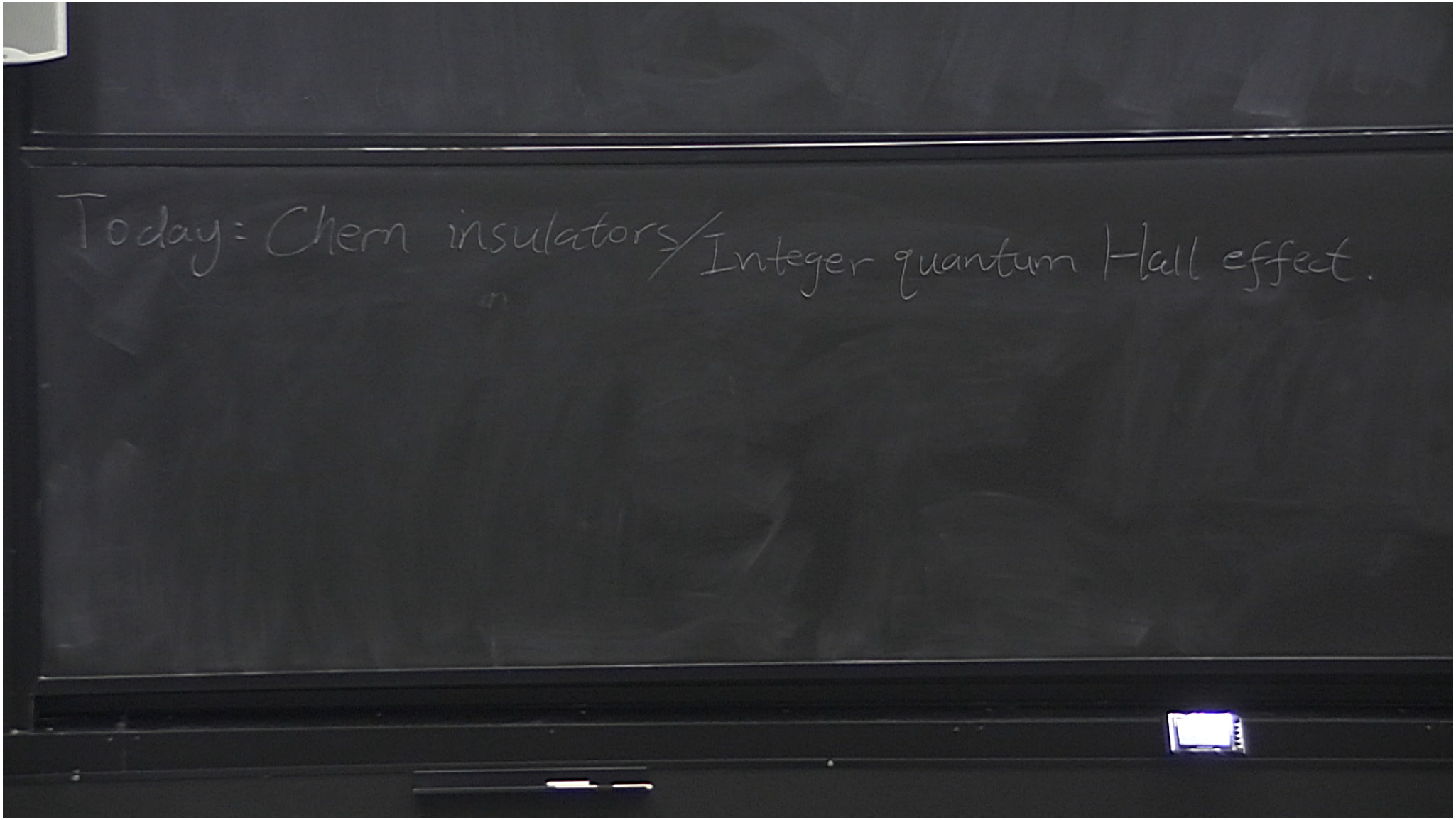
Title: PSI 2019/2020 - Condensed Matter (Wang) - Lecture 6

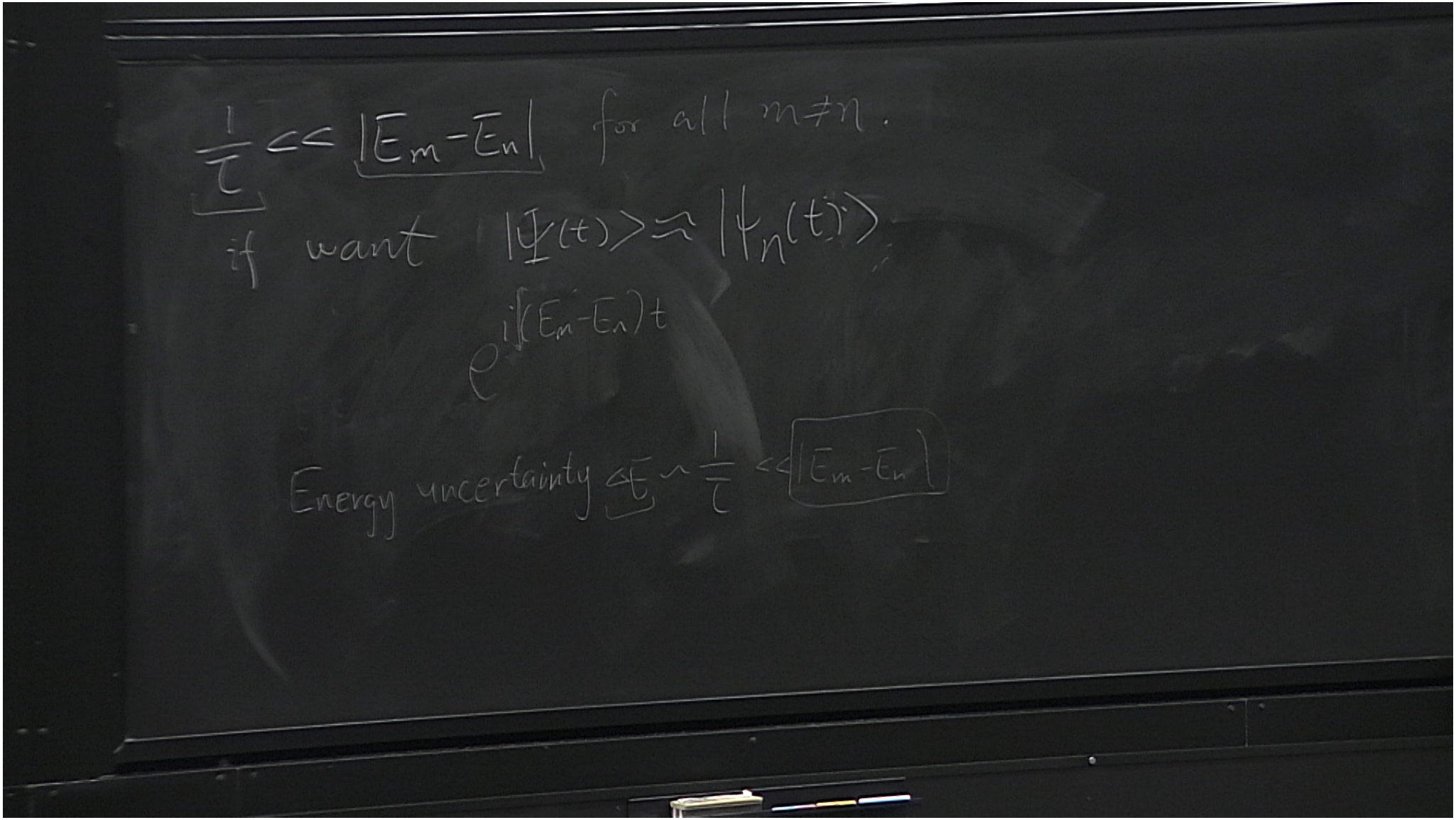
Speakers: Chong Wang

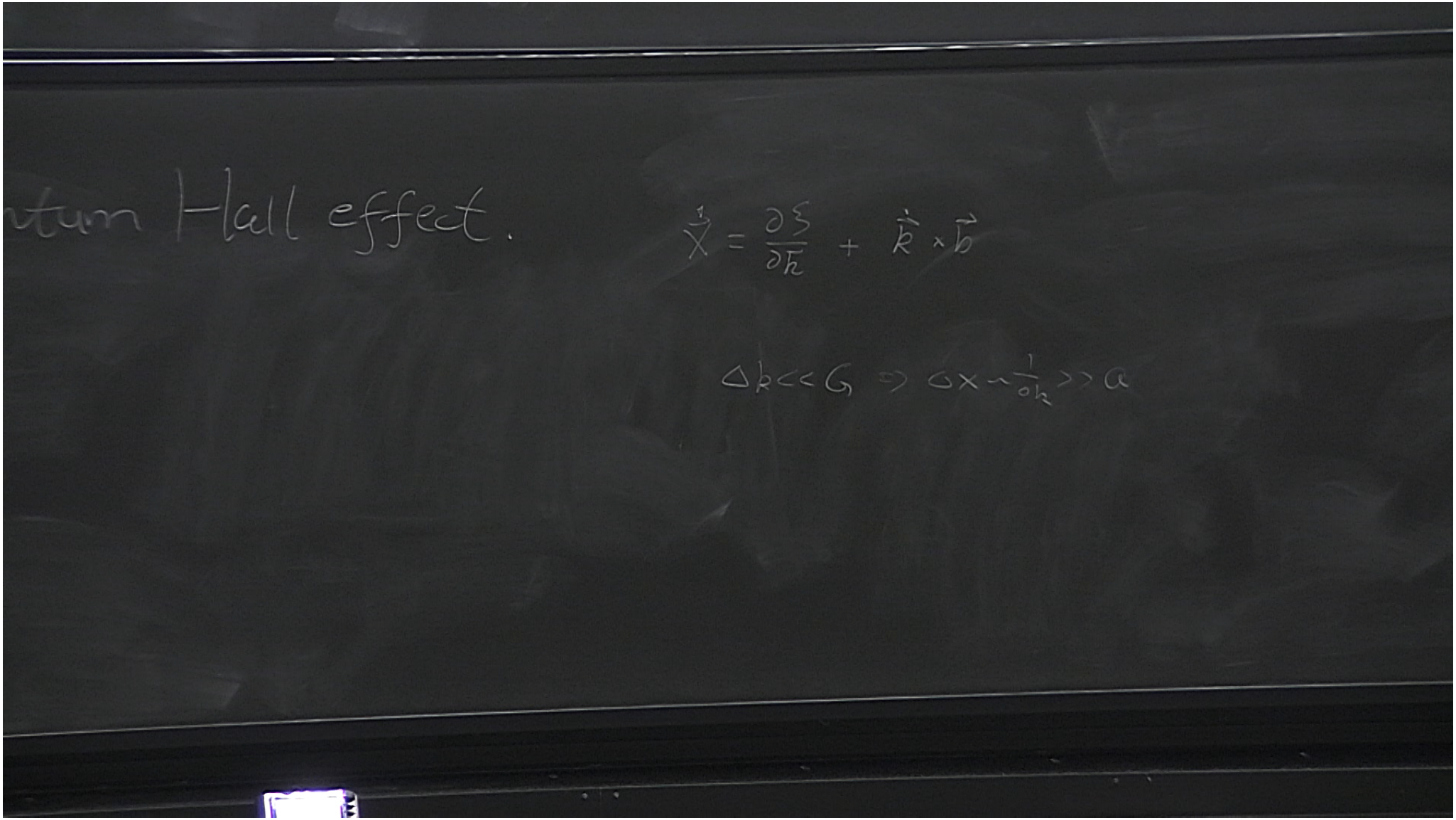
Collection: PSI 2019/2020 - Condensed Matter (Wang)

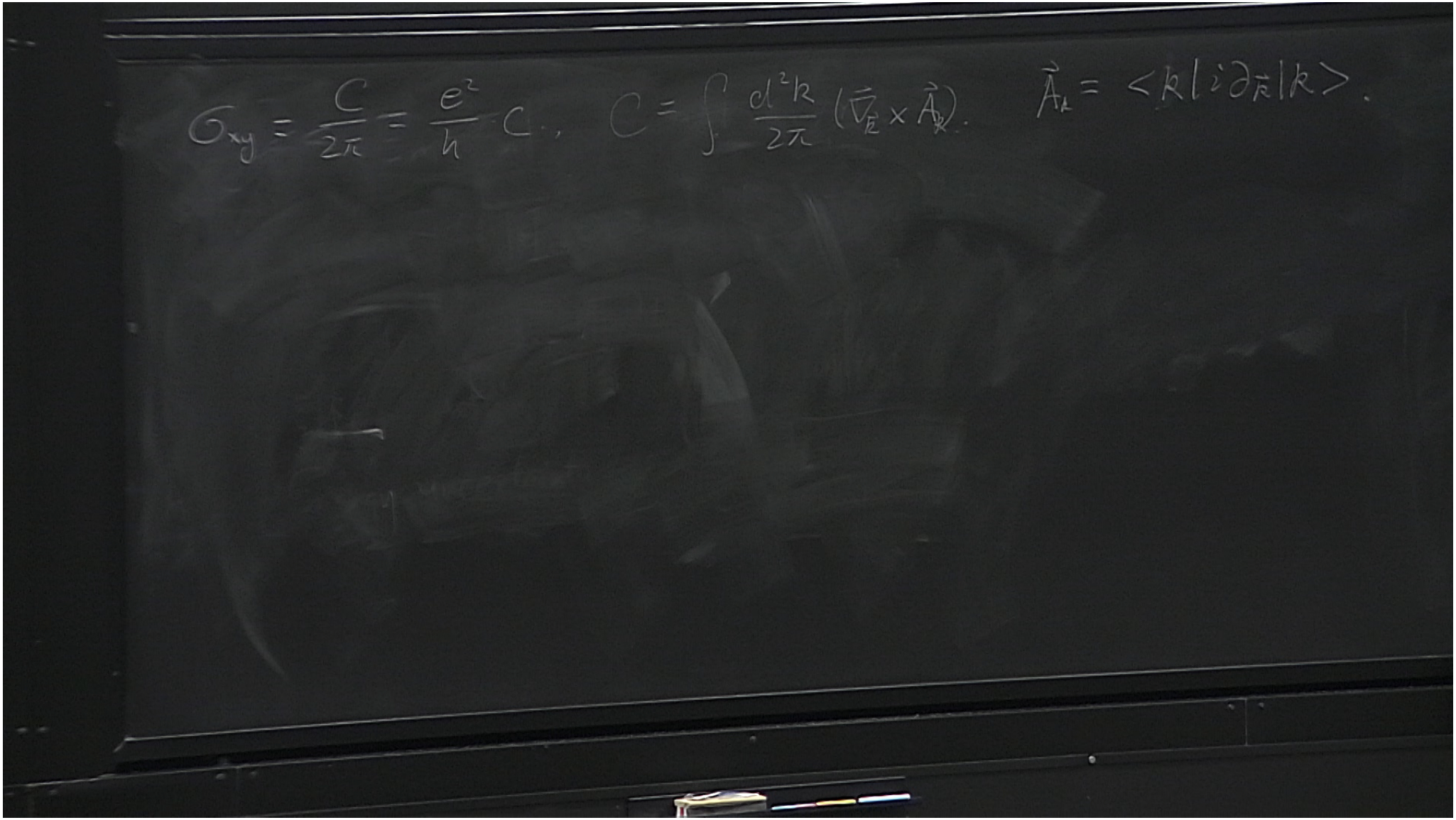
Date: November 19, 2019 - 10:45 AM

URL: http://pirsa.org/19110030


Today: Chern insulators / Integer quantum Hall effect.


$\frac{1}{\tau} \ll |E_m - E_n|$ for all $m \neq n$.
if want $|\Psi(t)\rangle \approx |\psi_n(t)\rangle$
$e^{i(E_m - E_n)t}$
Energy uncertainty $\Delta E \sim \frac{1}{\tau} \ll |E_m - E_n|$


tum Hall effect.
$\dot{\vec{X}} = \frac{\partial \xi}{\partial \vec{k}} + \dot{\vec{k}} \times \vec{b}$
$\Delta k << G \Rightarrow \Delta x \sim \frac{1}{\Delta k} >> a$

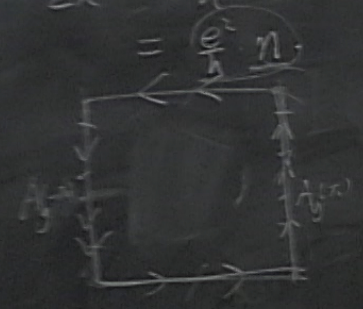

$\sigma_{xy} = \frac{C}{2\pi} = \frac{e^2}{h} C$, $C = \int \frac{d^2k}{2\pi} (\vec{\nabla}_{\vec{k}} \times \vec{A}_{\vec{k}})$. $\vec{A}_{\vec{k}} = \langle k | i \partial_{\vec{k}} | k \rangle$.

$\sigma_{xy} = \frac{C}{2\pi} = \frac{e^2}{h} C$.

$= \frac{e^2}{h} n_1$

$C = \int \frac{d^2k}{2\pi} (\vec{\nabla}_{\vec{k}} \times \vec{A}_{\vec{k}}) \cdot f(k)$ $\vec{A}_k = \langle k | i \partial_{\vec{k}} | k \rangle$.

$= \int_0^{2\pi} \frac{d\vec{k}}{2\pi} \cdot \vec{A} = n_1 \in \mathbb{Z}$.



$\vec{A}$ is not gauge invariant, so doesn't have to be single-valued.

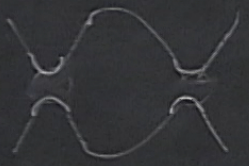


$\nabla \times \vec{A}$

$e^{i \oint_{\pi} \vec{A} \cdot d\vec{l}} = e^{i \oint_{-\pi} \vec{A} \cdot d\vec{l}}$

$\oint_{\pi} \vec{A} \cdot d\vec{l} = \oint_{-\pi} \vec{A} \cdot d\vec{l} + 2\pi n$

Graphene

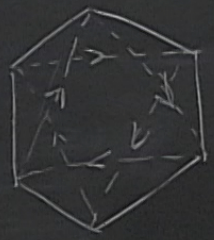


$$H = \sum_{k \sim K} \psi^{+} (\delta k_x \sigma_x + \delta k_y \sigma_y) \psi$$

$$k \sim K' \quad \psi^{+} (-\delta k_x \sigma_x + \delta k_y \sigma_y) \psi$$

$$= \psi^{+} (\delta k_x \tau_z \sigma_x + \delta k_y \sigma_y) \psi$$

$T: c_i \to c_i \quad i \to -i$

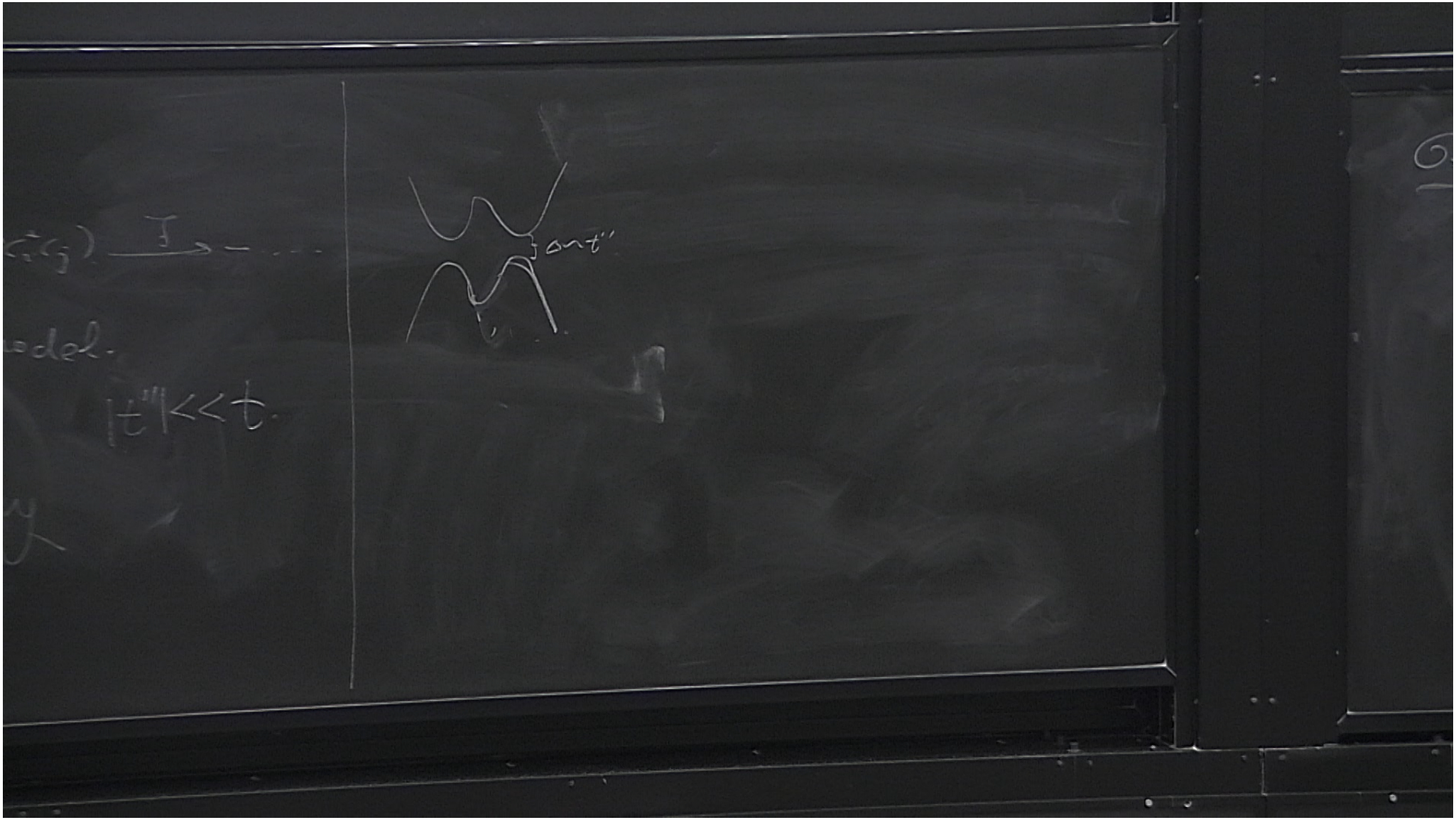


$i \to j = |t''|(i c_i^\dagger c_i - i c_i^\dagger c_j) \xrightarrow{T} -\ldots$

: Haldane model.

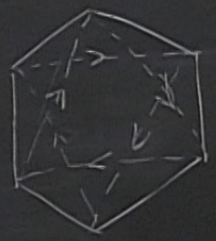
$|t''| << t$.

Break T, keep other symmetry


odel.
|t'|<<t.

$T: c_i \to c_i \quad i \to -i$

$i \to j = |t''|(e^{i\phi} c_j^\dagger c_i + e^{-i\phi} c_i^\dagger c_j) \xrightarrow{J} -\ldots$



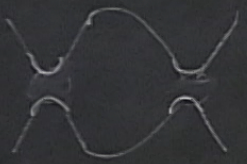
: Haldane model.

$|t''| << t$.

Break T, keep other symmetry

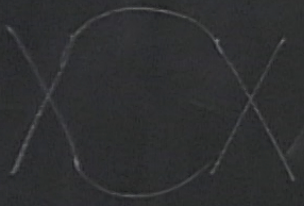
$$\sum_{\langle ij \rangle} t\, c_i^\dagger c_j$$

Graphene



$H = \psi^\dagger (\delta k_x \sigma_x + \delta k_y \sigma_y) \psi$

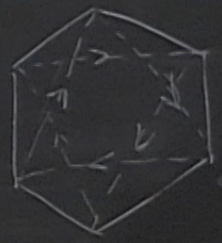
$k \sim K: \psi^\dagger (-\delta k_x \sigma_x + \delta k_y \sigma_y) \psi$



$= \psi^\dagger (\delta k_x \tau_z \sigma_x + \delta k_y \sigma_y) \psi$

$+ m \psi^\dagger \tau_z \sigma_z \psi$ mass term

$T: C_2 \to C_2$



Break T,

Time-reversal: $b(\vec{k}) \to -b(-\vec{k})$

$$\int \frac{d^3k}{(2\pi)^3}\, b = 0$$

$$A = \langle k | i\partial_k | k \rangle$$

$|k\rangle \to |-k\rangle$

$i \to -i$

$\partial_k \to -\partial_k$

$A(k) \to A(-k)$

$$b = \partial_k \times A$$

$$H(k) = C^{\dagger}(k)\,(2\times 2)\,C(k) = -(c^{\dagger}_{k,A}, c^{\dagger}_{k,B})\,\underbrace{\left(h_x(k)\sigma_x + h_y(k)\sigma_y + h_z(k)\sigma_z\right)}_{+\mathbb{1}}\begin{pmatrix} c_{k,A} \\ c_{k,B}\end{pmatrix}$$

$k \in B.Z.$

$$h_{x,y,z} = |h|\,\hat{h} \qquad E_k = \pm|h| = \pm\sqrt{h_x^2 + h_y^2 + h_z^2}$$

$$\hat{h} \in S^2$$

lowerband $|k\rangle$: eigenstate of $\hat{h}\cdot\vec{\sigma}$ with eigenvalue 1

$$|\hat{h}\rangle = \begin{pmatrix} \alpha(\hat{h}) \\ \beta(\hat{h})\end{pmatrix}$$

$$|k\rangle = \begin{pmatrix} \alpha c_{k,B} \\ \beta c_{k,A}\end{pmatrix} \qquad c_k = \sum_x \psi_k(x)\, c_x$$

$$C(k) = \begin{pmatrix} C_A(k) \\ C_B(k) \end{pmatrix}$$

$$C_{A,B}(k) = \sum e^{i k \cdot x} \; C_{A,B}(x)$$

$$|x, A\rangle$$

$$H(k) = C^{\dagger}(k)\,(2\times 2)\,C(k) = (c_{k\uparrow}^{\dagger}, c_{k\downarrow}^{\dagger})\,\underbrace{\left(h_x(k)\sigma_x + h_y(k)\sigma_y + h_z(k)\sigma_z\right)}_{\pm\Delta}\begin{pmatrix} |k\rangle_A \\ |k\rangle_B \end{pmatrix}$$

$k \in B.Z.$

$$h_{x,y,z} = |h|\,\hat{h} \qquad E_k = \pm|h| = \pm\sqrt{h_x^2 + h_y^2 + h_z^2}$$

$$\hat{h} \in S^2$$

lowerband $\begin{pmatrix} \alpha_k C_A(k) \\ \vdots \\ \beta_k C_B(k) \end{pmatrix}_k$ eigenstate of $\hat{h}\cdot\vec{\sigma}$ with eigenvalue -1

$$\begin{pmatrix} \alpha_k \\ \beta_k \end{pmatrix}$$

$$C_k = \sum_x \psi_k(x)\, c_x$$

$$\begin{pmatrix} |k\rangle_A \\ |k\rangle_B \end{pmatrix}$$

$$\pm\sqrt{h_x^2+h_y^2+h_z^2}$$

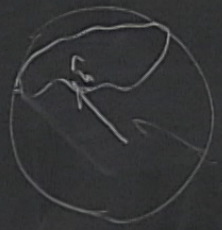
$$C(k) = \begin{pmatrix} C_A(k) \\ C_B(k) \end{pmatrix}.$$

$$C_{A,B}(k) = \sum e^{i k\cdot x}\, C_{A,B}(x)$$

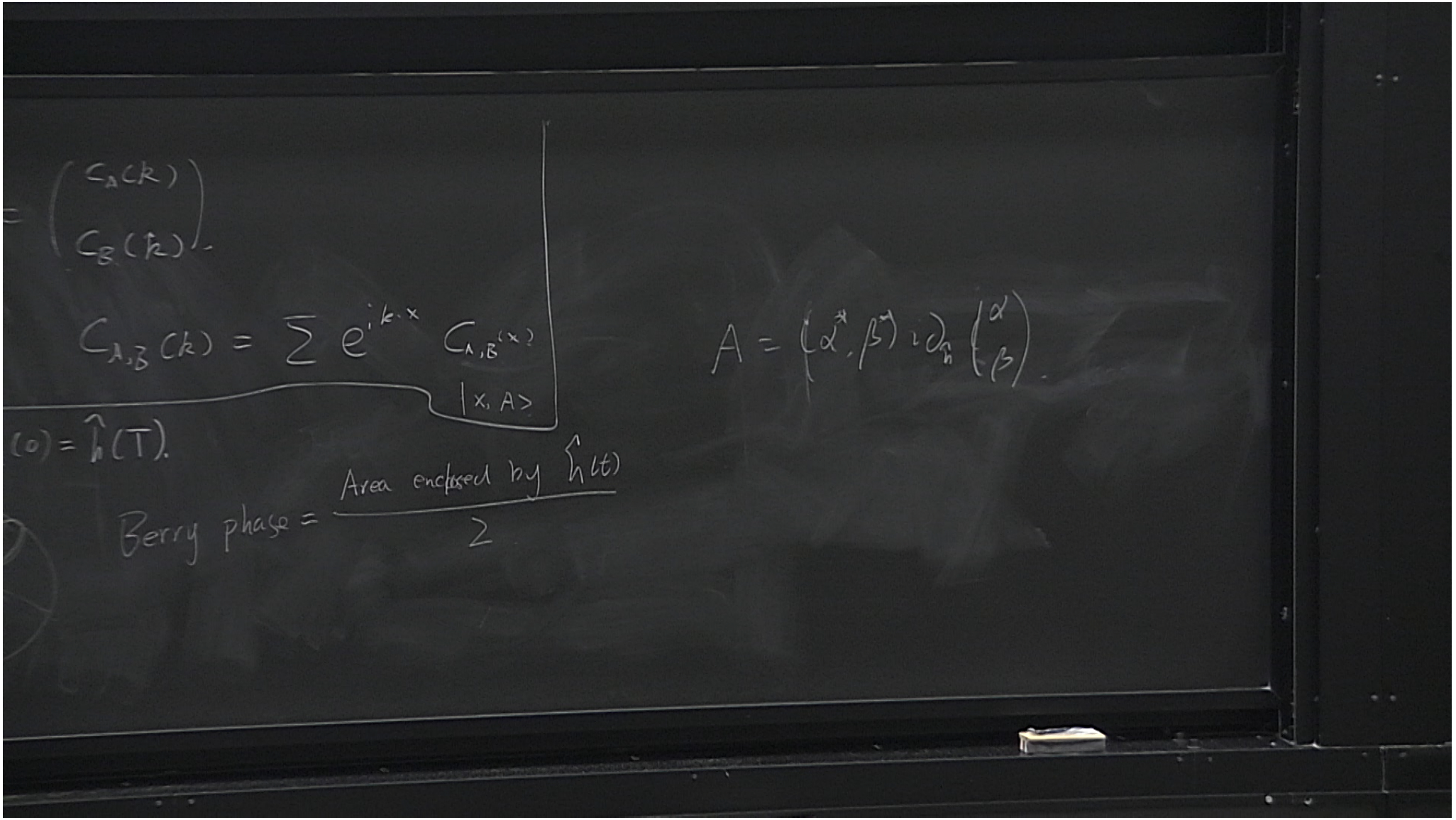
$|x, A\rangle$

eigenvalue 1

$\hat{h}(t)\cdot\vec{\sigma}$, $\hat{h}(0) = \hat{h}(T)$.



$$\text{Berry phase} = \frac{\text{Area enclosed by } \hat{h}(t)}{2}$$


$\begin{pmatrix} C_A(k) \\ C_B(k) \end{pmatrix}$
$C_{A,B}(k) = \sum e^{ik\cdot x} C_{A,B}(x)$
$|x, A\rangle$
$\hat{h}(0) = \hat{h}(T)$
Berry phase = $\frac{\text{Area enclosed by } \hat{h}(t)}{2}$
$A = (\alpha^*, \beta^*) \cdot \partial_n \begin{pmatrix} \alpha \\ \beta \end{pmatrix}$

$H = -t \sum_{\langle i,j \rangle} c_i^\dagger c_j - t'' \sum_{\langle\langle i,j \rangle\rangle} (i c_i^\dagger c_j - \text{h.c.})$

$(c_i^\dagger c_j + e^{-\frac{t}{\cdot}} c_i^\dagger c_j) \xrightarrow{J} - \ldots$

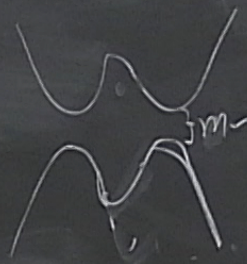
plane model.

$|t''| << t.$

symmetry

$c_i \to (-1)^{AB} c_i^\dagger$

$m \sim t'' << t.$



Far away (in momentum) from $K_+ - K_-$, t' doesn't do anything.

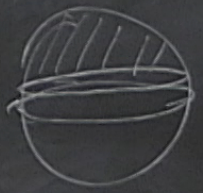
$\Rightarrow$ No contribution to $\int d^2k\, b$.



$V_0 \delta k >> m$

$\delta k_x \sigma_x + \delta k_y \sigma_y + m \sigma_z$

$h \approx \frac{\delta k_x}{|\delta k|} \hat{x} + \frac{\delta k_y}{|\delta k|} \hat{y} + m \hat{z}$



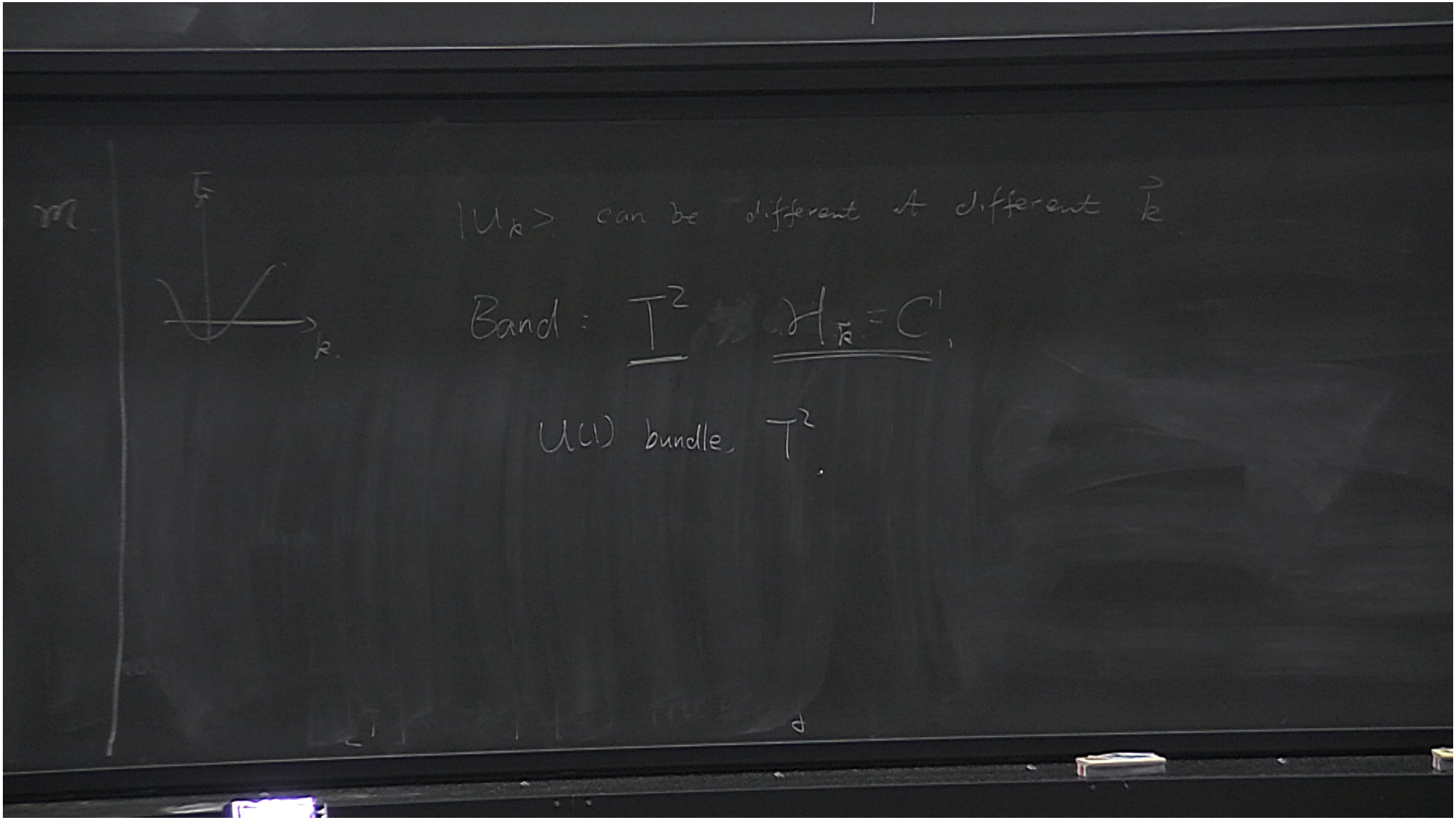
total Berry curvature from 1 Dirac cone with a mass m.

$$\int d^2k\ b = \mathrm{sgn}(m)\,\pi.$$

C_2 inversion symmetry: $b(k) \to b(-k)$

$\Rightarrow$ Same contribution from K, K'.

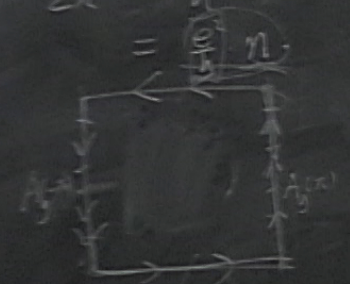
$$\Rightarrow C = \frac{2}{2\pi}\cdot \mathrm{sgn}(m)\,\pi = 1\cdot \mathrm{sgn}(m)$$


E
k
$|u_k\rangle$ can be different at different $\vec{k}$
Band: T^2 $\mathcal{H}_{\vec{k}} = C^1$
U(1) bundle, T^2

$$\sigma_{xy} = \frac{C}{2\pi} = \frac{e^2}{h} C, \qquad C = \int \frac{d^2k}{2\pi} (\vec{\nabla}_{\vec{k}} \times \vec{A}_{\vec{k}}) \cdot f(k) \quad \vec{A}_{\vec{k}} = \langle k | i \partial_{\vec{k}} | k \rangle .$$

$$= \frac{e^2}{h} n_1$$

$$= \int_0^{2\pi} \frac{d\vec{k}}{2\pi} \cdot \vec{A} = n_1 \in \mathbb{Z}.$$



$\vec{A}$ is not gauge invariant, so doesn't have to be single-valued



$$\vec{\nabla} \times \vec{A}$$

$$e^{i \oint_{\pi} \vec{A} \cdot d\vec{l}} = e^{i \oint_{-\pi} \vec{A} \cdot d\vec{l}}$$

$$\oint_{\pi} \vec{A} \cdot d\vec{l} = \int_{-\pi} \vec{A} \cdot d\vec{l} + 2\pi n$$