Title: PSI 2019/2020 - Condensed Matter (Wang) - Lecture 7

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Collection: PSI 2019/2020 - Condensed Matter (Wang)

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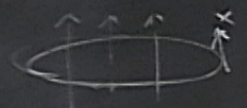
Today: Integer quantum Hall effect in Real space

- Flux-threading (Laughlin) argument
- Edge states.
- Landau level.

Digression: free particle on a ring

$\int dx\, e^{i2\pi \frac{x}{L}} |x\rangle = |p=1\rangle$

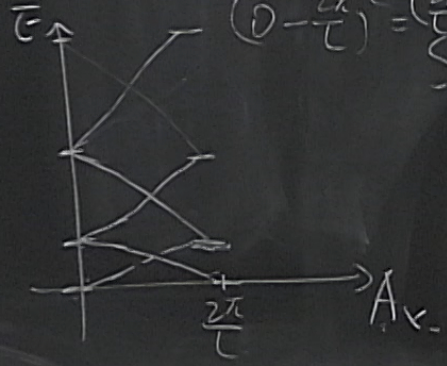


$$H = -\frac{(\partial_x - iA_x)^2}{2m} = \frac{(p - A_x)^2}{2m}, \quad p = \frac{2\pi n}{L}, \quad n \in \mathbb{Z}$$

(uniform A_x).

$x \in [0, L)$.

$\int A_x dx = \Phi = \int B d^2x$



Spectrum looks different from $A_x = 0$.

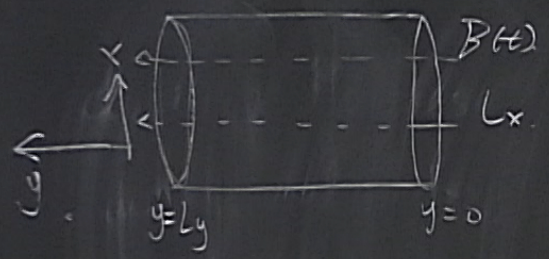
unless $A_x = \frac{2\pi}{L} \cdot m$, $m \in \mathbb{Z}$.

Large gauge transform

Reason: gauge transform: $|x\rangle \to e^{i2\pi m \frac{x}{L}} |x\rangle$

$$A_x \to A_x - \partial_x\left(2\pi m \frac{x}{L}\right) = 0.$$

$$\Phi = \oint A_x dx = 2\pi m = \frac{hc}{e} \cdot m = \Phi_0 m.$$



Bulk: insulator (gapped). Edge: ???

$B(t)$ changes slowly

$$\Phi(t) = \int B(t)\, d^2x = \oint A_x^{(t)}\, dx = A_x^{(t)} L_x$$

In general, system at different A_x are different.

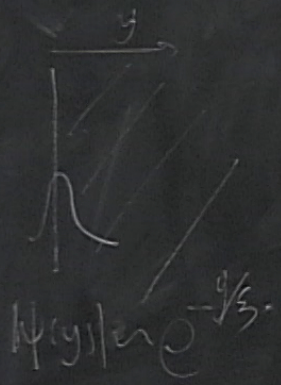
If $\Phi(t=0)=0$, $\Phi(t=T)=2\pi$, $t=0$, $t=T$ spectra identical

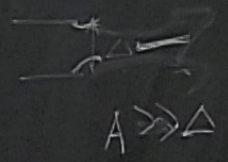
If entire system is gapped (bulk + edge), $|\Psi(T)\rangle = e^{i\theta}|\Psi(t=0)\rangle = |G.S.\rangle_0$

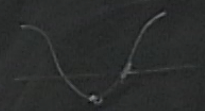
If edge gapless. $(A \sim \frac{1}{t_x})$.. $A_x \sim \frac{1}{L}$ could induce a transition out of ground state.

$|\Psi(t=T)\rangle$ = some state (possibly excited) of $H|(t=0)$.

Excitation should only live on boundary.







$\Psi(y)|_{\partial} \sim e^{-y/\xi}$.

Q: How much charge has flown from $y=0$ to $y=L_y$ in $\hat{y}$ direction?

$$\Delta Q_y = \int J_y \cdot dt = \int j_y \cdot dt\, dx = \int \sigma_{yx} E_x\, dt\, dx, \quad E_x = \frac{dA_x}{dt}.$$

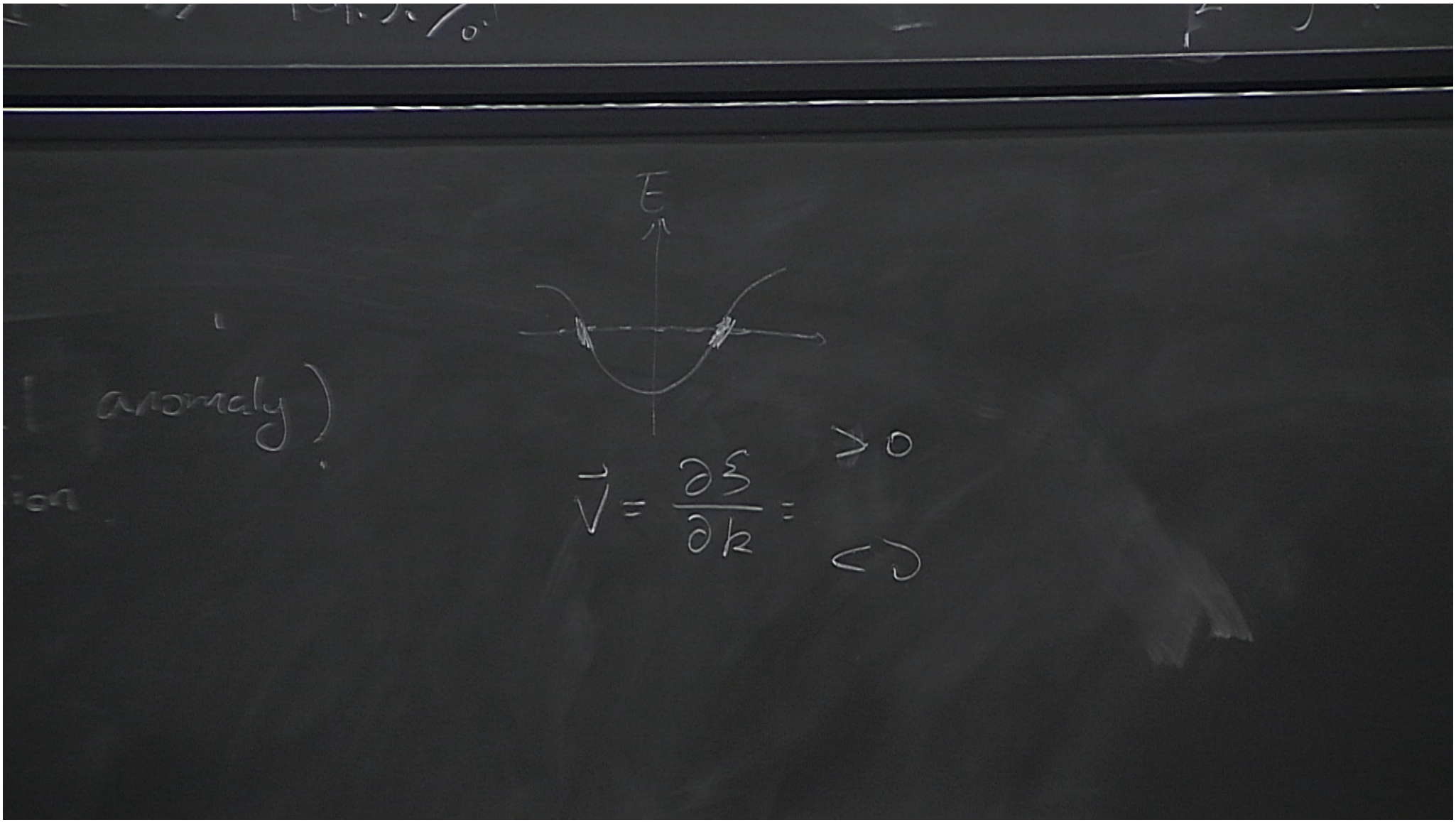
J_y = current in $\hat{y}$. $J_y = \int j_y dx$.

$$= -\sigma_{xy} L_x \int \frac{dA_x}{dt} dt = -\sigma_{xy} (L_x A_x)\Big|_0^T = -\sigma_{xy} \cdot 2\pi$$

If $\sigma_{xy} \neq 0$. $\Delta Q_y \neq 0$: charge transfer from right to left edge.

$\Delta Q_y \in \mathbb{Z}$. as long as all excitations carry integer charge (true for free fermion + weak interaction)

$$\Rightarrow \sigma_{xy} = \frac{n}{2\pi} = \frac{e^2}{h} \cdot n.$$


E
anomaly)
$\vec{V} = \frac{\partial \varepsilon}{\partial k} = \begin{matrix} > 0 \\ < 0 \end{matrix}$

If chiral system is gapped (bulk + edge), $|\Psi(T)\rangle = |\Psi(t=0)\rangle = |G\rangle$.

Edge state. $\sigma_{xy} = \frac{1}{2\pi}$. $y = L_y$ edge.

$\boxed{\Delta Q = -1}$ upon a 2π-flux threading. (a.k.a. chiral anomaly)

What is the simplest such state? – Chiral fermion (righ-moving)

$$H = v \sum_{k_x} c^{\dagger}_{k_x} (k_x - A_x) c_{k_x}$$

$$k_x = \frac{2\pi}{L_x} n$$

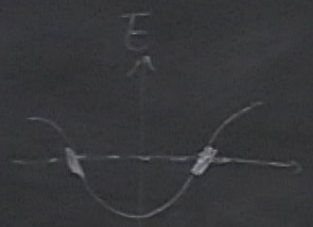
$y = 0$ edge.

$\Delta Q = +1$

left-moving fermion.

$\langle G.S. \rangle_0$

$$\Phi = \oint A_x dx = 2\pi m = \frac{hc}{e} \cdot m = \Phi_0 m.$$



$$\vec{v} = \frac{\partial \varepsilon}{\partial k} = \begin{cases} >0 \\ <0 \end{cases}$$

\# of left modes. − \# of right modes on $y=0$.

$=$ \# .. right .. − \# .. left .... on $y = L_y$

$$= 2\pi \sigma_{xy} \in \mathbb{Z}$$

(Bulk-boundary correspondence).

Edge state. $\sigma_{xy} = \frac{1}{2\pi}$. $y = L_y$ edge.

$\boxed{\Delta Q = -1}$ upon a 2π-flux threading. (a.k.a. chiral ano

What is the simplest such state? – Chiral fermion

$$H = -V \sum_{k_x} c^{\dagger}_{k_x} (k_x - A_x) c_{k_x}$$ (righ-moving)

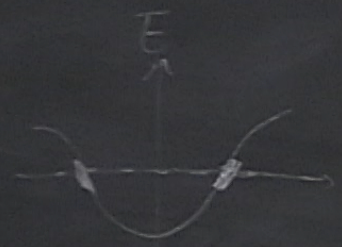
$$k_x = \frac{2\pi}{L_x} n$$

$y = 0$ edge.

$\Delta Q = +1$

lef-moving fe

$$\Phi = \oint A_x dx = 2\pi m = \frac{h}{e} \cdot m = \Phi_0 m.$$



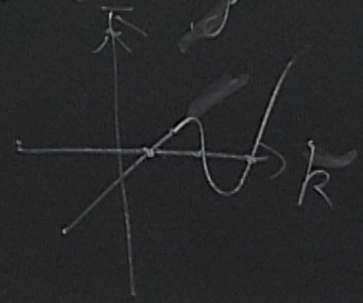
$$\vec{v} = \frac{\partial \varepsilon}{\partial k} = \begin{cases} >0 \\ <0 \end{cases}$$

# of left modes − # of right modes on $y=0$.

$=$ # .. right .. − # .. left .... on $y = L_y$

$= 2\pi \sigma_{xy} \in \mathbb{Z}$

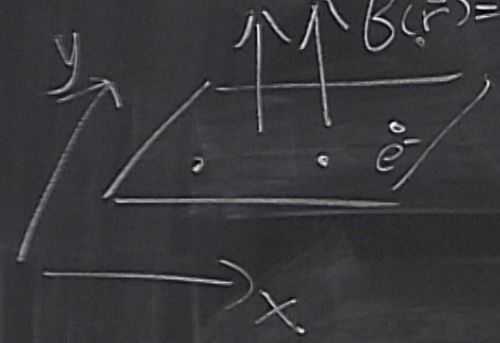
(Bulk-boundary correspondence)



Landau level as IQHE.

2D electron gas under B_z field.

$B(\vec{r}) = B$, $V(\vec{r}) = \mu$.



Landau gauge: $A_x = 0$, $A_y = Bx$.

Translation symmetry in $\hat{y}$ is kept.

label (partially) states by k_y.

$$\psi(x,y) = e^{ik_y y}\phi(x).$$

$$H = \frac{\hat{p}_x^2}{2m} + \frac{(\hat{p}_y - B\hat{x})^2}{2m} = \frac{\hat{p}_x^2}{2m} + \frac{B^2}{2m}\left(\hat{x} - \frac{k_y}{B}\right)^2.$$

Harmonic Oscillator! $\omega_c = \frac{B}{m}$: cyclotron frequency

Ground state: $\psi^{(0)}_{k_y}(x,y) = e^{ik_y y}\phi_0\left(x - \frac{k_y}{B}\right)$

$$\underbrace{E_0 = \frac{1}{2}\hbar\omega_c}_{\text{independent of } k_y}, \quad \langle x \rangle = \frac{k_y}{B}, \quad \sqrt{\langle \Delta x^2 \rangle} \propto \frac{1}{\sqrt{B}}$$

$$+ \frac{B^2}{2m}\left(\hat{x} - \frac{k_y}{B}\right)^2$$

$\omega_c = \frac{B}{m}$: cyclotron frequency

$$= e^{ik_y y}\, \phi_0\left(x - \frac{k_y}{B}\right)$$

$$\langle x \rangle = \frac{k_y}{B}, \quad \sqrt{\langle \Delta x^2 \rangle} \propto \frac{1}{\sqrt{B}}$$

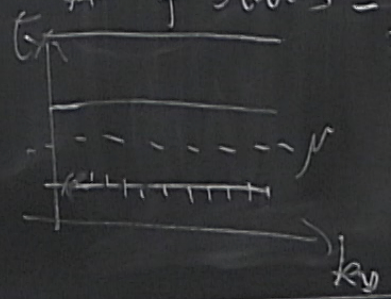
$L_x \times L_y$

# of states?

$$k_y = \frac{2\pi}{L_y} n_y \Rightarrow \langle x \rangle = \frac{2\pi}{B L_y} n_y \in [0, L_x)$$

$$\Rightarrow n_y \in \left[0, \frac{B L_x L_y}{2\pi}\right)$$

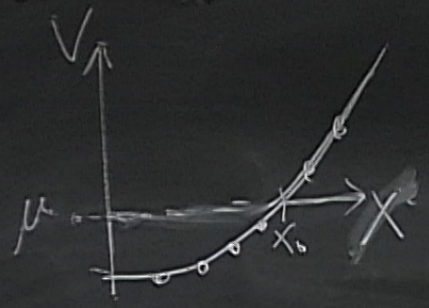
$$\# \text{ of states} = \frac{\Phi}{2\pi} = \Phi \Big/ \left(\frac{hc}{e}\right) = \frac{\Phi}{e^2} \Big/ \Phi_0$$



$$\boxed{\sigma_{xy} = \frac{1}{2\pi} = \frac{e^2}{h}}$$

TKNN, 1982

Edge: $V(\vec{r}) = V(x) - \mu$



$x = x_0 + \delta x$

$V(\vec{r}) = a\,\delta x = \underline{\frac{a}{B} k_y - \text{const}}$, chiral edge

but $x \sim \frac{k_y}{B}$ $\therefore$ $\delta x \sim \frac{k_y - k_y^{(0)}}{B}$.