Title: PSI 2019/2020 - Condensed Matter (Wang) - Lecture 3

Speakers: Chong Wang

Collection: PSI 2019/2020 - Condensed Matter (Wang)

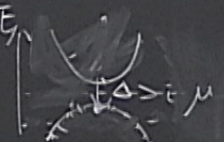
Date: November 14, 2019 - 10:45 AM

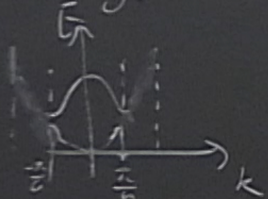
URL: http://pirsa.org/19110027

Today: More on Band Theory.

- General comments: Insulator vs. Metal vs. semi-metal.
- Stability and instability.
  1. Density waves: Metal $\longrightarrow$ Insulator
  2. Graphene revisited.

A crude classification of phase of matter at low Temperature.
according to how many excitations. (D.O.F)

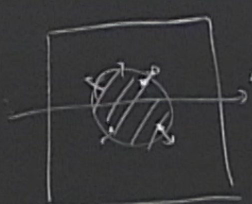
1) Nothing. Insulator. Gapped.  $T \ll \Delta$. $e^{-\frac{\Delta}{T}}$, $C = \frac{dE}{dT} \sim e^{-\frac{\Delta}{T}}$



$E-\mu$

$$N(E) \equiv \frac{\#\text{ of states with excitation energy between } E \sim E+dE}{\text{Volume} \times dE}$$

$= 0 \quad E < \Delta$

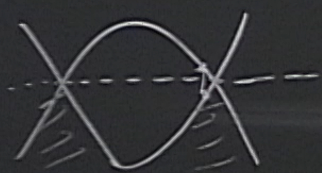
2). Lots of things. Metal.  $\Delta \sim O(\frac{1}{L})$

$N(E) = \underset{>0}{\text{constant}} + O(E)$.

$C \sim T$

3). Something. (Semimetal, quantum phase transitions)

(Graphene)



$\Delta \sim O(\frac{1}{L})$

$N(E) \propto E + O(E^2)$

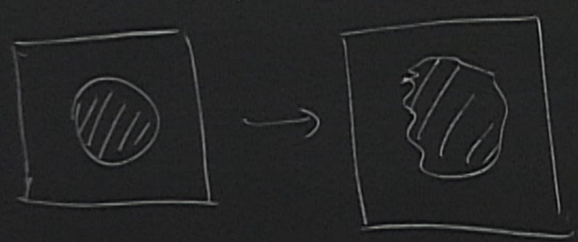
Vanishing, but only power law.

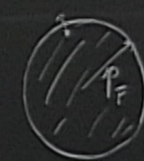
$C \sim T^2$

How robust are these?

1). Gapped. $\Delta > 0$. Some error in model. $\Delta \to \Delta + \delta\Delta$. $\Rightarrow$ Robust

2). Metal. if translation symmetry is preserved

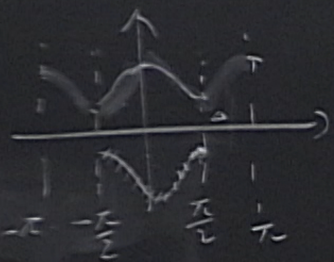




$$H = \frac{p^2}{2m} - \mu$$

$$n_p = \langle c_p^+ c_p \rangle = \begin{cases} 1 & \text{if } \frac{p^2}{2m} - \mu < 0 \\ 0 & \text{if } \cdots > 0 \end{cases}$$

$$p_F = \sqrt{2m\mu}$$



$\Delta \sim \delta$

Original translation is broken, $\mathbb{Z} \longrightarrow 2\mathbb{Z}$

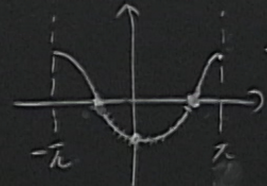
New unit cell. $\quad \tilde{a} = 2a, \quad a \equiv 1$

New reciprocal lattice vector. $\tilde{G} = \frac{2\pi}{\tilde{a}} = \frac{\pi}{a} = \pi$.

$$\boxed{k \sim k+\pi}$$

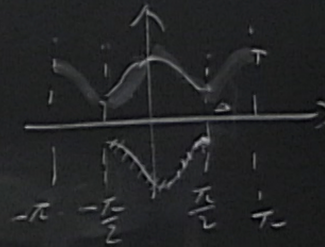
Ex. Density wave (translation breaking) $e^{iq\cdot x}$

1D:



$\} \; 4t \equiv W$.
Band width

$\boxed{e^{i\pi\cdot i}}\, c_i^\dagger c_i \longrightarrow$

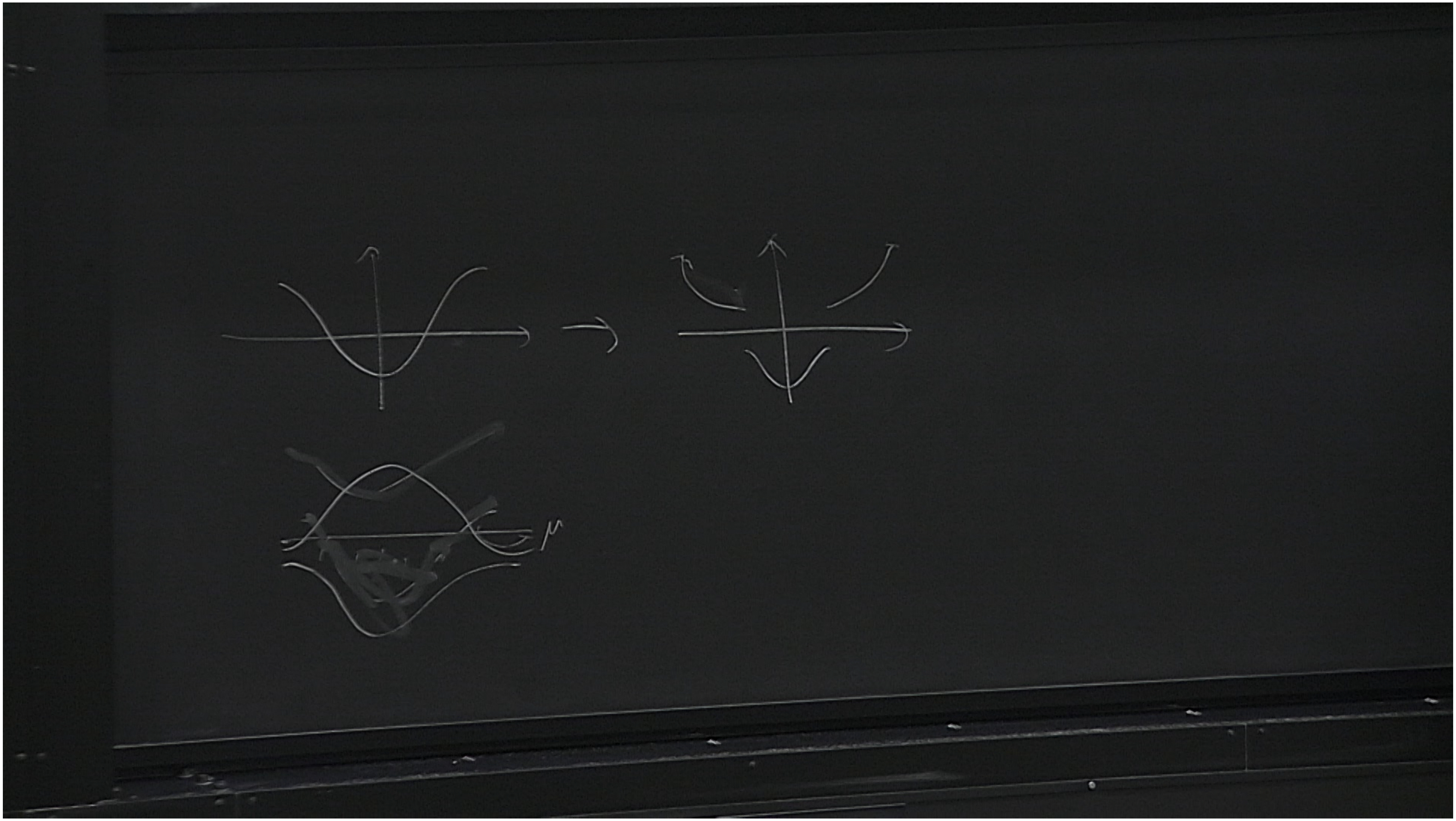


$$H = -t\sum_{i=1}^{N}\left(c_i^\dagger c_{i+1} + c_{i+1}^\dagger c_i\right) - \delta\sum_{j=1}^{N/2}\left(c_{2j}^\dagger c_{2j} - c_{2j+1}^\dagger c_{2j+1}\right)$$

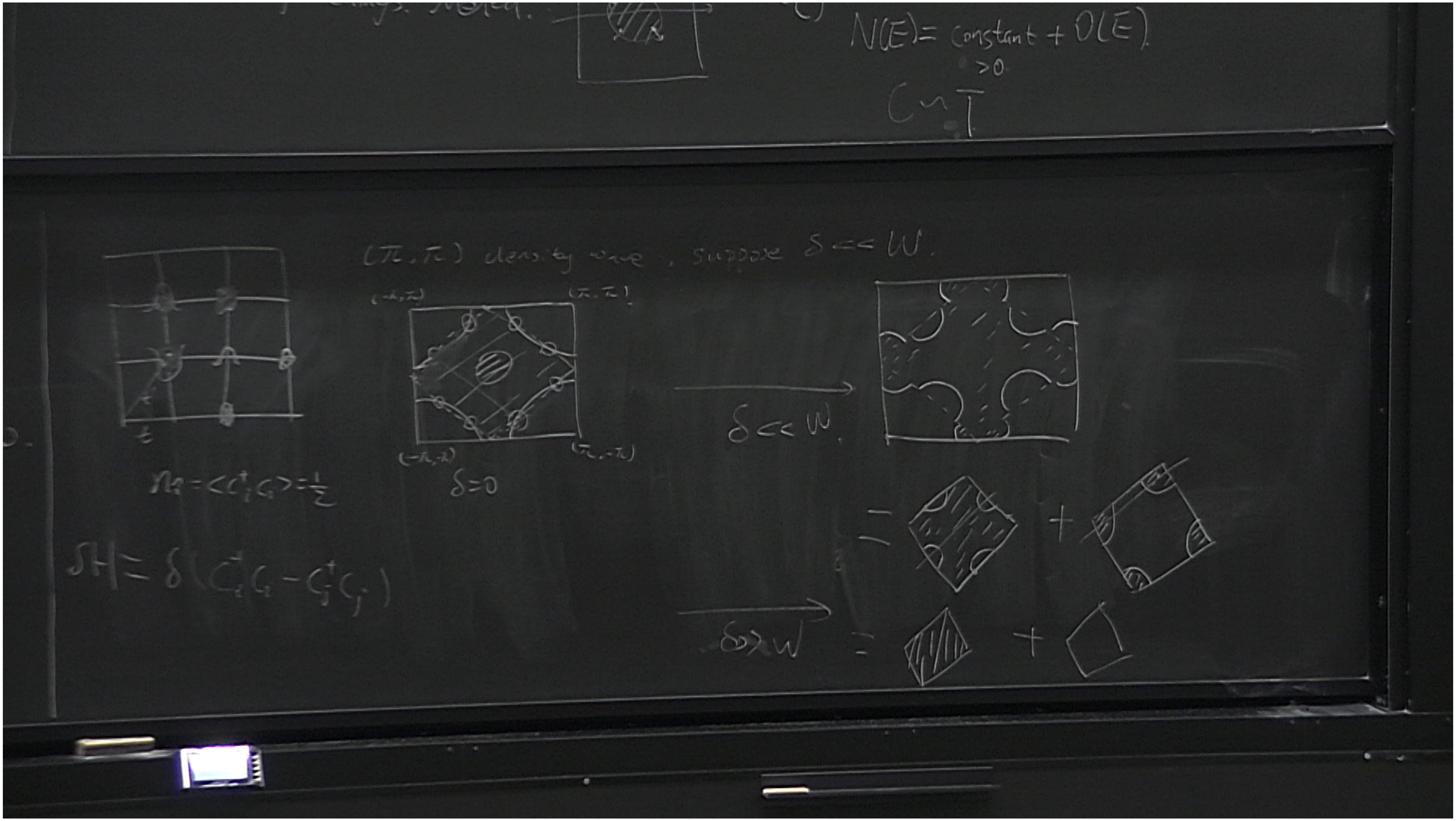
$$T \ll \Delta \sim \delta$$

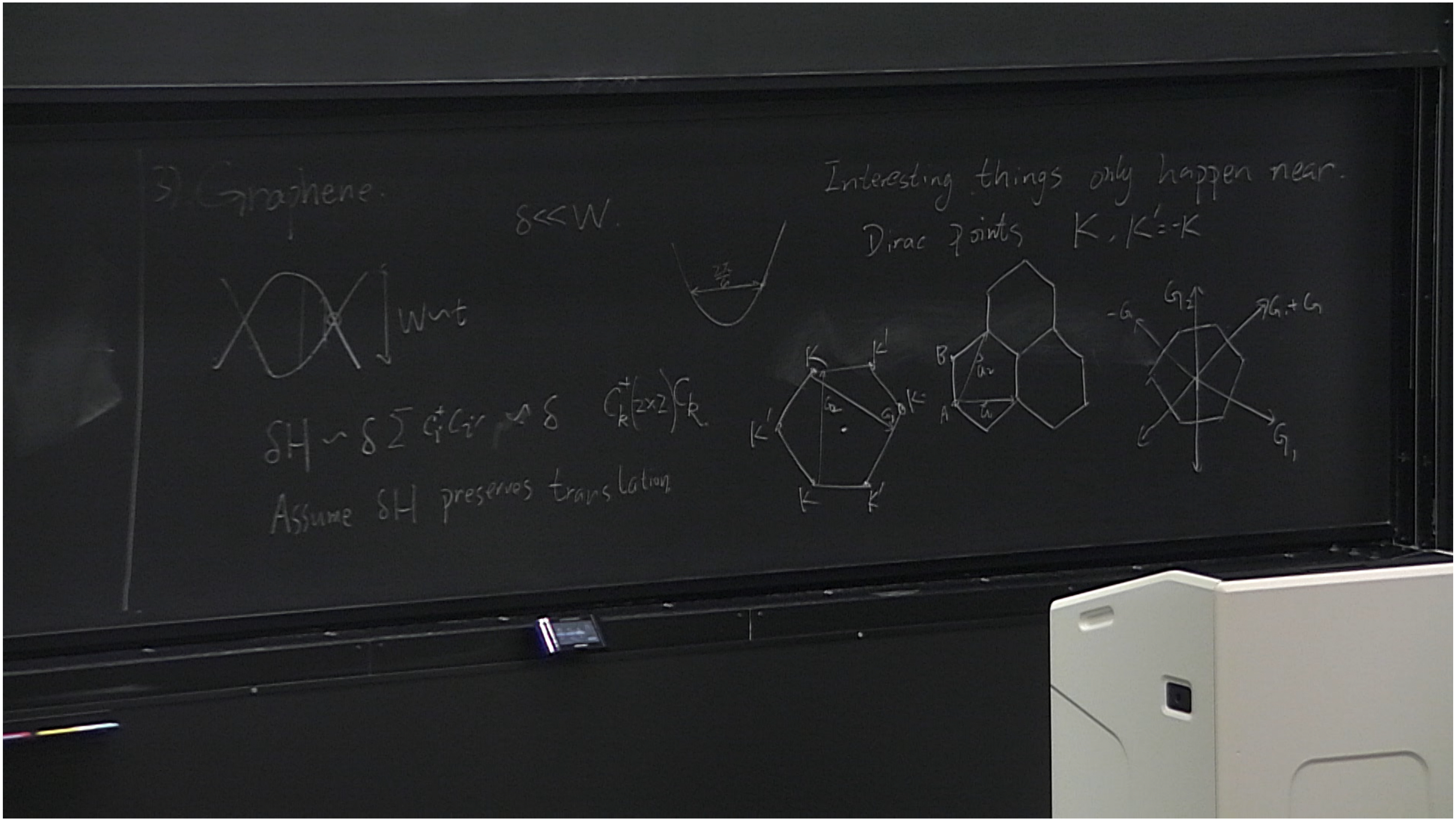
$$= -2t\sum_{k=-\pi}^{\pi}\cos k\, c_k^\dagger c_k - \delta\sum_k c_{k+\pi}^\dagger c_k$$

Suppose $\boxed{\delta \ll W \sim t}$



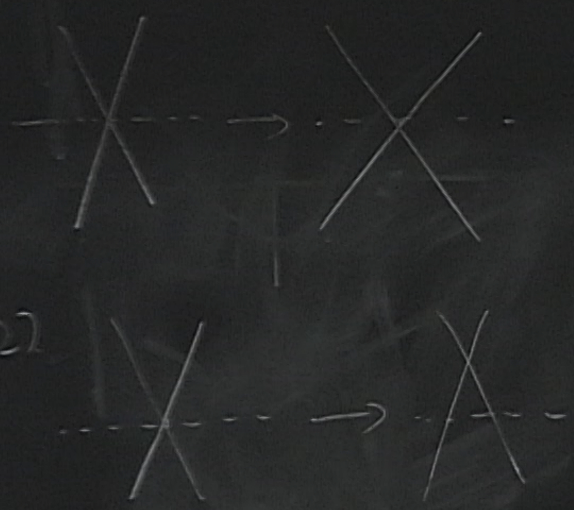
μ


N(E) = constant + O(E)
>0
C ~ T
(π, π) density wave , suppose δ << W.
(π, π)
(−π, −π)
(π, −π)
δ=0
δ<<W
n_i = <c_i† c_i> = 1/2
δH = δ(c_i† c_i − c_j† c_j)
δ>>W


3) Graphene.
δ<<W.
W~t
δH ~ δ Σ c_i^† c_i' ~ δ   c_k^† (2×2) c_k.
Assume δH preserves translation
Interesting things only happen near.
Dirac points K, K' = -K
K
K'
B
A
a_1
a_2
G_2
-G_1
G_1+G_2
G_1

(1)

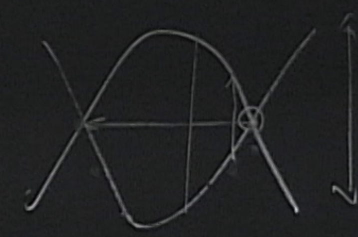
(2)



e.g.
Second-neighbor hopping

$$E(k)=\pm V\cdot\sqrt{\delta k_x^2+\delta k_y^2}+\delta$$

3) Graphene.



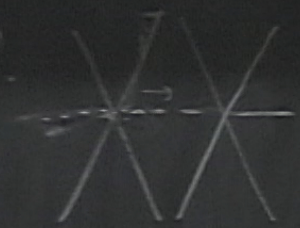
$\delta H \sim \delta \sum c_i^{\dagger} c_{i'}$

Assume δH preserve

$N(E) = \text{constant} + \nu(E).$

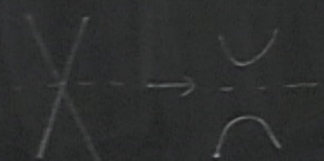
$>0.$

$C_n T$

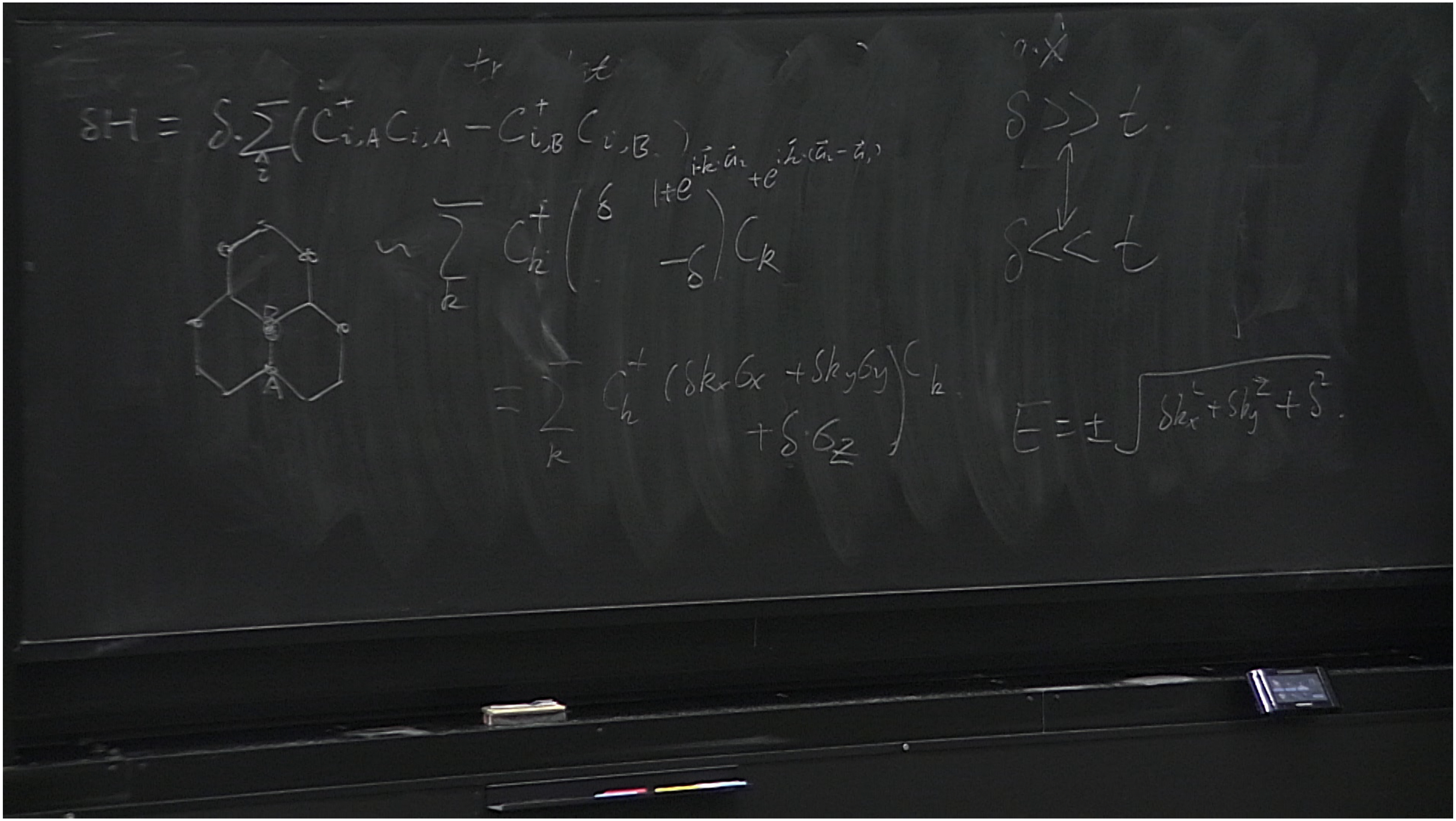
(3).  Dirac cone move away from $K \to K + \delta K$

Not possible if δH preserves C_3 rotation.

K & $K' = -K$ are invariant under C_3

(4)  Not possible if δH preserves C_2 & Time-reversal Symmetries.

$C_\xi \to C_\xi,\ i \to -i$


$\delta H = \delta \cdot \sum_i (C^\dagger_{i,A} C_{i,A} - C^\dagger_{i,B} C_{i,B})$
$\sim \sum_k C^\dagger_k \begin{pmatrix} \delta & 1+e^{i\vec{k}\cdot\vec{a}_2}+e^{i\vec{k}\cdot(\vec{a}_2-\vec{a}_1)} \\ & -\delta \end{pmatrix} C_k$
$= \sum_k C^\dagger_k (\delta k_x \sigma_x + \delta k_y \sigma_y + \delta \cdot \sigma_z) C_k$
$\delta \gg t$
$\delta \ll t$
$E = \pm\sqrt{\delta k_x^2 + \delta k_y^2 + \delta^2}$
B
A