Title: PSI 2019/2020 - Condensed Matter (Wang) - Lecture 8

Speakers: Chong Wang

Collection: PSI 2019/2020 - Condensed Matter (Wang)

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URL: http://pirsa.org/19110033

Landau's Fermi Liquid Theory

Refs. ① Anderson, "Basic Notions of Condensed Matter Physics".
Ch. 3 (a–c). (Conceptual & Insightful).

② Abrikosov, Gorkov, Dzyaloshinski (AGD), book.
Ch. 1 (Intro).
Ch. 4 (If you LOVE Feynman diagrams)

③ Re

ysics".

Next

③ Review articles on Fermi Liquid.

R. Shankar.

J. Polchinski.

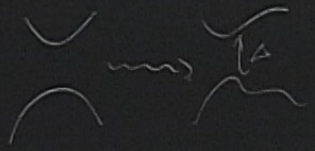
(Modern field theory & Renormalization Group)

Some comments on interaction.

$$H = U \sum_{\langle ij \rangle} c_i^\dagger c_i \, c_j^\dagger c_j \xrightarrow{\text{Fourier}} \int U(q) \, c_{k+q}^\dagger c_{k-q}^\dagger c_{k'} c_k \frac{dq}{(2\pi)^d} \frac{dk}{(2\pi)^d} \frac{dk'}{(2\pi)^d}$$

Imagine turning on U. slowly. $U(t)$ $\begin{cases} U(0) = 0 \\ U(T) = U \end{cases}$

① Gapped states (insulator)



As long as $\Delta > 0$ throughout

Two states are similar.

True if $U \ll \Delta$

May or may not be true if $U \gtrsim \Delta$.

Ch.4 (If you LOVE Feynman diagrams)

J. Polchinski (Modern field theory & Ren

② Semi-metal, e.g. Dirac fermion in Gaphene.

QFT/Renormalization Gp ⇒ $U \ll W$. won't change qualitative behavior if $d > 1$.

③ Metal?



Badly non-adiabatic??

Large density of states as $\xi \to 0$.

Ch.4 (If you LOVE Feynman diagrams)

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$|\Psi(t=0)\rangle = C_k^\dagger |\text{Fermi sea}\rangle$

$k > k_F$.

$\xi_k = \frac{k_F}{m}(k - k_F) \ll E_F = \frac{k_F^2}{2m}$

Abrikosov. Gorkov. Dzyaloshinski (AGD).
Ch.1 (Intro)
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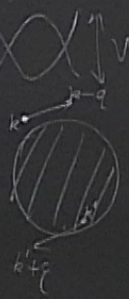
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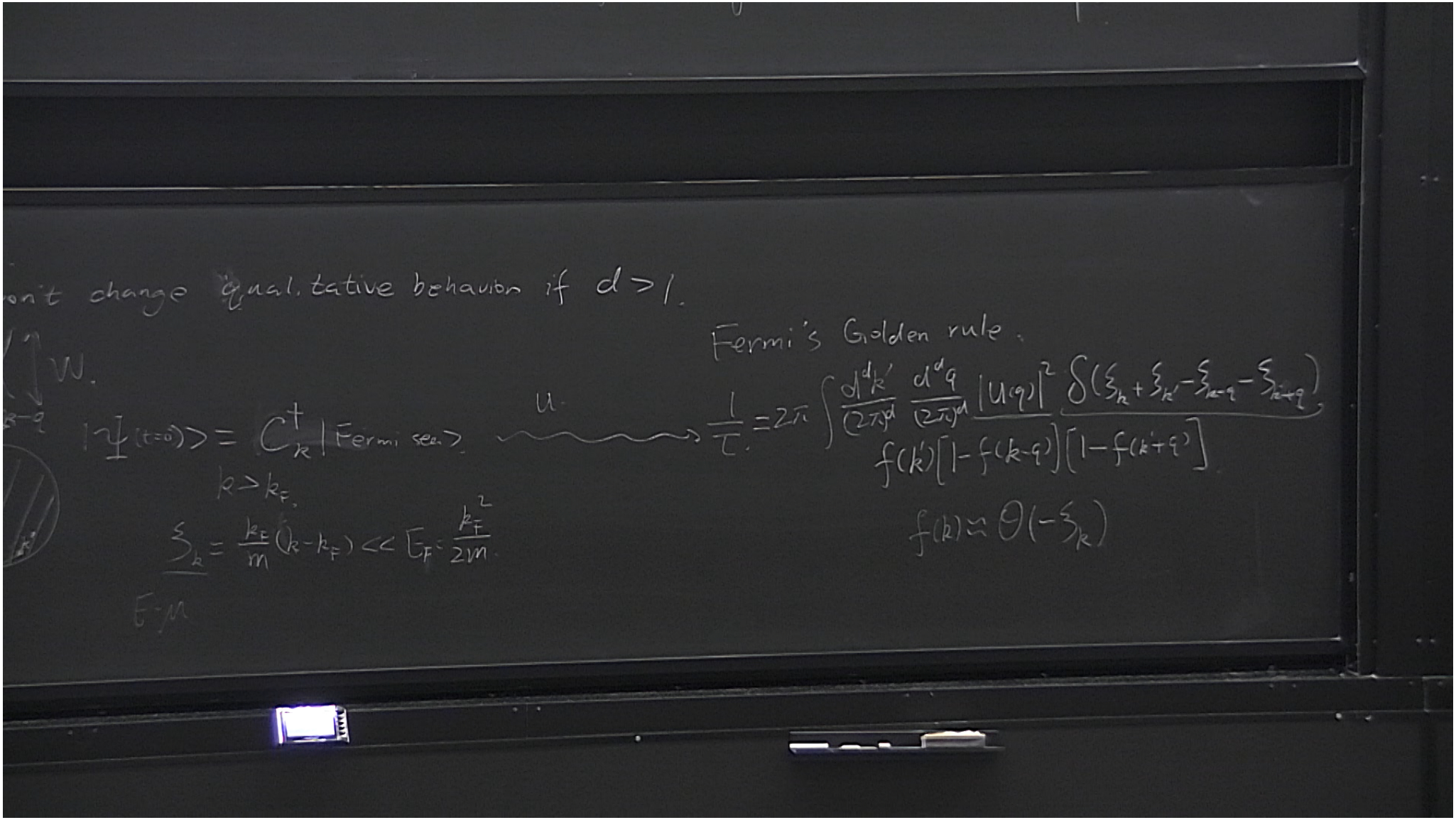
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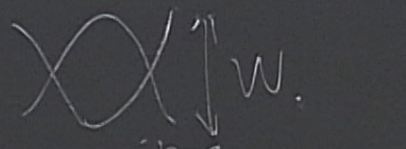
Fermi's Golden rule.

$U \rightsquigarrow \frac{1}{\tau} = 2\pi \int \frac{d^d k'}{(2\pi)^d} \frac{d^d q}{(2\pi)^d} |U_q|^2$


on't change qual. tative behavior if d > 1.
W.
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$k > k_F$.
$\xi_k = \frac{k_F}{m}(k - k_F) \ll E_F = \frac{k_F^2}{2m}$
$E = \mu$
U
Fermi's Golden rule.
$\frac{1}{\tau} = 2\pi \int \frac{d^d k'}{(2\pi)^d} \frac{d^d q}{(2\pi)^d} |U(q)|^2 \delta(\xi_k + \xi_{k'} - \xi_{k-q} - \xi_{k'+q})$
$f(k')[1 - f(k-q)][1 - f(k'+q)]$
$f(k) \simeq \Theta(-\xi_k)$

g. Dirac fermion in Graphene.

alization Gp $\Rightarrow$ $U \ll W$. won't change qualitative behavior if $d > 1$.

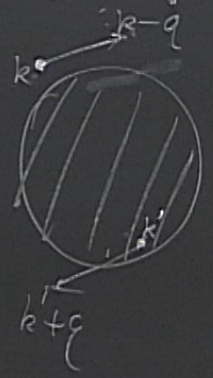


Ferm

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F-M

U. $\rightsquigarrow$ $\frac{1}{\tau} =$

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Fermi's Golden rule.

$c_k^\dagger |\text{Fermi sea}\rangle$ $\xrightarrow{u.}$ $\frac{1}{\tau} = 2\pi \int \frac{d^d k'}{(2\pi)^d} \frac{d^d q}{(2\pi)^d} |U(q)|^2 \delta(\xi_k + \xi_{k'} - \xi_{k-q} - \xi_{k'+q})$

$f(k')[1 - f(k-q)][1 - f(k'+q)]$

$> k_F$.

$\frac{k_F}{m}(k - k_F) \ll E_F = \frac{k_F^2}{2m}$.

$f(k) \simeq \Theta(-\xi_k)$

How many k', q actually contribute??

$\xi_k > 0$, $-\xi_{k'}$, ξ_{k-q}, $\xi_{k'+q} > 0$.

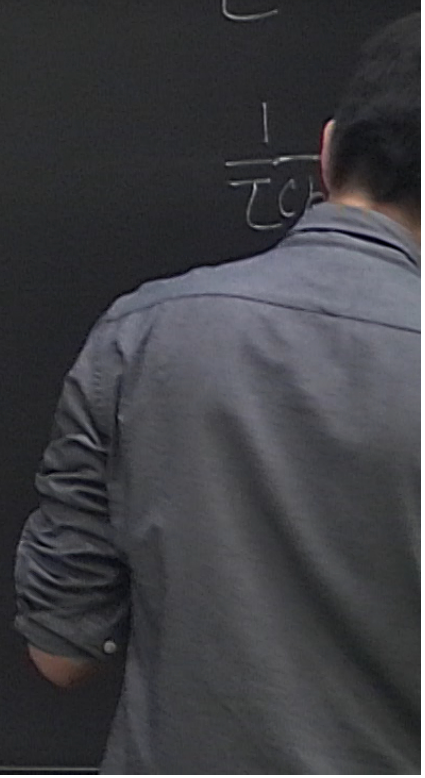
$\Rightarrow 0 < \xi_{k-q}, \xi_{k'+q}, -\xi_{k'} < \xi_k$.

$\Rightarrow 0 < k_F - |k'| < |k| - k_F$

$0 < |k+q| - k_F < |k| - k_F$

$\frac{1}{\tau} \propto$

$\frac{1}{\tau(\omega=$



$\xi_k = \frac{1}{m}(k - k_F) << E_F = \frac{1}{2m}$

k_{fc}

$E - \mu$

$\frac{1}{\tau} \propto (k - k_F)^2$.

$\frac{1}{\tau(k)} = \# \frac{u^2}{E_F^3} \xi_k^2 << \xi_k$ as long as ξ_k small enough.

depends on details

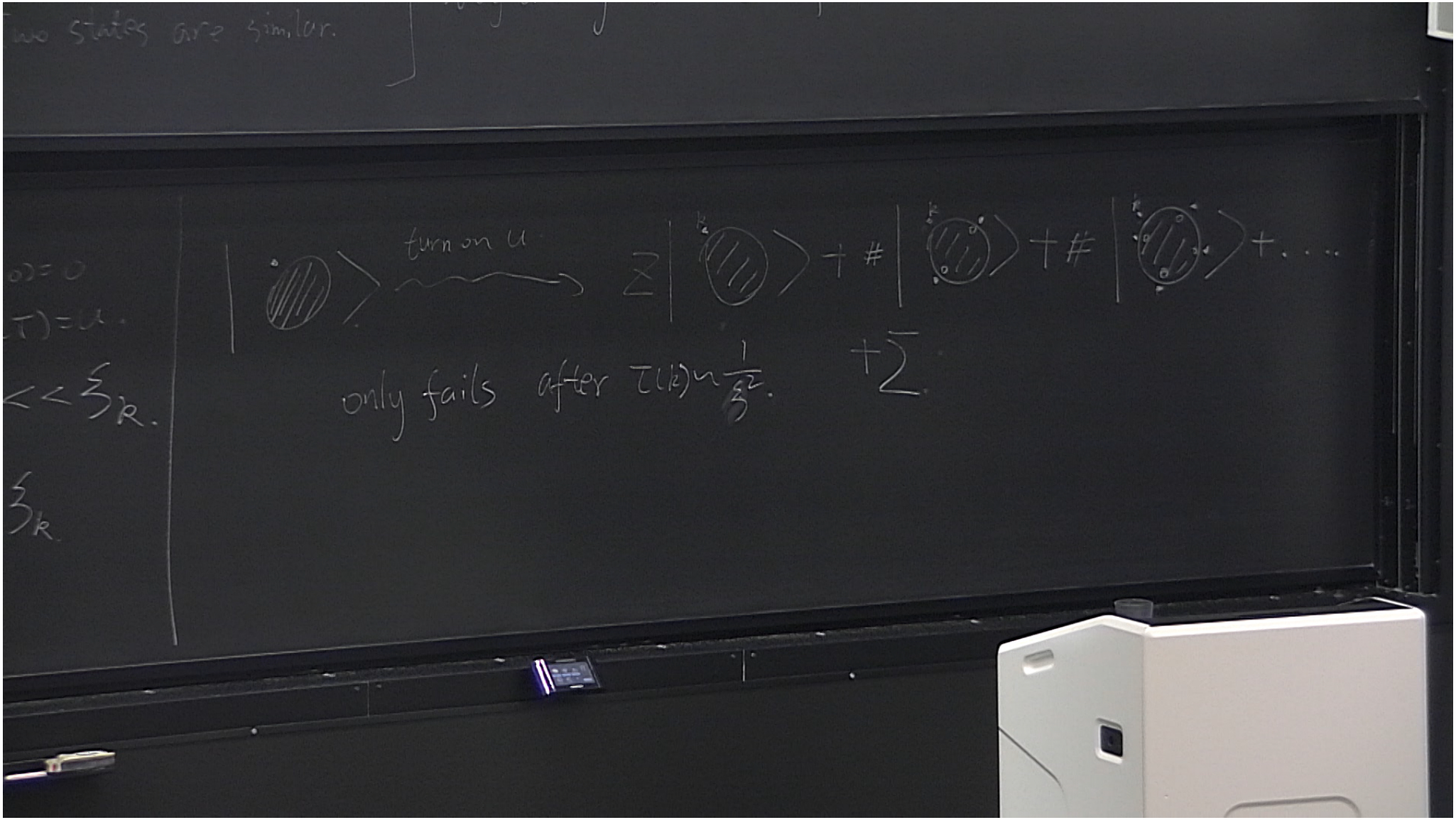
u(q), shape of Fermi, dimension...

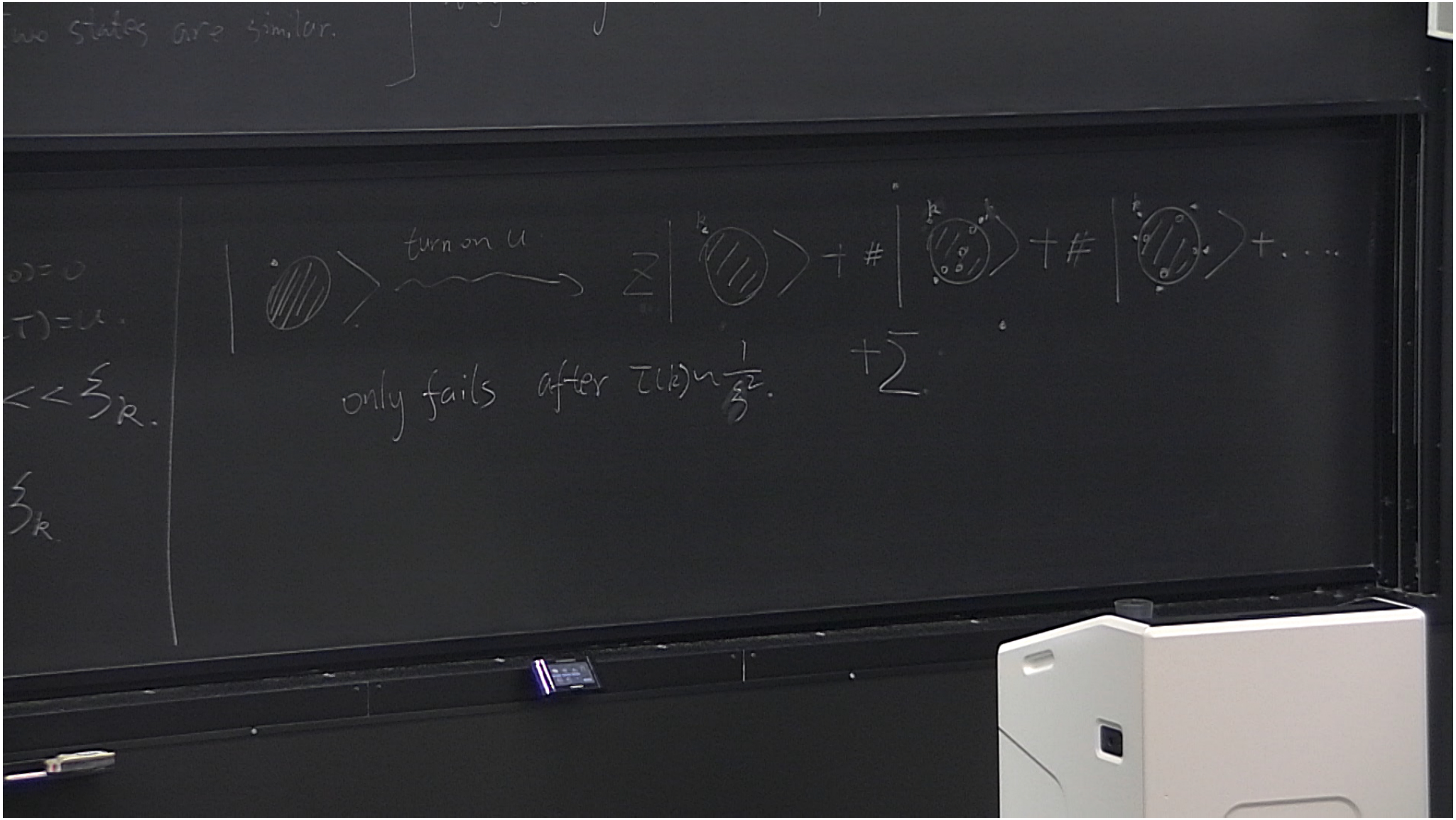
"Adiabatic" argument.

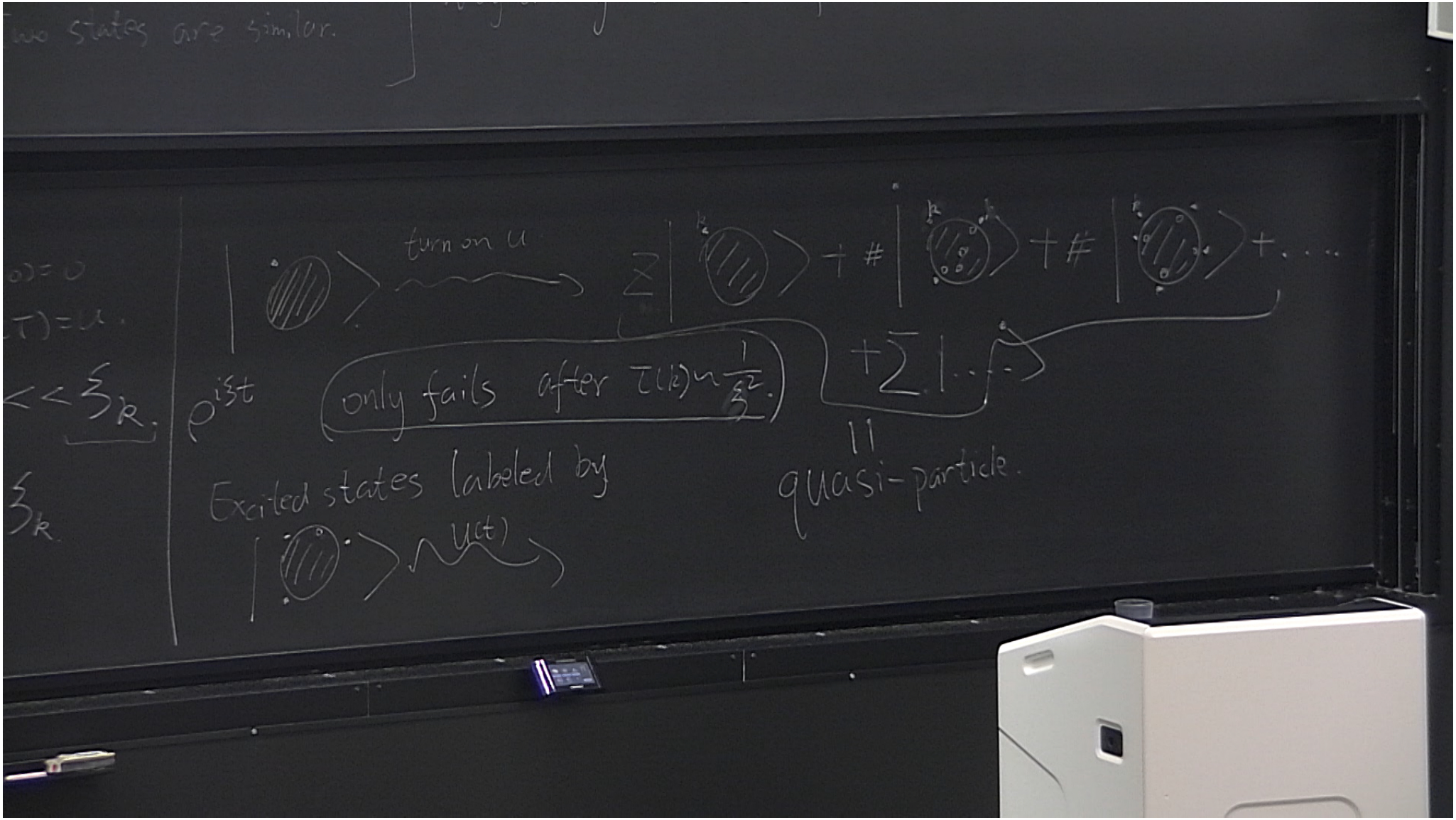
Turning on $u(t)$: $\begin{cases} u(0) = 0 \\ u(T) = u. \end{cases}$

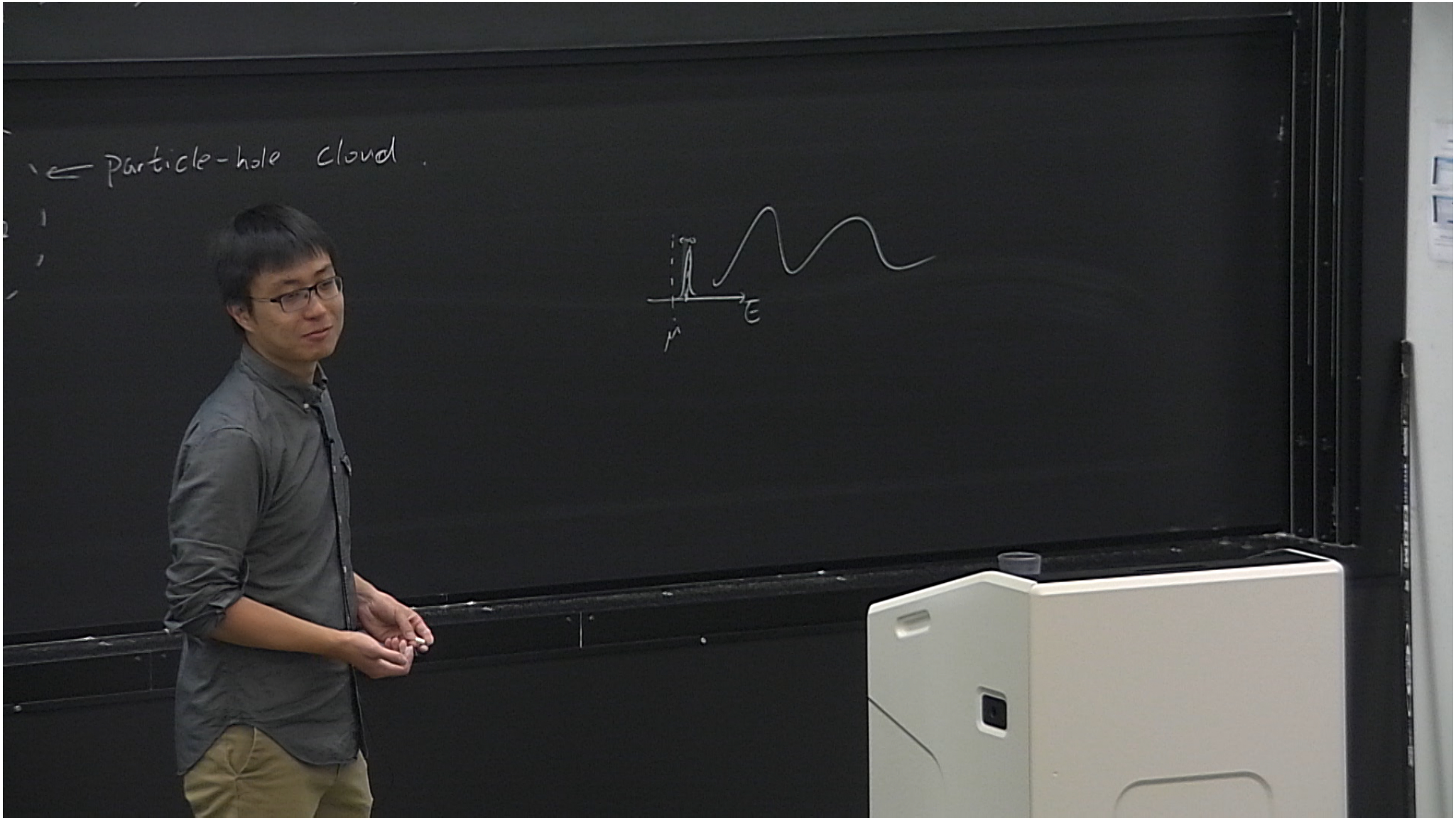
s.t. $\frac{1}{\Gamma(k)} << \frac{1}{T} << \xi_k.$

$\frac{1}{\Gamma(k)} \simeq \Delta\xi_k << \xi_k.$

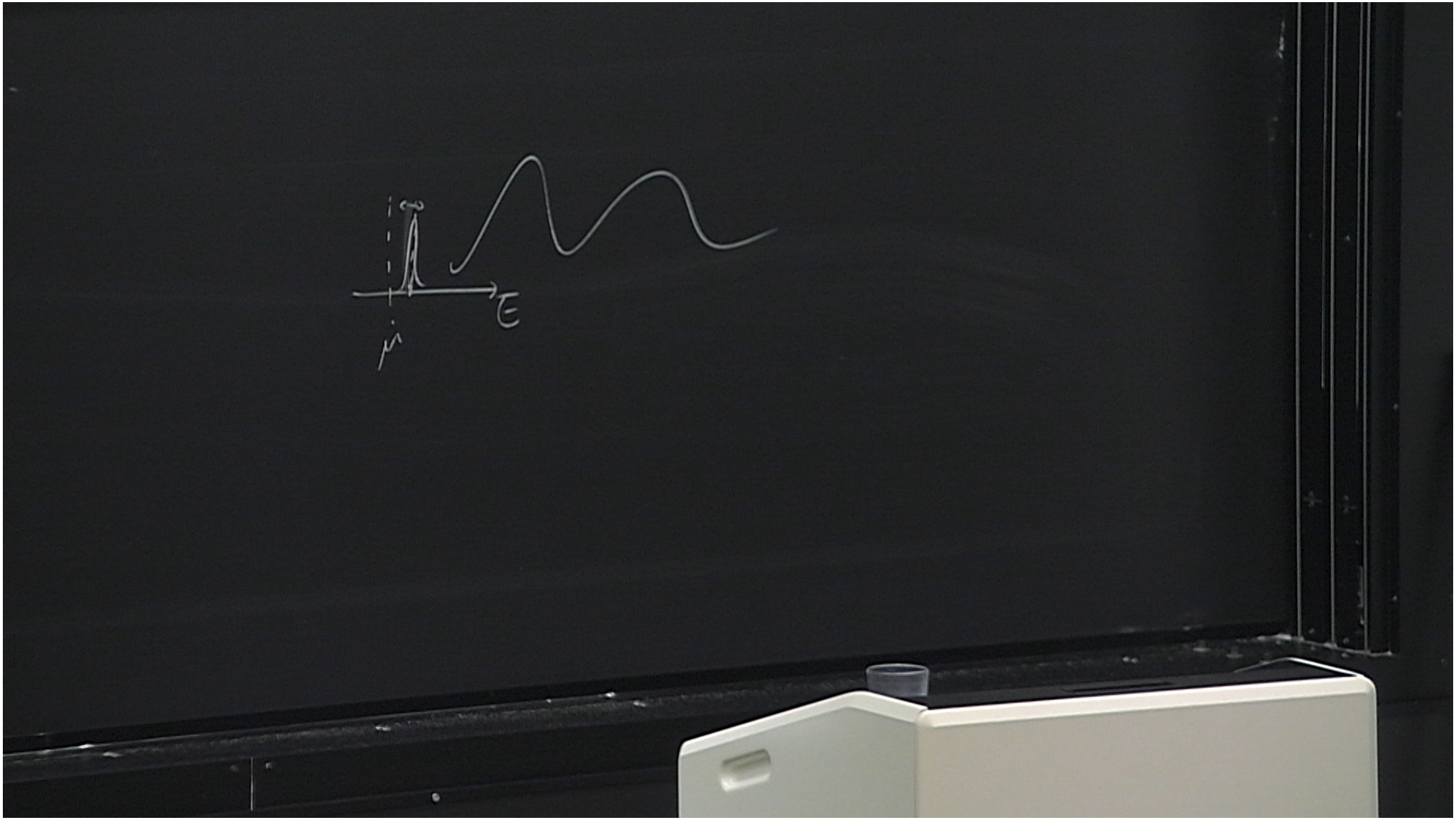

two states are similar.
turn on u
only fails after τ(k) ~ 1/ξ²
+Σ


turn on U
only fails after $\tau(k) \sim \frac{1}{\xi^2}$.
$\ll \xi_R$.
ξ_k


two states are similar.
turn on u
only fails after τ(k) ~ 1/ξ²
Excited states labeled by
quasi-particle.


← particle-hole cloud.
E
μ


Real space


E
μ

$$\xi_k = \frac{k_F}{m^*}(k - k_F)$$

effective mass.

Heavy Fermi liquid $m^* \sim 1000\, m_e$

Q: could k_F change?!

A: No!

Uniq). Shape of Fermi, dimension...

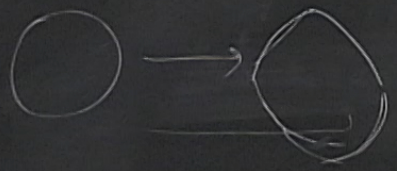
Luttinger Thm:

- Volume of Fermi sea does not change under adiabatic evolution

$$\frac{\text{Volume of Fermi sea}}{(2\pi)^d} = \text{Density of electron}$$

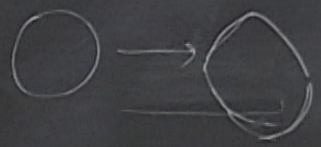
For Proof: AGD, Ch. 4

Oshikawa. "Topological approach to Luttinger's theorem ..."



Q: could k_F change?

A: No!



For Prof: AGD, Ch. 4

Oshikawa, "Topological ..."

Consequences:

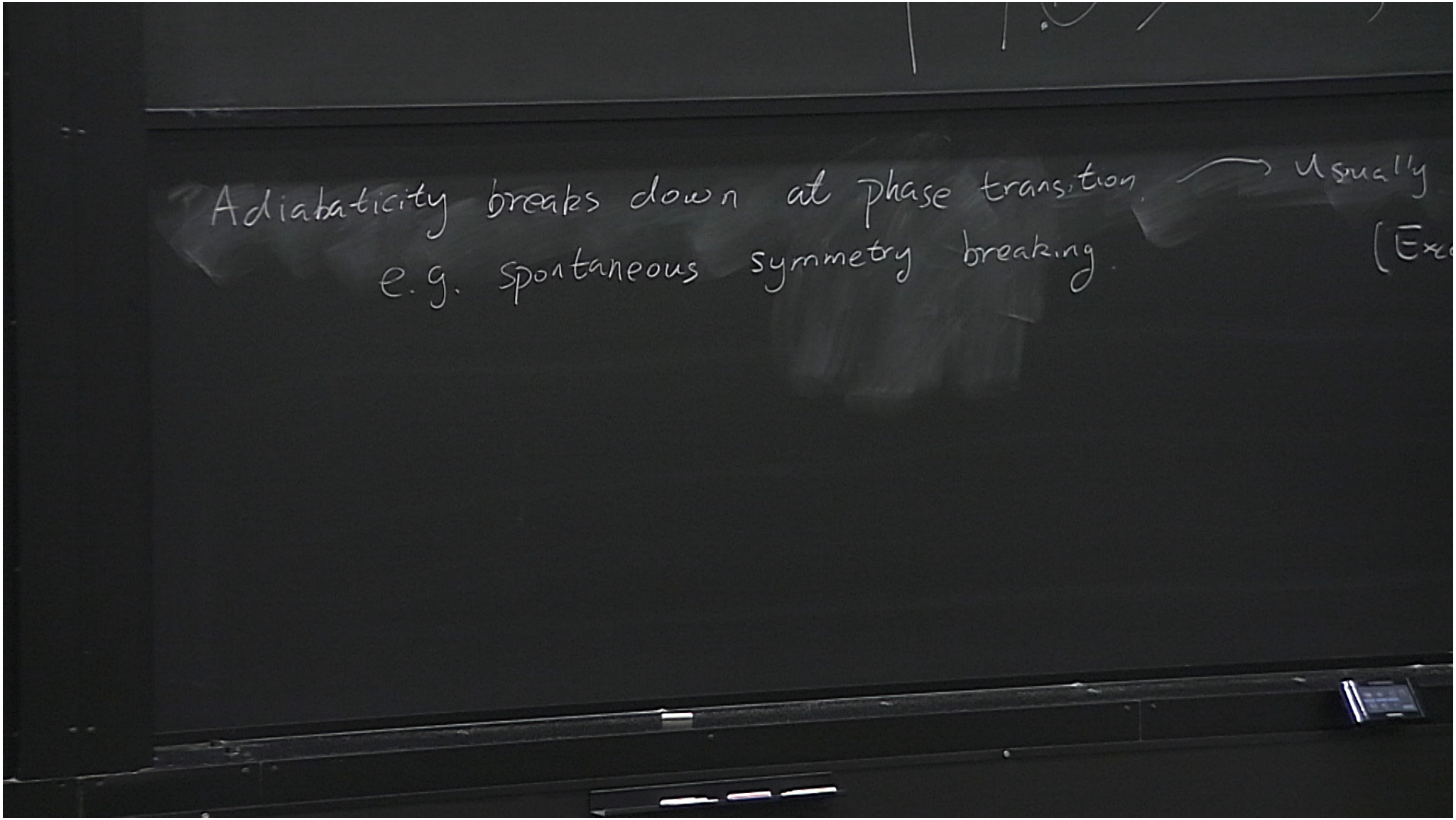
① Low temperature specific heat

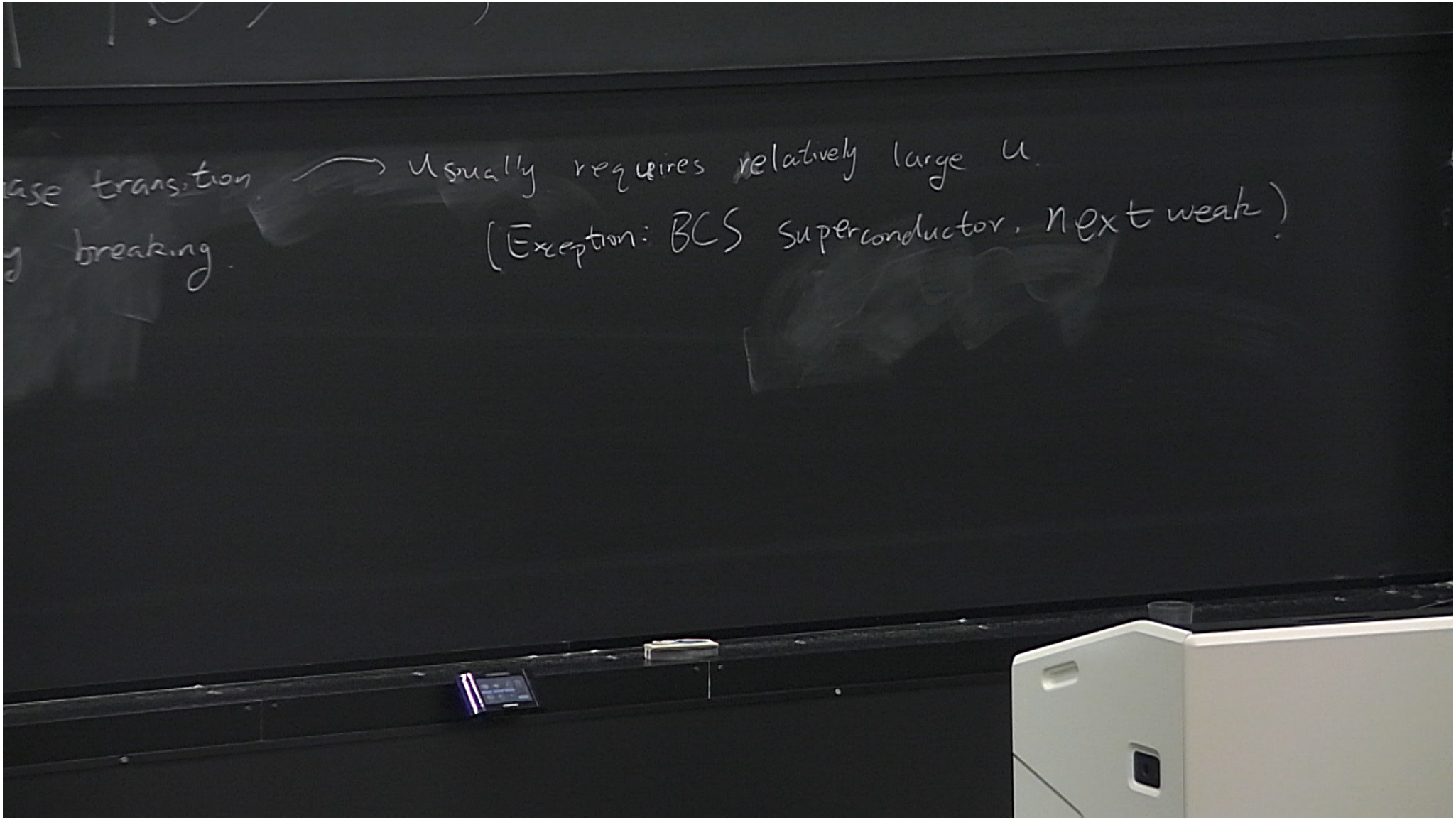
$C_v \propto m^* T$ (1) ~~no error bar~~ $+ T^2 \pm T^5$, at low enough T.

② quasi-particles have well-defined charge, momentum, energy (up to small uncertainty $\frac{1}{\tau} \propto \xi^2$), velocity $\vec{v} = \frac{\partial \xi}{\partial \vec{k}}$.

$\Rightarrow$ distribution function $f(\vec{k}, \vec{r}, t)$ well defined.

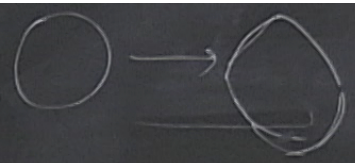
$\Rightarrow$ Boltzmann equation application


Adiabaticity breaks down at phase transition. → Usually
e.g. spontaneous symmetry breaking.
(Exc


ase transition → usually requires relatively large U
y breaking
(Exception: BCS superconductor, next week)



A: No!



Possible way to have non-Fermi liquid

① $\exists$ q.p. $|\psi_n\rangle = |n_1, n_2 \ldots\rangle$, but not adiabatically connected to a free fermion state.

$|n(k_1), n(k_2) \ldots\rangle$

Ex. 1D. $Z=0$, Luttinger liquid.

Fractional quantum Hall effect



② $\nexists$ q.p., Ex. 2D Fermi surface coupled to some gauge field.

$\frac{1}{\tau} \sim \xi_k^{\#<1} > \xi_k$ $\quad C_v \sim T^{2/3}$

SYK model

ot adiabatically connected to a free fermion state.

Ex. 1D. $Z=0$, Luttinger liquid.

Fractional quantum Hall effect

If $U \gg \bar{E}_F$



Wigner crystal.

Q.P., Ex. 2D Fermi surface coupled to some gauge field.

$\frac{1}{\tau} \sim \xi_k^{\#<1} > \xi_k$ $C_V \sim T^{2/3}$

SYK model