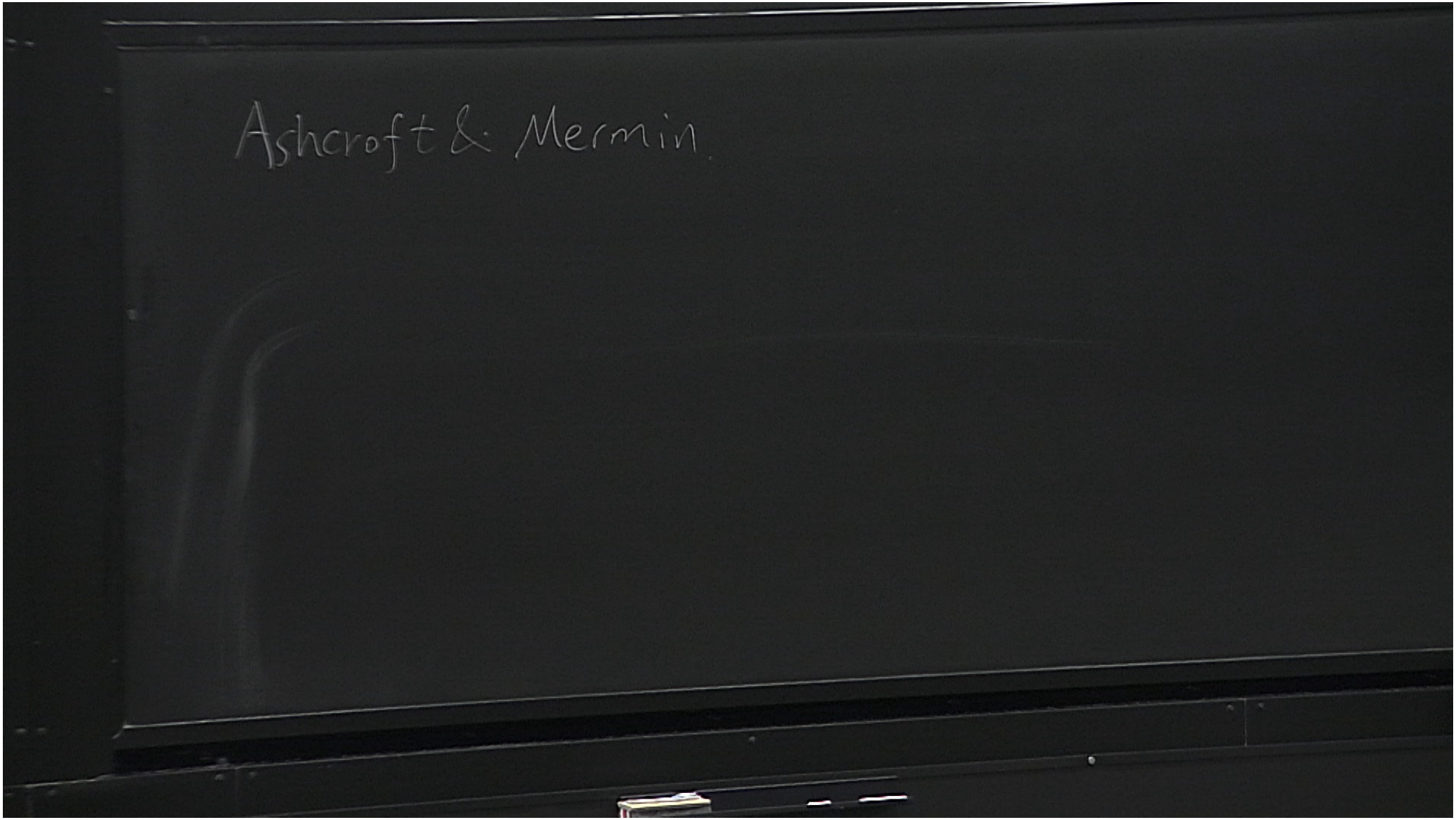
Title: PSI 2019/2020 - Condensed Matter (Wang) - Lecture 2

Speakers: Chong Wang

Collection: PSI 2019/2020 - Condensed Matter (Wang)

Date: November 12, 2019 - 10:45 AM

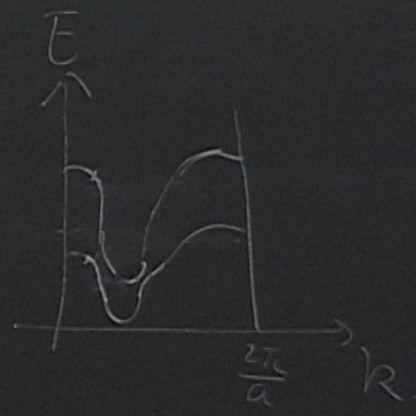
URL: http://pirsa.org/19110025


Ashcroft & Mermin

$\psi_n(k)$

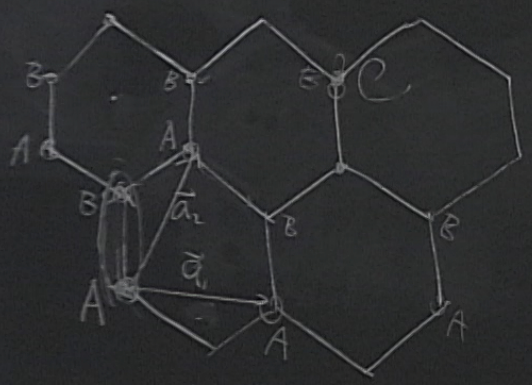
$k \in [0, \frac{2\pi}{a})$: continuous

n : discrete.



Graphene

C_{Ai}, C_{Bi}



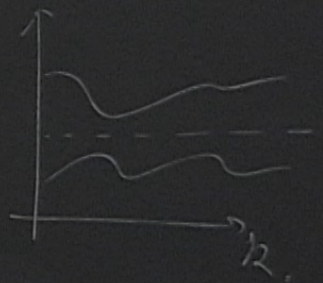
1 electron orble per site.

1 electron, but 2-spin states.

Half-filled

$$\langle C_{\uparrow}^{\dagger} C_{\uparrow} + C_{\downarrow}^{\dagger} C_{\downarrow} \rangle = 1$$

$$\langle C_{\uparrow}^{\dagger} C_{\uparrow} \rangle = \frac{1}{2}$$



$$\vec{x} = n_1 \vec{a}_1 + n_2 \vec{a}_2$$

$$c_{\vec{n},A}, \quad c_{\vec{n},B}$$



$$H = -\sum_{\langle i,j \rangle} t \, c_i^\dagger c_j + h.c.$$

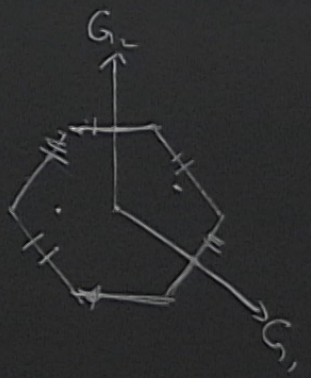
$$= -t \sum_{\vec{n}} c^\dagger_{\vec{n},A} c_{\vec{n},B} + c^\dagger_{\vec{n},B} c_{\vec{n},A} + c^\dagger_{\vec{n}+(0,1),A} c_{\vec{n},B} + c^\dagger_{\vec{n}+(-1,1),B} c_{\vec{n},A} + c^\dagger_{\vec{n}+(1,-1),A} c_{\vec{n},B} + c^\dagger_{\vec{n}+(0,-1),B} c_{\vec{n},A}$$

$$C_{\vec{n},S=A/B} = \frac{1}{\sqrt{N_1 N_2}} \sum_{\vec{k}} e^{-i\vec{k}\cdot(n_1\vec{a}_1 + n_2\vec{a}_2)} C_{\vec{k},S}$$

$$C_{\vec{k}} = \begin{pmatrix} C_{\vec{k},A} \\ C_{\vec{k},B} \end{pmatrix}$$

$$\vec{k} \sim \vec{k} + \vec{G}_i \,, \quad \vec{G}_i \cdot \vec{a}_j = 2\pi\delta_{ij}$$

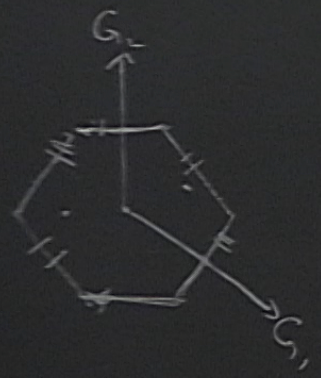
$$\vec{G}_i = \frac{\epsilon_{ij}\,\vec{a}_j \times \hat{z}}{(\vec{a}_1 \times \vec{a}_2)\cdot\hat{z}}$$



$H =$

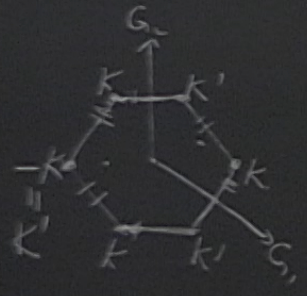
$$C_{\vec{k}} = \begin{pmatrix} C_{\vec{k},A} \\ C_{\vec{k},B} \end{pmatrix}$$

$$H = -t \sum_{\vec{k}} C_{\vec{k}}^{+} \begin{pmatrix} 0 & 1 + e^{i\vec{k}\cdot\vec{a}_2} + e^{i\vec{k}\cdot(\vec{a}_2 - \vec{a}_1)} \\ c.c. & 0 \end{pmatrix} C_{\vec{k}}$$



$$C_{\vec{k}} = \begin{pmatrix} C_{\vec{k},A} \\ C_{\vec{k},B} \end{pmatrix}$$

$$H = -t \sum_{\vec{k}} C_{\vec{k}}^{\dagger} \begin{pmatrix} 0 & 1 + e^{i\vec{k}\cdot\vec{a}_2} + e^{i\vec{k}\cdot(\vec{a}_2 - \vec{a}_1)} \\ c.c. & 0 \end{pmatrix} C_{\vec{k}}$$



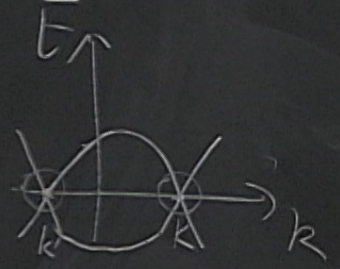
$$\begin{pmatrix} 0 & a \\ a^* & 0 \end{pmatrix} \quad \lambda = \pm |a| \qquad E(k) = 0 \text{ iff } \vec{k}\cdot\vec{a}_2 = -\vec{k}\cdot(\vec{a}_2 - \vec{a}_1) = \pm\frac{2\pi}{3} \mod 2\pi$$

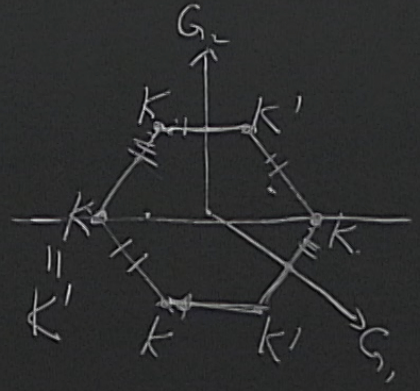
$$\sum_{\vec{k}} e^{-i\vec{k}\cdot(n_1\vec{a}_1+n_2\vec{a}_2)} C_{\vec{k},S}$$

$$C_k = \begin{pmatrix} C_{k,A} \\ C_{k,B} \end{pmatrix}$$

$$\vec{G}_i \cdot \vec{a}_j = 2\pi\delta_{ij}$$

$$= \frac{\epsilon_{ij}\,\vec{a}_j \times \hat{z}}{(\vec{a}_1 \times \vec{a}_2)\cdot\hat{z}}$$





$$H = -t\sum_{\vec{k}} C^{+}_{\vec{k}}$$

$$\begin{pmatrix} 0 & a \\ a^* & 0 \end{pmatrix}$$

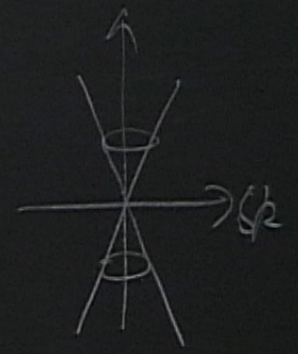
$$c_{\vec{k}} = \begin{pmatrix} c_{\vec{k},A} \\ c_{\vec{k},B} \end{pmatrix}$$

$$H = -t \sum_{\vec{k}} c^{+}_{\vec{k}} \begin{pmatrix} 0 & 1 + e^{i\vec{k}\cdot\vec{a}_2} + e^{i\vec{k}\cdot(\vec{a}_2 - \vec{a}_1)} \\ c.c. & 0 \end{pmatrix} c_{\vec{k}},$$



$$\begin{pmatrix} 0 & a \\ a^* & 0 \end{pmatrix} \quad \lambda_\pm = \pm |a| \qquad E(k) = 0 \text{ iff } \vec{k}\cdot\vec{a}_2 = -\vec{k}\cdot(\vec{a}_2 - \vec{a}_1) = \pm\frac{2\pi}{3} \text{ mod } 2\pi$$

$$E = \pm \frac{\sqrt{3}}{2} t \sqrt{\delta k_x^2 + \delta k_y^2} = \pm \frac{\sqrt{3}}{2} t |\delta k|$$



$\vec{k} = \pm K + \delta\vec{k}$

$$H \simeq \overbrace{\frac{\sqrt{3}}{2}t}^{v_D} \begin{pmatrix} 0 & \pm\delta k_x - i\delta k_y \\ \pm\delta k_x + i\delta k_y & 0 \end{pmatrix}$$

$E = \pm$

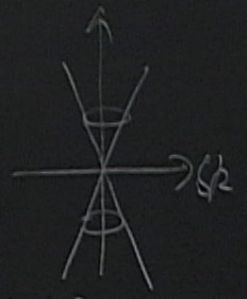
$$H_{\delta k} = v_D(\delta k_x \tau_z \sigma_x + \delta k_y \sigma_y)$$

Consider

valley.

Define "valley" index $\tau = \pm$ for modes near $\pm K$

$$E = \pm \frac{\sqrt{3}}{2} t \sqrt{\delta k_x^2 + \delta k_y^2} = \pm \frac{\sqrt{3}}{2} t \, |\delta k|$$



Consider: $\begin{pmatrix} \psi_+ \\ \psi_- \end{pmatrix}(\vec{x}) \equiv \int_{|\vec{k}| << \Lambda} \frac{d^2k}{(2\pi)^2} e^{i \vec{k} \cdot \vec{x}} \begin{pmatrix} C(K+k) \\ C(-K+k) \end{pmatrix}$.

$\Lambda << |\vec{G}|$

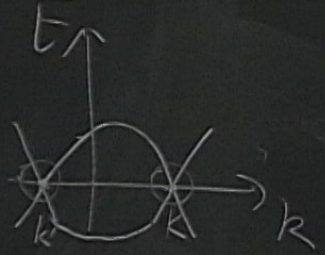
ear.
$\pm K$

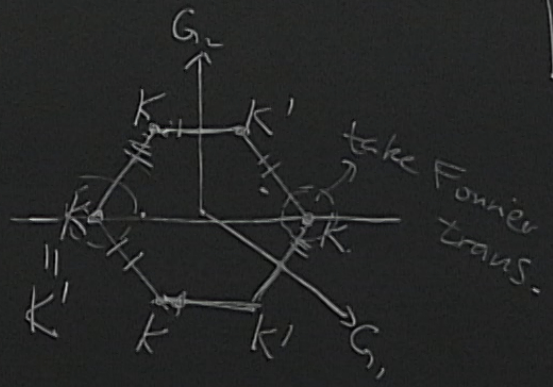
$$\frac{1}{\sqrt{N_1 N_2}} \sum_{\vec{k}} e^{-i\vec{k}\cdot(n_1\vec{a}_1 + n_2\vec{a}_2)} C_{\vec{k},S}$$

$$C_k = \begin{pmatrix} C_{k,A} \\ C_{k,B} \end{pmatrix}$$

$$\vec{k} + \vec{G}_i \,,\quad \vec{G}_i \cdot \vec{a}_j = 2\pi\delta_{ij}$$

$$\vec{G}_i = \frac{\epsilon_{ij}\,\vec{a}_j \times \hat{z}}{(\vec{a}_1 \times \vec{a}_2)\cdot\hat{z}}$$

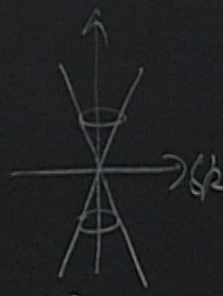




$$H = -t \sum_{\vec{k}} C^{\dagger}_{\vec{k}}$$

$$\begin{pmatrix} 0 & a \\ a^* & 0 \end{pmatrix} \lambda = \pm$$

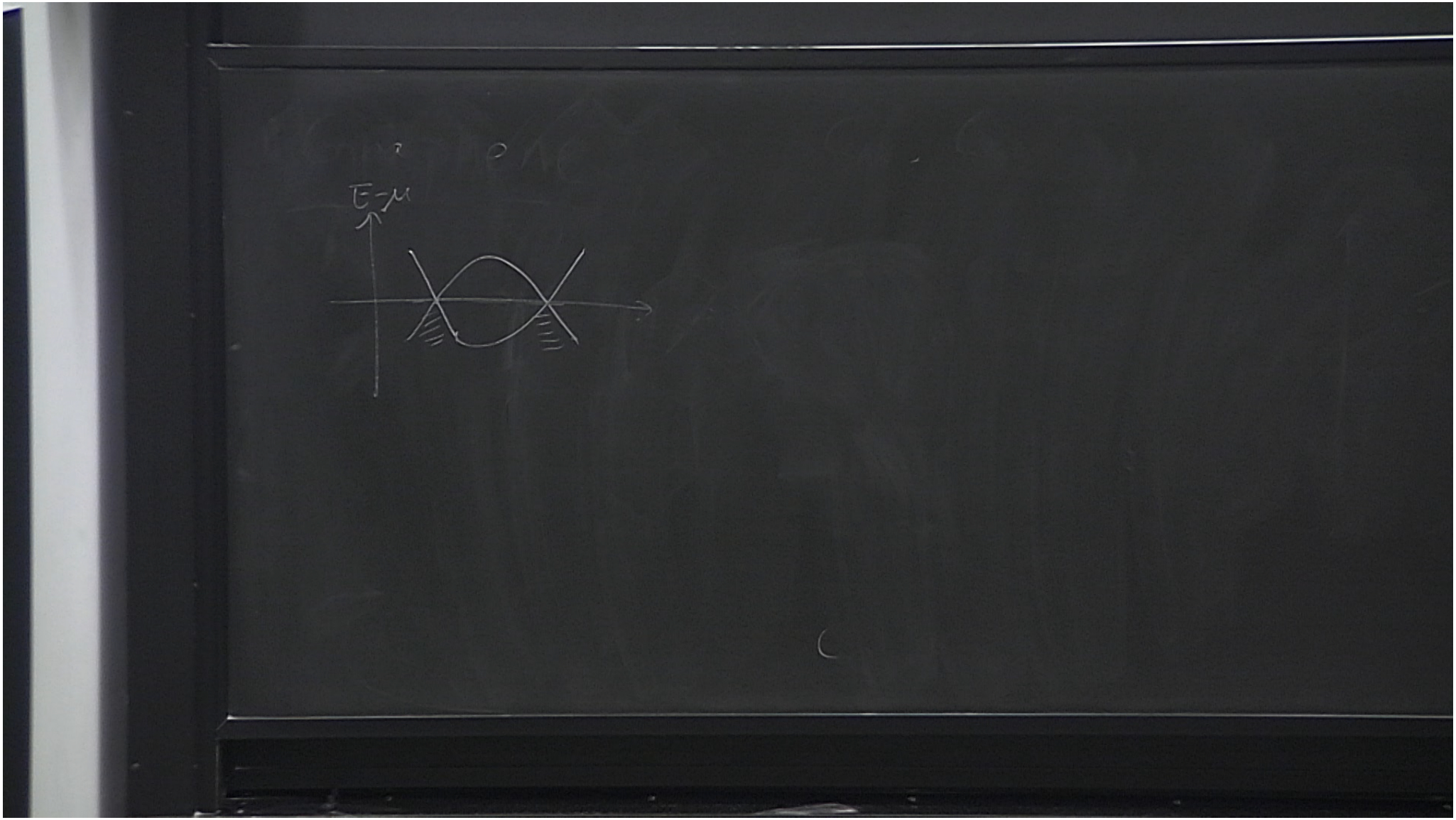
$$E = \pm \frac{\sqrt{3}}{2} t \sqrt{\delta k_x^2 + \delta k_y^2} = \pm \frac{\sqrt{3}}{2} t \, |\delta k|$$

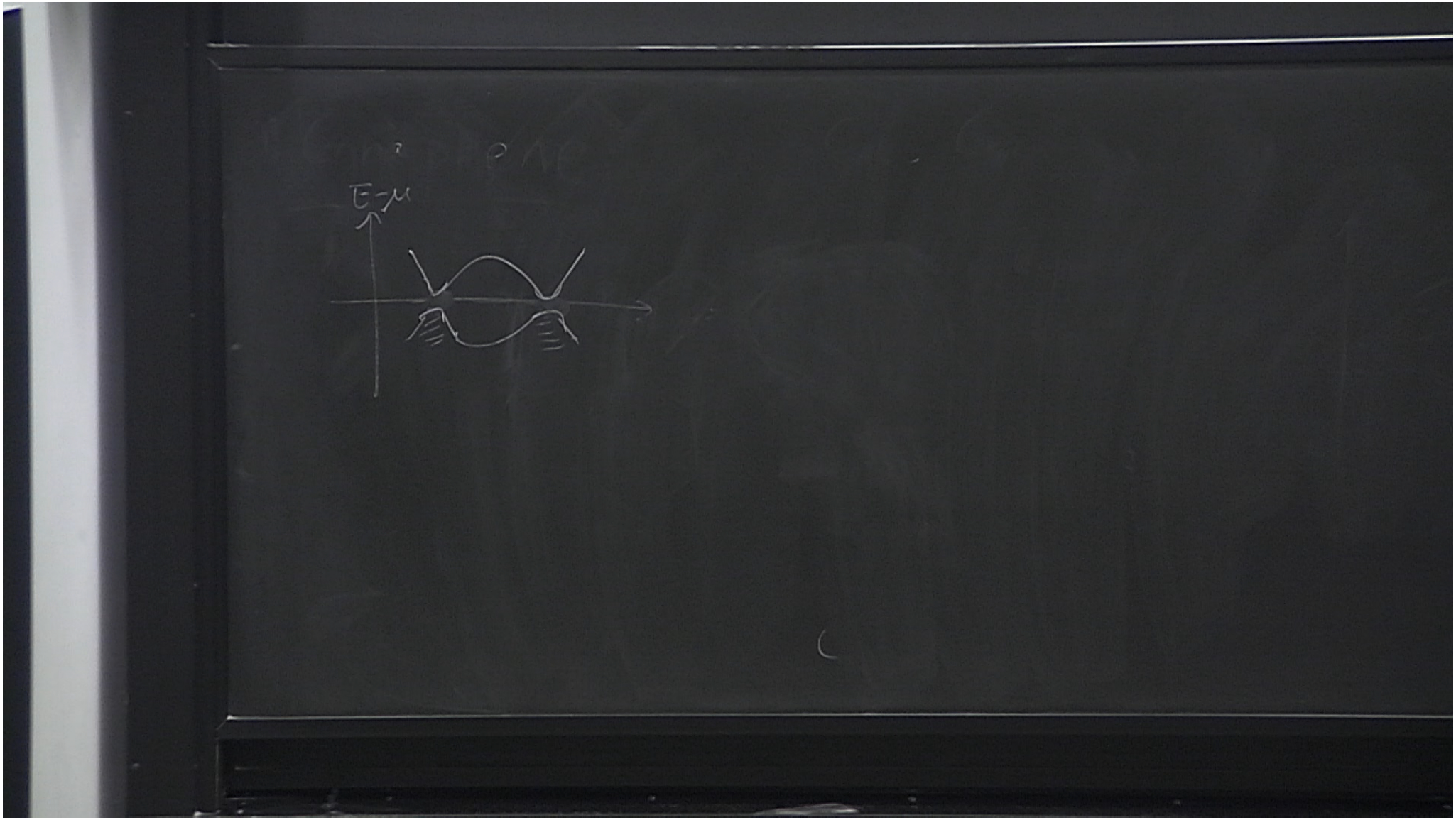


Consider: $\begin{pmatrix} \psi_+ \\ \psi_- \end{pmatrix}(\vec{x}) \equiv \int_{|\vec{k}| << \Lambda} \frac{d^2k}{(2\pi)^2} e^{i \vec{k} \cdot \vec{x}} \begin{pmatrix} c(K+k) \\ c(-K+k) \end{pmatrix}$.

$\Lambda << |\vec{G}|$

$H \sim v_0 \psi^\dagger (\tau_z G_x i\partial_x + G_y i\partial_y) \psi$ ← A UV completion of a QFT.


E-μ


E-μ

Mass term: $\psi^\dagger (\tau_{xy}\sigma_x, \sigma_z, \tau_z\sigma_z)\psi$

Symmetries.: Translation: $T_{\vec{a}_i}: \begin{pmatrix}\psi_+\\ \psi_-\end{pmatrix} \longrightarrow \begin{pmatrix} e^{i\vec{K}\cdot\vec{a}_i}\psi_+ \\ e^{-i\vec{K}\cdot\vec{a}_i}\psi_-\end{pmatrix} = e^{-i\frac{2\pi}{3}\tau_z}\psi$

$\vec{k} = \pm K + \delta\vec{k}$

$$H \simeq \underbrace{\frac{\sqrt{3}}{2} t}_{v_D} \begin{pmatrix} 0 & \pm\delta k_x - i\delta k_y \\ \pm\delta k_x + i\delta k_y & 0 \end{pmatrix}$$

$$E = \pm \frac{\sqrt{3}}{2} t \sqrt{\delta k_x^2 + \delta k}$$

$$H_{\delta k} = v_D (\delta k_x \tau_z \sigma_x + \delta k_y \sigma_y) + m\sigma_z$$

valley.

Consider. $\begin{pmatrix} \psi_+ \\ \psi_- \end{pmatrix}(\vec{x}) \equiv$

Define "valley" index $+,-$ for modes near $\pm K$

$E = \pm \sqrt{\quad}$

$H \simeq v_D \psi^\dagger (\tau_z \sigma_x$

$$H \simeq \underbrace{\frac{\sqrt{3}}{2}t}_{v_D} \begin{pmatrix} 0 & \pm\delta k_x - i\delta k_y \\ \pm\delta k_x + i\delta k_y & 0 \end{pmatrix}$$

$$E = \pm\frac{\sqrt{3}}{2}t\sqrt{\delta k_x^2 + \delta k_y^2}$$

$$H_{\delta k} = v_D(\delta k_x \tau_z \sigma_x + \delta k_y \sigma_y) + m\sigma_z$$

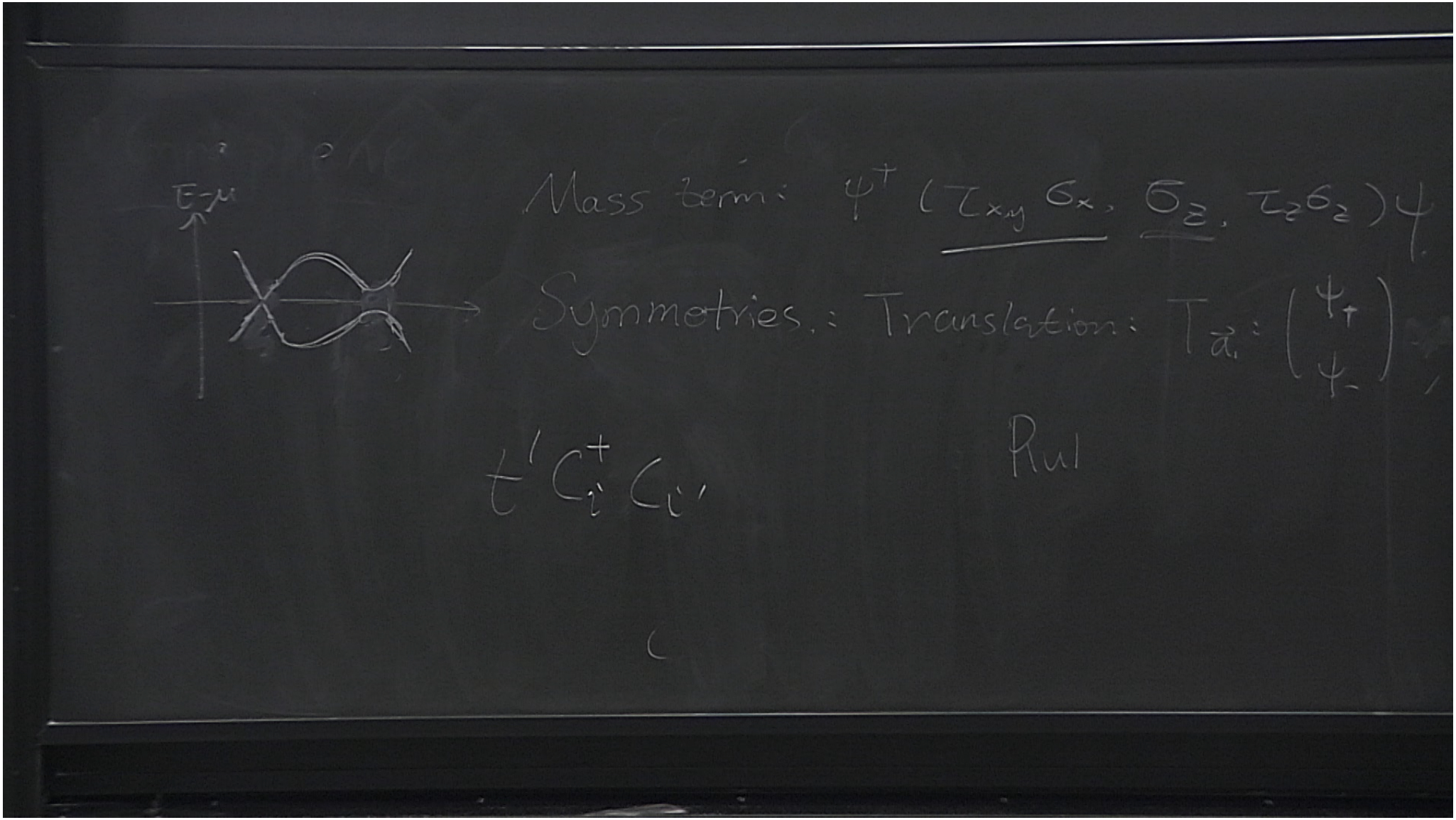
valley.

Consider: $\begin{pmatrix} \psi_+ \\ \psi_- \end{pmatrix}(\vec{x}) \equiv$

Define "valley" index $+,-$ for modes $\pm K$ near.

$$E = \pm\sqrt{v_D^2\delta k_x^2 + v_D^2\delta k_y^2 + m^2}$$

$$H \simeq v_D \psi^\dagger(\tau_z\sigma_x$$


E-μ
Mass term: ψ† (τ_xy σ_x, σ_z, τ_z σ_z) ψ
Symmetries.: Translation: T_a: (ψ↑ ψ↓)
t' c_i^+ c_i'
Rul

Mass term: $\psi^\dagger (\tau_x\sigma_x, \sigma_z, \tau_z\sigma_z)\psi$

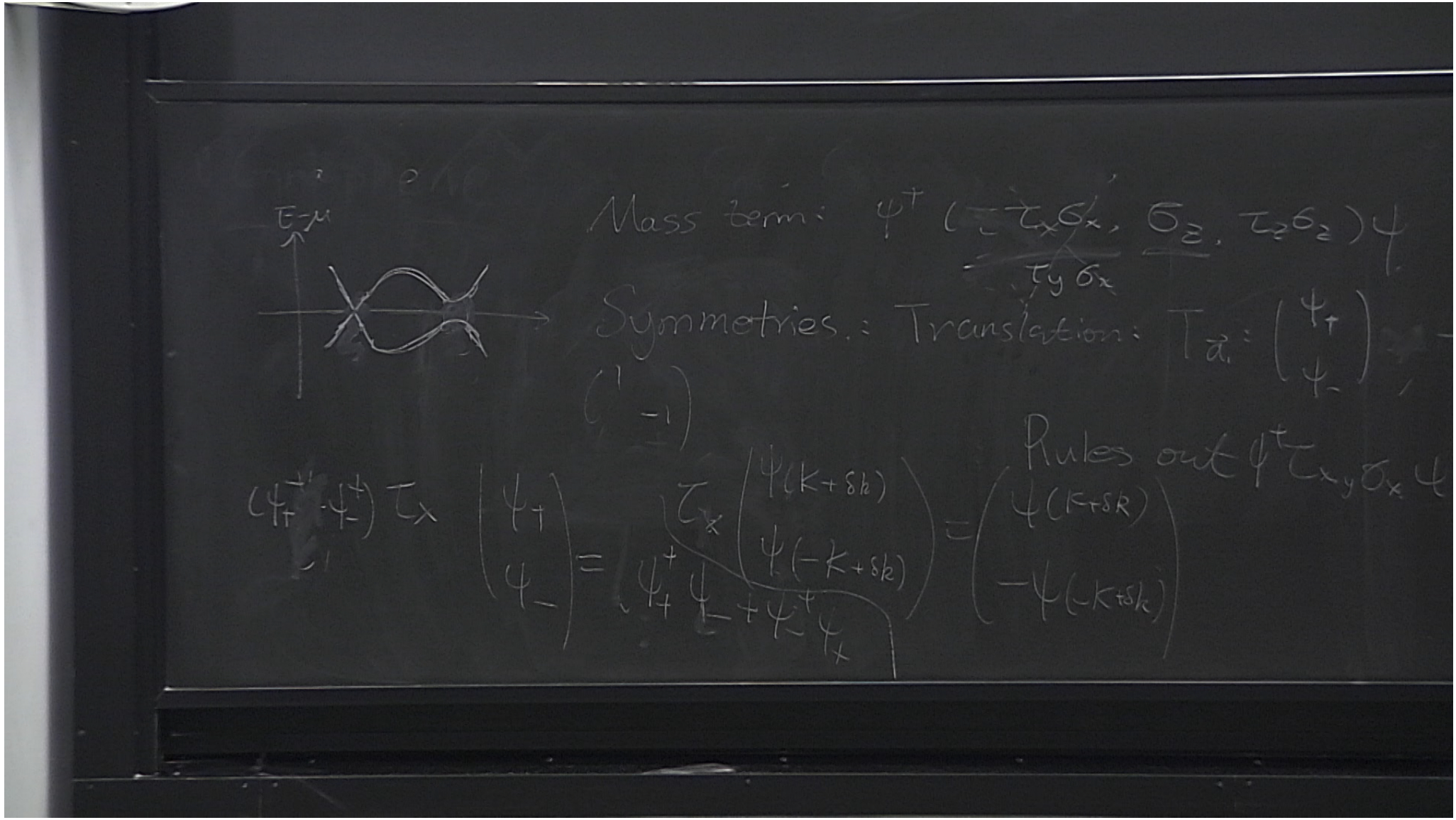
$\tau_y\sigma_x$

Symmetries.: Translation: $T_{\vec{a}_i}: \begin{pmatrix}\psi_+\\ \psi_-\end{pmatrix} \longrightarrow \begin{pmatrix} e^{i\vec{K}\cdot\vec{a}_i}\psi_+ \\ e^{-i\vec{K}\cdot\vec{a}_i}\psi_-\end{pmatrix} = e^{-i\frac{2\pi}{3}\tau_z}\psi$

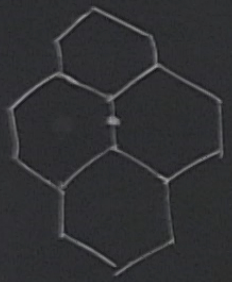
(-1)

$\tau_x\begin{pmatrix}\psi(K+\delta k)\\ \psi(-K+\delta k)\end{pmatrix} = \begin{pmatrix}\psi(\Gamma+\delta R)\\ -\psi(-K+\delta k)\end{pmatrix}$

Rules out $\psi^\dagger \tau_{xy}\sigma_x\psi \to \psi^\dagger e^{i\frac{2\pi}{3}\tau_z}\tau_{xy}\sigma_z e^{-i\frac{2\pi}{3}\tau_z}\psi$

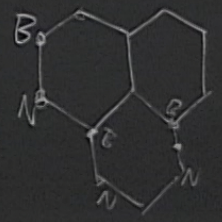

E-μ
Mass term: ψ† (τx σx, σz, τz σz) ψ
τy σx
Symmetries.: Translation: T_a: (ψ+ ψ-)
(1 -1)
Rules out ψ† τx-y σx ψ
(ψ+† ψ-†) τx (ψ+ ψ-) = ψ+† ψ- + ψ-† ψ+
τx (ψ(k+δk) ψ(-k+δk)) = (ψ(k+δk) -ψ(-k+δk))

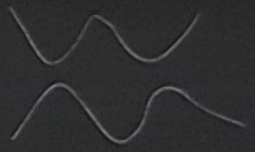
Space inversion C_2. $\begin{pmatrix}\psi_+\\ \psi_-\end{pmatrix}(x) \to \begin{pmatrix}\sigma_x\psi_-\\ \sigma_x\psi_+\end{pmatrix}(-x) = \sigma_x \mathcal{I}_x \psi(-x)$



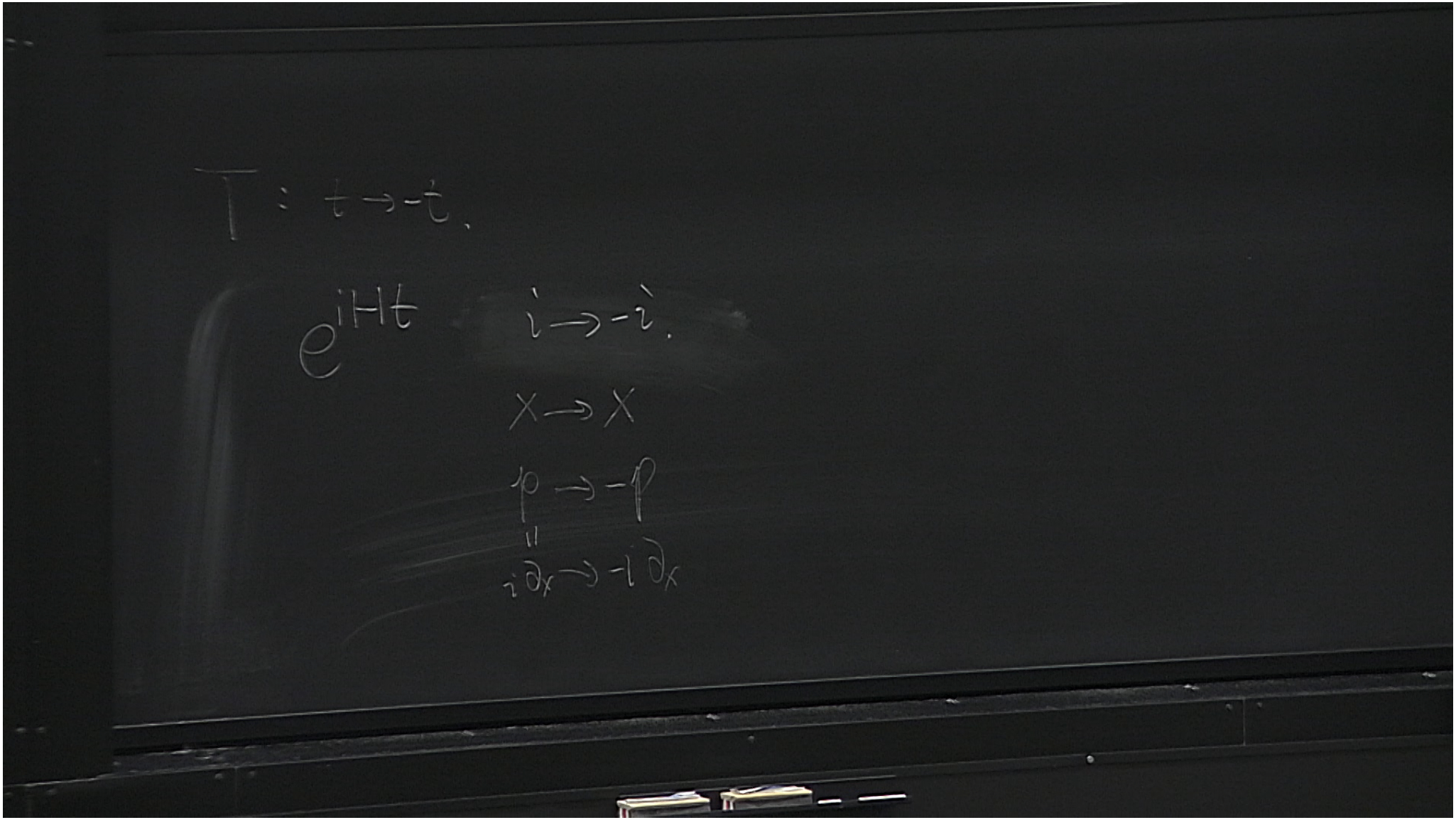
Rules out $\psi^\dagger \sigma_z \psi$

Hexagonal Boron Nitride (h-BN)

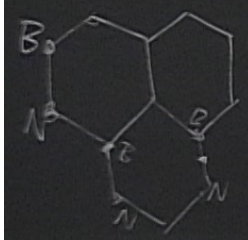


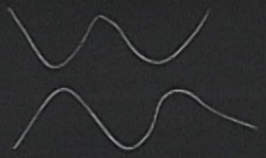


$s\mu\left(c_z^\dagger c_B - c_N^\dagger c_N\right)$


T : t → -t.
e^{iHt}
i → -i.
X → X
p → -p
i∂_x → -i∂_x

$\tau_x \tau_x \psi(-x)$





$s\mu(c_B^\dagger c_B + c_N^\dagger c_N)$

Time-reversal (anti-unitary)

$T: \; i \to -i$

$\begin{pmatrix} \psi_+ \\ \psi_- \end{pmatrix} \to \begin{pmatrix} \psi_- \\ \psi_+ \end{pmatrix} = \tau_x \psi.$

$H = \psi^\dagger (\underset{(-)(-)}{\tau_z} \sigma_x i\partial_x + \underset{(-)(-)}{\sigma_y} i\partial_y) \psi$

Rules out $\psi^\dagger \tau_z \sigma_z \psi$

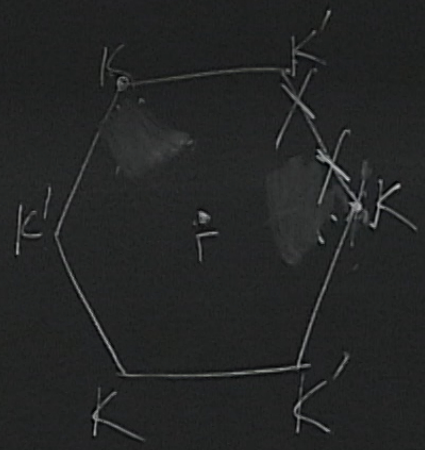
$T : t \to -t$.

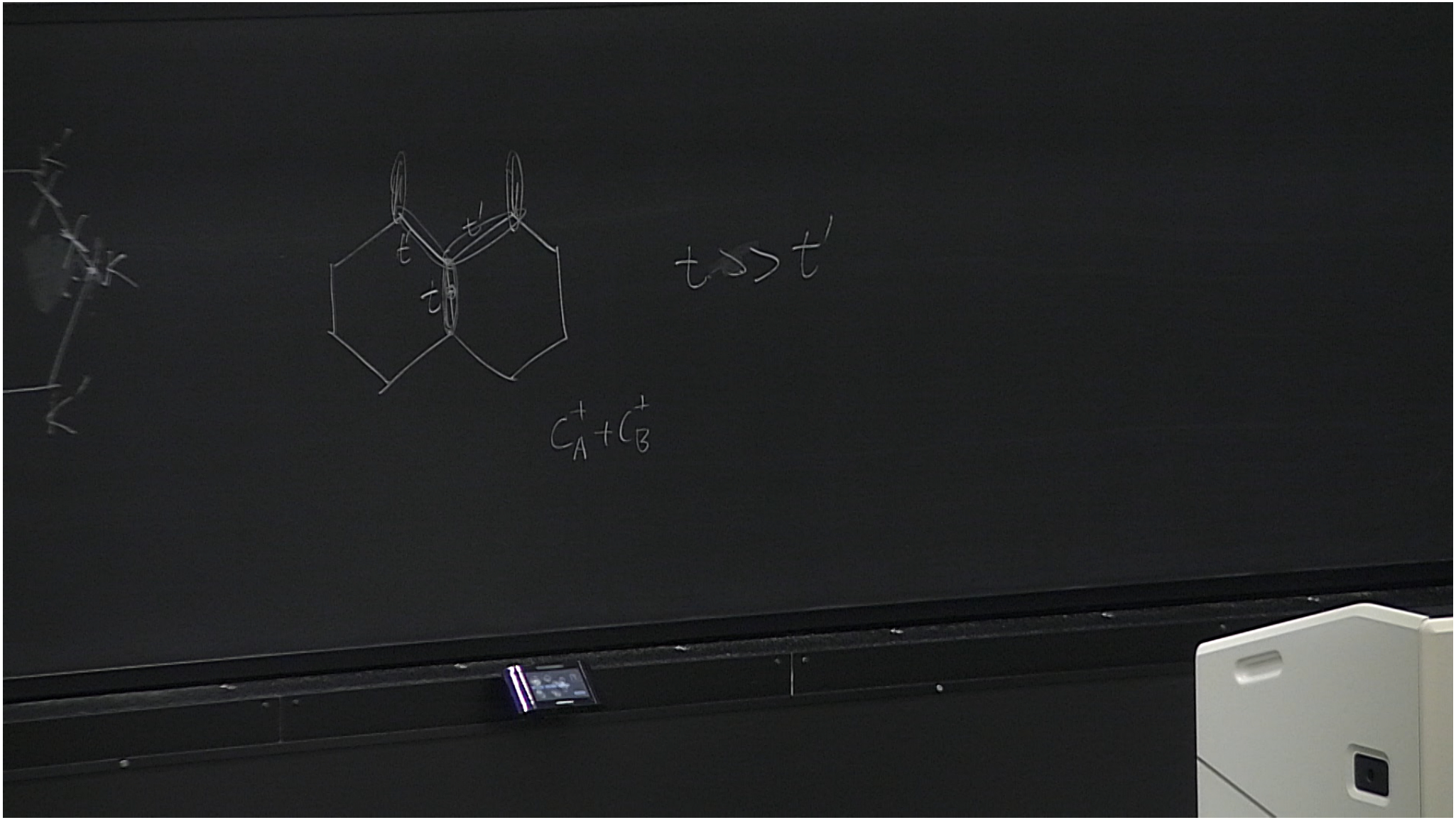
e^{iHt} $\quad i \to -i$.

$X \to X$

$p \to -p$

$i\partial_x \to -i\partial_x$




t
t'
t'
t
$t >> t'$
$c_A^\dagger + c_B^\dagger$