

Title: PSI 2019/2020 - Condensed Matter (Wang) - Lecture 1

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(Quantum) Many-body physics.

$$10^{23} \quad \text{Dim}(\mathcal{H}) \sim 2^{10^{23}}$$

1). Computation issue

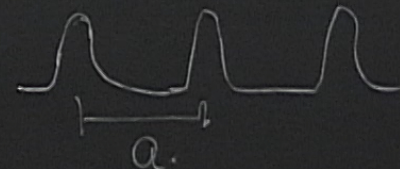
2). What is "understanding"?

Sphererical chickens $\rightarrow$ Universality.

Free electrons.

Band theory: free electrons on lattice.

$$H|\psi_E\rangle = E|\psi_E\rangle$$



Discrete lattice translation

Operator: $T_{na}|x\rangle = |x+na\rangle$: unitary

Symmetry. $T_{na}H = H T_{na}$

lattice

$$T_a |\psi_E\rangle = e^{ika} |\psi_E\rangle \text{ for some } k, \quad k \in [0, \frac{2\pi}{a}]$$

Finite system, periodic B.C. N . $T_{Na} = (T_a)^N = 1$

$$k = \frac{2\pi n}{aN}, \quad n \in \mathbb{Z} \quad e^{ikaN} = 1$$

$\mathbb{Z}$

$$|\psi_k\rangle = e^{ikx} |u_k\rangle, \quad \langle x | u_k \rangle \text{ periodic, } |u_k\rangle \text{ defined over 1 unit cell.}$$

a.k.a Bloch Thm, $|u_k\rangle$: Bloch wavefunction

Ex. 1. Tight-binding

$i \in \mathbb{Z}$, One electron per site $|\hat{i}\rangle$

$$H = -t \sum (|\hat{i}+1\rangle \langle \hat{i}| + |\hat{i}\rangle \langle \hat{i}+1|)$$

$$|k\rangle = \sum_{\hat{i}} \frac{e^{i(k \cdot \hat{i})}}{\sqrt{N}} |\hat{i}\rangle, \quad |\hat{i}\rangle = \sum_{k=\frac{2\pi n}{N}, n=0}^{N-1} \frac{e^{-i(k \cdot \hat{i})}}{\sqrt{N}} |k\rangle$$

eqy. $\text{Tr} H \text{Tr} a = H \text{Tr} a$

Bloch $|u_m\rangle, |u_k\rangle$: Bloch wavefunction

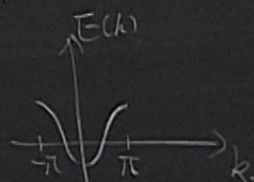
$$H = - \sum_i \sum_k \sum_{k'} \frac{t}{N} (e^{i(k'-k)\hat{i}} e^{ik'} |k\rangle \langle k'| + h.c.)$$

$$\sum_i \frac{e^{i(k'-k)\hat{i}}}{N} = \delta_{k,k'}$$

$$= -t \sum_k (e^{ik} + e^{-ik}) |k\rangle \langle k|$$

$$= -2t \sum_k \cos k |k\rangle \langle k|$$

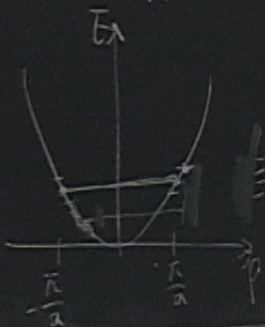
$$= \sum_k E(k) |k\rangle \langle k|$$



Ex. 2. Almost free space

$$H = \frac{p^2}{2m} + V(x), \quad V(x) = V(x+a) = \sum_{n \neq 0} V_n \cos \frac{2\pi n}{a} x$$

Suppose $|V_n| \ll \frac{1}{2m} \left(\frac{2\pi n}{a} \right)^2$



Consider $|p = \pm \frac{\pi}{a} + \delta p\rangle, |\delta p| \ll \frac{\pi}{a}$

$$H_0 | \pm \frac{\pi}{a} + \delta p \rangle = \frac{1}{2m} \left(\frac{\pi}{a} \right)^2 | \pm \frac{\pi}{a} + \delta p \rangle \pm \frac{\pi}{2ma} \delta p$$

$$\frac{e^{i \frac{2\pi n}{a} x} + e^{-i \frac{2\pi n}{a} x}}{2}$$

$$\langle -\frac{\pi}{a} + \delta p | V | \frac{\pi}{a} + \delta p \rangle = \frac{1}{2} V_1$$

$$\langle \frac{\pi}{a} | V | -\frac{\pi}{a} \rangle = \frac{1}{2} V_1$$

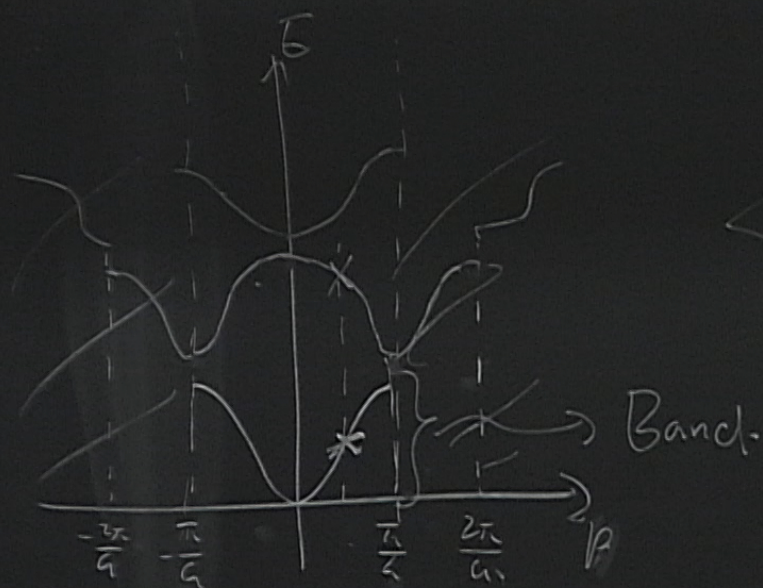
$H \approx \text{Const} +$

$$H \approx \text{const} + \begin{pmatrix} \frac{\pi}{ma} \delta p & \frac{1}{2} V_1 \\ \frac{1}{2} V_1 & -\frac{\pi}{ma} \delta p \end{pmatrix}$$

$$= \frac{\pi}{ma} \delta p \sigma_z + \frac{1}{2} V_1 \sigma_x, \quad E = \pm \sqrt{\left(\frac{\pi}{ma}\right)^2 \delta p^2 + \left(\frac{V_1}{2}\right)^2}$$

$$+ \delta p > \frac{1}{2} V_1$$

$$> -\frac{1}{2} V_1$$



$$\langle x | \psi_n(k) \rangle = e^{ikx} \underbrace{\sum_{l \in \mathbb{Z}} u_l^{(n)} e^{i \frac{2\pi l}{a} x}}_{\parallel \langle x | u_k^{(n)} \rangle}$$

Ex 1 & 2 are not that different.

If introduce Energy cut-off Λ in Ex. 2.

Ex. 1 = finite $\text{Dim}(\mathcal{H})$ per volume.

Many-electron, non-interacting (free)

Ground state

$$\langle C_k^\dagger C_k \rangle = 1$$

$$C_i, C_i^\dagger$$

$$\{C_i, C_j\} = \{C_i^\dagger, C_j^\dagger\} = 0$$

$$\{C_i, C_j^\dagger\} = \delta_{ij}$$

$$\frac{\epsilon(k)}{\hbar}$$

$$H = -t \sum_i (C_{i+1}^\dagger C_i + C_i^\dagger C_{i+1}) - \mu \sum_i C_i^\dagger C_i = - \sum_k (2t \cos k + \mu) C_k^\dagger C_k$$

$$C_k = \frac{1}{\sqrt{N}} \sum_i e^{ik\hat{i}} C_i$$

...ing. ...ing. $\ln H \left[\ln a - \frac{1}{2} \right] \ln a$

ernating (free)

$$\{c_i^+, c_j\} = 0$$

$$\{c_i^+, c_j^+\} = \delta_{ij}$$

$$H = -\mu \sum_i c_i^+ c_i = -\sum_k (2t \cos k + \mu) c_k^+ c_k$$

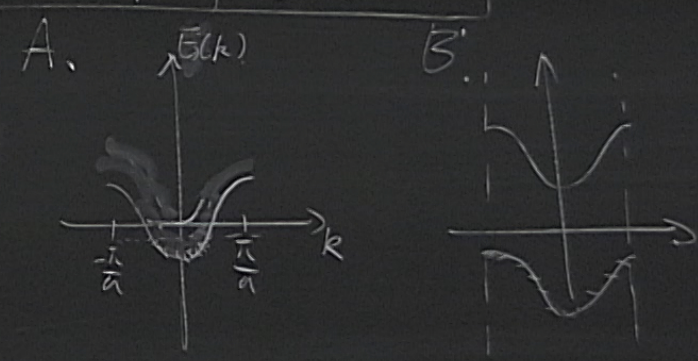
$$= \sum_k \xi(k) c_k^+ c_k$$

Ground State

$$\langle c_k^+ c_k \rangle = 1 \text{ if } \xi(k) < 0$$

$$= 0 \text{ if } \xi(k) > 0$$

$$\langle n_E \rangle = \frac{1}{e^{\frac{E}{T}} + 1}$$



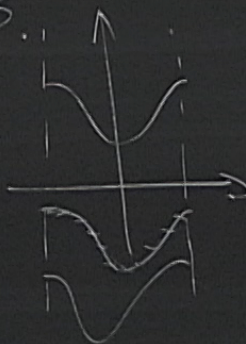
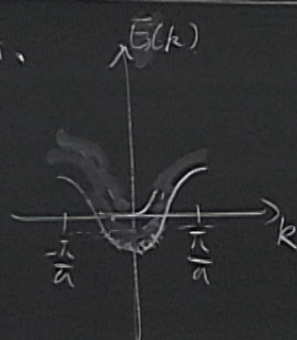
band state

$$\langle C_k^\dagger C_k \rangle = 1 \text{ if } \xi(k) < 0$$

$$= 0 \text{ if } \xi(k) > 0$$

$$\langle n_E \rangle = \frac{1}{e^{\frac{E}{T}} + 1}$$

A. B.



To have insulator:

Charge density/unit cell $\in \mathbb{Z}$.

$$\langle C_i^\dagger C_i \rangle = \frac{1}{N} \sum_k \langle C_k^\dagger C_k \rangle = 1$$

Real-life electron have spin-1/2.

Insulator, $\rho \in \mathbb{Z}$

$T_{n_1 \vec{a}_1 + n_2 \vec{a}_2 + n_3 \vec{a}_3}$ is a symmetry,

$\vec{a}_{1,2,3}$ are linearly independent

$n_{1,2,3} \in \mathbb{Z}$.

$$\begin{aligned} \mathbb{Z}^{\otimes 3}. \quad T_{\vec{n}} |\psi_E\rangle &= e^{i(\vec{k}_1 n_1 + \vec{k}_2 n_2 + \vec{k}_3 n_3)} |\psi_E\rangle \\ &= e^{i \vec{k} \cdot (n_1 \vec{a}_1 + n_2 \vec{a}_2 + n_3 \vec{a}_3)} |\psi_E\rangle. \end{aligned}$$

$$T^{-1} H T = H$$

$$\vec{k}_i \sim \vec{k}_i + 2\pi$$

$$\vec{k} \sim \vec{k} + \vec{G}_i,$$

1. $\vec{G}$ defines a lattice. $\leftarrow$ Reciprocal lattice.

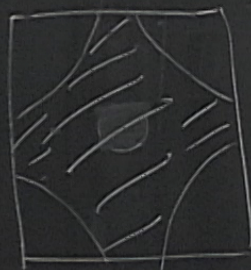
$\vec{k}$ live in a unit cell of this lattice.

Brillouin zone (B.Z.)

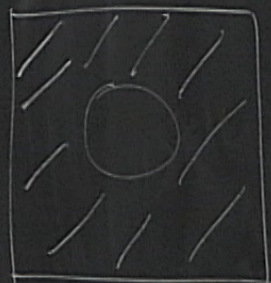
$\vec{G}$ defines a lattice. $\leftarrow$ Reciprocal lattice.

$\vec{k}$ live in a unit cell of this lattice.

Brillouin zone (B.Z.)



$pcc 1$



C_i, C_i^+

$\tilde{C}_i = C_i^+, \tilde{C}_i^+ = C_i$

hole fermi-pockets

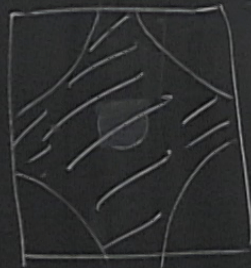
$$\frac{\vec{a}_j \times \vec{a}_k}{\vec{a}_l \cdot (\vec{a}_j \times \vec{a}_k)}$$

$$(\vec{a}_i + n_j \vec{a}_j) = m 2\pi$$

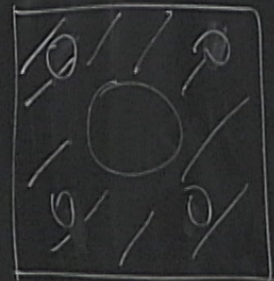
$\vec{G}$ defines a lattice. $\leftarrow$ Reciprocal lattice.

$\vec{k}$ live in a unit cell of this lattice.

Brillouin zone (B.Z.)



$\approx$



$p < 1$

C_i, C_i^+

$\tilde{C}_i = C_i^+, \tilde{C}_i^+ = C_i$

hole fermi-pocket

$$\frac{\vec{a}_j \times \vec{a}_k}{\vec{a}_l (\vec{a}_j \times \vec{a}_k)}$$

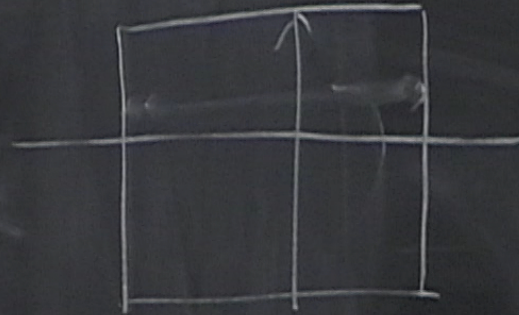
$$(\vec{a}_i + n_j \vec{a}_j) = m 2\pi$$

Velocity: $V_i^{(n)} \equiv \frac{\partial E(n, \vec{k})}{\partial k_i}$

"Effective mass"

$$\left(\frac{1}{m^*}\right)_{ij} = \frac{\partial^2 E}{\partial k_i \partial k_j}$$

Fermi energy $\sim k_F v_F \sim 10^4 - 10^5 \text{ K}$



$$E = p_x^2 + p_y^2 .$$

