Title: PSI 2019/2020 - Condensed Matter (Wang) - Lecture 4

Speakers: Chong Wang

Collection: PSI 2019/2020 - Condensed Matter (Wang)

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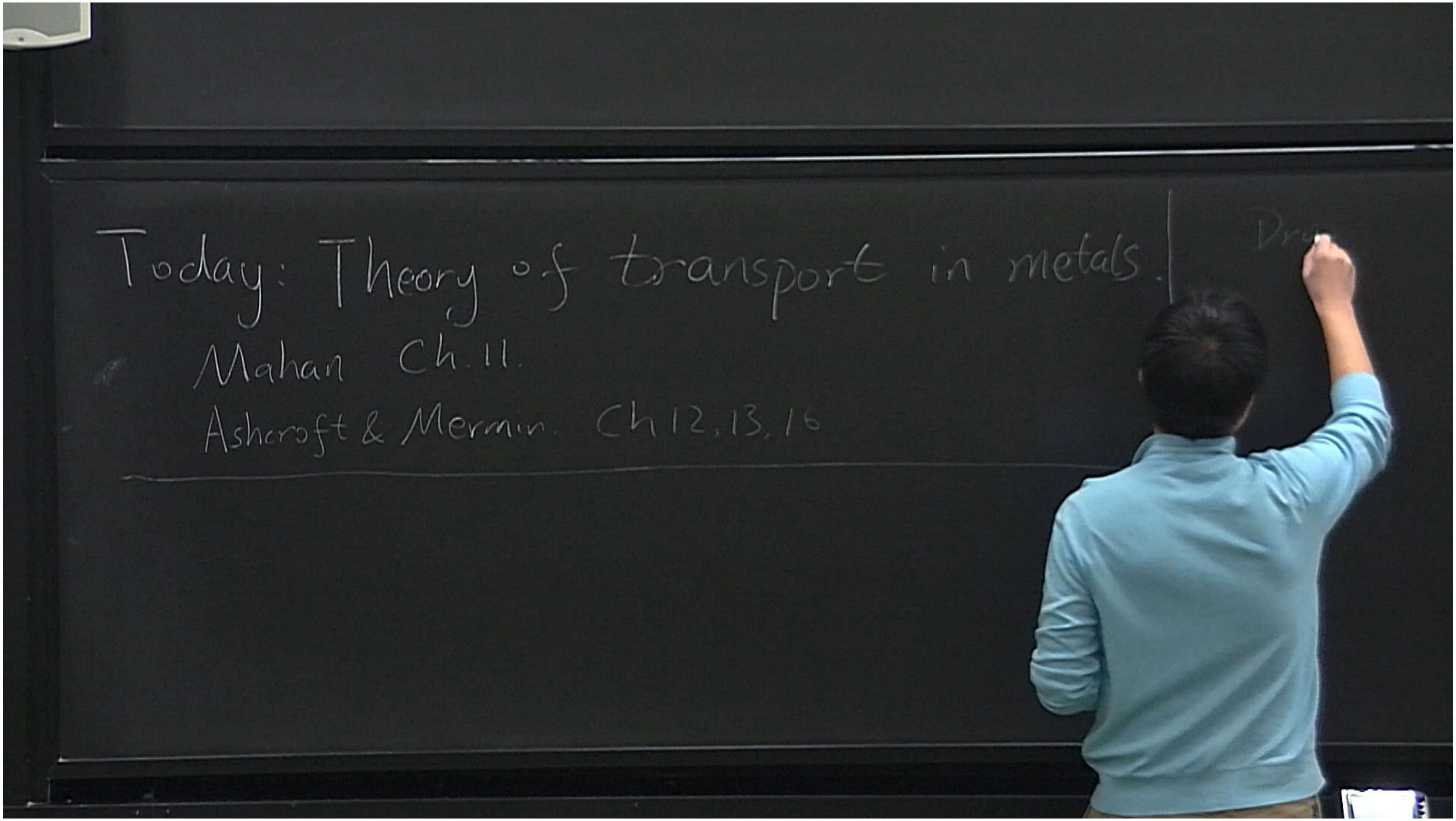

Today: Theory of transport in metals.
Mahan Ch. 11.
Ashcroft & Mermin Ch 12, 13, 16

Drude Theory

"Classical metal", continuum space, with random scattering

$$m^* \frac{d}{dt}\vec{v} = \vec{F} - \frac{m^*}{\tau}\vec{v}$$

$\vec{F} = e\vec{E}$

τ ↑ random scattering (impurity)

Drude Theory

"Classical metal", continuum space, with random scattering

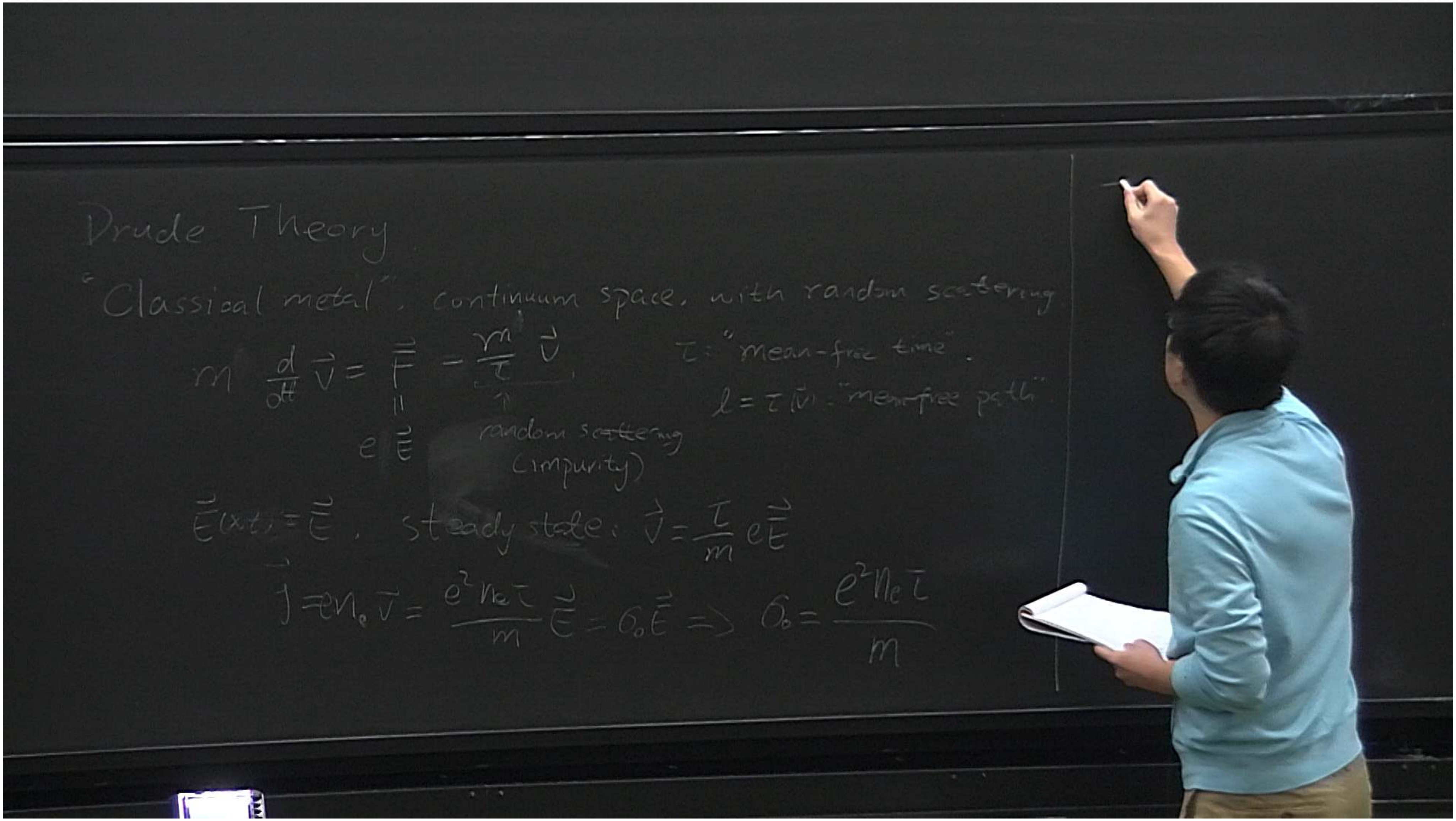
$$m \frac{d}{dt}\vec{v} = \vec{F} - \frac{m}{\tau}\vec{v}$$

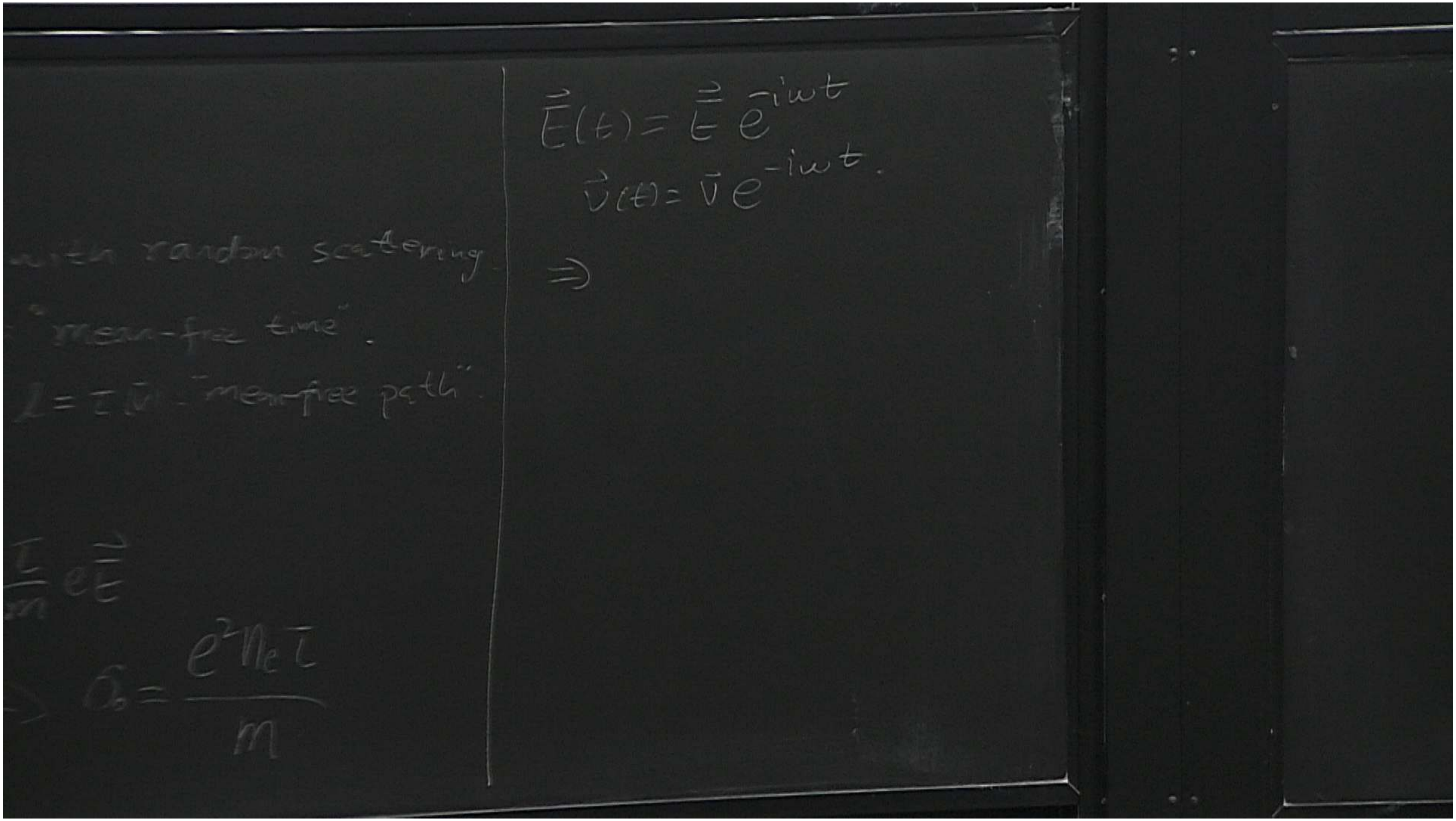
$\vec{F} = e\vec{E}$

$\frac{m}{\tau}\vec{v}$: random scattering (impurity)

τ: "mean-free time".

$l = \tau|\vec{v}|$: "mean-free path".


Drude Theory
"Classical metal", continuum space, with random scattering
$m \frac{d}{dt}\vec{v} = \vec{F} - \frac{m}{\tau}\vec{v}$
$\vec{F} = e\vec{E}$
random scattering (impurity)
τ: "mean-free time".
$l = \tau|\vec{v}|$: "mean-free path"
$\vec{E}(x,t) = \vec{E}$, steady state: $\vec{v} = \frac{\tau}{m} e\vec{E}$
$\vec{j} = e n_e \vec{v} = \frac{e^2 n_e \tau}{m}\vec{E} = \sigma_0 \vec{E} \Rightarrow \sigma_0 = \frac{e^2 n_e \tau}{m}$



$\vec{E}(t) = \vec{E} e^{-i\omega t}$
$\vec{v}(t) = \vec{v} e^{-i\omega t}$.
with random scattering.
$\Rightarrow$
"mean-free time".
$l = \tau |\bar{v}|$ "mean-free path".
$\sigma_0 = \frac{e^2 n_e \tau}{m}$

with random scattering

τ "mean-free time".

$l = \tau \bar{v}$ "mean-free path"

$\frac{\tau}{m} e\vec{E}$

$\Rightarrow \boxed{\sigma_0 = \frac{e^2 n_e \tau}{m}}$

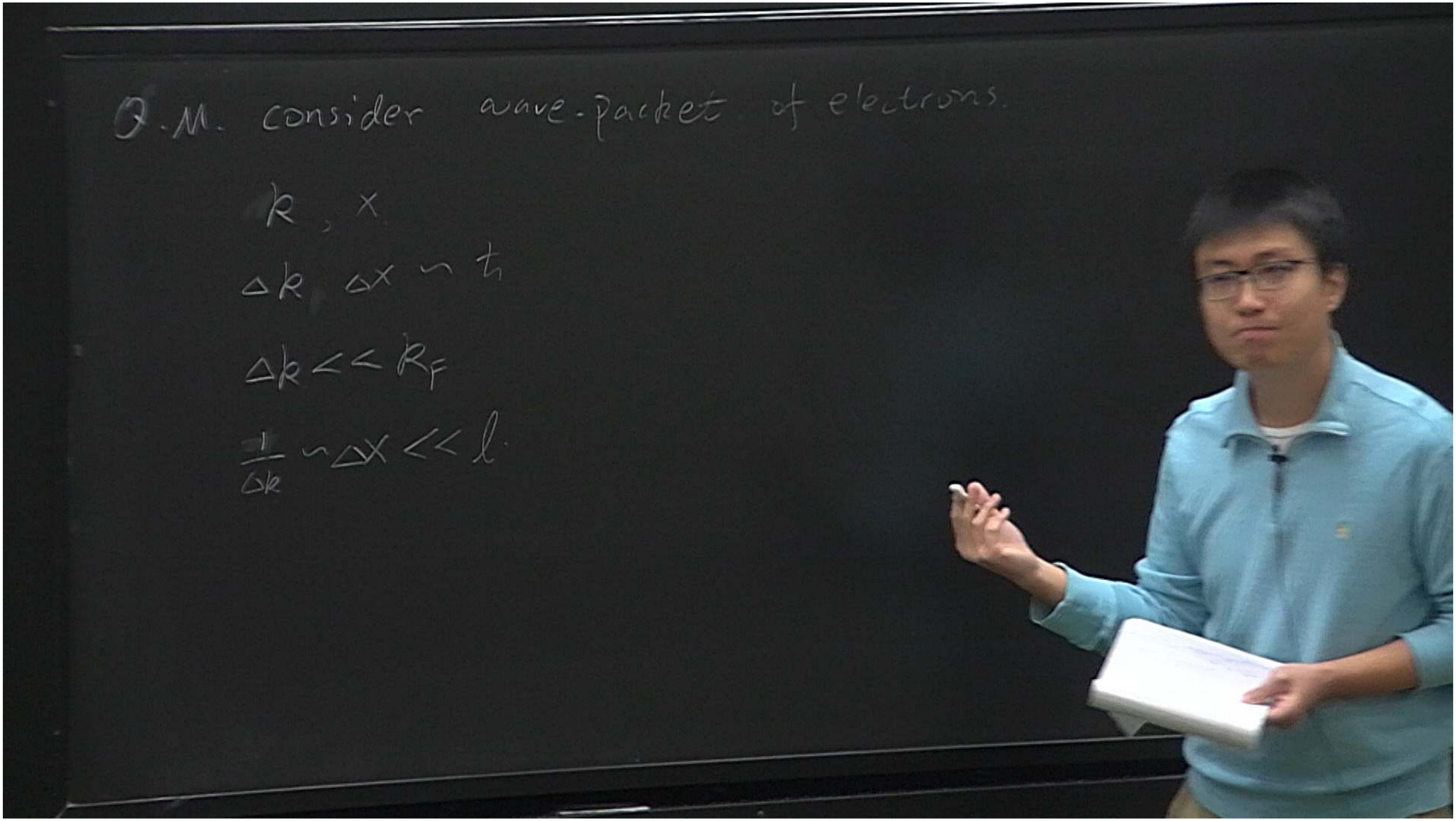
$$\vec{E}(t) = \vec{E} e^{-i\omega t}$$

$$\vec{v}(t) = \vec{v} e^{-i\omega t}$$

$$\Rightarrow \vec{v} = \frac{e\vec{E}}{\frac{m}{\tau} - m \cdot i\omega}$$

$$\boxed{\sigma = \frac{\sigma_0}{1 - i\omega\tau}}$$

Q.M.


Q.M. consider wave-packet of electrons.
k, x
Δk, Δx ~ ħ
Δk << k_F
1/Δk ~ Δx << l.

Q.M. consider wave-packet of electrons.

k, x

$\Delta k \, \Delta x \sim 1$,

$\Delta k << k_F$

$\frac{1}{\Delta k} \sim \Delta x << l$.

Makes sense if $k_F l >> 1$.

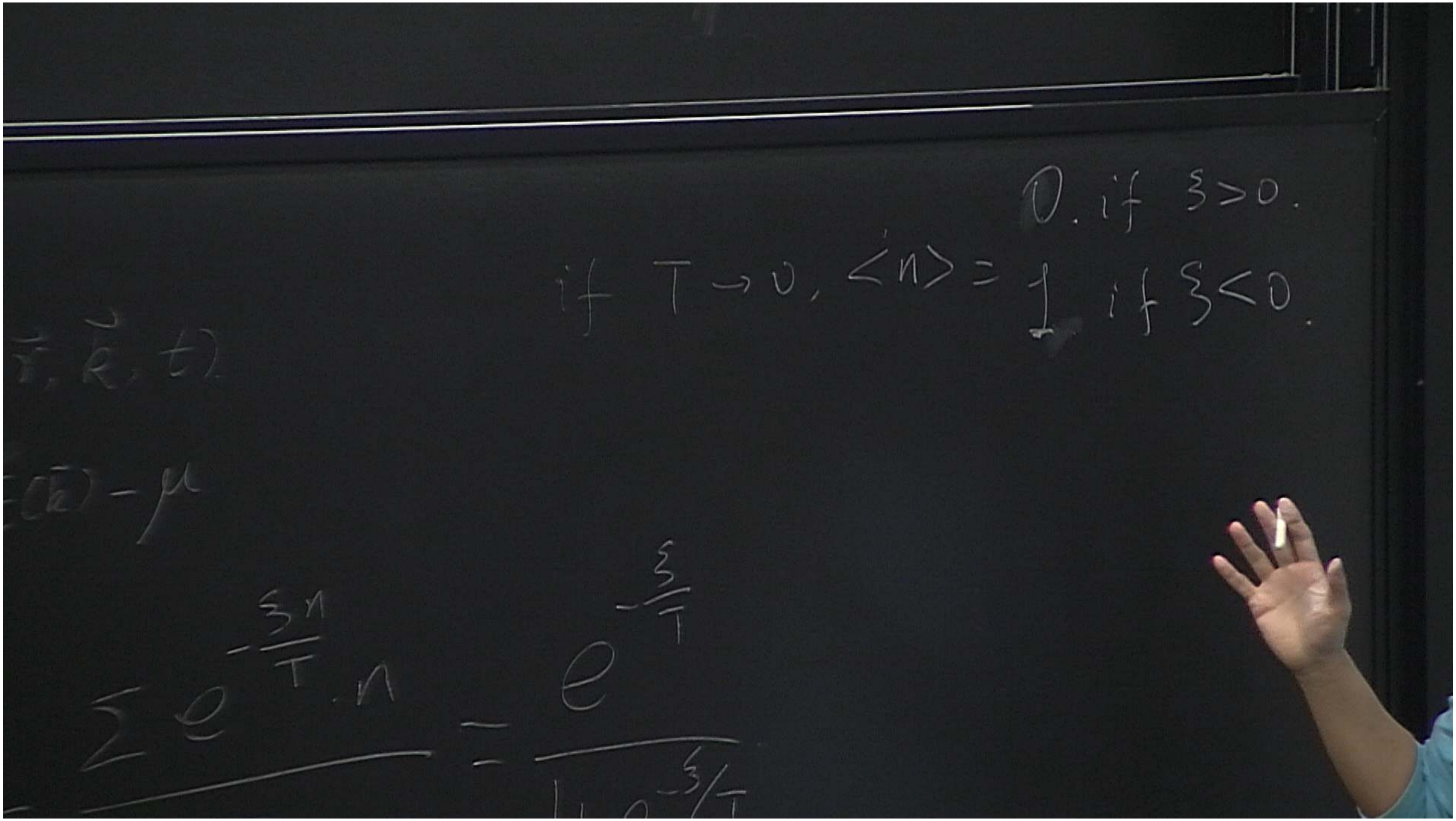
Boltzmann Equation.

Electron distribution function $f(\vec{r}, \vec{k}, t)$

Zero field. $f^{(0)}(\vec{r}, \vec{k}, t) = \dfrac{1}{e^{\frac{\xi(k)}{T}} + 1}$ $\xi(k) = E(k) - \mu$

$$Z = \sum_{n=0,1} e^{-\frac{(E-\mu)n}{T}} = 1 + e^{-\xi/T}, \quad \langle n \rangle = \frac{\sum e^{-\frac{\xi n}{T}} \cdot n}{Z} = \frac{e^{-\frac{\xi}{T}}}{1 + e^{-\xi/T}}.$$

if $T \to 0$, $\langle n \rangle =$

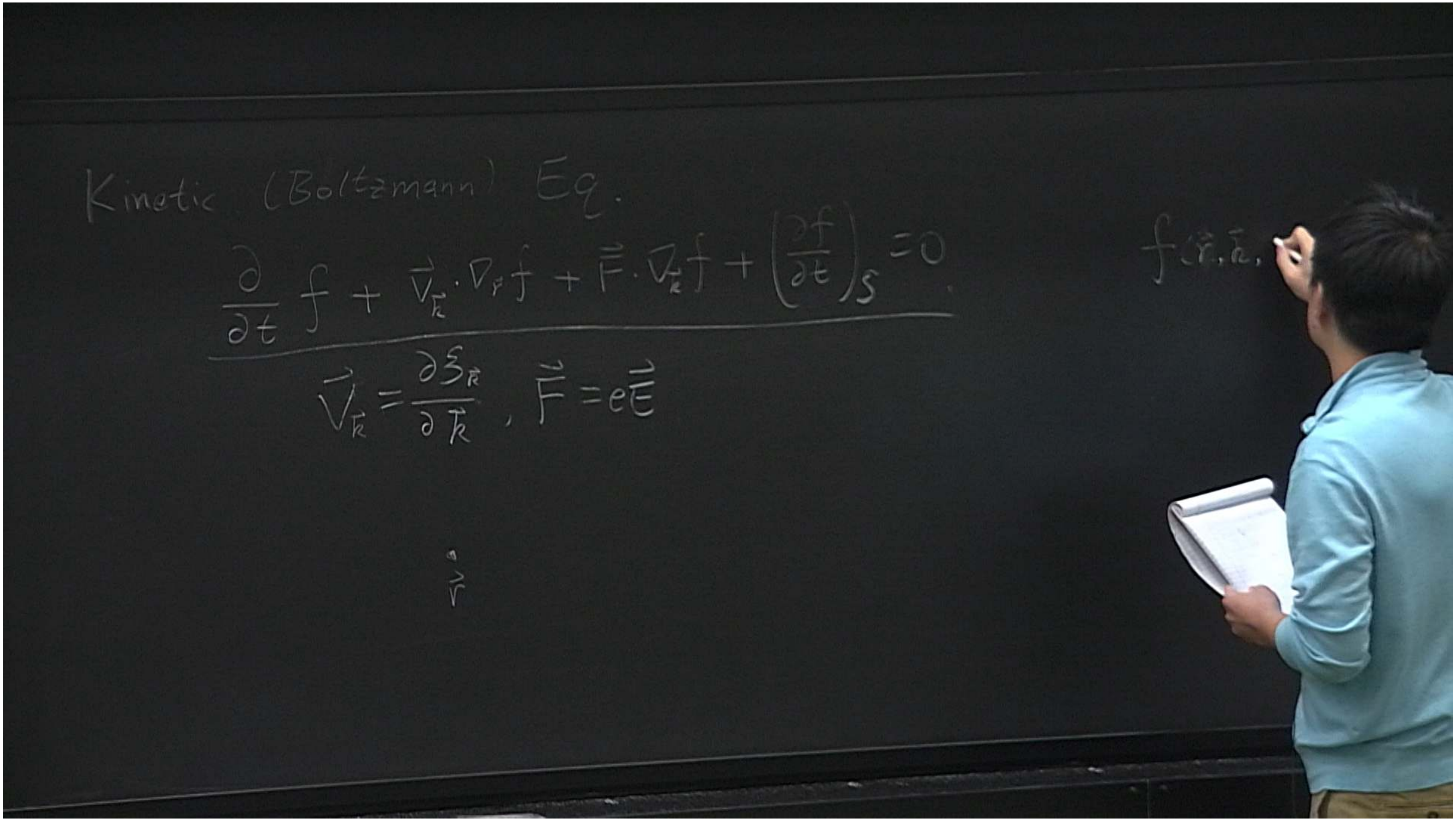

0. if ξ>0.
if T→0, <n> = 1 if ξ<0.

$$\text{if } T \to 0, \quad \langle n \rangle = \begin{cases} 0 & \text{if } \xi > 0 \\ 1 & \text{if } \xi < 0 \end{cases}$$

$(\vec{r}, \vec{k}, t)$

$\xi(k) - \mu$

$$= \frac{\sum e^{-\frac{\xi n}{T}} \cdot n}{Z} = \frac{e^{-\frac{\xi}{T}}}{1 + e^{-\xi/T}}.$$


Kinetic (Boltzmann) Eq.
$\frac{\partial}{\partial t} f + \vec{v}_{\vec{k}} \cdot \nabla_{\vec{r}} f + \vec{F} \cdot \nabla_{\vec{k}} f + \left(\frac{\partial f}{\partial t}\right)_s = 0$
$\vec{v}_{\vec{k}} = \frac{\partial \xi_{\vec{k}}}{\partial \vec{k}}, \quad \vec{F} = e\vec{E}$
$\vec{r}$
$f(\vec{r}, \vec{k},$

$$f(\vec{r},\vec{k},t) = \frac{\#\text{ of particles in } {[\vec{r};\, \vec{r}+\delta\vec{r}] \atop [\vec{k};\, \vec{k}+\delta\vec{k}]}}{|\delta\vec{r}|^{d} \cdot |\delta k|^{d}}$$

Kinetic (Boltzmann) Eq.

$$\boxed{\frac{\partial}{\partial t} f + \boxed{\vec{v}_{\vec{k}} \cdot \nabla_{\vec{r}} f} + \underline{\vec{F} \cdot \nabla_{\vec{k}} f} + \underline{\left(\frac{\partial f}{\partial t}\right)_s} = 0}$$

$f(\vec{r}, \vec{k},$

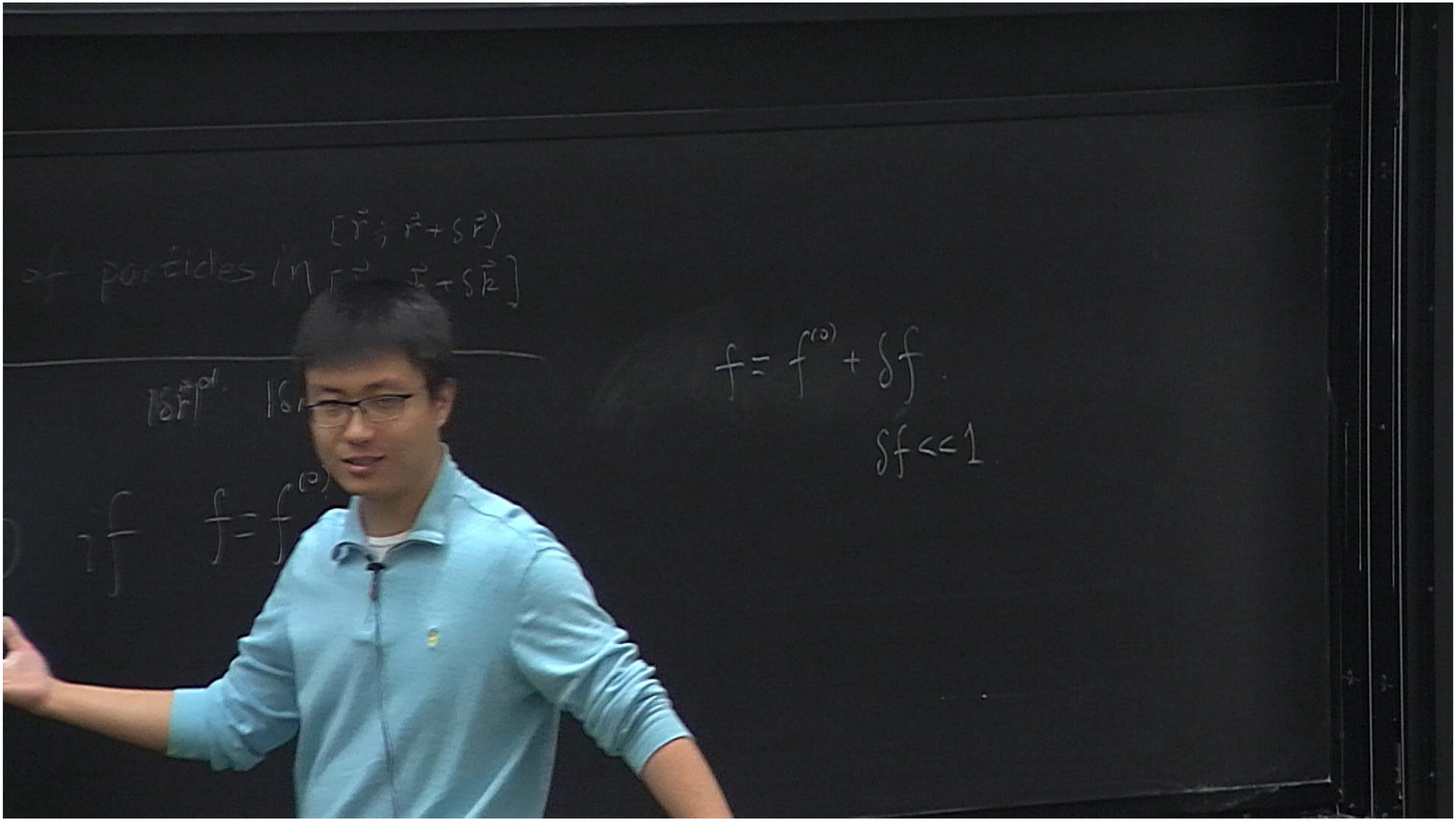
$$\vec{v}_{\vec{k}} = \frac{\partial \xi_{\vec{k}}}{\partial \vec{k}} = \frac{d\vec{r}}{dt}, \quad \vec{F} = e\vec{E} = \frac{d\vec{k}}{dt}$$

Kinetic (Boltzmann) Eq.

$$\boxed{\frac{\partial}{\partial t} f + \boxed{\vec{v}_{\vec{k}} \cdot \nabla_{\vec{r}} f} + \vec{F} \cdot \nabla_{\vec{k}} f + \underbrace{\left(\frac{\partial f}{\partial t}\right)_s}_{\text{collision/scattering term}} = 0}$$

$f(\vec{r}, \vec{k},$

$$\vec{v}_{\vec{k}} = \frac{\partial \xi_{\vec{k}}}{\partial \vec{k}} = \frac{d\vec{r}}{dt}, \quad \vec{F} = e\vec{E} = \frac{d\vec{k}}{dt}$$


of particles in
$f = f^{(0)} + \delta f$
$\delta f \ll 1$

Keep term 1st order in E, δf.

Assume $\vec{E}(\vec{r},t) = \vec{E}(t)$, T uniform

T = temperature

$\nabla_r f = 0$

$e\vec{E}\cdot\nabla_{\vec{k}} f = e\vec{E}\cdot\nabla_k f^{(0)} =$

Keep term 1st order in E, δf.

Assume $\vec{E}(\vec{r},t) = \vec{E}(t) = \vec{E}\, e^{-i\omega t}$, T uniform

T = temperature

$\nabla_r f = 0$

$$e\vec{E}\cdot\nabla_{\vec{k}} f = e\vec{E}\cdot\nabla_k f^{(0)} = e\,\vec{E}\cdot\underbrace{\frac{\partial \xi}{\partial \vec{k}}}_{=\vec{v}_k}\frac{\partial f^{(0)}}{\partial \xi} = e\vec{E}\cdot\vec{v}_k\,\frac{df^{(0)}}{d\xi}$$

$\partial_t f = \partial_t \delta f$

$$\boxed{\delta f(t) = \delta f\, e^{-i\omega t}}$$

$$\left(\frac{\partial f}{\partial t}\right)_c = \frac{\delta f}{\tau}$$

$$\Rightarrow \delta f_{(k)} = \frac{\tau}{1-i\omega\tau} \, \vec{v}_k \cdot e\vec{E} \left(-\frac{df^{(0)}}{d\xi}\right)\Bigg|_{\xi(k)}$$

$$\Rightarrow \delta f_{(k)} = \frac{\tau}{1 - i\omega\tau}\, \vec{v}_k \cdot e\vec{E} \left(- \frac{d f^{(0)}}{d\xi} \right)\Bigg|_{\xi(k)}$$

If $\omega = 0$, $T = 0$

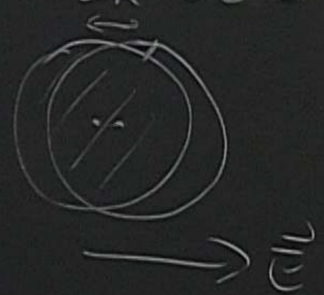
$\Delta k = e\vec{E}\,\tau$

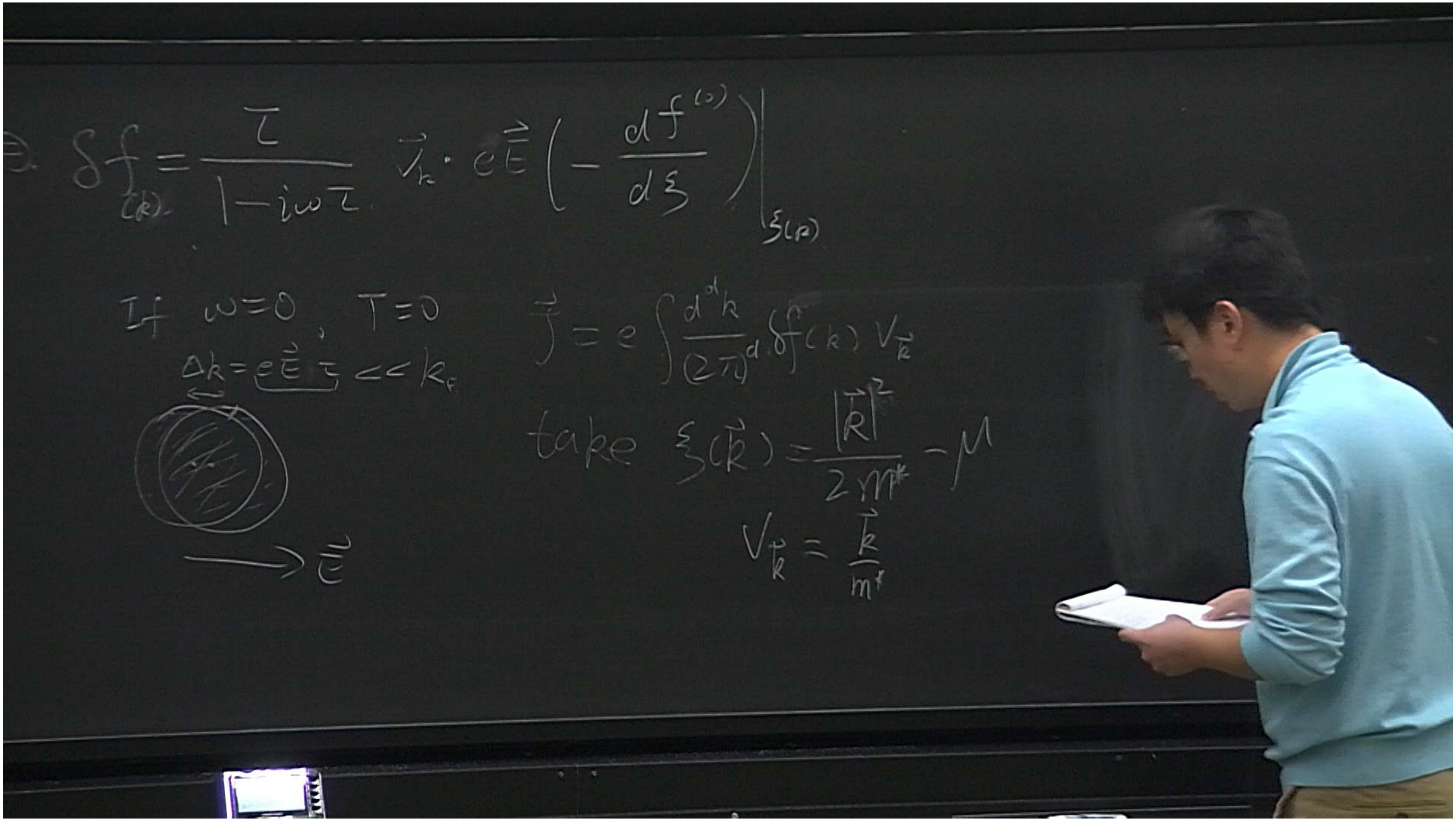
$$\Rightarrow \delta f_{(k)} = \frac{\tau}{1-i\omega\tau}\, \vec{v}_k \cdot e\vec{E} \left(-\frac{df^{(0)}}{d\xi}\right)\Bigg|_{\xi(k)}$$

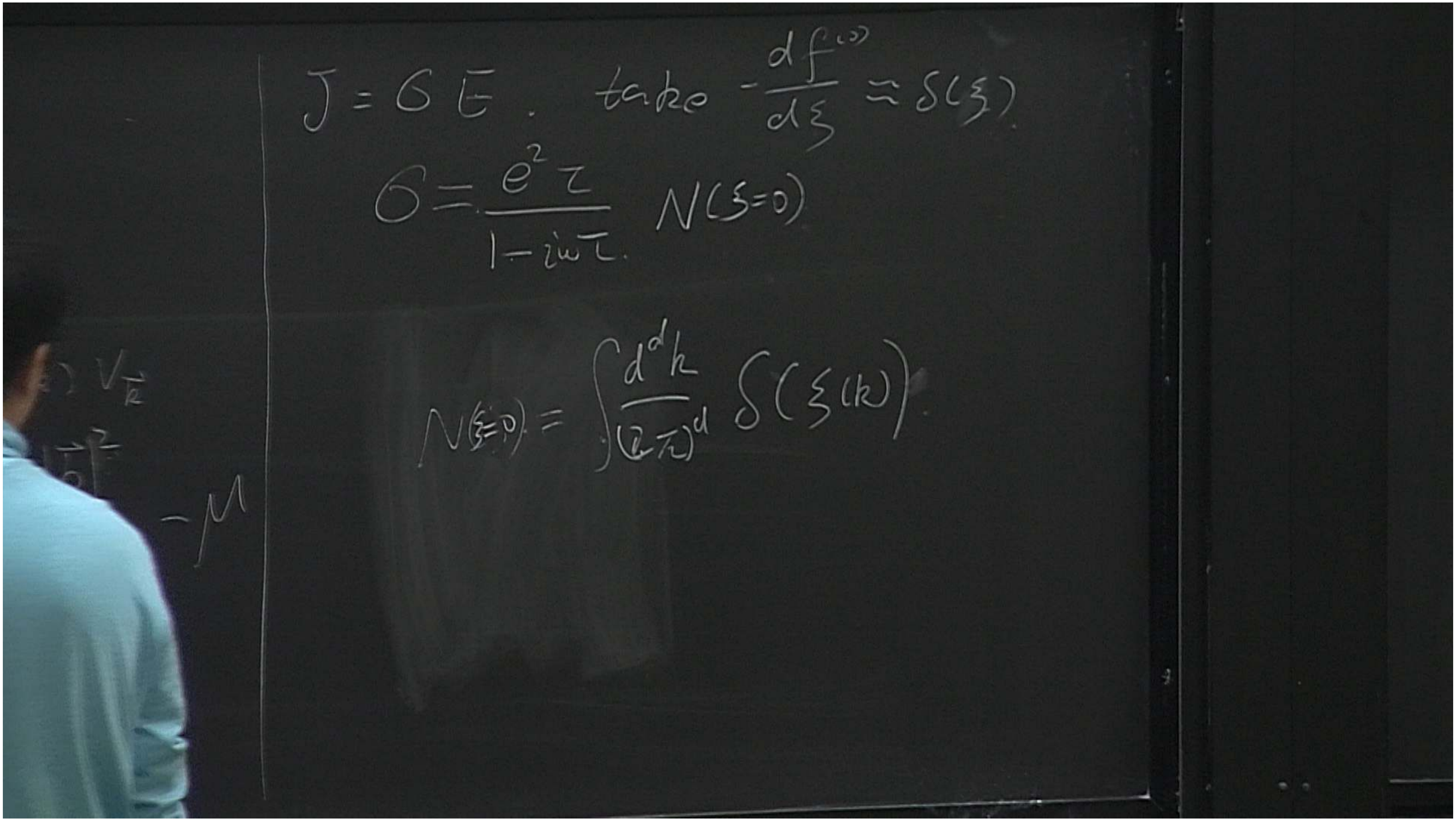
If $\omega=0$, $T=0$

$\Delta k = e\vec{E}\tau$

$$\vec{j} = e\int \frac{d^dk}{(2\pi)^d}\, f(k)\, v_{\vec{k}}$$




$\delta f_{(k)} = \frac{\tau}{1-i\omega\tau} \vec{v}_k \cdot e\vec{E} \left(-\frac{df^{(0)}}{d\xi}\right)\Big|_{\xi(k)}$
If $\omega = 0$, $T = 0$
$\Delta k = e\vec{E}\tau << k_F$
$\vec{E}$
$\vec{j} = e\int \frac{d^dk}{(2\pi)^d} \delta f(k) V_{\vec{k}}$
take $\xi(\vec{k}) = \frac{|\vec{k}|^2}{2m^*} - \mu$
$V_{\vec{k}} = \frac{\vec{k}}{m^*}$


$J = \sigma E$. take $-\frac{df^{(0)}}{d\xi} \simeq \delta(\xi)$
$\sigma = \frac{e^2 \tau}{1 - i\omega\tau} N(\xi=0)$
$N(\xi=0) = \int \frac{d^d k}{(2\pi)^d} \delta(\xi(k))$

$(k)\, V_{\vec{k}}$

$= \frac{|\vec{k}|^2}{2m^*} - \mu$

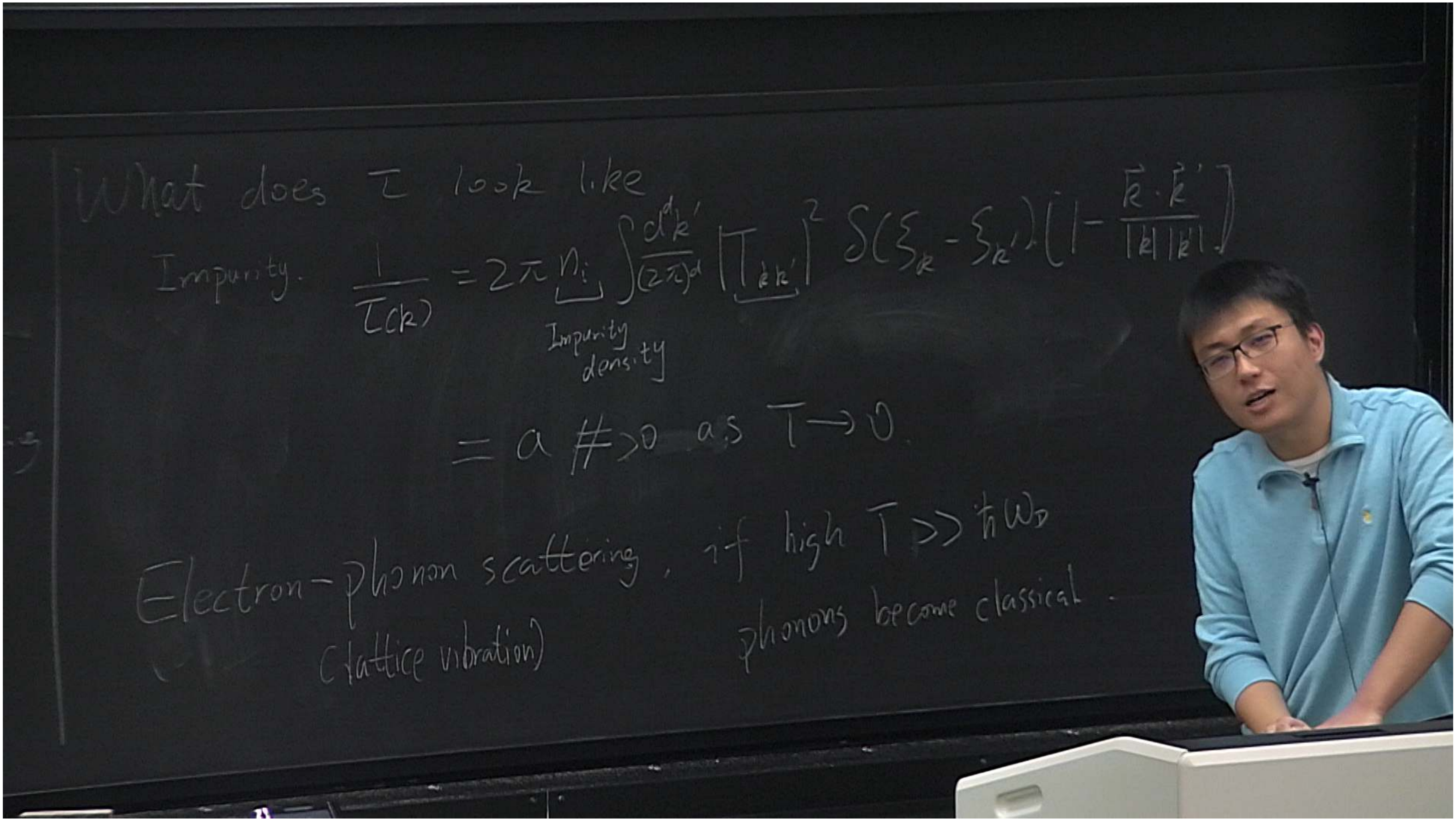
$= \vec{k}$

$J = \sigma E$. take $-\frac{df^{(0)}}{d\xi} \simeq \delta(\xi)$

$$\sigma = \frac{e^2 \tau}{1 - i\omega\tau} N(\xi = 0) \langle V_x^2 \rangle$$

average over Fermi surface

$$= \frac{e^2 n_e \tau}{m^* (1 - i\omega\tau)}$$


What does τ look like
Impurity. $\frac{1}{\tau(k)} = 2\pi \underbrace{n_i}_{\text{Impurity density}} \int \frac{d^dk'}{(2\pi)^d} |\underbrace{T_{kk'}}|^2 \delta(\xi_k - \xi_{k'}) \left(1 - \frac{\vec{k}\cdot\vec{k}'}{|k||k'|}\right)$
$= a \# > 0$ as $T \to 0$
Electron-phonon scattering, if high $T >> \hbar\omega_D$
(lattice vibration)
phonons become classical.

ook like

$$2\pi \underbrace{n_i}_{\text{Impurity density}} \int \frac{d^d k'}{(2\pi)^d} \left|\underbrace{T_{kk'}}\right|^2 \delta(\xi_k - \xi_{k'}) \left(1 - \frac{\vec{k}\cdot\vec{k}'}{|k||k'|}\right)$$

$= a \; \# > 0$ as $T \to 0$.

scattering, if high $T \gg \hbar\omega_D$

phonons become classical

ation)

Equipartition.

$\langle n_{\text{phonon}} \rangle \propto k_B T.$

$\frac{1}{\tau} \propto T.$

Boltzmann Eq.

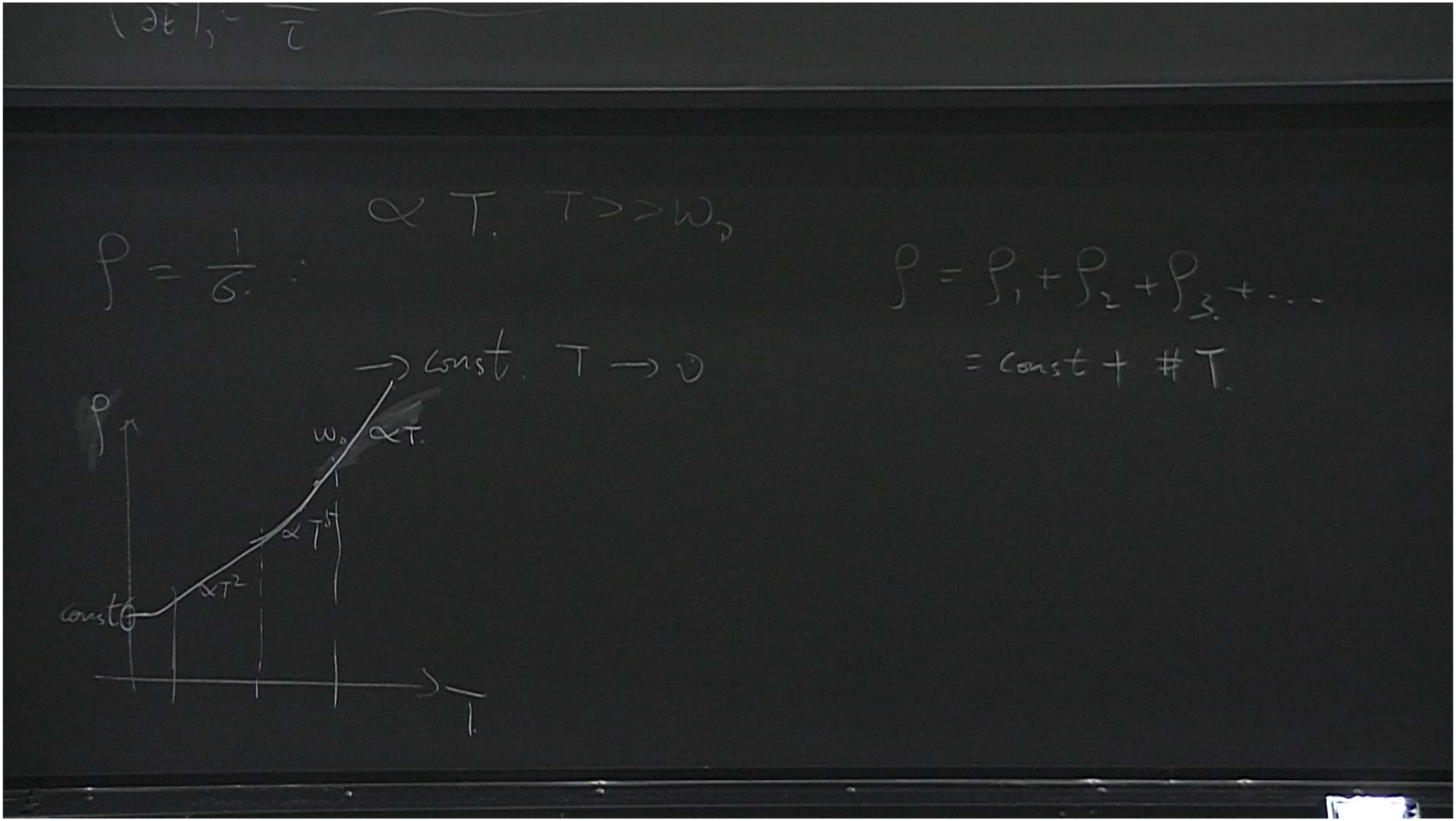
$$\frac{\partial}{\partial t} f + \boxed{\vec{v}_{\vec k} \cdot \nabla_{\vec r} f} + \underline{\vec F \cdot \nabla_{\vec k} f} + \underbrace{\left(\frac{\partial f}{\partial t}\right)_s}_{\text{collision/scattering term}} = 0$$

$$\vec{v}_{\vec k} = \frac{\partial \varepsilon_{\vec k}}{\partial \vec k} = \frac{d\vec r}{dt}, \quad \vec F = e\vec E = \frac{d\vec k}{dt}$$

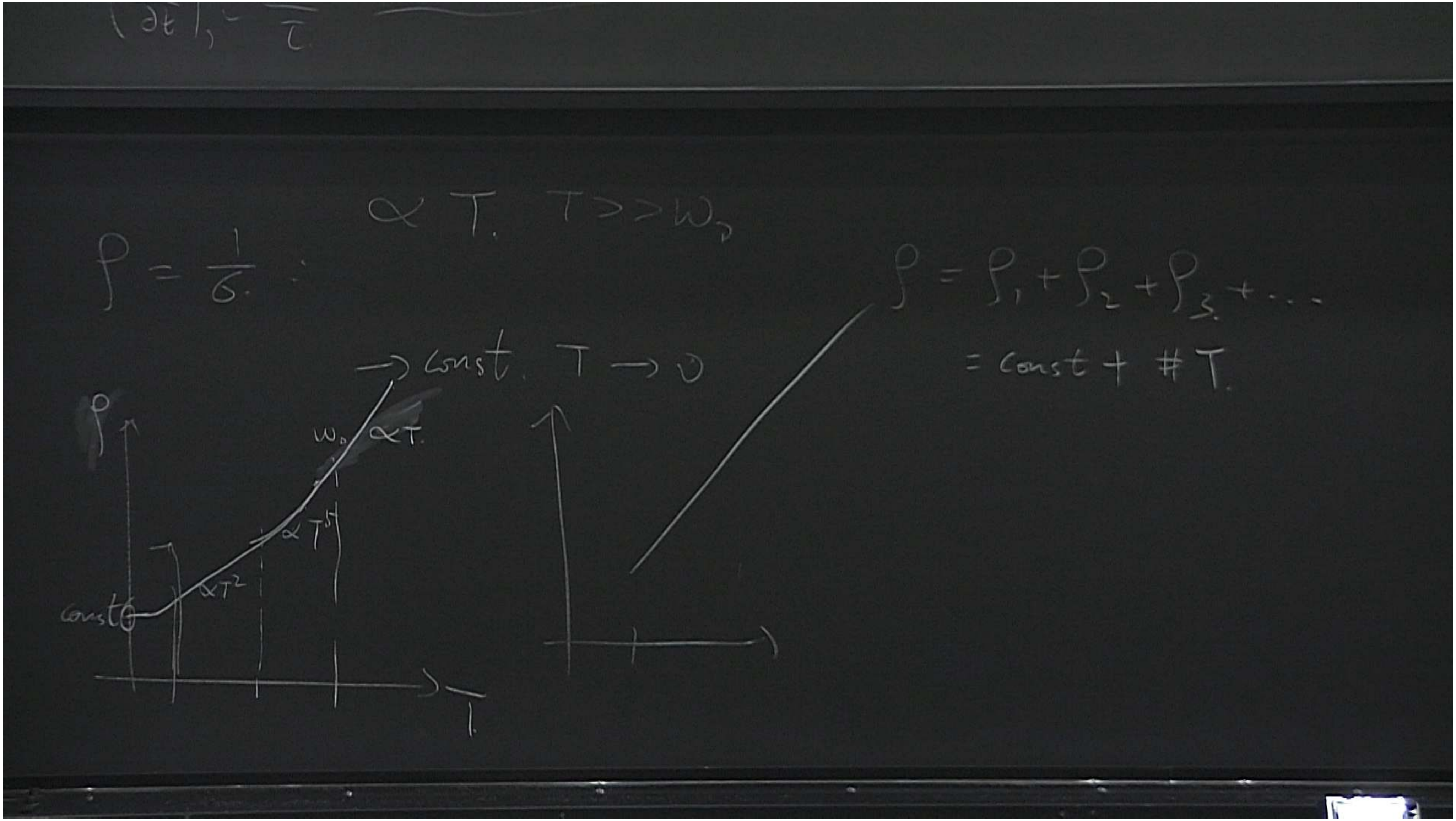
Equipartition.

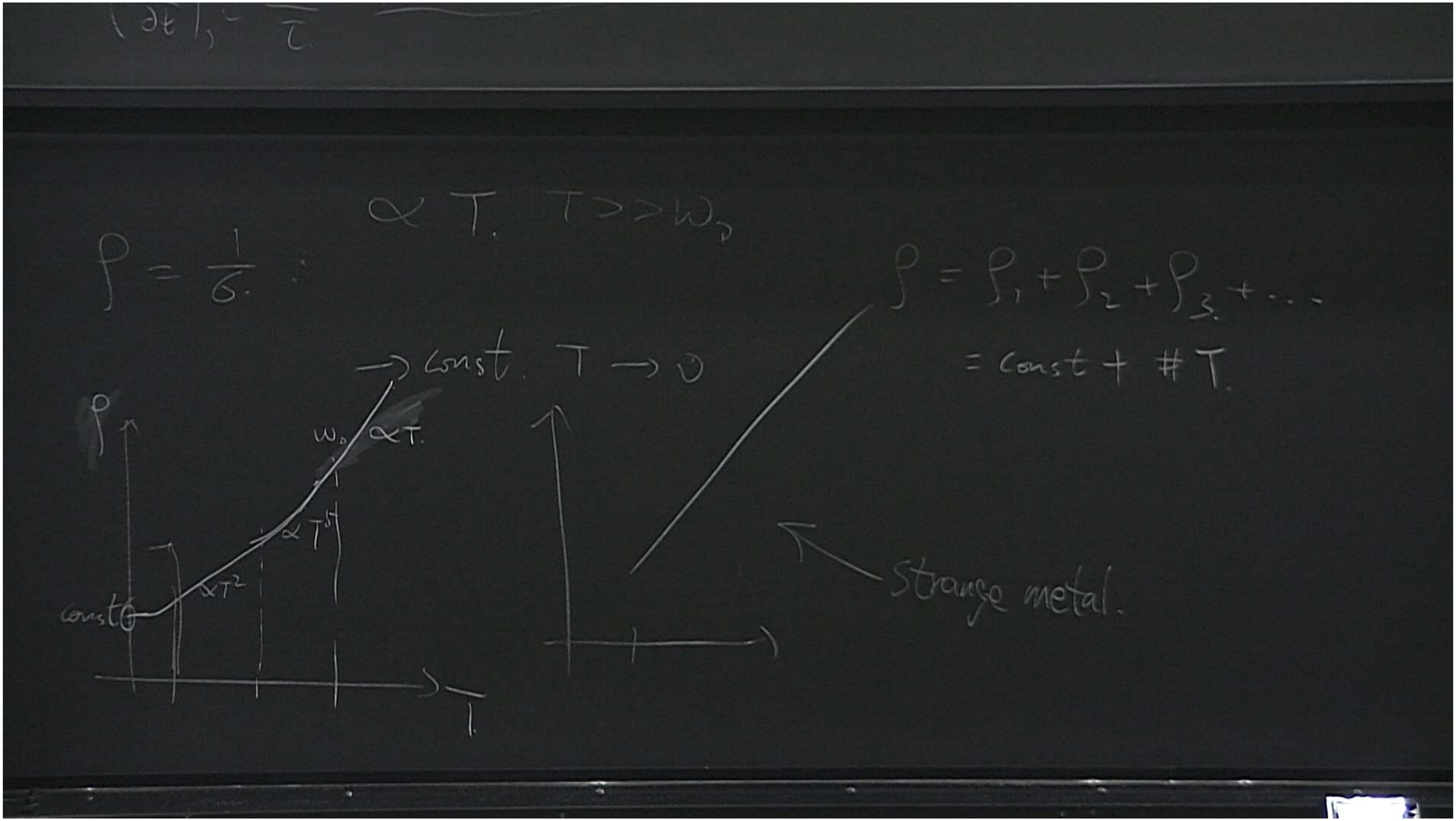
$$\langle n_{phonon} \rangle \propto k_B T.$$

$$\frac{1}{\tau} \propto T$$



$\propto T. \quad T >> \omega_D$
$\rho = \frac{1}{\sigma}$
$\rho = \rho_1 + \rho_2 + \rho_3 + \dots$
$\to const. \quad T \to 0$
$= const + \# T.$
ρ
ω_D
$\propto T.$
$\propto T^5$
$\propto T^2$
const
T


$\rho = \frac{1}{\sigma}$;
$\propto T$. $T >> \omega_D$
$\rho = \rho_1 + \rho_2 + \rho_3 + \dots$
$= \text{Const} + \# T$.
$\to$ Const. $T \to 0$
ρ
Const
$\propto T^2$
$\propto T^5$
ω_D
$\propto T$.
T

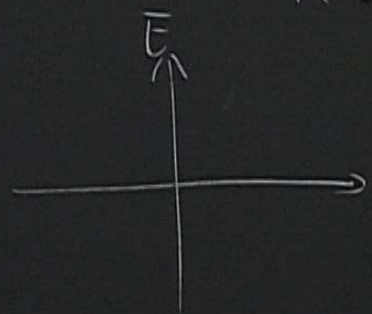


$\rho = \frac{1}{\sigma}$
$\propto T$. $T >> \omega_D$
$\rightarrow$ const. $T \rightarrow 0$
$\rho = \rho_1 + \rho_2 + \rho_3 + \ldots$
$= \text{const} + \# T$.
ρ
ω_D
$\propto T$.
$\propto T^5$
$\propto T^2$
const
T.
Strange metal.

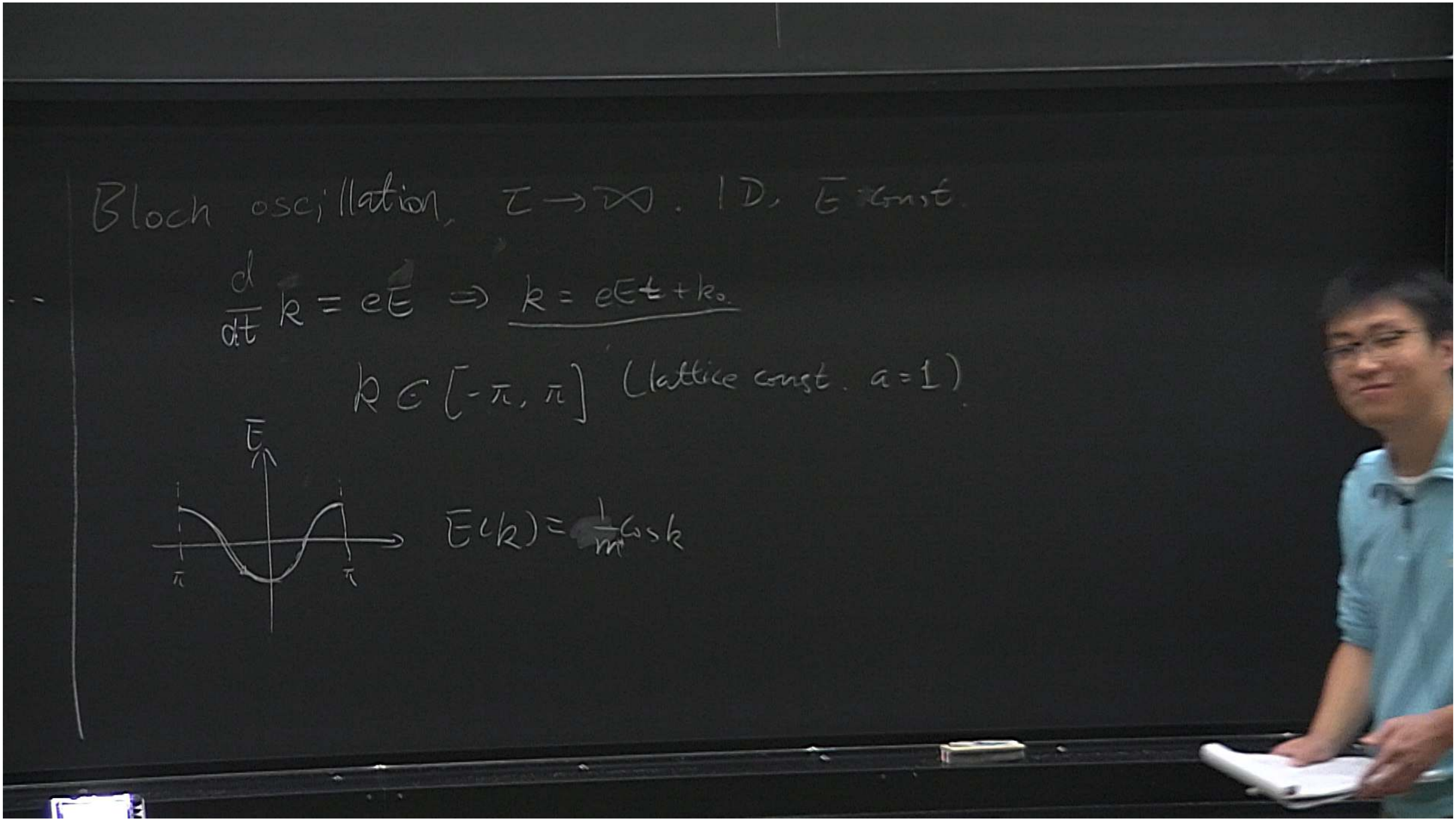
Bloch oscillation, $\tau \to \infty$. 1D, E const.

$$\frac{d}{dt} k = eE \Rightarrow k = eEt + k_0$$

$k \in [-\pi, \pi]$ (lattice const. $a = 1$)



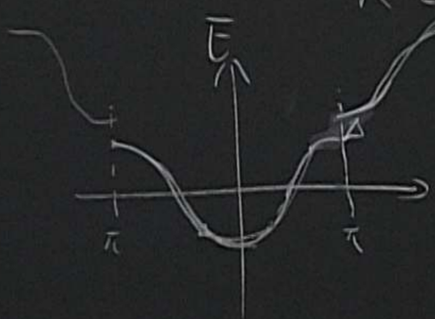
$E(k) = \frac{1}{m} \cos k$


Bloch oscillation, τ → ∞ . 1D, E const.
d/dt k = eE ⇒ k = eEt + k₀
k ∈ [-π, π] (lattice const. a = 1)
E
π
π
E(k) = 1/m cos k

Bloch oscillation, $\tau \to \infty$. 1D, E const.

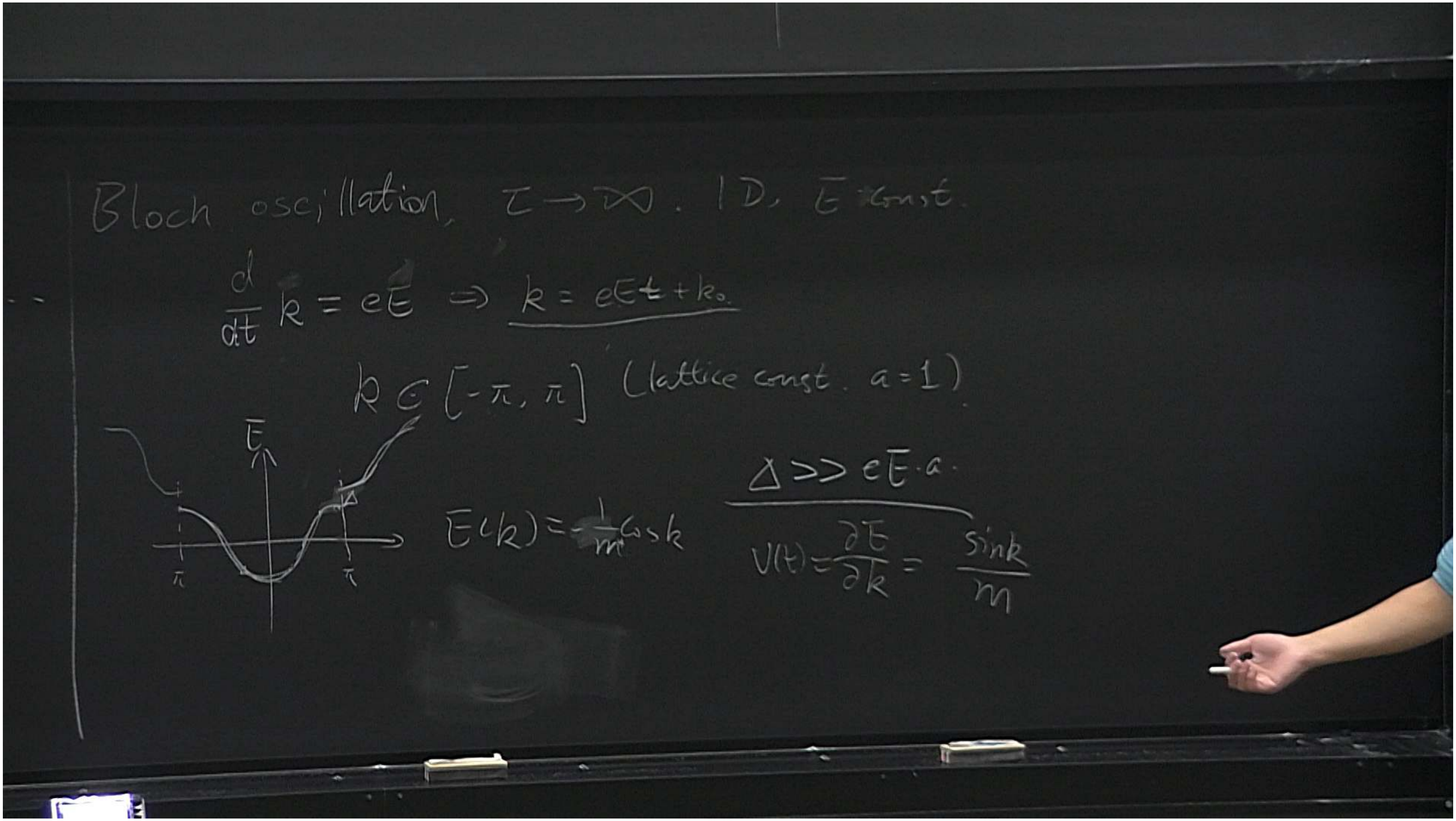
$$\frac{d}{dt} k = eE \Rightarrow \underline{k = eEt + k_0}$$

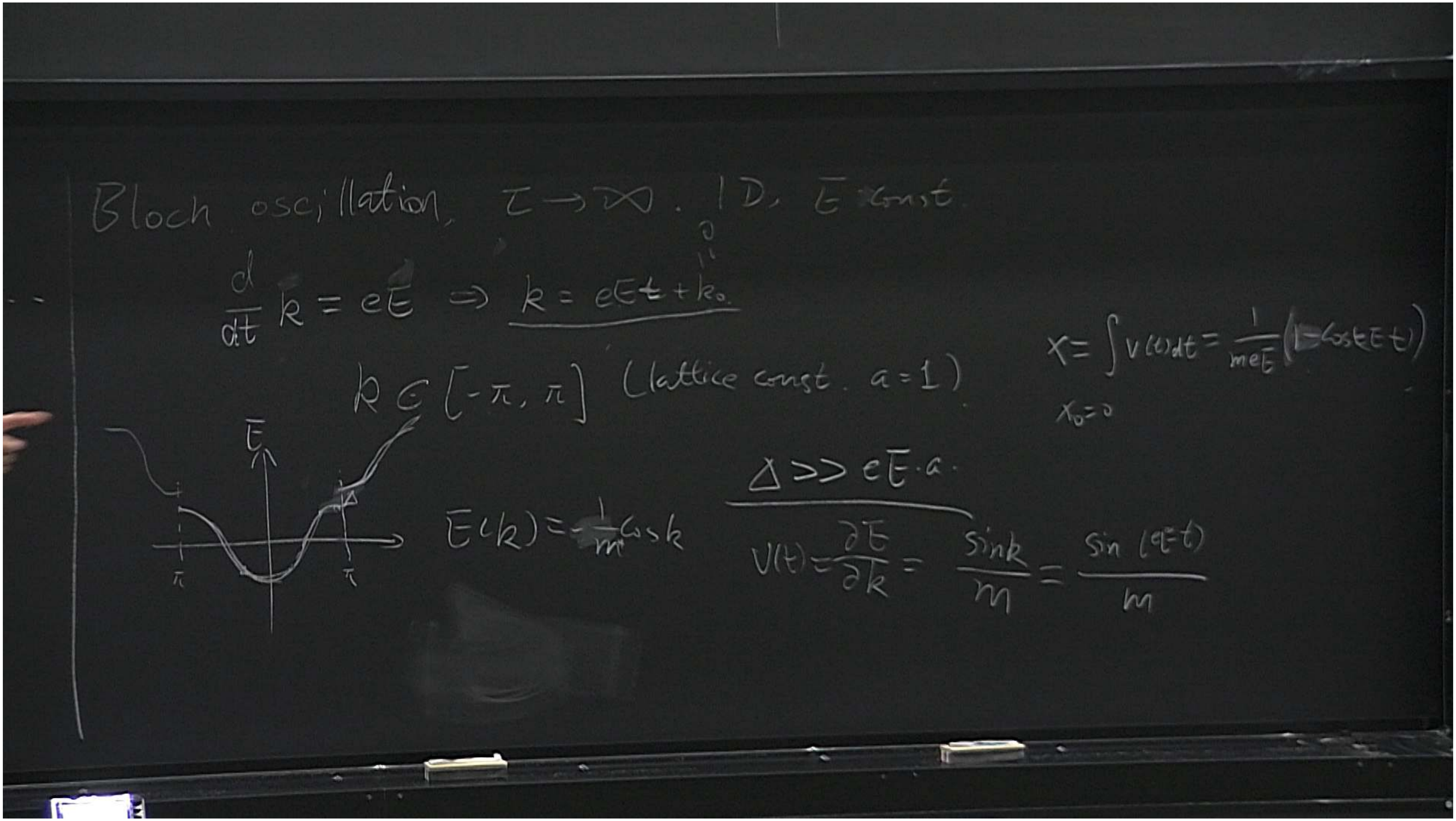
$$k \in [-\pi, \pi] \quad \text{(lattice const. } a = 1\text{)}$$

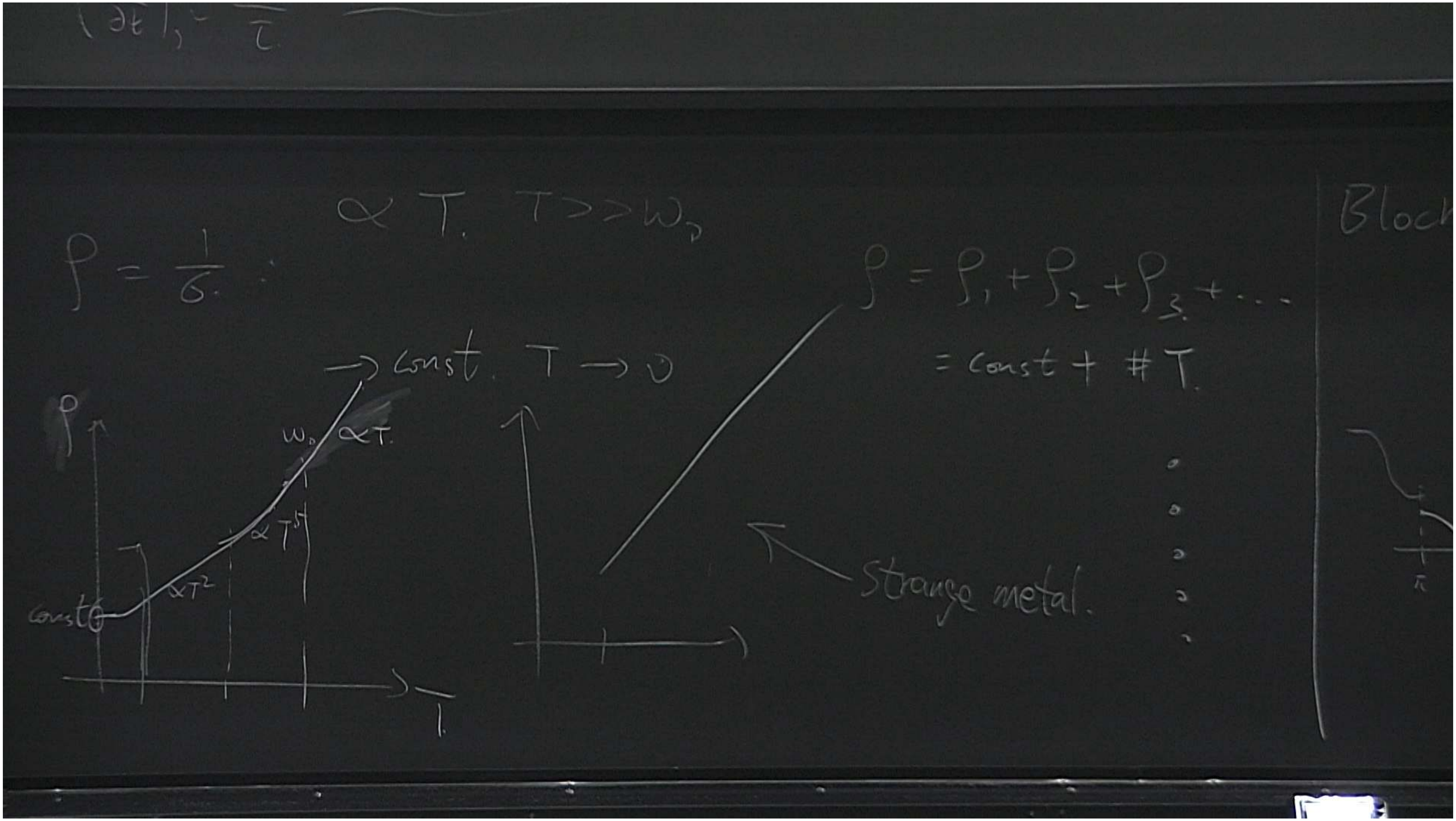


$$E(k) = \frac{1}{m}\cos k$$

$$\Delta \gg eE \cdot a.$$


Bloch oscillation, $\tau \to \infty$. 1D, E const.
$\frac{d}{dt} k = eE \Rightarrow k = eEt + k_0$
$k \in [-\pi, \pi]$ (lattice const. $a = 1$)
E
Δ
π
π
$E(k) = -\frac{1}{m} \cos k$
$\Delta >> eE \cdot a$.
$V(t) = \frac{\partial E}{\partial k} = \frac{\sin k}{m}$


Bloch oscillation, $\tau \to \infty$. 1D, E const
$\frac{d}{dt} k = eE \Rightarrow k = eEt + k_0$
$k \in [-\pi, \pi]$ (lattice const. $a=1$)
$x = \int v(t) dt = \frac{1}{meE}(1 - \cos eEt)$
$x_0 = 0$
$\Delta >> eE \cdot a$
$E(k) = -\frac{1}{m} \cos k$
$v(t) = \frac{\partial E}{\partial k} = \frac{\sin k}{m} = \frac{\sin(eEt)}{m}$


ρ = 1/σ
∝ T. T>>ω_D
→ const. T → 0
ρ = ρ_1 + ρ_2 + ρ_3 + ...
= const + # T.
ρ
ω_D
∝ T.
∝ T^5
∝ T^2
const
T
Strange metal.
Bloch