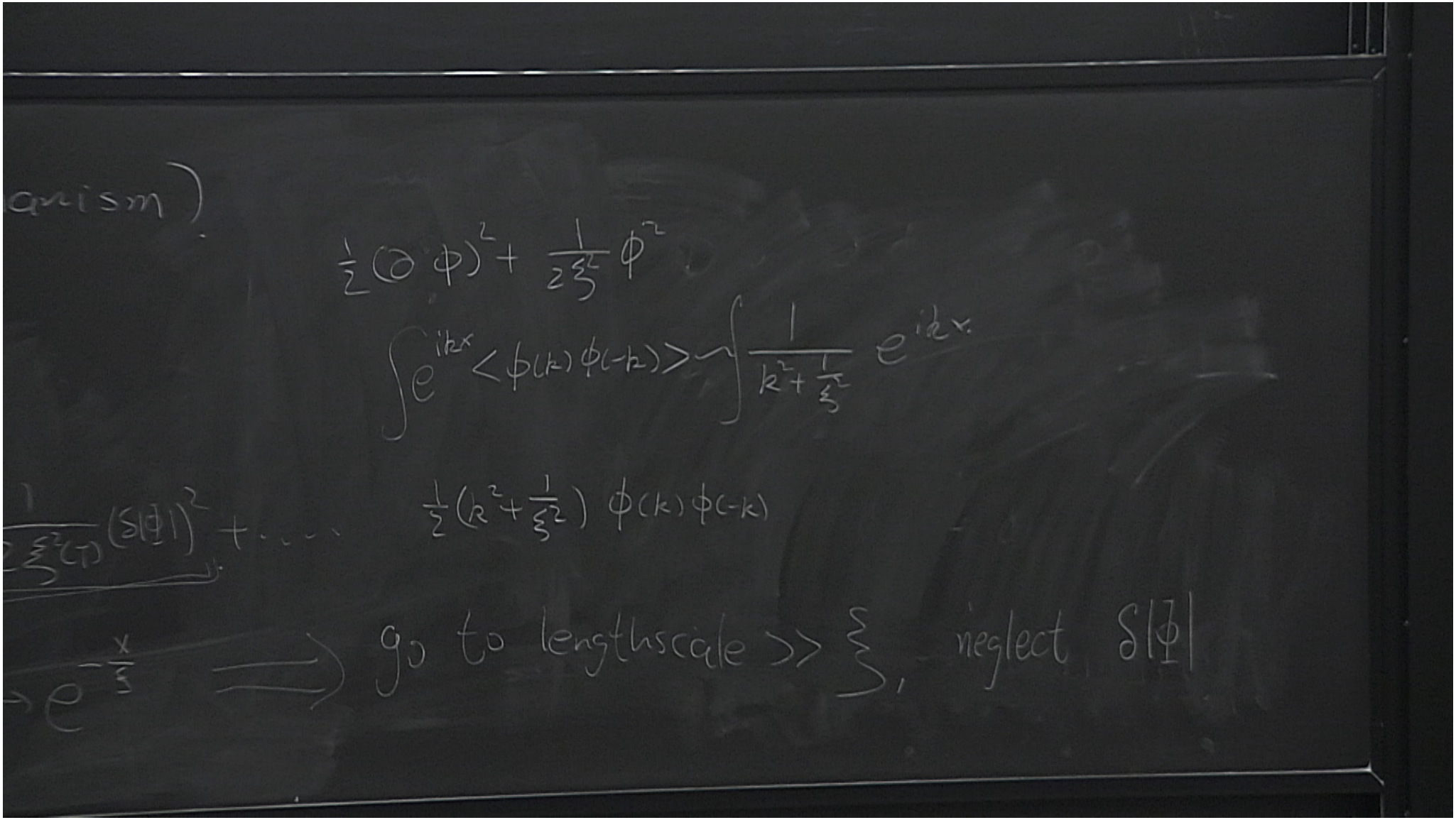
$$|\Phi_0|^2 \propto \frac{1}{2\xi^2 b} \propto (T_c - T)$$

$$\Phi = (|\Phi_0| + \delta|\Phi|)e^{i\theta}$$

$$f = \frac{1}{2}\rho_s(\vec{\partial}\theta - 2\vec{A})^2 + \frac{1}{2}(\vec{\partial}\delta|\Phi|)^2 + \frac{1}{2\xi^2(T)}(\delta|\Phi|)^2 + \dots$$

$$\rho_s = |\Phi_0|^2 \propto T_c - T$$

$$\langle \delta|\Phi|(x)\, \delta|\Phi|(0)\rangle \to e^{-\frac{x}{\xi}}$$


$\frac{1}{2}(\partial\phi)^2 + \frac{1}{2\xi^2}\phi^2$
$\int e^{ikx} \langle \phi(k)\phi(-k) \rangle \sim \int \frac{1}{k^2 + \frac{1}{\xi^2}} e^{ikx}$
$\frac{1}{2}(k^2 + \frac{1}{\xi^2})\,\phi(k)\phi(-k)$
$e^{-\frac{x}{\xi}}$
go to lengthscale >> ξ, neglect $\delta|\phi|$

$$f[\theta, A] = \frac{1}{2} \rho_s (\vec{\partial}\theta - 2\vec{A})^2$$

ρ_s: phase stiffness.

Minimize w.r.t. θ (Eq of motion)

$$\frac{\delta f}{\delta \theta} = 0 \Rightarrow \vec{\nabla} \cdot (\vec{\nabla}\theta - 2\vec{A}) = 0.$$

$$f[\theta, A] = \frac{1}{2} \rho_s (\vec{\partial}\theta - 2\vec{A})^2$$. ρ_s : phase stiffness

Minimize w.r.t. θ (Eq of motion)

$$\frac{\delta f}{\delta \theta} = 0 \Rightarrow \boxed{\vec{\nabla} \cdot (\vec{\nabla}\theta - 2\vec{A}) = 0}$$

Electric current

$$j_e \equiv -\frac{\delta f}{\delta A} \Rightarrow \boxed{j_e = 2\rho_s (\vec{\nabla}\theta - 2\vec{A})}$$

$$f[\theta, A] = \frac{1}{2}\rho_s(\vec{\partial}\theta - 2\vec{A})^2$$

ρ_s: phase stiffness

Minimize w.r.t θ (Eq of motion)

$$\frac{\delta f}{\delta \theta} = 0 \Rightarrow \vec{\nabla}\cdot(\vec{\nabla}\theta - 2\vec{A}) = 0$$

Electric current

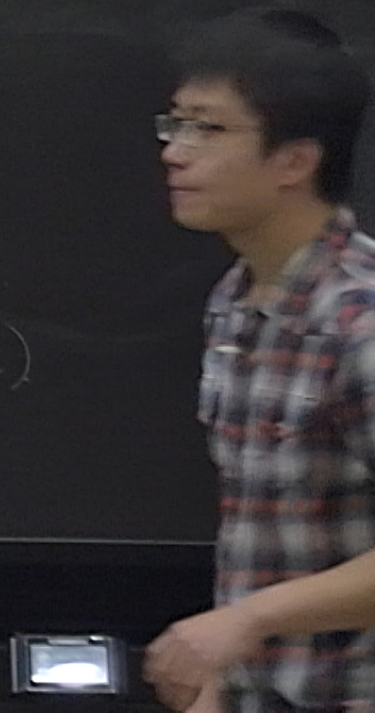
$$j_e \equiv -\frac{\delta f}{\delta A} \Rightarrow j_e = 2\rho_s(\vec{\nabla}\theta - 2\vec{A})$$

Pick a gauge

$$\check{A} = A - \frac{1}{2}\nabla\theta$$

$$j_e = -4\rho_s\check{A}$$ (London eq.)

$$\vec{\nabla}\cdot\check{A} = 0$$ (Coulomb gauge)



Uniform AC Electric field, $E(t) = E e^{-i\omega t} = \partial_t \vec{A}$ $\quad i\omega \vec{A} = E$.

$$j_e = \frac{4\rho}{i\omega} E \Rightarrow \sigma(\omega) = \frac{4\rho_s}{i\omega}$$ non-dissipative, diverges as $\omega \to 0$.

Uniform AC Electric field, $E(t) = E e^{-i\omega t} = \partial_t \vec{A}$ $\quad i\omega \vec{A} = E$.

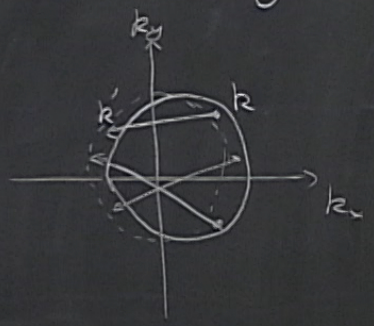
$$j_e = \frac{\rho}{i\omega} E \Rightarrow \sigma(\omega) = \frac{\rho_s}{i\omega}$$ non-dissipative, diverges as $\omega \to 0$.

$$m \frac{dV}{dt} = E + \frac{m}{\tau} V \qquad \sigma(\omega) = \frac{n_e}{m} \cdot \frac{1}{\frac{1}{\tau} + i\omega}$$

$$j = n_e V = \frac{n_e}{m} \int dt\, E$$

Weak disorder can change ρ_s but will not change qualitative structure.

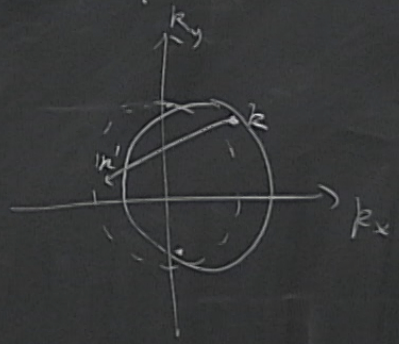
Another argument.



Normal metal

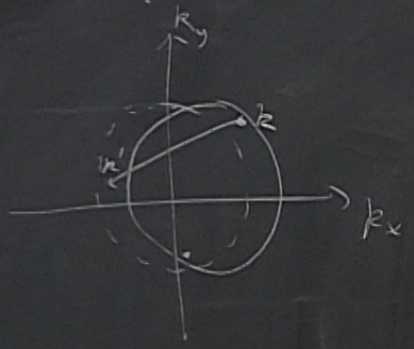
$\xi(k) - \xi(k')$ won't be large

Superconductor.



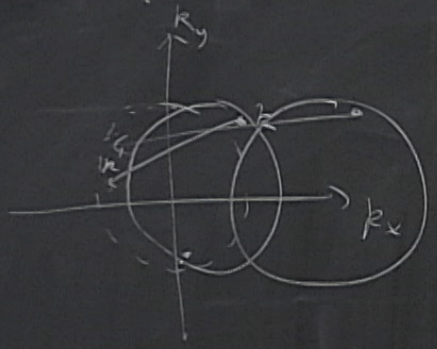
$k \to k'$ scattering requires pair-breaking, energy barrier Δ

Superconductor.



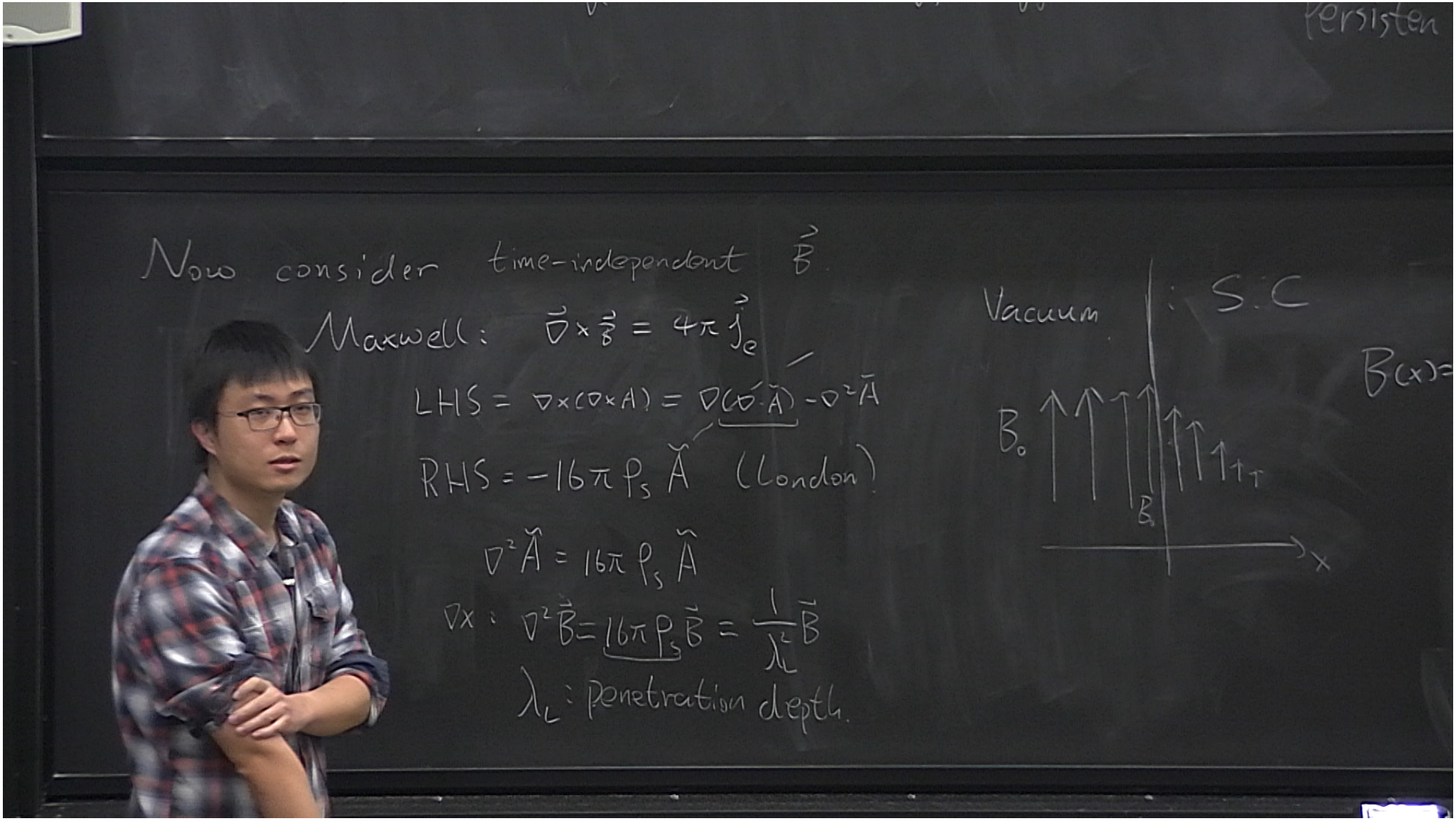
$k \to k'$ scattering requires pair-breaking, energy barrier Δ $\longrightarrow$ System decides not to relax.

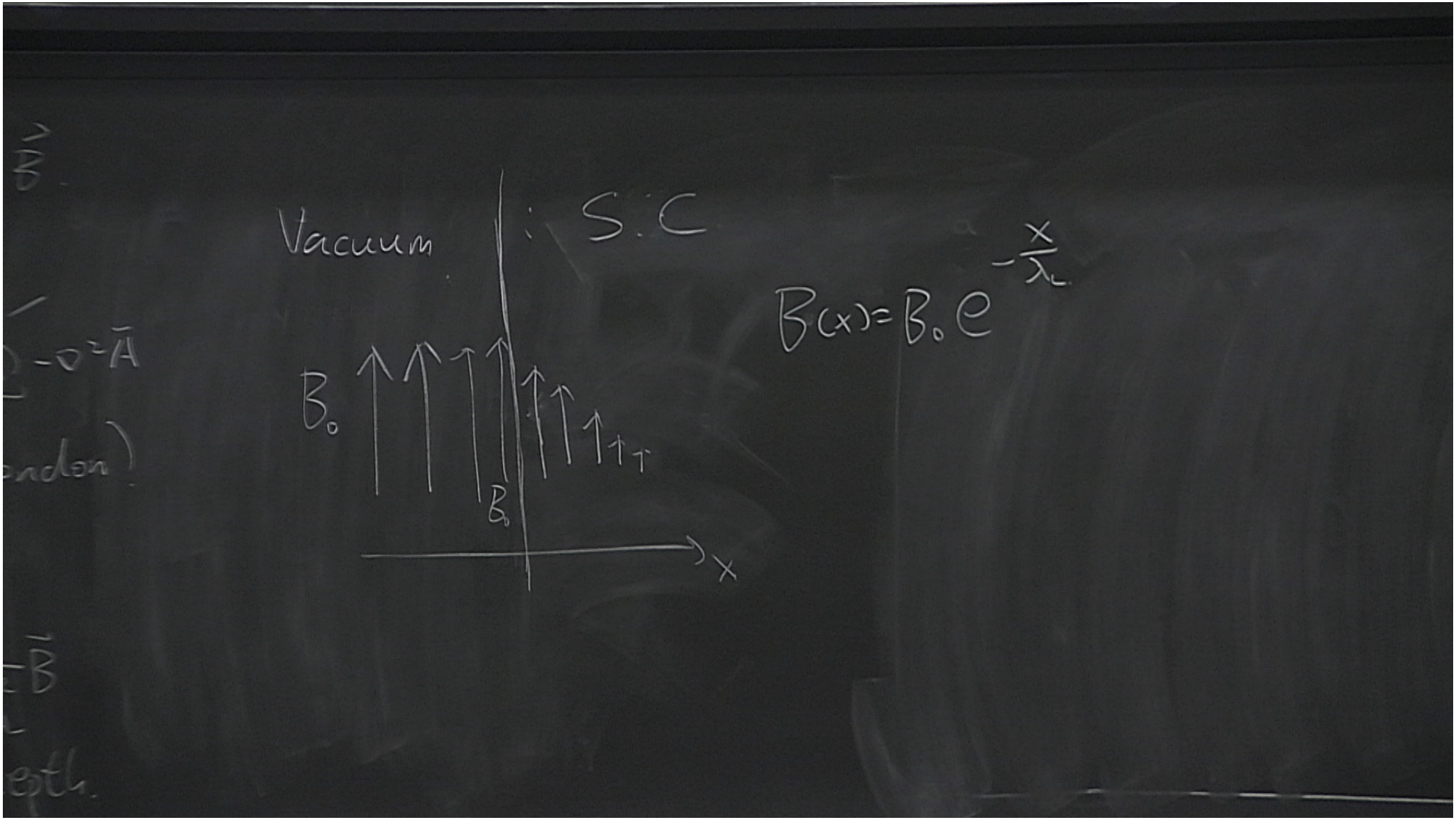
Superconductor. $\Rightarrow \exists$ "upper critical current" j_c.

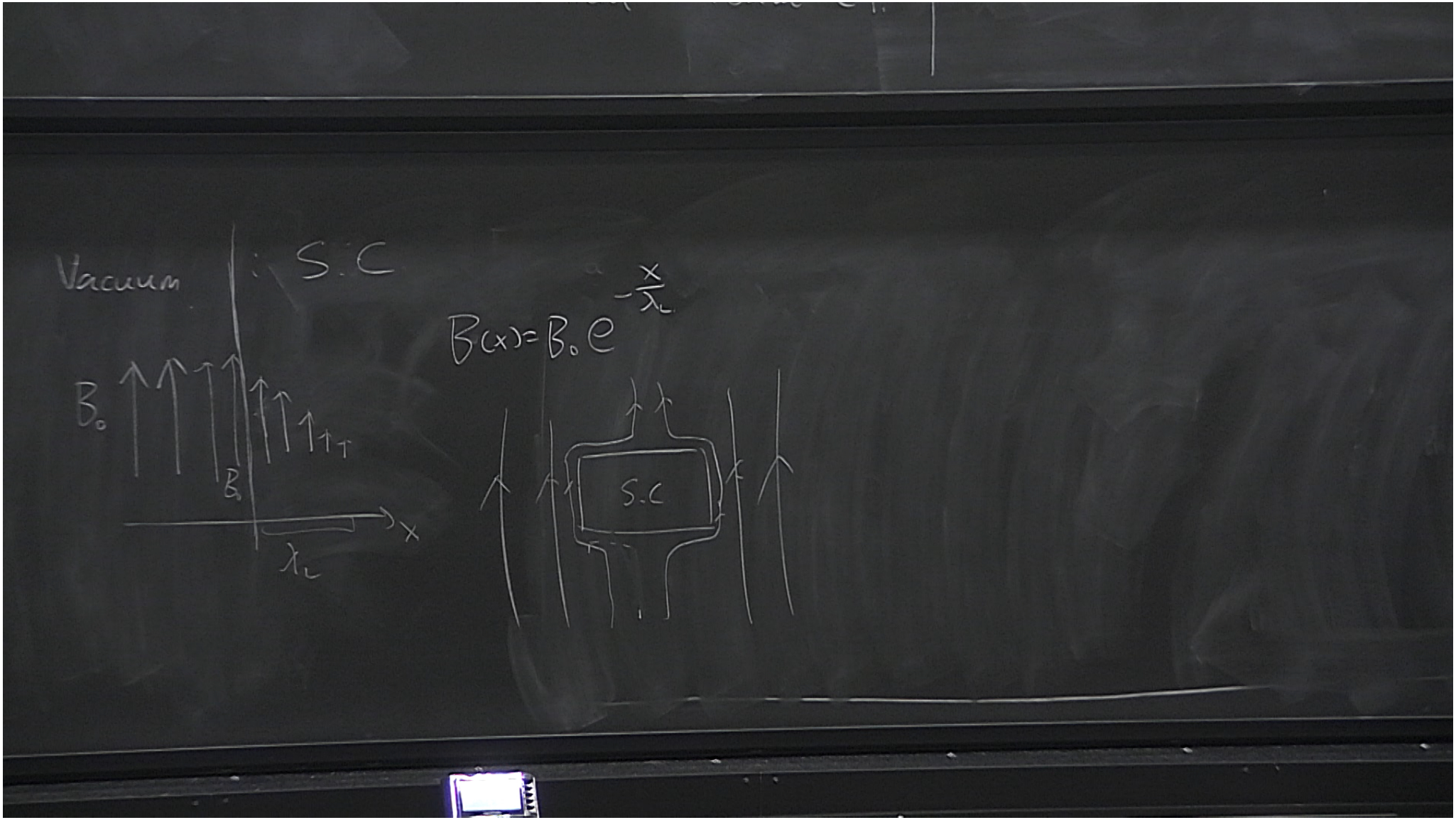


$k \to k'$ scattering requires pair-breaking, energy barrier Δ $\longrightarrow$ System decides not to relax.

Persisten current without $\vec{E}$!


persisten
Now consider time-independent $\vec{B}$
Maxwell: $\vec{\nabla} \times \vec{B} = 4\pi \vec{j}_e$
LHS = $\nabla \times (\nabla \times A) = \nabla(\nabla \cdot \vec{A}) - \nabla^2 \vec{A}$
RHS = $-16\pi \rho_s \vec{A}$ (London)
$\nabla^2 \vec{A} = 16\pi \rho_s \vec{A}$
$\nabla \times$: $\nabla^2 \vec{B} = 16\pi \rho_s \vec{B} = \frac{1}{\lambda_L^2} \vec{B}$
λ_L : penetration depth.
Vacuum : S.C
B_0
B_0
x
B(x)=


Vacuum
S.C
$B(x) = B_0 e^{-\frac{x}{\lambda_L}}$
B_0
B_0
x


Vacuum
S.C
$B(x) = B_0 e^{-\frac{x}{\lambda_L}}$
B_0
B
x
λ_L
S.C

Persisten current without $\vec{E}$!

$\vec{B}$

$\nabla \cdot \vec{A}) - \nabla^2 \vec{A}$

(London)

$\frac{1}{\lambda^2} \vec{B}$

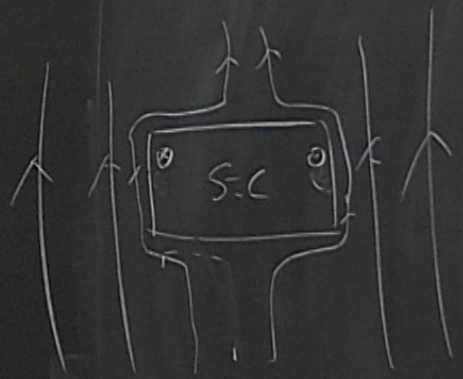
depth.

eddy current.

Vacuum | S.C

B_0 B_0 B_T x λ_L

$B(x) = B_0 e^{-\frac{x}{\lambda_L}}$



Meissner effect → pe

"Perfect diamagnet"

$|\bar{\Psi}|^2 \propto T_c - T$ ↓ $\langle \delta|\Psi|(x)\, \delta|\Psi|(0)\rangle \to e^{-\frac{x}{\xi}} \Longrightarrow$ go to lengthscale $\gg \xi$, neglect

eddy current

Vacuum | S.C

B_0

B_T

B_0

λ_L

x

$B(x) = B_0 e^{-\frac{x}{\lambda_L}}$

Meissner effect ⟶ persistent current

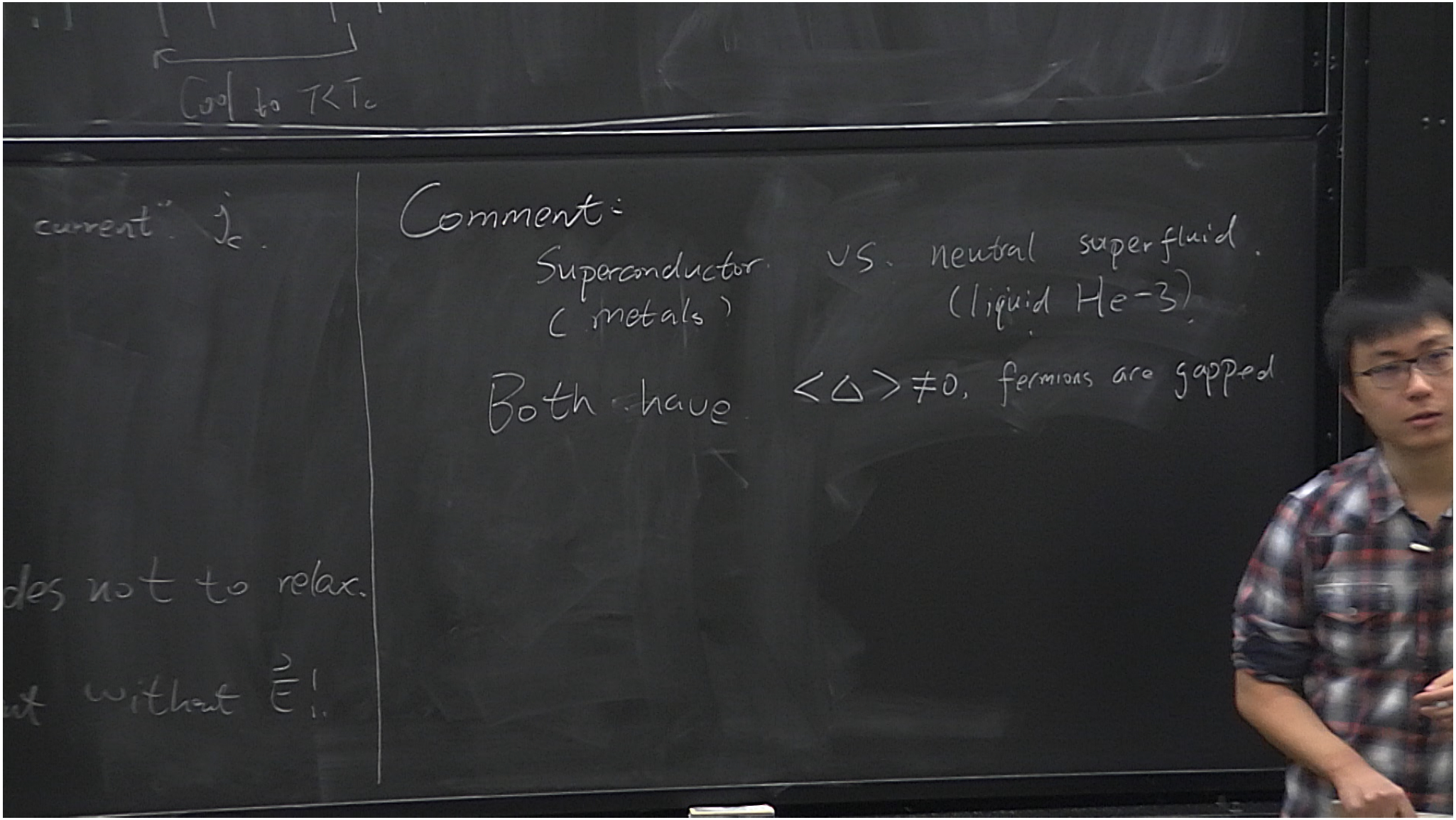
Perfect diamagnet

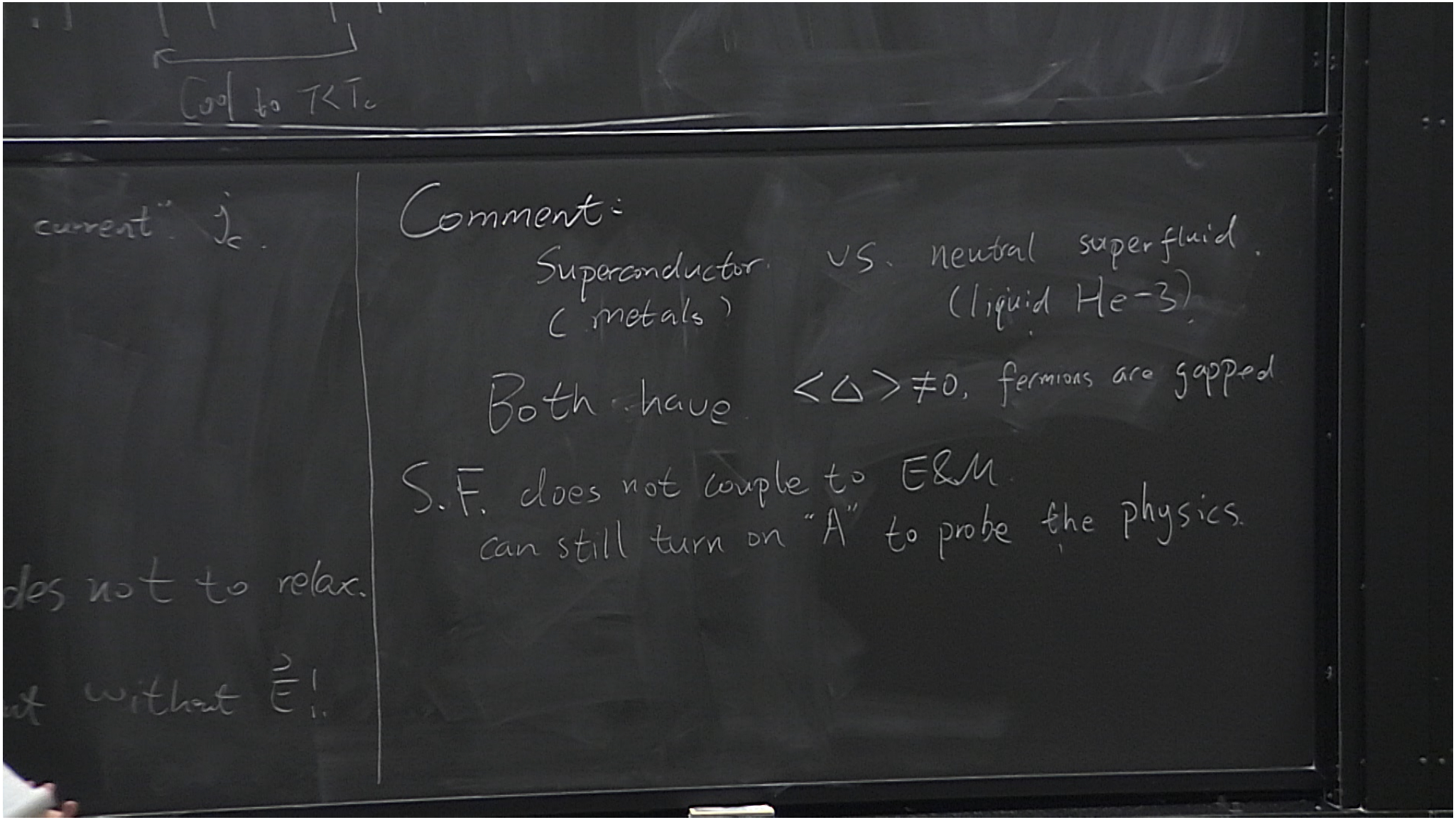
S.C

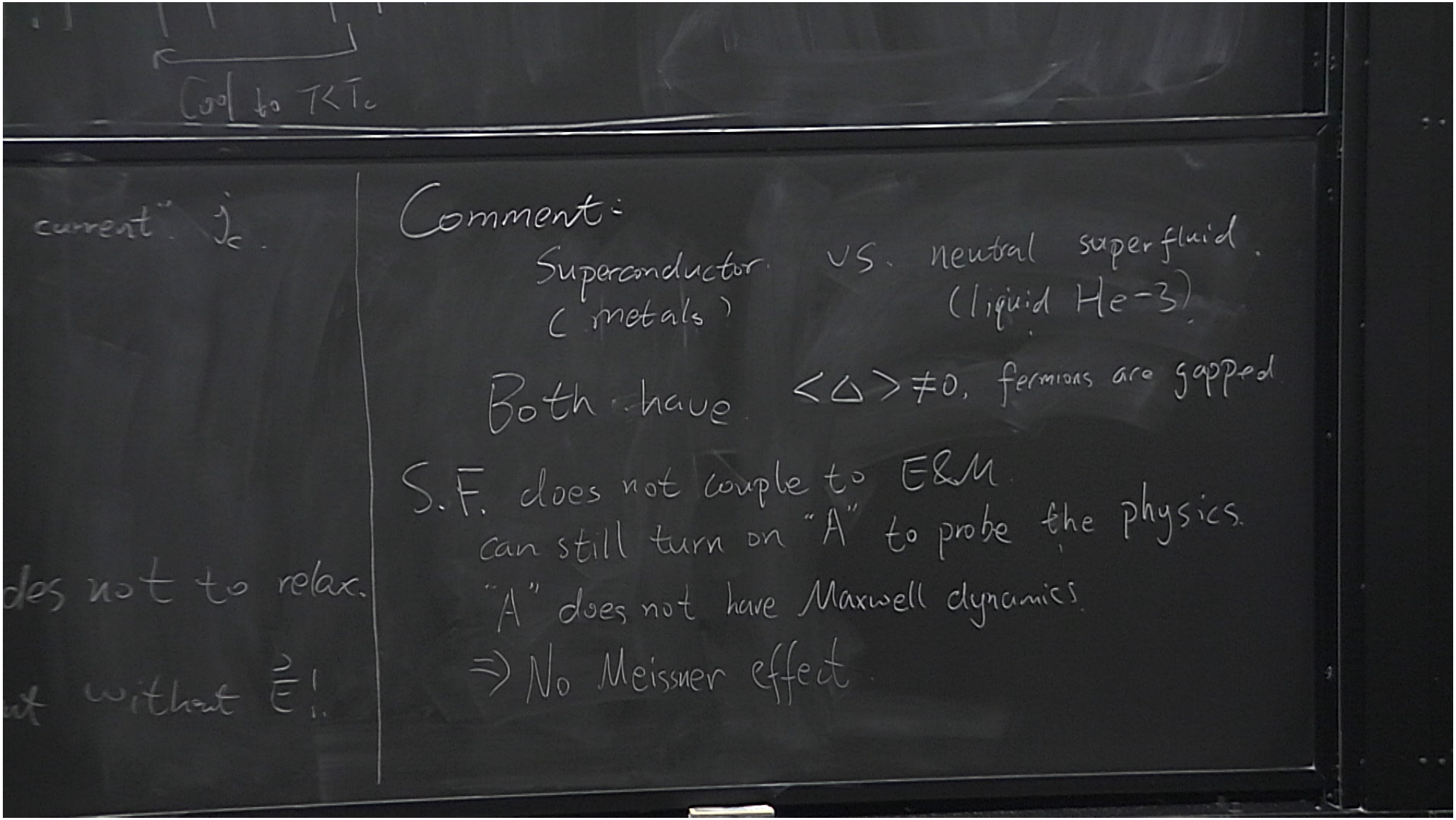
$T \geq T_c$

Cool to $T < T_c$

⇒ ∃ "upper critical current" j_c.


Cool to $T < T_c$
current j_c.
Comment:
Superconductor (metals) vs. neutral superfluid (liquid He-3).
Both have $\langle \Delta \rangle \neq 0$, fermions are gapped
does not to relax.
without $\vec{E}$!


Cool to $T < T_c$
current j_c.
Comment:
Superconductor (metals) vs. neutral superfluid (liquid He-3).
Both have $\langle \Delta \rangle \neq 0$, fermions are gapped
S.F. does not couple to E&M
can still turn on "A" to probe the physics.
does not to relax.
t without $\vec{E}$!


Cool to T<Tc
current j_c.
Comment:
Superconductor (metals) vs. neutral superfluid (liquid He-3).
Both have $\langle\Delta\rangle \neq 0$, fermions are gapped
S.F. does not couple to E&M
can still turn on "A" to probe the physics.
"A" does not have Maxwell dynamics
⇒ No Meissner effect
does not to relax.
ut without $\vec{E}$!

Fluctuation in $\theta \Rightarrow$ a soft mode

$$f = \frac{1}{2}\rho_s(\nabla\theta)^2 = \frac{1}{2}\rho_s k^2|\theta|^2$$

energy cost is arbitrily small as $k \to 0$

"Massless boson" if viewed as $(2+1)d$ field theory in Euclidean

Fluctuation in θ $\Rightarrow$ a soft mode

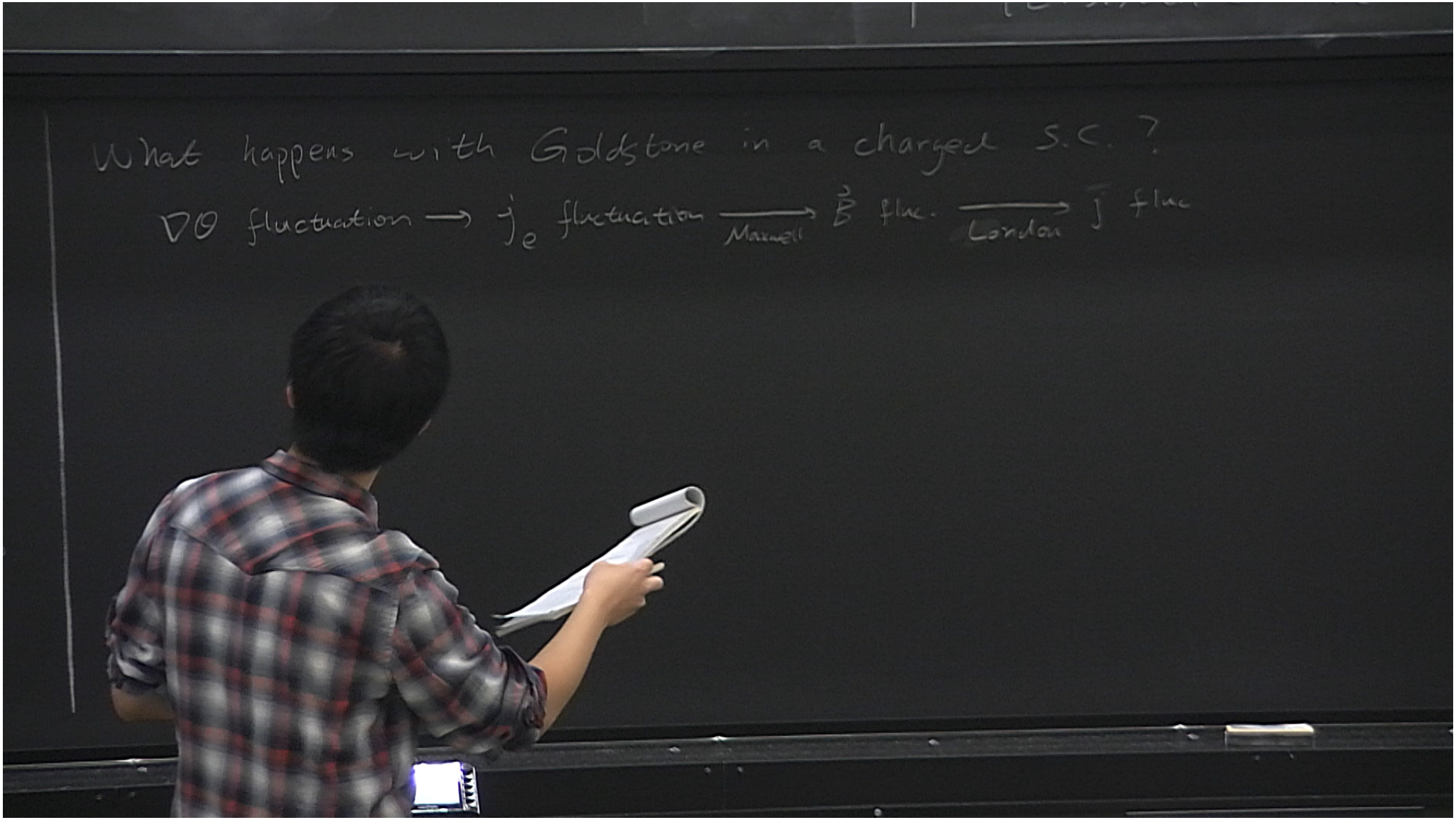
$$f = \frac{1}{2}\rho_s(\nabla\theta)^2 = \frac{1}{2}\rho_s k^2|\theta|^2$$

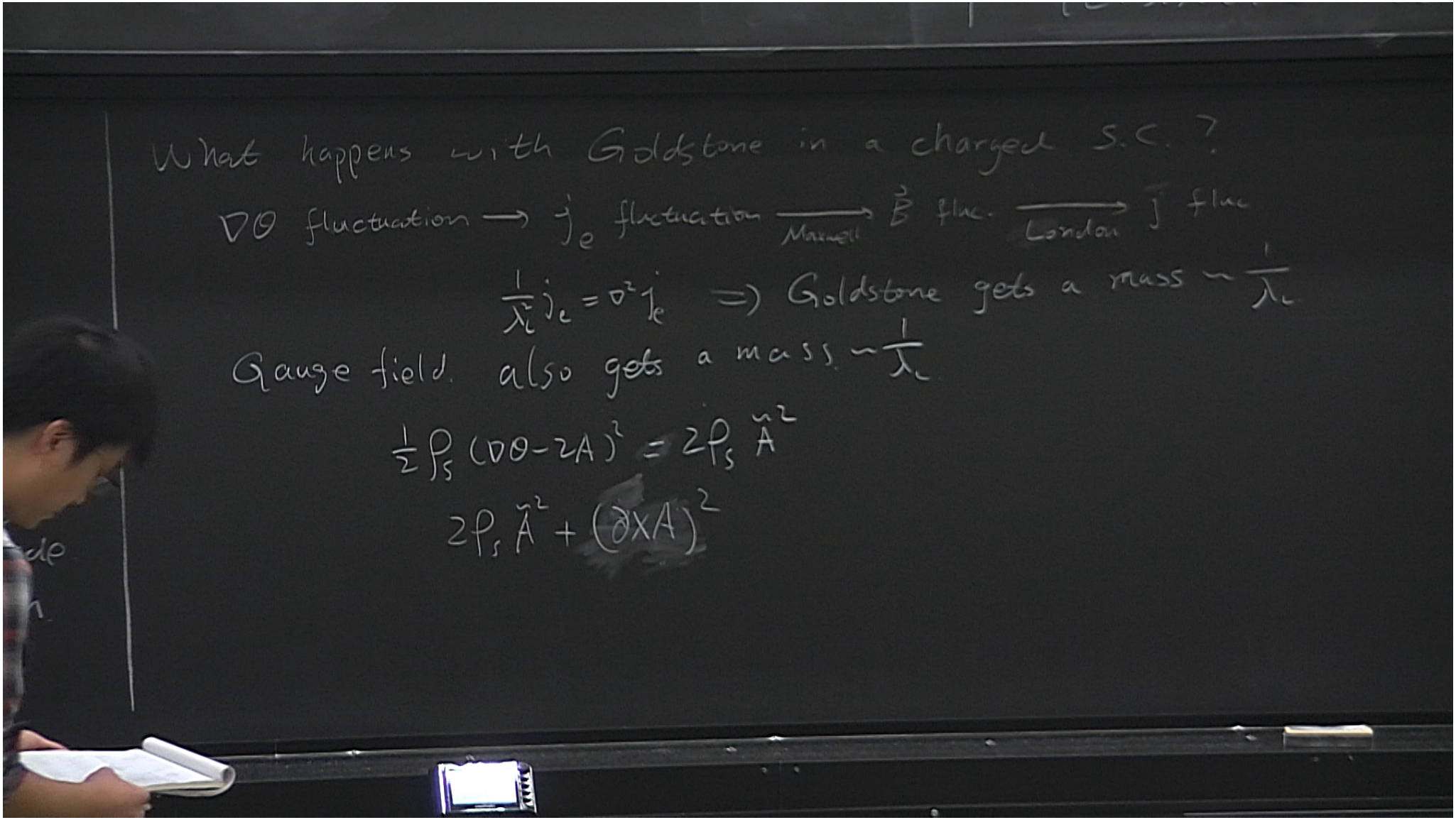
energy cost is arbitrily small as $k \to 0$

"Massless boson" if viewed as (2+1)d field theory in Euclidean $\Rightarrow$ Goldstone boson/mode

A General feature of SSB of continuous symm.




What happens with Goldstone in a charged S.C. ?
DO fluctuation → j_e fluctuation → $\vec{B}$ fluc. → $\vec{j}$ fluc
Maxwell
London


What happens with Goldstone in a charged S.C.?
∇θ fluctuation → j_e fluctuation —Maxwell→ $\vec{B}$ fluc. —London→ $\vec{J}$ fluc
$\frac{1}{\lambda_L^2} j_e = \nabla^2 j_e$ ⇒ Goldstone gets a mass ~ $\frac{1}{\lambda_L}$
Gauge field also gets a mass ~ $\frac{1}{\lambda_L}$
$\frac{1}{2}\rho_s(\nabla\theta - 2A)^2 = 2\rho_s \vec{A}^2$
$2\rho_s \vec{A}^2 + (\partial \times A)^2$

Persisten current without $\vec{E}$!

$\Rightarrow$ No Meissner effect

Persisten current exists $\Rightarrow$ Superfluid.

What happens with Goldstone in a charged S.C.?

$D\theta$ fluctuation $\longrightarrow$ j_e fluctuation $\xrightarrow[\text{Maxwell}]{}$ $\vec{B}$ fluc. $\xrightarrow[\text{London}]{}$ J fluc

$\frac{1}{\lambda_L^2} j_e = \nabla^2 j_e \Rightarrow$ Goldstone gets a mass $\sim \frac{1}{\lambda_L}$

Gauge field also gets a mass $\sim \frac{1}{\lambda_L}$

$\frac{1}{2}\rho_s (D\theta - 2A)^2 = 2\rho_s \tilde{A}^2$

$2\rho_s \tilde{A}^2 + (\partial \times A)^2 \Rightarrow$ a massive vector boson

Anderson–Higgs mechanism.

Works for other gauge group.

Persisten current without $\vec{E}$!

$\Rightarrow$ No Meissner effect

Persisten current exists $\Rightarrow$ Superfluid.

What happens with Goldstone in a charged S.C.?

$\theta\theta$ fluctuation $\longrightarrow$ j_e fluctuation $\xrightarrow[\text{Maxwell}]{}$ $\vec{B}$ fluc. $\xrightarrow[\text{London}]{}$ J fluc.

$\frac{1}{\lambda_L^2} j_e = \nabla^2 j_e$ $\Rightarrow$ Goldstone gets a mass $\sim \frac{1}{\lambda_L}$

Gauge field also gets a mass $\sim \frac{1}{\lambda_L}, \frac{1}{\xi}$

$\frac{1}{2}\rho_s (\nabla\theta - 2A)^2 = 2\rho_s \vec{A}^2$

$2\rho_s \vec{A}^2 + (\nabla \times A)^2$ $\Rightarrow$ a massive vector boson

Anderson–Higgs mechanism.

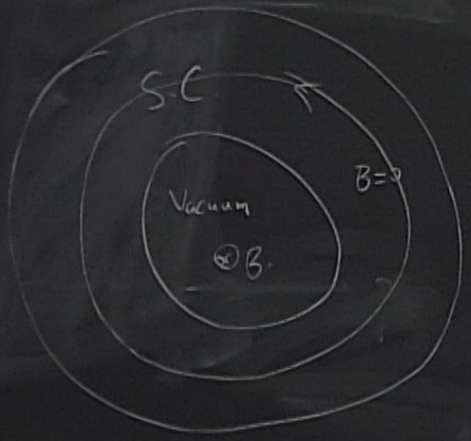
Works for other gauge group.

Same mode.

Flux quantization.

$\frac{1}{2}\rho_s(\nabla\theta - 2A)^2$

Vacuum. $\nabla\theta - 2A = 0$



$$\int_{\text{hollow}} B\, d^2x = \oint \vec{A}\, d\vec{l} = \oint \frac{1}{2}\vec{\nabla}\theta \cdot dl = \frac{1}{2}\cdot 2\pi \cdot n, \quad n \in \mathbb{Z}.$$

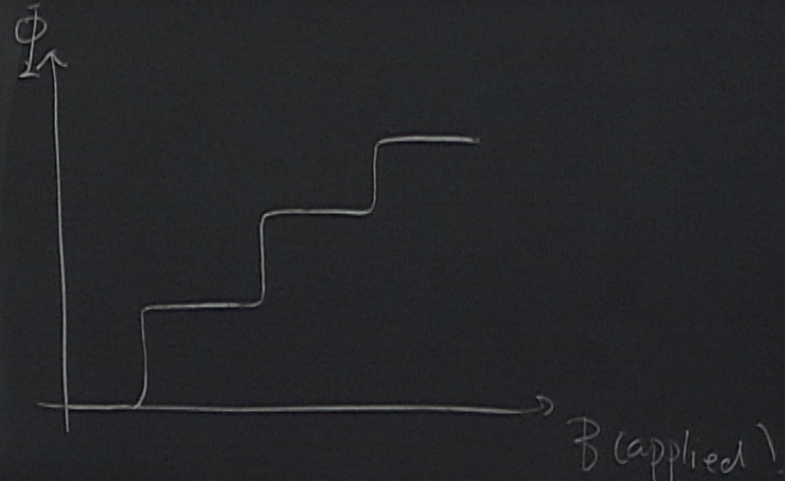
$$= \frac{1}{2}\Phi_0 n$$

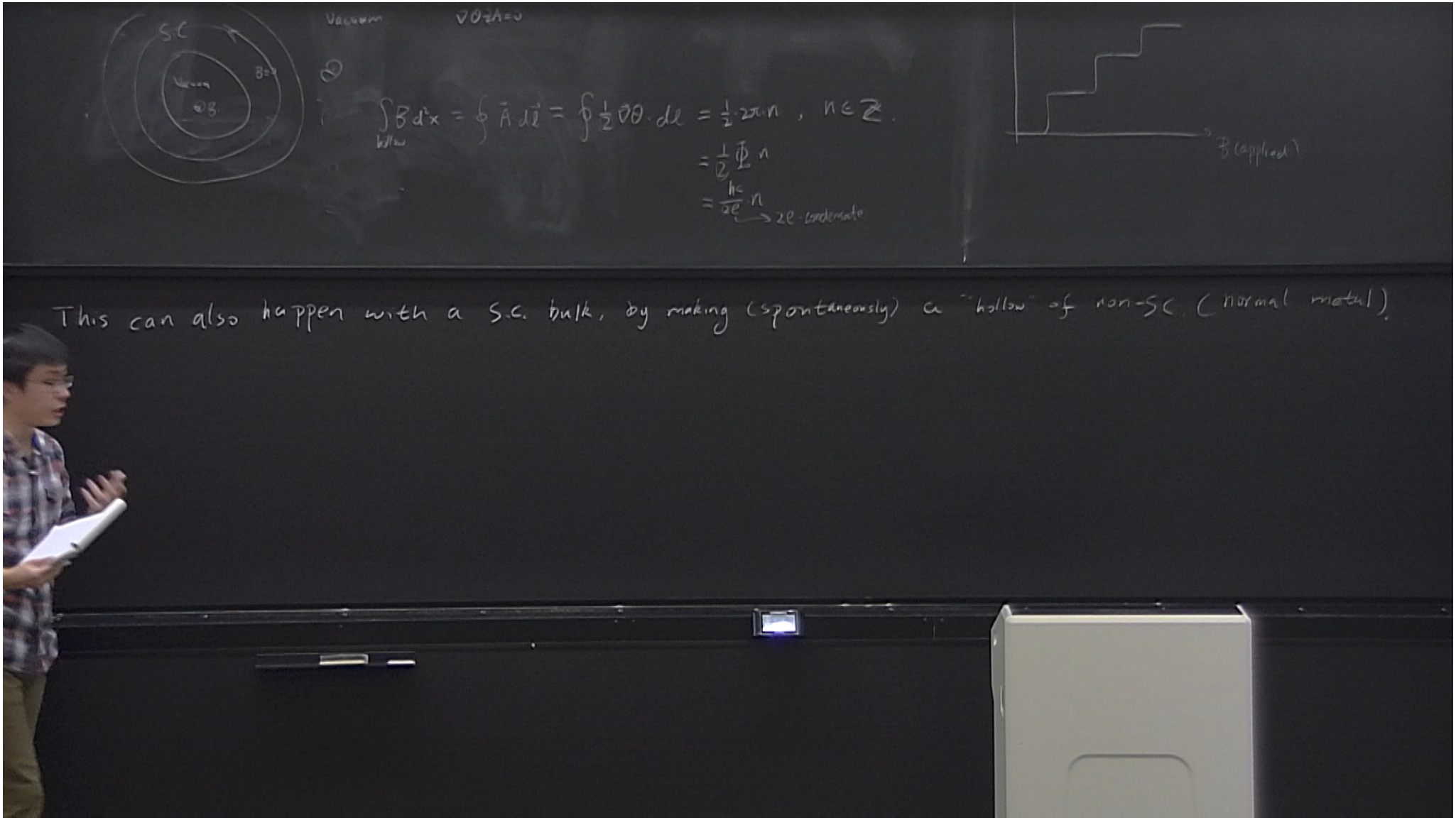
$$= \frac{hc}{2e} n$$

$$= \oint \frac{1}{2} \vec{\nabla}\theta \cdot d\ell = \frac{1}{2} \cdot 2\pi \cdot n \,, \quad n \in \mathbb{Z}.$$

$$= \frac{1}{2} \Phi_0 \, n$$

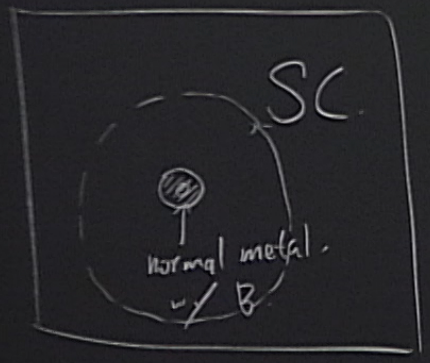
$$= \frac{hc}{2e} \cdot n \longrightarrow 2e\text{-condensate}$$




S.C
Vacuum
B=0
∇θ-A=0
$\int_{hollow} B\,d^2x = \oint \vec{A}\,d\vec{l} = \oint \frac{1}{2}\nabla\theta \cdot dl = \frac{1}{2}\cdot 2\pi n$, $n \in \mathbb{Z}$.
$= \frac{1}{2}\Phi_0 n$
$= \frac{hc}{2e} \cdot n$
2e - condensate
B (applied)
This can also happen with a S.C. bulk, by making (spontaneously) a "hollow" of non-SC (normal metal).

$= \frac{hc}{2e} \cdot n$ → 2e-condensa

This can also happen with a S.C. bulk, by making (spontane

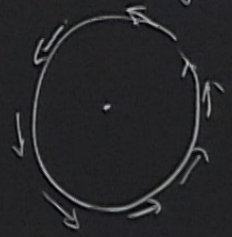


$$\Phi = \int B d^2x = \frac{\Phi_0}{2} n$$

$$\int dl \cdot \vec{\nabla}\theta = 2\pi \cdot n$$

a vortex (degree-n), "topological defect".



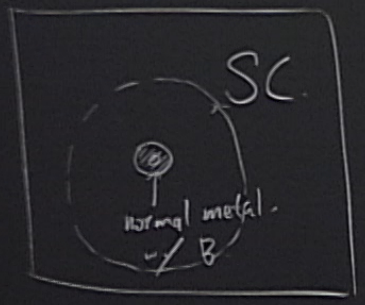


stable.

$= \frac{1}{2} \Phi_0 \, n$

$= \frac{hc}{2e} \cdot n$ → 2e - condensate

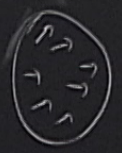
This can also happen with a S.C. bulk, by making (spontaneously) a "hollo

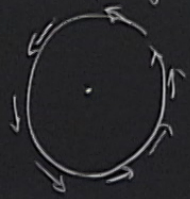


$\Phi = \int B \, d^2x = \frac{\Phi_0}{2} n$

$\int dl \cdot \vec{\nabla} \theta = 2\pi \cdot n$

a vortex (degree-n), "topological defect".





stable, θ not defined at vortex core, $|\Phi| = 0$